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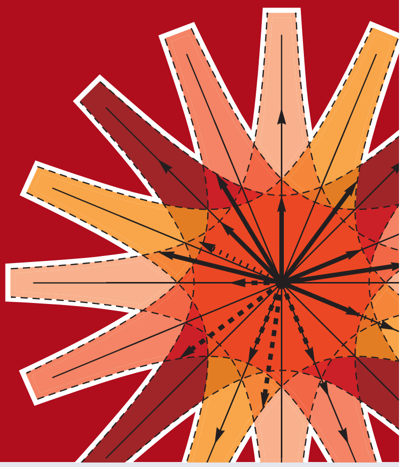

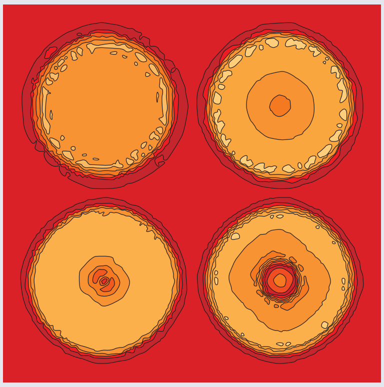

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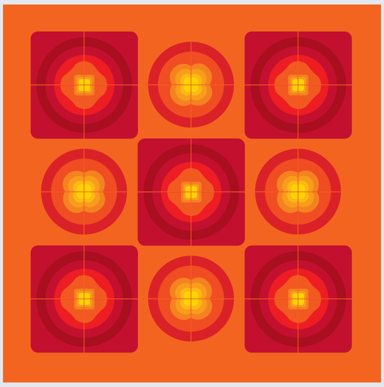

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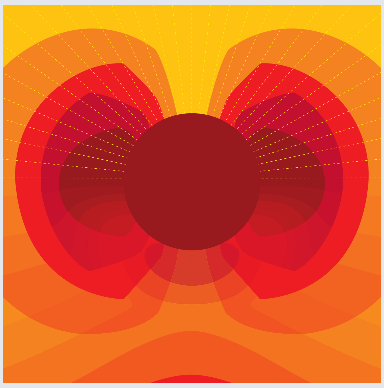

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Explosive synchronization on co-evolving networks

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Abstract – Many realistic dynamics are based on complicated networks such as the co-evolving networks with mutual correlation, in contrast to the recent focus on single- or multi-layered networks. We here study this kind of realistic dynamics by presenting a network model consisting of two interdependent subnetworks, which have the same power-law degree distribution. We focus on the dynamics of explosive synchronization. We show that the explosive synchronization can exist for a large class of co-evolving networks with scale-free distributions, thus extending the condition of explosive synchronization from strong correlation with $\omega_i = k_i$ to weak correlation with $\omega_i = f(k_i)$.

Introduction. – The study of dynamics on complex networks is the fundamental issue in network science [1–14]. It is found that the network structure can drastically influence the dynamics on it. Two such examples are epidemic spreading and the explosive synchronization (ES). For the former, there exists an epidemic threshold in a random network and the epidemic will outbreak only when its contagious rate is greater than a certain threshold. Surprisingly, it is found that the threshold will become zero for SF networks [15–19]. For the latter, it refers to the extremely abrupt synchronization transition recently reported in certain networked oscillators systems, such as the generalized Kuramoto model and coupled Rössler oscillators [8–14]. In refs. [8,9], it was shown that ES occurs when the network is scale free (SF) and the intrinsic frequency ω_i of phase oscillator is taken as the degree k_i of the node on which it sits. That is, the condition for the appearance of an ES transition is the strong correlation between the intrinsic frequency and the node's degree. This finding comes from the understanding of the microscopic mechanisms of explosive percolation, which represents an abrupt percolation transition in random [20] and SF networks [21,22] and has recently attracted a lot of attention [23–26].

The above important findings are mainly focused on single-layered networks. However, many realistic dynamics are based on multi-layered networks rather than single-layered networks [27–36]. For examples, the proper functioning of the Internet relies on the power network, thus forming a double-layered network [29]. Traffic networks constructed based on flights, trains, and coaches are other typical multi-layered networks [37]. The topologies of these networks have obvious two or more layers. The studies on them have provided important insights for some phenomena in complex networks, such as the catastrophic cascading failure, *i.e.*, a failure in the power grid leading to a failure of some nodes in the Internet [29], and the accelerating spreading of epidemic by airline traffic [38], etc.

More complicated networks are the co-evolving interdependent networks where the mutual influence exists not only between the network and dynamics but also among their subnetworks. The dynamics on these networks is more realistic and definitely worthy to be studied. A characteristic feature of this dynamics is that it usually shows a balance between competition and cooperation such as ecological systems in which numerous species interact via predation, herbivory, mutualistic support, competition, cooperation, and so on. A prototypical example is the plant-animal networks where the mutualistic relation between them is beneficial for the survival and reproduction of both of them such as animal pollinators and flowering plants. It is pointed out that for the mutualistic plant-pollinator networks, the degree distributions of animals are close to power laws, while those of plants are of truncated power-law, exponential, or stretched-exponential form [39]. This kind of intrinsic balance exists not only in the ecological systems but also in social networks and coupled oscillators networks. For example, it is well known that individuals have crisis awareness, *e.g.*, if one realizes that there are infected people around him, he will spontaneously take some measures to protect himself so that the risk will be reduced at most. The crisis awareness depends on the obtained information such as the news, advertisements, and rumors, etc. Except for the infectious contact, the obtained information will jointly influence the epidemic spreading [40–46]. Thus, the contact network will combine with the information network together to influence the epidemic spreading. How to model these systems and understand their dynamics is an open problem.

Motivated by these evidences, we here present a network model of co-evolving interdependent networks, in which both subnetworks have the same power-law degree distribution and their mutual correlation can be adjusted. Then we study ES on the correlated co-evolving networks. We challenge whether the condition of strong correlation between the intrinsic frequency and the node's degree is necessary for the appearance of ES. Interestingly, we find that the ES can exist for a large class of co-evolving networks with SF distributions, thus extending the condition of ES from strong correlation to weak correlation.

The paper is organized as follows. In the second section, we present the model of co-evolving interdependent networks. In the third section, we discuss the ES on the co-evolving network. Finally, discussions and conclusions are given in the fourth section.

The co-evolving network model of interdependent networks. – This kind of network consists of several subnetworks and is usually not static but in principle evolving. During the evolution, the subnetworks are not only growing by themselves but also influence each other. For simplicity, we here focus on two interdependent subnetworks. In particular, we assume that at each evolutionary step, the probability of connectivity of each subnetwork will depend on both its own degree and the degree of the other. The resulting two subnetworks will be mutually correlated.

Figure 1 shows the schematic figure of interdependent networks, where the upper left (G_1) and upper right (G_2) panels denote the two subnetworks, respectively, and the bottom panel represents the interdependent networks. In general, we call the two subnetworks mutually correlated if each node in network G_1 is correlated to one and only one node of network G_2 and vice versa, according to specific rules. In this case, each pair of correlated nodes can be merged into a common node but with two sets of degrees. Let $k_{1,i}$ be the degree of node i in G_1 and $k_{2,i}$ the degree of the correlated node i in G_2. Then $(k_{1,i}, k_{2,i})$ will be the two sets of degrees at the common node i, see the bottom of fig. 1.

During the evolutionary process, we add one node to the co-evolving network at each time step. The added node will have two sets of links, *i.e.*, one set of links connects to m existing nodes of network G_1 and another set of links connects to m existing nodes of network G_2. Similarly to the Barabási and Albert (BA) model [47], we take the preferential attachment. But each link is preferential to both G_1 and G_2, in contrast to the single network of the BA model. In detail, the probability for an existing node i to get a new link is taken as $P_{1,i} \sim \upsilon_{11}k_{1,i} + \upsilon_{12}k_{2,i}$ in G_1 and $P_{2,i} \sim \upsilon_{21}k_{1,i} + \upsilon_{22}k_{2,i}$ in G_2, where $\upsilon_{11}, \upsilon_{12}, \upsilon_{21}$ and υ_{22} are the coefficients. As the total existing links are $2mt$ at the time step t in both G_1 and G_2, $P_{1,i}$ and $P_{2,i}$ can be normalized as $P_{1,i} = (\upsilon_{11}k_{1,i} + \upsilon_{12}k_{2,i})/2mt$ and $P_{2,i} = (\upsilon_{21}k_{1,i} + \upsilon_{22}k_{2,i})/2mt$. Thus, the mean-field equations of the model can be given as

$$\begin{aligned}\frac{\partial k_{1,i}}{\partial t} &= m\left[\upsilon_{11}\frac{k_{1,i}}{2mt} + \upsilon_{12}\frac{k_{2,i}}{2mt}\right] = \upsilon_{11}\frac{k_{1,i}}{2t} + \upsilon_{12}\frac{k_{2,i}}{2t},\\ \frac{\partial k_{2,i}}{\partial t} &= m\left[\upsilon_{21}\frac{k_{1,i}}{2mt} + \upsilon_{22}\frac{k_{2,i}}{2mt}\right] = \upsilon_{21}\frac{k_{1,i}}{2t} + \upsilon_{22}\frac{k_{2,i}}{2t}.\end{aligned} \tag{1}$$

For simplicity, we take $\upsilon_{11} + \upsilon_{12} = 1$ and $\upsilon_{21} + \upsilon_{22} = 1$. Eq. (1) becomes

$$\begin{aligned}\frac{\partial k_{1,i}}{\partial t} &= (1-\alpha_1)\frac{k_{1,i}}{2t} + \alpha_1\frac{k_{2,i}}{2t},\\ \frac{\partial k_{2,i}}{\partial t} &= \alpha_2\frac{k_{1,i}}{2t} + (1-\alpha_2)\frac{k_{2,i}}{2t},\end{aligned} \tag{2}$$

where $\upsilon_{11} = 1-\alpha_1, \upsilon_{12} = \alpha_1, \upsilon_{21} = \alpha_2, \upsilon_{22} = 1-\alpha_2$. Equation (2) is the model of co-evolving interdependent networks and can be represented by $S(\alpha_1, \alpha_2)$, *i.e.*, the connectivity probability of a subnetwork depends on not only the degree of itself but also its counterpart. Different pairs of (α_1, α_2) will give different correlation of degrees. Specially, it will become the BA model when $\alpha_1 = \alpha_2 = 0$, and the "opposite determined" network when $\alpha_1 = \alpha_2 = 1$.

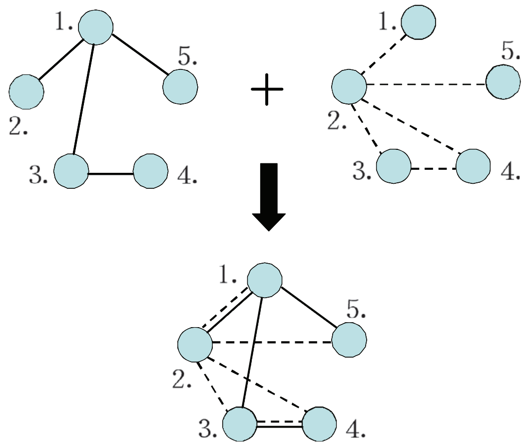

Fig. 1: (Color online) Schematic diagram of interdependent networks. Top left: the subnetwork G_1; top right: the subnetwork G_2; bottom: the co-evolving network with two sets of degrees $(k_{1,i}, k_{2,i})$.

The solutions of eq. (2) can be figured out by rewriting it as follows:

$$\begin{aligned}\frac{\partial k_{1,i}}{\partial(\ln t)} &= (1-\alpha_1)\frac{k_{1,i}}{2}+\alpha_1\frac{k_{2,i}}{2},\\ \frac{\partial k_{2,i}}{\partial(\ln t)} &= \alpha_2\frac{k_{1,i}}{2}+(1-\alpha_2)\frac{k_{2,i}}{2}\end{aligned} \qquad (3)$$

The characteristic equation of eq. (3) is

$$\lambda^2-\frac{2-\alpha_1-\alpha_2}{2}\lambda+\frac{1-\alpha_1-\alpha_2}{4}=0 \qquad (4)$$

with two roots $\lambda_1=1/2$ and $\lambda_2=(1-\alpha_1-\alpha_2)/2$. Since they are simple real roots, we obtain two specific solutions of eq. (3) as $(k_{1,i}^{(1)},k_{2,i}^{(1)})=(A_1e^{\lambda_1\ln t},A_2e^{\lambda_1\ln t})$ and $(k_{1,i}^{(2)},k_{2,i}^{(2)})=(B_1e^{\lambda_2\ln t},B_2e^{\lambda_2\ln t})$, where A_1,A_2,B_1,B_2 are constants. By substituting them into eq. (2), respectively, we obtain $A_2=A_1$ and $B_2=-\frac{\alpha_2}{\alpha_1}B_1$. Hence, the general solution is $(k_{1,i},k_{2,i})=(A_1t^{1/2}+B_1t^{(1-\alpha_1-\alpha_2)/2},A_2t^{1/2}-\frac{\alpha_2}{\alpha_1}B_1t^{(1-\alpha_1-\alpha_2)/2})$. Suppose a node i is added at time $t=t_i$, then we have $k_{1,i}(t_i)=k_{2,i}(t_i)=m$. Substituting them into the general solution, we get $(1+\frac{\alpha_2}{\alpha_1})B_1=0$. Since both α_1, α_2 are positive, we obtain $B_1=0$ and $A_1=m/t_i^{1/2}$. Thus, the exact solution of eq. (2) is

$$k_{1,i}(t)=m\left(\frac{t}{t_i}\right)^{\frac{1}{2}},\quad k_{2,i}(t)=m\left(\frac{t}{t_i}\right)^{\frac{1}{2}} \qquad (5)$$

for any $0\le\alpha_{1,2}\le 1$. The corresponding degree distribution is

$$P(k)\sim k^{-\gamma} \qquad (6)$$

for both $k_{1,i}$ and $k_{2,i}$, where $\gamma=3$. It is noticed that the parameters α_1, α_2 do not show up in eq. (6), indicating that no matter how a subnetwork specifically depends on another, its degree distribution remains the same. To confirm it numerically, we take $m=3$ and the network size $N=1000$, which gives the average degree $\langle k\rangle=6$. Figure 2 shows the simulation results for different pairs of (α_1,α_2). For comparison, we also consider the case of the BA model there. It is obvious to see that all the cases are overlapped, confirming the theoretical prediction given in eq. (6).

To characterize how the parameters (α_1,α_2) influence the mutual correlation between the two subnetworks, we calculate the inter-degree-degree correlation (IDDC) [48,49]. The IDDC describes the correlation between $k_{1,i}$ and $k_{2,i}$ and is defined as

$$r_{12}=\frac{1}{\sigma_q^2}\sum_{jk}jk(e_{jk}-q_j^1q_k^2), \qquad (7)$$

where q_l^m represents the probability of a node with degree l in the G_m network, e_{jk} is the joint probability of a pair sets of degrees (k_1,k_2) with $k_1=j$, $k_2=k$, respectively, and $e_{jk}=q_j^1q_k^2$ when there are no correlations between k_1 and k_2. The variance $\sigma_q^2=\sum_k k^2q_k-[\sum_k kq_k]^2$ represents the maximum IDDC for $q_k^1=q_k^2=q_k$. r_{12} will be in the range $[-1,1]$ with the value 1 for a system with maximum IDDC, 0 for no IDDC, and -1 for maximum anti-IDDC. Equation (7) can be rewritten as

$$r_{12}=\frac{N^{-1}\sum_i j_ik_i-[N^{-1}\sum_i\frac{1}{2}(j_i+k_i)]^2}{N^{-1}\sum_i\frac{1}{2}(j_i^2+k_i^2)-[N^{-1}\sum_i\frac{1}{2}(j_i+k_i)]^2}, \qquad (8)$$

where j_i, k_i are the degrees of the i-th node in G_1 and G_2, respectively, N is the total number of nodes. r_{12} shows the similarity of interdependent networks. Although $k_{1,i}(t)$ and $k_{2,i}(t)$ in eq. (5) have the same expression, their linked nodes may be quite different. When $r_{12}=1$, the linked nodes of $k_{1,i}(t)$ and $k_{2,i}(t)$ will be exactly the same, while for $r_{12}=0$, the linked nodes of $k_{1,i}(t)$ and $k_{2,i}(t)$ will be extremely different.

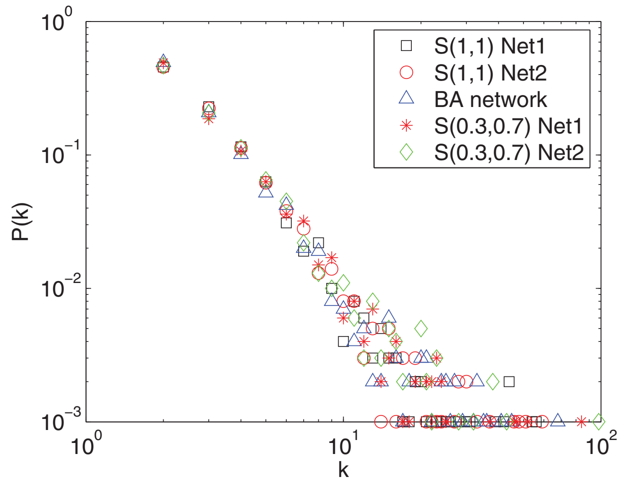

Fig. 2: (Color online) $P(k)$ *vs.* k for the two networks G_1 and G_2 with the network size $N=1000$ and average degree $\langle k\rangle=6$ where the "squares" and "circles" denote the case of $(\alpha_1=1,\ \alpha_2=1)$ for G_1 and G_2, respectively, the "asterisks" and "diamonds" denote the case of $(\alpha_1=0.3,\ \alpha_2=0.7)$ for G_1 and G_2, respectively, and the "triangles" denote the case of the BA network.

Figure 3 shows the dependence of r_{12} on the parameters α_1 and α_2. From fig. 3 we have $r_{12}=0.62$ for the case of $\alpha_1=\alpha_2=0$, which represents two independent SF networks. This r_{12} is consistent with the result obtained in ref. [48] where the inter-similarity is obtained as $r_{12}=0.6$. From fig. 3 we also see that r_{12} increases with both α_1 and α_2, indicating that a larger value of r_{12} implies a stronger correlation between G_1 and G_2. In sum, the constructed G_1 and G_2 are independent but with high mutual correlation.

Explosive synchronization on the co-evolving network. – ES, *i.e.*, an extremely abrupt synchronization transition, has been shown in SF networks, as a consequence of a positive correlation between the heterogeneity of the connections and the natural frequencies of the oscillators [8]. The used model can consist in the Kuramoto

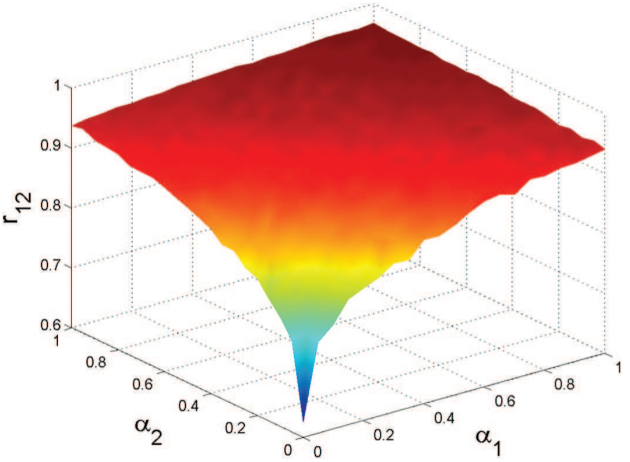

Fig. 3: (Color online) Dependence of r_{12} on the parameters α_1 and α_2.

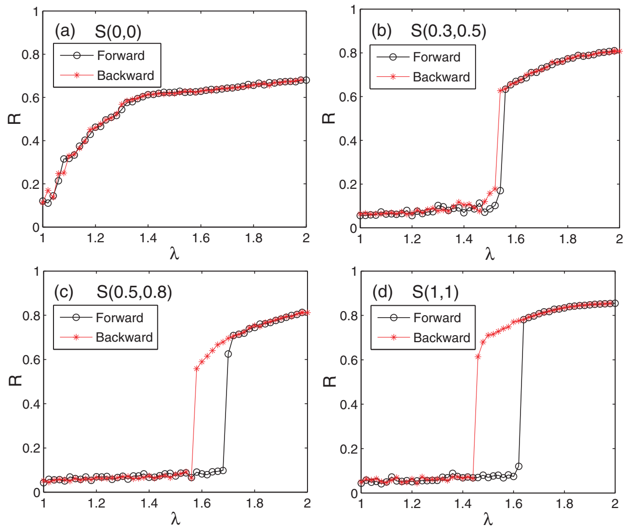

Fig. 4: (Color online) R *vs.* λ for different pairs of (α_1, α_2) where (a) to (d) represent the cases of $S(0,0), S(0.3,0.5), S(0.5,0.8)$ and $S(1,1)$, respectively.

oscillators as follows:

$$\dot{\theta}_i = \omega_i + \lambda \sum_{j=1}^{N} A_{ij} \sin(\theta_j - \theta_i), \quad i = 1, \dots, N, \tag{9}$$

where ω_i stands for the natural frequency of oscillator i, λ accounts for the coupling strength, A_{ij} denotes the adjacency matrix of the network with $A_{ij} = 1$ when oscillators i and j are connected, while $A_{ij} = 0$ otherwise. Reference [8] considers the case of $\omega_i = k_i$. A key question is: should $\omega_i = k_i$ be a necessary condition for the occurrence of ES? That is, is it still possible for us to observe the ES by taking $\omega_i = f(k_i)$, where $f(k_i)$ is a general function of k_i?

To answer the question, we investigate the dynamics of eq. (9) on the constructed co-evolving network. We choose the subnetwork G_1 as the coupling network, *i.e.*, the matrix A_{ij} is taken by the connections of G_1. We noticed that in many two-layer networks, the local dynamics on one layer might not be governed or controlled by the topology of the layer itself, but instead by the connectivity of the other layer. Taking an epidemic outbreak as a rough example, the virus spreads in a human contact network, but the human behaviors could be strongly affected by the communication network, especially during the key initial stage of the outbreak. Therefore, instead of taking $\omega_i = k_{1,i}$, in the present work we take $\omega_i = k_{2,i}$. That is, the natural frequency ω_i is only a property of G_2. In this way, the natural frequency ω_i will be a function of $k_{1,i}$ and the function can be represented by the inter-degree-degree correlation r_{12}. Following [8], we introduce the order parameter R to measure the degree of synchronization among the N oscillators as follows:

$$R(t)e^{i\Psi(t)} = \frac{1}{N} \sum_{j=1}^{N} e^{i\theta_j(t)}. \tag{10}$$

Simple algebra gives $\Psi(t) = \frac{1}{N} \sum_{j=1}^{N} \theta_j$ and $R(t) = \frac{1}{N} \sqrt{(\sum_{j=1}^{N} \cos\theta_j)^2 + (\sum_{j=1}^{N} \sin\theta_j)^2}$. $R(t)$ will reach unity when the system is fully synchronized, while $R(t) = 0$ for the totally incoherent solution.

Figure 4 shows the results of numerical simulations for different pairs of (α_1, α_2) where (a) to (d) represent the cases of $S(0,0), S(0.3,0.5), S(0.5,0.8)$ and $S(1,1)$, respectively. We see that R exhibits a continuous transition in fig. 4(a) but becomes an abrupt transition in figs. 4(b) to (d), indicating that the ES has been induced by the parameters α_1 and α_2. More interestingly, we find that there is a hysteresis loop between the forward and backward changing of the coupling strength λ and the hysteresis loop becomes larger with the increase of α_1 and α_2. This finding implies a proportional relation between the mutual correlation r_{12} and the size of the hysteresis loop, indicating that a larger IDDC has more influence on the dynamics of the network.

Discussions and conclusions. – Except for the ES, the co-evolving network model can be used for other cases with dynamics on complicated networks such as the epidemic spreading. The epidemic spreading has been well addressed recently on both the single-layered networks [15–19] and the two-layered networks [40,41,50], but it is still open for the correlated co-evolving network. As pointed out in the introduction, people will take some prevention measures to protect themselves if they are informed of the risk around, *i.e.*, crisis awareness. This kind of information may come from Internet, newspapers, TV, friends, etc., and forms an information network. Thus, we face a two-layered network: contact network and information network. Considering that information will make people change their connections and then conversely change the news paths, the contact and information subnetworks thus form a correlated co-evolving network. We have checked the epidemic spreading on this correlated co-evolving network and found that both the contact and information subnetworks will influence the epidemic spreading.

The present co-evolving network model provides an alternative approach of the BA model to produce the degree distributions of the power law, and thus it extends the BA model to a broad class of systems by varying the parameters (α_1, α_2). As we know, a network structure is not fixed for a fixed degree distribution but can be adjusted by changing its clustering coefficient and assortativity, etc. The co-evolving network model can be thus considered as a third way to change the network structure with fixed degree distribution. On the other hand, from eq. (2) we can easily find that the total degree of node i, *i.e.*, $k_{1,i}+k_{2,i}$, also satisfies the same power law. This is an interesting result as it shows a possible similarity between the local and global networks, thus deserves to be further studied.

In conclusion, we have presented a co-evolving interdependent network model consisting of two subnetworks, which comes from the observation that most realistic dynamics occur in complicated networks such as the co-evolving networks with mutual correlation. We show that its power-law degree distribution is not influenced by the parameters α_1 and α_2, which suggests that the BA model may be also rooted in inter-organized processes, in contrast to the self-organized processes. Based on this model, we have discussed how the model of co-evolving interdependent networks induces the ES. We find that the ES transition exists in a broad class of $\omega_i = f(k_i)$ with weak correlation between ω_i and k_i, in contrast to the previous condition of strong correlation $\omega_i = k_i$. Thus, this finding extends the condition of ES from strong correlation to weak correlation.

* * *

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