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ANALYSIS OF TRAFFIC FLOW ON COMPLEX NETWORKS

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We propose a new routing strategy for controlling packet routing on complex networks. The delivery capability of each node is adopted as a piece of local information to be integrated with the load traffic dynamics to weight the next route. The efficiency of transport on complex network is measured by the network capacity, which is enhanced by distributing the traffic load over the whole network while nodes with high handling ability bear relative heavier traffic burden. By avoiding the packets through hubs and selecting next routes optimally, most travel times become shorter. The simulation results show that the new strategy is not only effective for scale-free networks but also for mixed networks in realistic networks.

Keywords: Complex networks; routing strategy; network capacity.

1. Introduction

Transportation networks are large-scale networks with complex structures and represent a very common kind of networks in the real-world networks. Examples include vehicles on roads, passengers between airports, packet flows in the Internet, and electrical power transmitted in power grids. Free, uncongested traffic flows on these networks are critical for their normal and efficient function. The explosive increase in the numbers of users led to a continuous growth of transportation networks. Thus, efficiently handling and delivering packets, e.g., improving network capacity and avoiding undesired congestion, has naturally attracted more and more attentions not only in practice, but also in theoretical research. There have been many previous works in this direction.[1–15]

A model of information flow is characterized by two parameters: the information generation rate R and the delivery capacity C, which is the capability of the nodes to process information. C is often either a constant or related to the network structure. Usually, the efficiency of transport is measured by the network capacity R_c, which is the maximum number of packets created that the network can support. R_c is the critical rate at which a phase transition occurs from free to congested traffic flow, and generally larger R_c implies more efficient transport. In particular, when the created packet is below a certain value, $R < R_c$, the number of created and delivered packets are balanced. A steady state, or free flow of traffic on network can be maintained. For $R > R_c$, congestions occur in the sense that the number of accumulated packets increases with time, due to the fact that the delivery capacity of each node is limited. Intuitively, network capacity could be significantly enhanced or without limit with very large average degree of connectivity and/or node delivery capability. However, this may not be feasible because of the potentially high cost. Recent works have shown that traffic congestion depends sensitively on the network structure.[2] It is found that scale-free networks and random networks are more tolerant of traffic congestion than regular networks and Cayley trees, while scale-free (SF) networks are more prone to congestion than random networks. Generally, it is hard to redesign the network structure. The delivery capability of each node is also pre-designed and limited by network performance.

Besides the network structure of the underlying network, the efficiency of a traffic network depends on the routing strategy, which determines how a packet will be delivered from the source to its destination. Compared with extensively redesigning or modifying the topology, it is easy to have better routing strategies in practice. The aim of this paper is to address the improvement via routing protocol. The shortest path routing is a commonly used routing strategy. Here, the shortest path means the path with the smallest number of links. However, it is found that routing by the shortest path on a SF network, the load distribution follows a power law.[16] This indicates that hub nodes should be heavily loaded and congested due to the large amount of packets passing along the shortest paths. To prevent such congestion, it is desirable that the traffic loads can be distributed to the whole network while hub nodes take on a relatively heavy load. On the other hand, finding the shortest path requires global topology information, which is usually inaccessible in most networks. Therefore, the previous studies suggested that the actual path finding strategy should be based on local information.[10–15] A basic assumption in these studies is that each node performs a local search among its neighbors. Each node has only local structure and/or traffic information. In this paper, we propose an optimal local routing strategy based on delivery capacity and traffic flow. The delivery capability of each node is adopted as a guide to weight the next route. We are interested in enhancing the network capacity while reducing the average transit time.

In order to better avoid traffic congestion, we study traffic-flow dynamics on complex networks in a balanced setting based on the original network structure. Our routing strategy is to make the nodes with high processing ability as powerful and as efficient as possible for processing the information, while balancing the queue length of each node. Simulation results show that the network capacity is enhanced by distributing the traffic load over the whole network and the travel times become shorter by optimal path finding. The optimal strategy is not only effective for scale-free networks but also for mixed networks in realistic networks.

In Sec. 2, we describe our traffic model and routing strategy. In Sec. 3, we present the simulation results. A brief summary is given in Sec. 4.

2. Traffic Model and Routing Strategy

Once the network is constructed, we seek a path through which a packet is transmitted from source to its destination. We model the traffic flow of packets on complex networks as follows.

(1) Packet generation: at each time step, there are R packets generated in the network with randomly selected source nodes and destinations which are different from the source. Once a packet is generated, it is placed at the end of the node queue if this node already has several packets waiting to be delivered. The existing packets may be generated at some previous time steps or they are transmitted from other nodes.

(2) Packet delivery: at each time step, a node i can deliver at most C_i packets toward its next node according to routing protocol and placed at the end of the queues of the selected nodes. When the node cannot transfer all the packets accumulated in its queue, first-in-first-out queuing discipline is applied. If it has less than C_i packets in its queue, all packets in the queue are forwarded in one step. Once the packet reaches its destination, it will be removed from the network. This procedure applies to every node at the same time. As a result, the delivering time that a packet needs to reach its destination is related not only to the number of nodes passed through, but also to the number of existing packets along its path.

(3) Packet routing: at each time step, all nodes perform a parallel local search among their immediate neighbors. We design a traffic routing strategy based on the local traffic load and delivery capability. Assume that node s holds a packet that should be delivered to node t. If t is found within the immediate neighbor nodes of s, the packet will be delivered from s directly to its target t and then removed from the system. Otherwise, the node forwards the packet to one of its neighbors i with the shortest waiting time. The weight of waiting time is defined as

$$H_i = \frac{Q_i/C_i}{\sum_{j\in g(i)} Q_j/C_j}\,, \tag{1}$$

where $g(i)$ is the set of all the neighbor nodes of s, Q_i is the number of packets in the queue of node i and C_i is the delivery capability of that node. Q_i/C_i is the estimated waiting time at node i. The queue length is changed dynamically at each time step according to the local traffic load dynamics. After computing the weight of each neighbor node i of node s, we select the next router node among the neighbors which has the minimum weight. If there is more than one node with the minimum weight, select one of them randomly. At each time step, the weight H_i will be calculated dynamically according to the current traffic information in the network.

In real-world networks, different nodes may have the same or different abilities to forward packets. It has been usually assumed that each node has same capability, where C_i is a constant for all node, or is proportional to its degree so that highly connected node has higher delivery capability. In our traffic model, the delivery capability C_i is not preassigned for each node. It is a local information that should be known by neighbor nodes.

In contrast to previous works taking the node degree as the major local static information, in our routing strategy we adopt node delivery capability as the key consideration factor. The reason is explained as follows. First we observe that the newly generated packets will be uniformly distributed on every node, independent of the network topology. From the packet delivery point of view, in the case that the delivering capacities for each node are the same constant, the topology does not help to handle the packets, and the hub nodes have higher probability of being delivered a packet simply because they have a larger number of connections. If each node has a different ability to forward packets, then when the queue lengths are short, the higher delivery capacity nodes will dominate and packets will tend to be transmitted to highly connected nodes. However, when packets accumulate in the queue, our strategy selects an optimal path based on a weight that emphasizes the delivery capacity of each nodes, thereby enhancing the overall system performance.

We note that the topology (e.g., SF or otherwise) does not appear explicitly in our weighting function. However, this does not indicate that the optimal routing strategy is independent of the network structure. In fact, the structure of the set of all the neighbor nodes $g(i)$ should represent the (local) topology of the network. The network topology indirectly affects the optimal route path finding. The higher the degree a node has, the higher probability a packet will be sent to it. It should be noticed that strictly scale-free networks are idealized through recent works, which reveal that many networks including the Internet are complex with scale-free features. Realistic networks however always contain both scale-free and random components. This “mixed” characteristic is the case for many transportation networks. The new routing strategy does not specify the topology of the network, so it is robust in the real-world network application.

The new routing strategy is consistent with what is stated in Ref. 13. They assumed identical node delivery capability and proposed that the probability of a neighbor node i to which the packet will be delivered is $p_i = k(q+1)^\beta / \sum_j k(q+1)^\beta$. The optimal choice of β is -3. It can be seen that the number of packets in the queue carries greater weight than the degree does. When R is not large, there is a small number of packets in the queue. The forwarding probability is mainly determined by the degree of the node, and the hub nodes will bear heavier traffic load. When R approaches the phase transition point, the queues are becoming longer, and the forwarding probability is then mainly determined by the number of packets in the queue. The traffic burden on nodes with different degree is almost the same. In our weighted time optimal routing strategy, the node delivery capability is not fixed, and therefore, it has general applicability.

3. Simulation Results and Discussion

Since many of the realistic networks have SF properties, it is thus interesting to study the traffic flow on an example of a SF network. SF networks can be constructed with various methods. The first model is the BA model[17,18] which is constructed based on the ideas of growth and preferential attachment. Starting with a small number m_0 of fully connected nodes, we add a new node with m edges that links to m different nodes in the existing graph according to preferential attachment. The probability that a new node will be connected to node i is proportional to the degree k_i of the node i, that is, $\Pi(k_i) = k_i / \sum_j k_j$, where the sum runs over all existing nodes. The SF network shows power-law behavior in the degree distribution. The following simulation are performed for $m_0 = 4$, $m = 3$, and network size $N = 1000$.

We first investigate the network capacity which is qualified by the critical generating rate R_{c}. We use the following order parameter to characterize the phase transition,[1]

$$\eta = \lim_{t \to \infty} \frac{\langle \Delta W \rangle}{R \Delta t}\,, \tag{2}$$

where $\Delta W = W(t+\Delta t) - W(t)$ with $\langle \ldots \rangle$ indicating the average over time windows of width Δt, $W(t)$ is the total number of packets in the network at time t. The order parameter η represents the ratio between the outflow and the inflow of packets. η equals zero in the free-flow state and becomes positive when packets accumulate within the network. Given a specific network model, the maximum packet generation rate R is defined as the critical traffic load R_{c}. When $R < R_{\text{c}}$, the network is in the steady free-flow state, $\Delta W \approx 0$ and $\eta \approx 0$; when $R > R_{\text{c}}$, ΔW increases with Δt. Therefore, sharp jump of η from 0 to positive value indicates the congestion happened.

We set the packet delivery capacity C_i for each node as:

$$C_i = \lfloor \alpha + \beta k_i \rfloor\,, \tag{3}$$

where $\alpha > 0$, $0 < \beta < 1$ are control parameters, k_i is the degree of node i. When $\beta = 0$, each node has the same delivery capacity $C_i = \alpha$. When $\beta > 0$, C_i is proportional to its degree. We perform the simulation by setting $\alpha = 1$, and changing β. Specially, in the case that $C_i > 1$, in order to avoid a long queue suddenly, a collection of C_i packets is not transmitted together but instead delivered one by one to the next router at each time step. The next router is selected from the updated

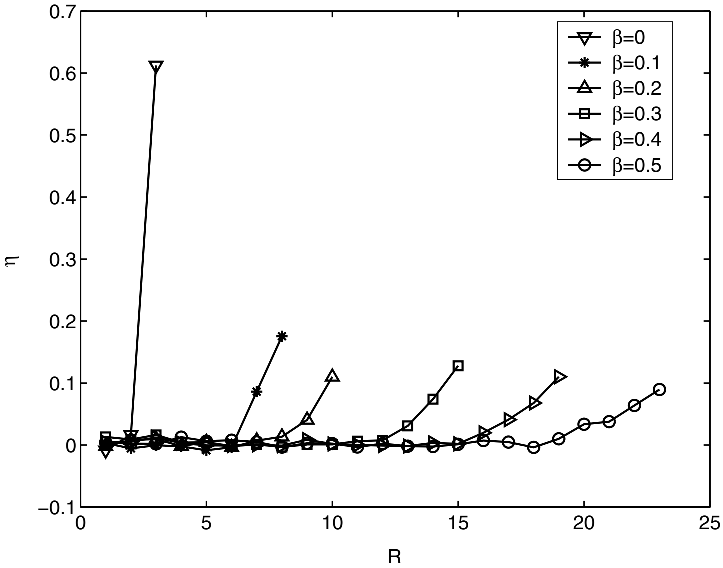

Fig. 1. The order parameter η versus R: the packet-generating rate for SF network. Results are shown for different delivery capability C. $\alpha = 1$, $\beta = 0, 0.1, 0.2, 0.3, 0.4, 0.5$, respectively. $N = 1000$, $m_0 = 4$, $m = 3$.

weight table H_i. Within each time step, we recalculate the weight of the node that just has one packet sent to and update the weight table to find the next router until all C_i packets are sent out. That means H_i is possibly dynamically changed within each time step.

In Fig. 1, we show the simulation results of optimal network capacities for different parameters β. The simulation time is 5000 time steps and only use the data of the last 1000 time steps when the network traffic flow can be viewed as stable. η represents an average over an ensemble of 10 network realizations. We can see that, for all β, η is approximately zero when R is small; it suddenly increases when R is larger than a critical value R_{c}. And as expected, the capacity of the system increases with β.

In Fig. 2, we show the simulation results of optimal network capacities for two different networks, SF network (circles) and random network (triangles). The random network is constructed in the same way as the SF network except that the probability that a new node is connected to node i is the same for all existing nodes. It can be seen clearly that the network capacity for SF network is larger than that for random network. However, as shown in Ref. 2, with shortest path routing, the packet transmission routes are better distributed for random networks than for SF networks. The efficiency of the transportation, or the network capacity is improved significantly for SF network after adopting the optimal routing strategy. This can be understood from the fact that with our new routing strategy, packet loads of the nodes with high degrees are reduced and congestion triggered by these

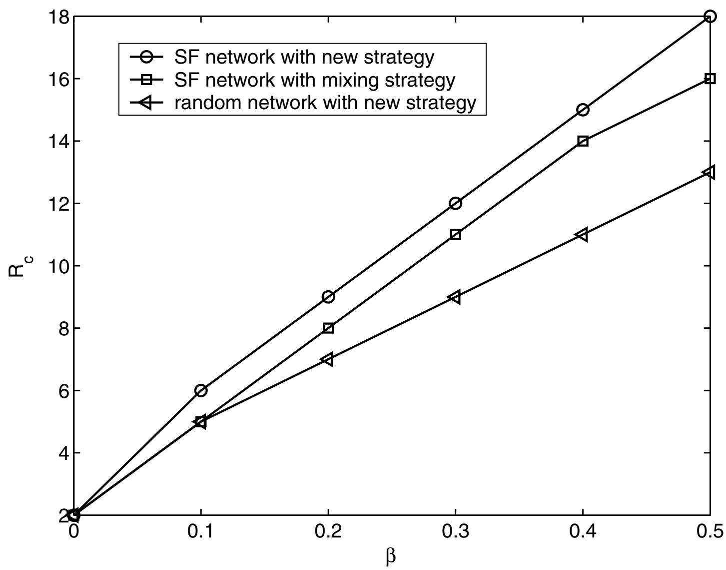

Fig. 2. $R_{\rm c}$ versus β for SF network with new strategy (circles), random network with new strategy (triangles), and SF network with mixing strategy proposed in Ref. 13 (squares). For each networks, $N = 1000$, $m_0 = 4$, $m = 3$.

nodes is delayed. Thus, the network capacity is effectively enhanced. To provide a comparison, we also show the simulation result of network capacity by using the mixing strategy proposed in Ref. 13 for the SF network in Fig. 2 (squares). It can be seen that the network capacity is improved by our proposed new strategy.

Next, we investigate the distribution of the loads on each node as a function of its degree in two cases: each node has the same ability to process packet and different delivery capabilities. As expected, the results demonstrate that the packets avoid the hub nodes. In Fig. 3, we show the simulation results of load distributions for $C_i = 5$ in two states, i.e., the stable state $R = 4$ and in critical state $R = R_{\rm c} = 9$. n_k is the number of load on the node with degree k. We can see that the traffic loads get distributed over the network while the large degree nodes bear relatively heavier traffic burden. This is because that constant delivery capacity leads to the routing weight is determined by the queue length. Meanwhile, the highly connected nodes have larger probability to be chosen. In Fig. 4, simulation results for $C_i = 1 + 0.5k_i$ are shown. At low traffic density $R = 4$, queuing occurs rarely and delivery capability determines the packet routing. Thus, the higher degree nodes have larger numbers of packets waiting in the queue. Traffic loads show power-law properties. At the intermediate density $R = 13$, waiting time is related to both the number of packets in the queue and the process rate, so traffic loads on the large degree nodes begin to distribute to the low degree nodes. In the critical state $R = 18$, queuing becomes more important and consequently the load distribution goes to the whole network — traffic loads do not accumulate on the hub nodes but spread

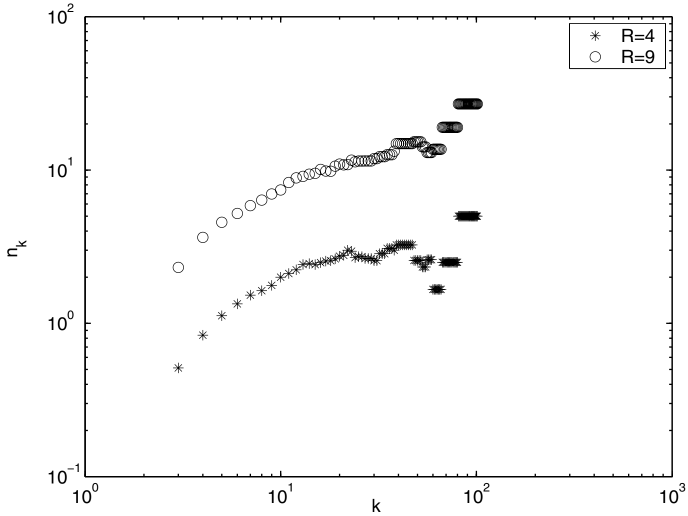

Fig. 3. The distribution of traffic loads with constant delivery capacity, $C_i = 5$. The stars represent network capacity $R = 4$, and the circles represent network capacity $R = 9$.

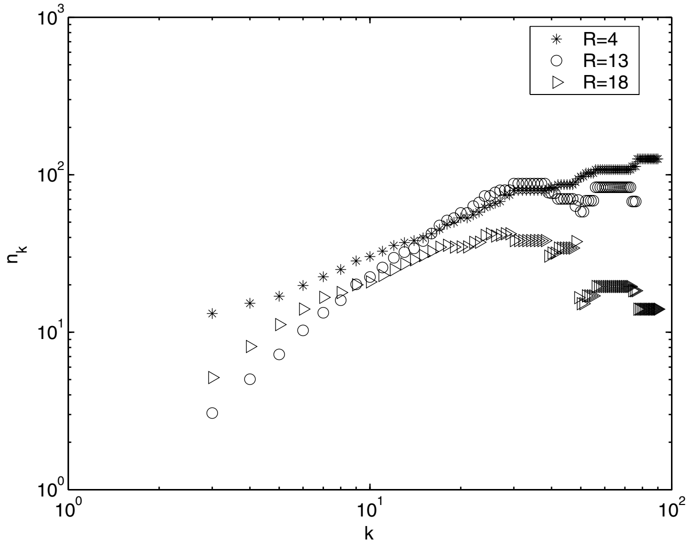

Fig. 4. The distribution of traffic loads with different delivery capacity, $C_i = 1 + 0.5k_i$. The stars represent network capacity $R = 4$, the circles represent network capacity $R = 13$, and the triangles represent network capacity $R = 18$.

to the whole network. These effects are the results of the fact that when there are less packets in the queue, the high degree nodes have the ability to process more packets and thus more packets are transmitted to them. When queues are building up, packets are finding a more optimal path to pass through.

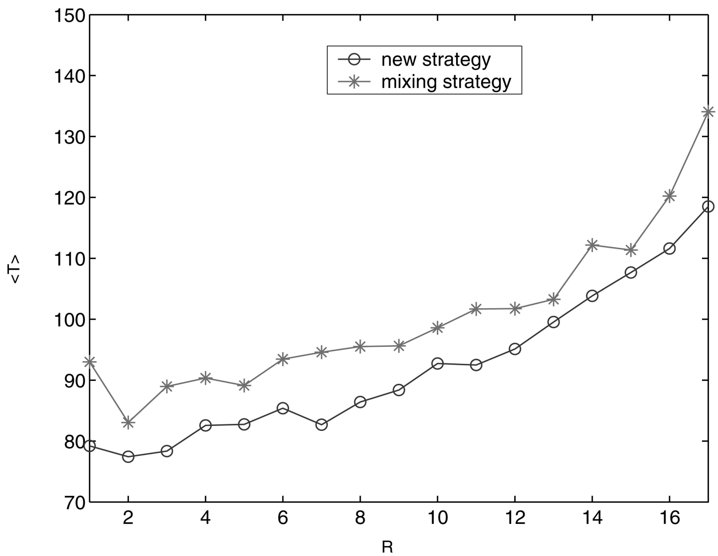

Fig. 5. Comparsion on the transit time for SF netwrok, $C_i = 1 + 0.5k_i$.

The traffic load distribution gives a further insight into the effect of the new strategy. Packets are transmitted to the short queue node to avoid packet accumulation and to the high process ability node to improve delivery rate, resulting in maximum network capacity, regardless of how the node delivery ability is defined.

In addition, the other important performance index is the communication velocity, which can be estimated by the average packets travel time from source nodes to destinations over a long time period. The total time spent along the path, i.e., the transit time, depends both on the number of nodes the packets pass through and the time the packets spent waiting in the queues along that path. With an efficient routing strategy, transit time can be short. In our model, we do not take into account the time delay of packet transfer at each node or edge, so all the packets are delivered in unit time, regardless of the distance between any two nodes. By avoiding queue, in our model most travel times become shorter, in spite of the fact that the routes pass through more nodes. In Fig. 5, we compare the transit times $\langle T \rangle$ using the new routing strategy and using Ref. 13 mixing strategy with the same network structure. Clearly, the new protocol improves the communication velocity.

4. Conclusion

In conclusion, we studied the problem of transport on complex network and proposed here the optimal routing strategy. The advantage of the new strategy is that it makes the high process ability nodes as powerful and as efficient as possible for processing information, while it also balances the queue length of each node. The large network capacity is due to the exploitation of the delivery capability, which reflects the process rate of each node, and the dynamical traffic information, which reflects the traffic burden on each node. With this as the basis in determining the optimal routing, packets can find their ways with the shortest waiting time and with higher probability pass through the larger connected nodes, resulting in faster transmission. In addition, we note that real-world networks are generally mixed networks. It always contain both scale-free and random components. Our new routing strategy does not specify the topology of the networks, so it is robust in realistic network applications.

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