Synchronizability of network ensembles with prescribed statistical properties

Shuguang Guan,[1,2] Xingang Wang,[1,2] Kun Li,[1,2] Bing-Hong Wang,[3,4] and C.-H. Lai[2,5]
[1]*Temasek Laboratories, National University of Singapore, 117508 Singapore*
[2]*Beijing-Hong Kong-Singapore Joint Center of Nonlinear and Complex Systems, National University of Singapore, 117508 Singapore*
[3]*Department of Modern Physics University of Science and Technology of China, Hefei, Anhui 230026, People's Republic of China*
[4]*Shanghai Academy of System Science, Shanghai 200093, People's Republic of China*
[5]*Department of Physics, National University of Singapore, 117543 Singapore*

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It has been shown that synchronizability of a network is determined by the local structure rather than the global properties. With the same global properties, networks may have very different synchronizability. In this paper, we numerically studied, through the spectral properties, the synchronizability of ensembles of networks with prescribed statistical properties. Given a degree sequence, it is found that the eigenvalues and eigenratios characterizing network synchronizability have well-defined distributions, and statistically, the networks with extremely poor synchronizability are rare. Moreover, we compared the synchronizability of three network ensembles that have the same nodes and average degree. Our work reveals that the synchronizability of a network can be significantly affected by the local pattern of connections, and the homogeneity of degree can greatly enhance network synchronizability for networks of a random nature. © *2008 American Institute of Physics*. [DOI: 10.1063/1.2841198]

Recently, investigations of synchronization have been extended to the area of complex networks. For example, the synchronization on small-world networks, scale-free networks, weighted networks, gradient networks, and modular networks has been extensively studied. One of the main purposes of such studies is to explore how network topologies, such as degree distributions and the average network distance, affect the dynamics on the network. However, degree distribution and other global network properties are statistical quantities. With the same global network properties, networks may have a different propensity for synchronization. In the present work, we investigate the synchronizability of network ensembles in which networks have the same global network properties. The statistical properties of the eigenvalues and eigenratios, which characterize the synchronizability of networks, are analyzed and compared for different network ensembles.

I. INTRODUCTION

Synchronization on complex networks has been extensively investigated in the past decade.[1–27] The central issue in these studies is the interplay between network topology and the dynamics on it. This prompts the important and interesting question about the synchronizability of networks.[9,21,26,27] Here, the synchronizability of network refers to the synchronization that can be achieved for the larger set of parameter values, e.g., for a larger Lyapunov exponent of local dynamics or for a wider possible range of coupling strengths.[4,6,27] For the synchronization of identical oscillators, the master stability function (MSF) theory has shown that the influence of the network topology on synchronization can be inferred through the spectra of the coupling matrix.[4,6] Let us denote the adjacent matrix characterizing the network topology by A. Then the Laplacian matrix can be defined by $L=D-A$, where D is the diagonal matrix of node degrees. If A is symmetric, L is also a symmetric and positive semidefinite matrix, and all its eigenvalues are real and non-negative. From small to large, the eigenvalues can be ordered as $\lambda_1 = 0 \leq \lambda_2 \leq \cdots \leq \lambda_N$. The smallest eigenvalue is always zero because all the row sums of L are zero. The MSF theory shows that the synchronizability of a network can be measured by the eigenratio $r=\lambda_N/\lambda_2$, i.e., the ratio of the maximum eigenvalue λ_N to the smallest nonvanishing eigenvalue λ_2. The smaller the ratio, the better the synchronizability of the network, and vice versa.[9,27]

Complex networks are usually classified by degree distributions and other global network properties. However, networks with the same degree distribution or global properties may have very different synchronizability.[20,21] This can be understood by an analogy in statistical mechanics: the degree distribution is like a macro state (global property), and it corresponds to many micro states which have different local structures, and thus have different synchronizability. What is of particular interest is the distributions of network spectral properties for a network ensemble with the same degree distribution or degree sequence. What are the shapes of the distributions for network spectral properties? Are they well-defined, and do they converge to the average values with the increase of the ensemble size? How many networks in the ensemble have extremely poor synchronizability? These are interesting and important questions relevant to the synchro-

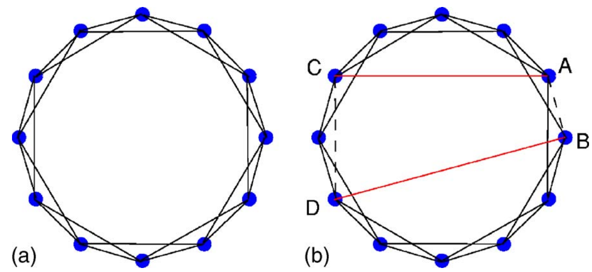

FIG. 1. (Color online) (a) Schematic illustration of a regular network with $N=12$ and $k_d=4$. (b) A variant network is generated by applying the link switching operation once from network (a). The link switching operation is applied as follows: first, arbitrarily choose two links AB and CD; then break these two links, and instead connect nodes A with C, and B with D. In this way, networks (a) and (b) have exactly the same degree sequence but different local pattern of connection.

nizability of networks. Recently, Kim and Motter studied the ensemble averageability in network spectra.[22] Their results show that for scale-free network ensembles, the eigenvalues relevant to network synchronizability have well-defined ensemble distributions, and statistically, the networks with extremely poor synchronizability are actually rare. Motivated by the above questions and the work in Ref. 22, we investigated in this work the synchronizability of network ensembles with prescribed statistical properties. This paper is organized as follows. In Sec. II, we construct three types of network ensembles. The networks in each ensemble all have the same statistical properties. In Sec. III, the distributions of the network spectral properties measuring the network synchronizability are investigated. In Sec. IV, we studied the correlation between the network synchronizability and two important network properties, i.e., the average network distance and the largest relative betweenness. For network ensembles consisting of networks with the same size and the same average degree, we particularly compare their synchronizability in Sec. V. Finally, the paper ends with a conclusion.

II. NETWORK ENSEMBLES

In this section, we construct three network ensembles with prescribed statistical properties. Each ensemble consists of one million networks. The specific construction of the network ensembles is as follows.

As we know, one important statistical property of complex networks is the degree distribution or degree sequence in a strict sense. The degree sequence of a network is a series of integer numbers, $(k_1, k_2, \cdots, k_i, \cdots)$, where k_i is the degree of node i. In order to construct a network ensemble with the same degree sequence, here we apply the technique of link switching,[21,23] which enables us to change the neighbors of a node without altering its degree. In other words, the link switching operation is degree-preserving. This operation has been schematically shown in Fig. 1. It should be pointed out that although applying link switching to a node does not change its degree, this operation changes its concrete neighbors, i.e., the local connection pattern. As we will see later, this change can significantly affect the network synchronizability. First of all, we construct a network ensemble as follows:

Ensemble A. This ensemble consists of networks with exactly the same degree sequence. As shown in Fig. 1, we start from a regular network with N nodes forming a ring. Each node in the network has k_d links to its nearest neighbors. Then the link switching operations are applied successively, and each time a variant of the starting network is generated. Since the starting network is a regular network, the degree sequences of all networks in this ensemble are regular, i.e., $(k_d, k_d, \cdots, k_d)$. Note that the link switching operation may break down a connected network into two isolated networks. To avoid this situation, we can monitor the second smallest eigenvalue of the variant network generated. If the network breaks into two parts, its second smallest eigenvalue will become zero (the smallest eigenvalue is always zero). If this happens, we just remove the corresponding variant network from the ensemble, and go back one step in the program to apply the link switching again.

In practice, the network ensemble with exactly the same global properties, such as the degree sequence, is seldom used. Frequently, we study complex networks in terms of network ensembles with the same statistical properties, such as the degree distribution or the average degree, etc. For example, the scale-free network generated by the Barabási–Albert algorithm has a power-law degree distribution with an exponent -3.[2] Since the local connection patterns of randomly generated networks with the same statistical properties are different, it is thus very important to investigate their synchronizability in the sense of an ensemble rather than an individual network. For this purpose, we further construct another two network ensembles as follows:

Ensemble B. In this case, an ensemble of random networks is generated. All the networks in the ensemble have the same average degree.

Ensemble C. In this case, an ensemble of randomly generated small-world networks is constructed. Each small-world network is generated by the WS approach:[1] first a regular network of N nodes is generated. The nodes form a ring and each node has k_d connections to its nearest neighbors. Then a small-world network is obtained by rewinding k_r links.

III. DISTRIBUTIONS OF SPECTRAL PROPERTIES

In this section, we analyze the spectral properties for *Ensemble A*. Since networks in this ensemble all have exactly the same regular degree sequence, what is of particular interest is the distributions of the eigenvalues λ_2, λ_N, and the eigenratio $r=\lambda_N/\lambda_2$, which are relevant to network synchronizability.

In Fig. 2, the distributions of r, λ_N, and λ_2 for *Ensemble A* with $N=100$, $k_d=6$ are plotted. It is seen that the distributions are all bell-shaped, though with long tails. Remarkably, the eigenratios r are distributed over quite a wide range that covers almost three orders of magnitude. The existence of very large eigenratios implies that there are networks having very poor synchronizability in the ensemble, though all the

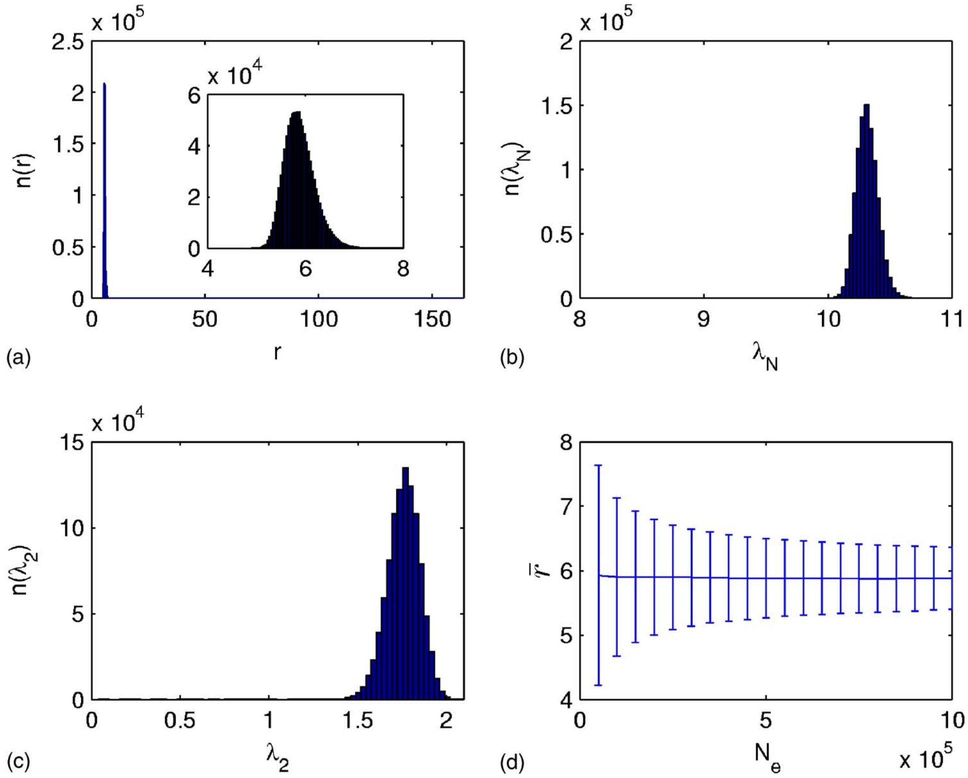

FIG. 2. (Color online) (a)–(c) The distributions of network spectral properties r, λ_N, and λ_2 for *Ensemble A* with $N=100$, $k_d=6$. The scope of the horizontal axis corresponds approximately to the range of the distributions. This rule is kept for all the following figures. The inset in (a) enlarges the peak of the distribution. (d) The average of r and its variance vs the ensemble size N_e for *Ensembles A* with $N=100$, $k_d=6$.

networks in *Ensemble A* have the same regular degree sequence (6, 6, ⋯, 6). On the other hand, although the distribution of r is very board, most of the eigenratios concentrate in a small interval between 5 and 7 as shown in the inset of Fig. 2(a). This shows that the variant networks with extremely poor synchronizability are actually very rare in *Ensemble A*. We have also studied other examples with different N and k_d. The distributions of network spectral properties are similar to that shown in Fig. 2.

Do the above distributions of the eigenvalues and eigenratios converge with the increase of ensemble size? Or, in other words, do the eigenvalues and eigenratios have fixed probability density functions when the ensemble size approaches infinity? The answers to these questions are positive. For example, in Fig. 2(d), we plot the averages of r and its variances versus the ensemble size N_e. It is seen that with the increase of the size of the ensemble, the eigenratio r converges to its average value, showing that the network spectral properties can be well represented by the ensemble averages. The same results have been found for the distributions of λ_N and λ_2. We noticed that this important issue was first studied in Ref. 22, and similar results have been reported for a different class of networks, i.e., the random scale-free networks. Here, our work further extends previous findings in scale-free network ensembles to network ensembles with the same regular degree sequence.

To understand the distributions of r, λ_N, and λ_2, we plot in Fig. 3 their time series according to the time steps when the link switching is operated. Since the link switching operations can change network properties, such as the average network distance and the cluster coefficient of the network, we also plot the time series of these two relevant quantities in Fig. 3. A careful examination of the time series in Fig. 3

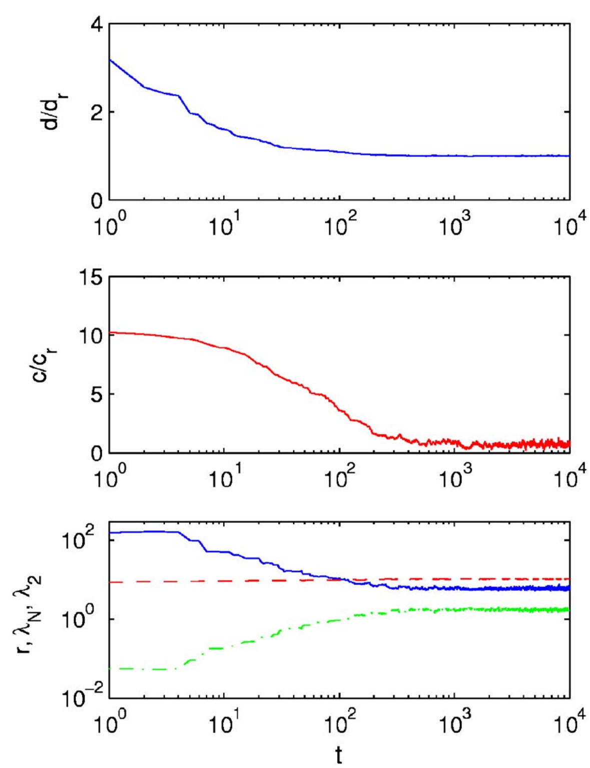

FIG. 3. (Color online) (a),(b) The time series of normalized average network distance d/d_r (a) and the normalized network cluster coefficient (b) for networks generated from a regular network with $N=100$, $k_d=6$ by link switching operations. d_r and c_r correspond, respectively, to the average network distance and the network cluster coefficient for random networks with $N=100$, $\bar{k}_d=6$. Here, the bar denotes the average degree. Both d and c approximately converge to d_r and c_r after $t>200$. (c) The time series of network spectral properties r (solid line), λ_N (dashed line), and λ_2 (dot-dashed line).

reveals that there exists a transient stage during the process of link switching. In the initial 200 time steps, both the average network distance and the cluster coefficient of the network decrease drastically. After this period, they fluctuate with small amplitudes around the averages, which are comparable with the counterparts of random networks. These results reveal that the repeating of the link switching operation has changed the nature of the starting regular network. Specifically, the networks generated in the transient stage show a small-world effect since they have smaller average network distances than that of regular networks and larger cluster coefficients than that of random networks. After the transient stage, the networks generated show properties of random networks. Since *Ensemble A* contains one million networks and there are only several hundred variant networks showing small-world effect, essentially it can be regarded as an ensemble of networks with features of random networks. Of course, they are very special "random" networks because they have twofold characteristics: i.e., their degree sequences are the same as regular network, while their network properties are like that of random networks. For this reason, we refer to the networks in *Ensemble A* as special homogeneous random networks.

From Fig. 3(c), it can be seen that the spectral properties of networks in *Ensemble A* are consistent with the changes of the average network distance and the cluster coefficient. During the transient stage, λ_2 increases by almost two orders of magnitude, while λ_N is almost constant. This demonstrates that λ_2 is sensitive to the changes of the local connection pattern induced by the link switching operations, but λ_N does not. As a result, the eigenratios also decrease significantly in the transient stage. According to the above analysis, we can conclude that for networks with exactly the same regular degree sequence, their synchronizabilities vary a lot. However, the network ensemble does have well-defined average spectral properties that converge with the increase of the ensemble size. We further understand that the networks with very poor synchronizability (corresponding to very large r values) in *Ensemble A* are those generated at the initial stage of the link switching process, i.e., in *Ensemble A*, a very small part of networks with large average distance and large cluster coefficient exhibit poor synchronizability.

IV. CORRELATION ANALYSIS

Structure determines function. The central task in the study of synchronizability of networks is to find how network structure affects its synchronizability. Previously, many network properties, such as degree distribution, the average degree, the average network distance, the node betweenness, and the degree heterogeneity, have been shown to be related to the synchronizability of different classes of networks. For example, in many networks, it has been shown that the smaller average network distance leads to the better synchronization of dynamics on networks. In other cases, it is reported that the betweenness is more relevant to the network synchronizability.[9,27] We notice that in most existing works the study of the relation between network synchronizability and network topological properties is usually based on limited network samples. Usually, the number of network samples is less than 100. In our opinion, it is desirable to carry out such studies based on a network ensemble with a large number of certain networks. For this purpose, in the present work, we propose to apply correlation analysis between network synchronizability and two important network topological properties: the average network distance d, and the largest relative betweenness of network l_m.[27]

In Fig. 4, the distributions of d and l_m for typical examples of three network ensembles are plotted. It can be seen that they are all bell-shaped. Especially, the probability density functions of d and l_m are very smooth for *Ensemble B*. We notice that although the averages of d and l_m for network samples have been extensively used in the previous works on complex networks, their statistical properties have not been

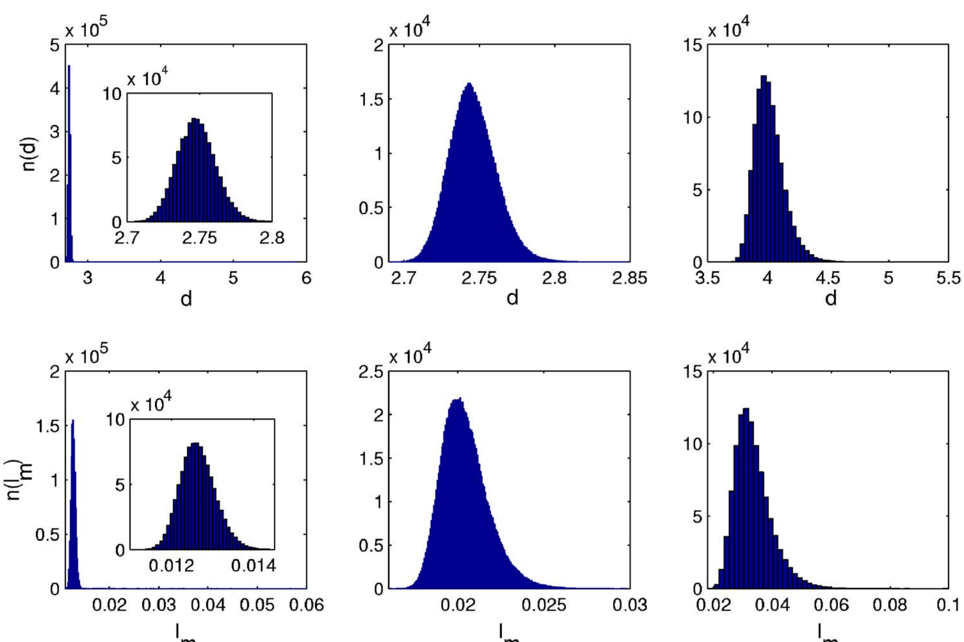

FIG. 4. (Color online) The distributions of network topological properties d and l_m for typical examples of three network ensembles. d is the average network distance, and l_m is the largest relative node betweenness, respectively. Left column: *Ensemble A* with $N=100$ and $k_d=6$. Middle column: *Ensemble B* with $N=100$ and $\bar{k}_d=6$. Right column: *Ensemble C* with $N=100$, $k_d=6$, and $k_r=20$. The insets enlarge the peaks of the corresponding distributions.

TABLE I. The correlation coefficients c_{rd} and c_{rl} for typical examples of three network ensembles. r,d,l denote the eigenratio, the average network distance, and the largest relative betweenness of the network, respectively.

	Ensemble A	Ensemble B	Ensemble C		
	$k_d=6$	$\bar{k}_d=6$	$k_d=6,\ k_r=10$	$k_d=6,\ k_r=20$	$k_d=6,\ k_r=30$
c_{rd}	0.381	0.439	0.820	0.771	0.742
c_{rl}	0.197	0.000	0.361	0.392	0.387

TABLE II. The averages of network spectral properties r, λ_N, λ_2, and network topological properties d for examples of three network ensembles.

	Ensemble A	Ensemble B	Ensemble C		
	$k_d=6$	$\bar{k}_d=6$	$k_d=6,\ k_r=10$	$k_d=6,\ k_r=20$	$k_d=6,\ k_r=30$
$\bar{r}$	5.881	19.431	77.964	46.603	33.631
$\bar{\lambda}_N$	10.315	13.786	9.661	10.082	10.401
$\bar{\lambda}_2$	1.759	0.745	0.130	0.224	0.318
$\bar{d}$	2.748	2.746	4.747	4.012	3.674

investigated before. Here, we show that, for several types of network ensembles, the network topological properties d and l_m have well-defined ensemble averages. This result justifies the use of averages for these two quantities in previous studies. Since the network synchronizability is characterized by its eigenratio r, we investigate the correlations between r and the network topological properties d as well as l_m. The linear correlation coefficient between two series x and y can be calculated as follows: $c_{xy}=l_{xy}/\sqrt{l_{xx}l_{yy}}$, where $l_{xy}=\Sigma_{i=1}^{N}(x_i-\bar{x})(y_i-\bar{y})$, $l_{xx}=\Sigma_{i=1}^{N}(x_i-\bar{x})^2$, $l_{yy}=\Sigma_{i=1}^{N}(y_i-\bar{y})^2$, and $\bar{x},\bar{y}$ are the averages of x and y, respectively. According to this formula, we calculate the correlation coefficients c_{rd} and c_{rl} for some network ensembles. The results are listed in Table I. First, the correlation coefficients c_{rd} for examples of *Ensemble C* are generally large, showing that the synchronizability of small-world networks is highly related to the average network distance. Secondly, for *Ensemble A* and *Ensemble B*, the correlation coefficients c_{rd} are smaller compared with that of *Ensemble C*, showing that the synchronizability of random networks is less correlated to the average network distance than the small-world networks. Thirdly, compared with c_{rd}, c_{rl} is generally smaller for each network ensemble, especially $c_{rl}\approx 0$ for *Ensemble B*. This implies that for random networks and small-world networks, the network synchronizability is weakly correlated to the largest relative betweenness. This result is reasonable since the quantity of largest relative betweenness is more suitable to describe networks with important hubs, such as scale-free networks. For random networks and small-world networks, their degrees satisfy exponential distribution, i.e., their degrees are approximately homogeneous, thus the largest relative betweenness might not be a suitable quantity to describe these types of networks.

V. SYNCHRONIZABILITY OF NETWORK ENSEMBLES

Usually, when we discuss the synchronizability of certain types of complex networks, actually what we mean is the synchronizability of an ensemble of networks. Therefore, it is very interesting to compare the synchronizability of network ensembles constructed in Sec. II. For this purpose, we make all networks in three ensembles have the same size N and the same average node degree $\bar{k}_d$.

In Table II, we list the averages of network spectral properties r, λ_N, λ_2, and network topological property d, for several typical examples of network ensembles. We have several findings here. First, according to the eigenratios, it can be found that among the three network ensembles, *Ensemble C*, i.e., the ensemble with small-world networks, has the worst synchronizability, while *Ensemble A*, i.e., the ensemble with *homogeneous random networks*, has the best synchronizability in the sense of ensemble average. From Table II, we can see that $\bar{\lambda}_N$ of three network ensembles are of the same order of magnitude for all examples, but $\bar{\lambda}_2$ vary greatly for different network ensembles. This suggests that $\bar{\lambda}_2$ is more sensitive to the change of local connection pattern, which affects the network synchronizability. Secondly, we further compare the synchronizability of two random network ensembles, i.e., *Ensemble A* and *Ensemble B*. We can find that the former has much better synchronizability than the latter since the eigenratio $\bar{r}$ for *Ensemble A* is one order magnitude smaller than that of *Ensemble B*. This finding is remarkable if we notice that *Ensemble A* and *Ensemble B* have almost the same average network distance. Therefore, our result strongly suggests that the homogeneity of degree is in favor of synchronization even on random networks. Previously, similar results have been shown for scale-free networks in Ref. 27. Here, we further show that for networks of random nature, besides the average network distance, the homogeneity of degree is another factor enhancing synchronization on networks.

We have systematically compared the synchronizability of *Ensembles A, B*, and *C* with other parameter settings, for example, $k_d=4,6,8,10,12,14,16$, $k_r=10,20,30$, and similar results are obtained. In Fig. 5, we plot the averages of r, λ_N, λ_2, and d versus the average degree $\bar{k}_d$. From Fig. 5(a), it can be seen that with the increase of $\bar{k}_d$, $\bar{r}$ decreases showing the network ensembles having better synchronizability with larger $\bar{k}_d$. This is due to the fact that the average network distance generally decreases with larger average degree, as shown in Fig. 5(d). Comparing the synchronizability of three network ensembles, it is clearly seen that *Ensemble A* has the best synchronizability while *Ensemble C* has the worst synchronizability. From Fig. 5(d), it is be found that for all $\bar{k}_d$, *Ensembles A* and *B* have almost the same average network distance, but they have significantly different synchronizability as shown in Fig. 5(a). In addition, *Ensemble C* has a relatively larger average network distance than that of *Ensembles A* and *B*. Based on the results shown in Fig. 5, we

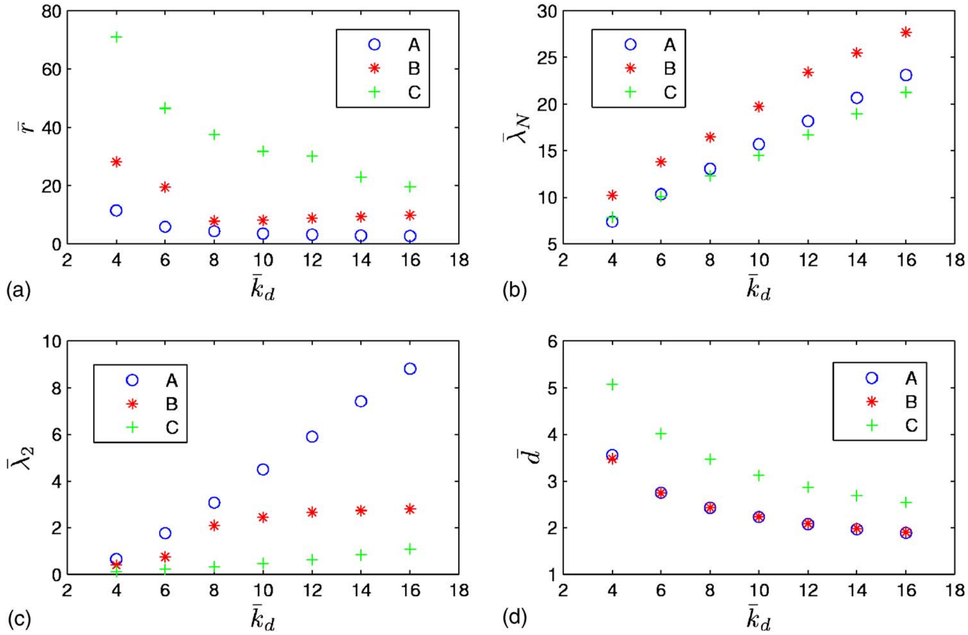

FIG. 5. (Color online) The averages of r, λ_N, λ_2, and d vs parameter $\bar{k}_d$. $N=100$ for all network ensembles, and $k_r=20$ for *Ensemble C*. The legends A, B, and C denote *Ensembles A, B*, and *C*, respectively.

can conclude that the homogeneous random networks have much better synchronizability than both usual random networks and small-world networks.

VI. CONCLUSION

To conclude, we carry out statistical analysis to the synchronizability of three types of network ensembles: *Ensemble A* contains networks generated from a regular network by link switching operations, and *Ensembles B* and C consist of networks with the same average degree. Our results show that with the same degree sequence, the eigenvalues and eigenratios, which characterize the network synchronizability, have well-defined distributions, i.e., the distributions converge to fixed probability density functions with the increase of ensemble size. Equivalently, this is to say that the network spectral properties can be reasonably represented by the ensemble averages. Statistically, in this ensemble, the networks with extremely poor synchronizability, i.e., those with very large eigenratios, are very rare. Our statistical analysis reveals that besides the degree of node, the local connection pattern can significantly affects the network synchronizability. We also investigate the correlation between the eigenratio and two network topological properties. It is shown that for networks with exponential degree distributions, such as random networks and small-world networks, the average distance is more correlated to the network synchronizability than the largest relative betweenness. Finally, we compare the synchronizability of three network ensembles. Interestingly, it is found that the ensemble of homogeneous random networks, i.e., the networks that have regular degree sequence but show properties of random network, has the best synchronizability among the three ensembles. Our results suggest that the homogeneity of degree can greatly enhance the network synchronizability for networks of random nature.

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