



Outline

- Introduction
- Signal, random variable, random process and spectra
- Analog modulation
- Analog to digital conversion
- Digital transmission through baseband channels
- Signal space representation
- Optimal receivers
- Digital modulation techniques
- Channel coding
- **Synchronization**
- Information theory



Synchronization

- Synchronization is one of the most critical functions of a communication system with coherent receiver. To some extent, it is the basis of a synchronous communication system.
- Three kinds of synchronization: Carrier synchronization, Symbol/Bit synchronization, and Frame synchronization.
- Carrier synchronization (载波同步): Receiver needs estimate and compensate for **frequency** and **phase** differences between a received signal's **carrier wave** and the receiver's **local oscillator** for the purpose of coherent demodulation, no matter it is analog or digital communication systems.

Chapter 8.8-8.9

韩声栋等, 《通信原理》



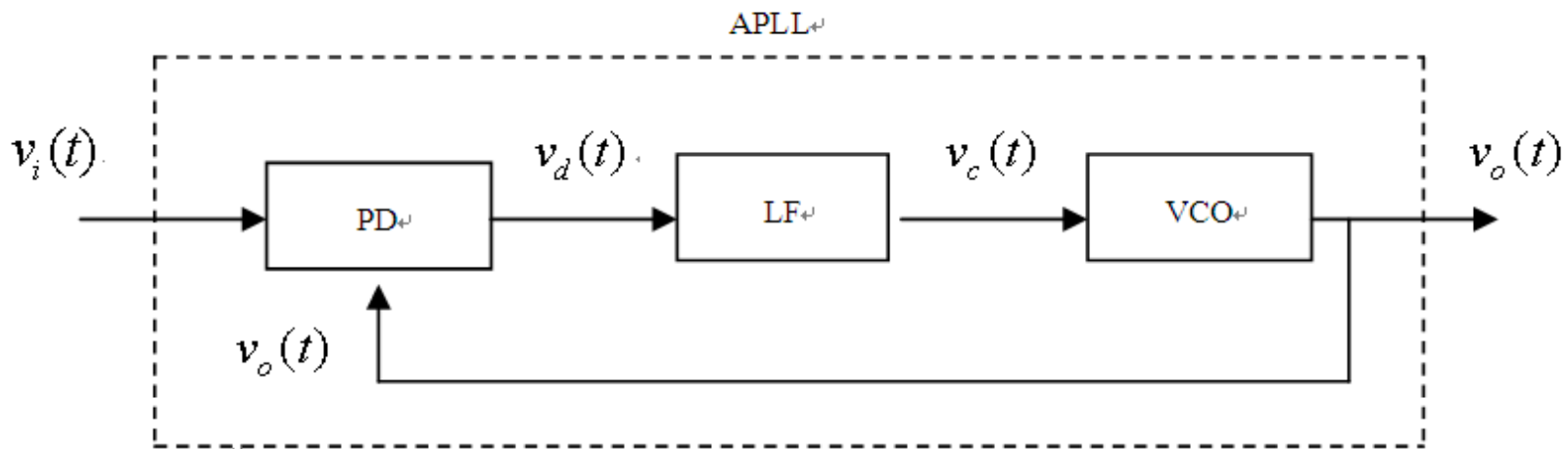
Synchronization

- Symbol/Big synchronization (符号/位同步): In digital systems, the output of the receiving filter (i.e. matched filter) must be sampled at the symbol rate and at the **precise sampling time instants**. Hence, we require a clock signal. The process of extracting such a **clock signal** at the receiver is called symbol/bit synchronization.
- Frame synchronization (帧同步): In frame-based digital systems, receiver also needs to estimate the starting/stopping time of a **data frame**. The process of extracting such a clock signal is called frame synchronization.



Synchronization

- Phase-Locked Loop (PLL, 锁相环)
 - PLL is often used in carrier syn. and symbol syn. It is a closed-loop control system consisting of
 - Phase detector (**PD**): generate the phase difference of $v_i(t)$ and $v_o(t)$.
 - Voltage-controlled oscillator (**VCO**): adjust the oscillator frequency based on this phase difference to eliminate the phase difference. At steady state, the output frequency will be exactly the same with the input





Synchronization

- Phase-Locked Loop (PLL, 锁相环)

$$v_i(t) = v_i \sin[\omega_0 t + \phi(t)]$$

$$v_o(t) = v_o \cos[\omega_0 t + \hat{\phi}(t)]$$

➤ A PD contains a multiplier and a low-pass filter. The output of PD is:

$$v_d(t) = K_d \sin[\phi(t) - \hat{\phi}(t)] = K_d \sin \phi_e(t)$$

➤ LF is also a LPF. The output of the LF is (where $F(p)$ is the transfer function)

$$v_c(t) = F(p)v_d(t)$$



Synchronization

- Phase-Locked Loop (PLL, 锁相环)
 - The output of VCO can be a sinusoid or a periodic impulse train. The differentiation of the output frequency are largely proportional to the input voltage.

$$\frac{d\hat{\phi}(t)}{dt} = K_v v_c(t)$$

- If $F(p)=1$, then

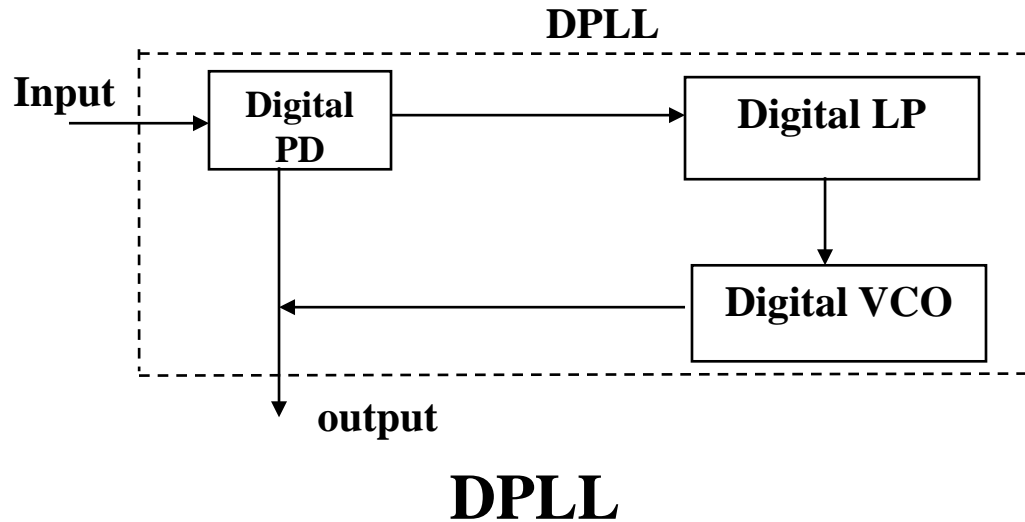
$$\frac{d\hat{\phi}(t)}{dt} = K \sin \phi_e(t)$$

The first kind of loop!



Synchronization

- Phase-Locked Loop (PLL, 锁相环)
 - Digital PLL.





Synchronization

- Phase-Locked Loop (PLL, 锁相环)
 - In a coherence system, a PLL is used for:
 1. PLL can track the **input frequency** and generate the output signal with small phase difference.
 2. PLL has the character of **narrowband filtering** which can eliminate the noise introduced by modulation and reduce the additive noise.
 3. Memory PLL can sustain the coherence state for enough time.
 - **CMOS-based integrated PLL** has several advantages such as ease of modification, reliable and low power consumption, therefore are widely used in coherence system.



Carrier synchronization

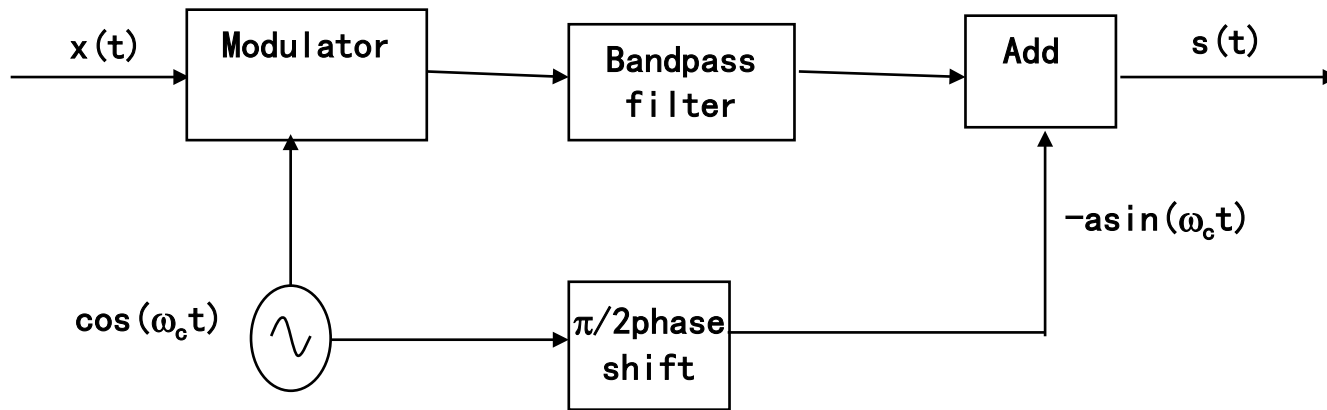
- To extract the carrier:
 - **Pilot-tone insertion** method: Sending a carrier component at specific spectral-line along with the signal component. Since the inserted carrier component has high frequency stability, it is called **pilot** (导频).
 - **Direct extraction** method: Directly extract the synchronization information from the received signal component.



Carrier synchronization

- **Pilot-tone insertion method:**

➤ Insert pilot to the modulated signal



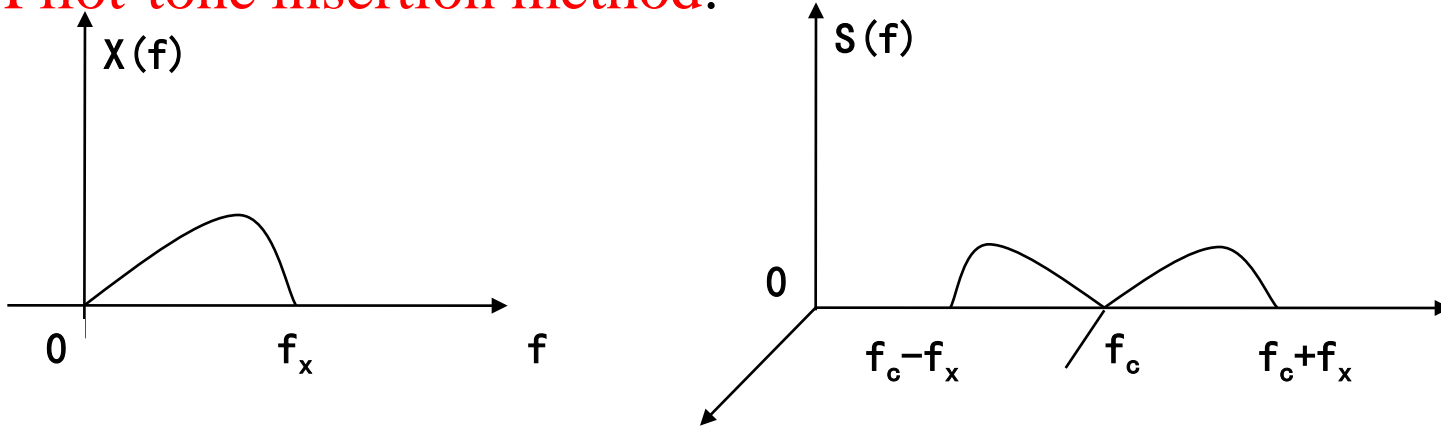
➤ The pilot signal is generated by shift the carrier by 90° and decrease by several dB, then add to the modulated signal. Assume the modulated signal has 0 DC component, then the pilot is

$$s(t) = f(t)\cos\omega_c t - a\sin\omega_c t$$



Carrier synchronization

- Pilot-tone insertion method:



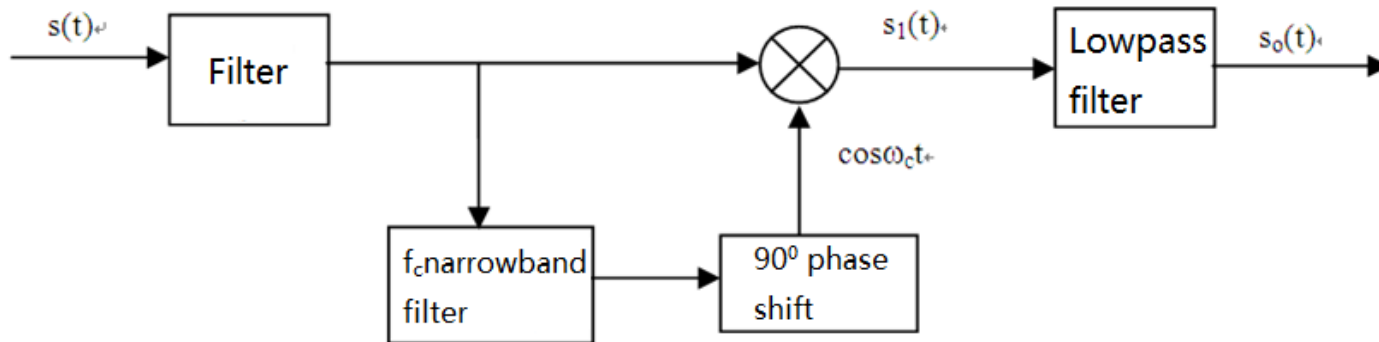
➤ The receiver uses a narrowband filter with central frequency f_c to extract the pilot $a \sin \omega_c t$ and then the carrier $a \cos \omega_c t$ can be generated by simply shifting 90° .



Carrier synchronization

- **Pilot-tone insertion method:**

- Narrowband filter receiver structure



$$\begin{aligned} s_1(t) &= s(t) \cdot \cos \omega_c t = f(t) \cos^2 \omega_c t - a \sin \omega_c t \cos \omega_c t \\ &= \frac{1}{2} f(t) + \frac{1}{2} f(t) \cos 2\omega_c t - \frac{1}{2} a \sin 2\omega_c t \end{aligned}$$

$$\text{After the LPF } s_0(t) = \frac{1}{2} f(t)$$

- DSB, SSB and PSK are all capable of pilot-tone insertion method. VSB can also apply pilot-tone insertion method but with certain modification.



Carrier synchronization

- **Pilot-tone insertion method:**
 - The drawback of narrowband filter receiver includes:
 1. The pass band is not narrow enough
 2. f_c is fixed, cannot tolerate any **frequency drift** with respect to the central frequency
 3. Can be replaced by PLL
 - Pilot-tone insertion method is suitable for DSB, SSB, VSB and 2PSK



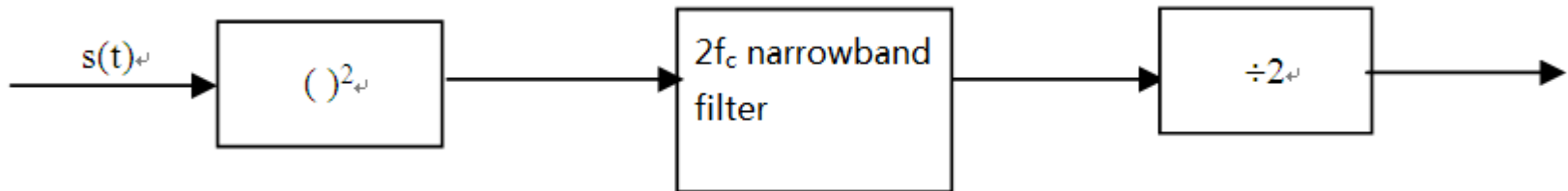
Carrier synchronization

- **Direct extraction method:**
 - If the spectrum of the received signal already contains carrier component, then the carrier component can be extracted simply by a **narrowband filter** or a **PLL**
 - If the modulated signal suppresses the carrier component, then the carrier component may be extracted by performing **nonlinear transformation** or using a PLL with specific design.



Carrier synchronization

- **Nonlinear transformation based method:**
 - Square transformation



Example: a DSB signal $s(t) = f(t)\cos\omega_c t$

If $f(t)$ has 0 DC component, then $s(t)$ does not have carrier component

square transformation: $s^2(t) = \frac{1}{2}f^2(t) + \frac{1}{2}f^2(t)\cos 2\omega_c t$

now $f^2(t)$ contains DC component, let it be α so: $f^2(t) = \alpha + f_m(t)$

then $s^2(t) = \frac{1}{2}\alpha + \frac{1}{2}f_m(t) + \frac{1}{2}\alpha\cos 2\omega_c t + \frac{1}{2}f_m(t)\cos 2\omega_c t$



Carrier synchronization

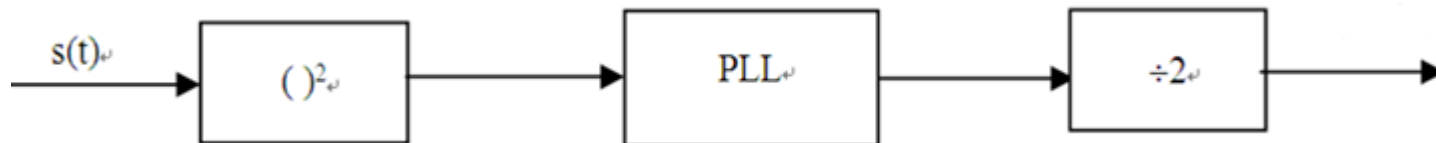
- **Nonlinear transformation based method:**

- **Square transformation**

$$s^2(t) = \frac{1}{2}\alpha + \frac{1}{2}f_m(t) + \frac{1}{2}\alpha \cos 2\omega_c t + \frac{1}{2}f_m(t) \cos 2\omega_c t$$

The first term is the DC component. The second term is the low frequency component. The third term is the $2\omega_c$ component. The 4th term is the frequency component symmetrical distributed of $2\omega_c$ —modulation noise. After narrowband filtering, only the 3rd term and a small fraction of 4th term left, then the carrier component can be extracted by frequency division.

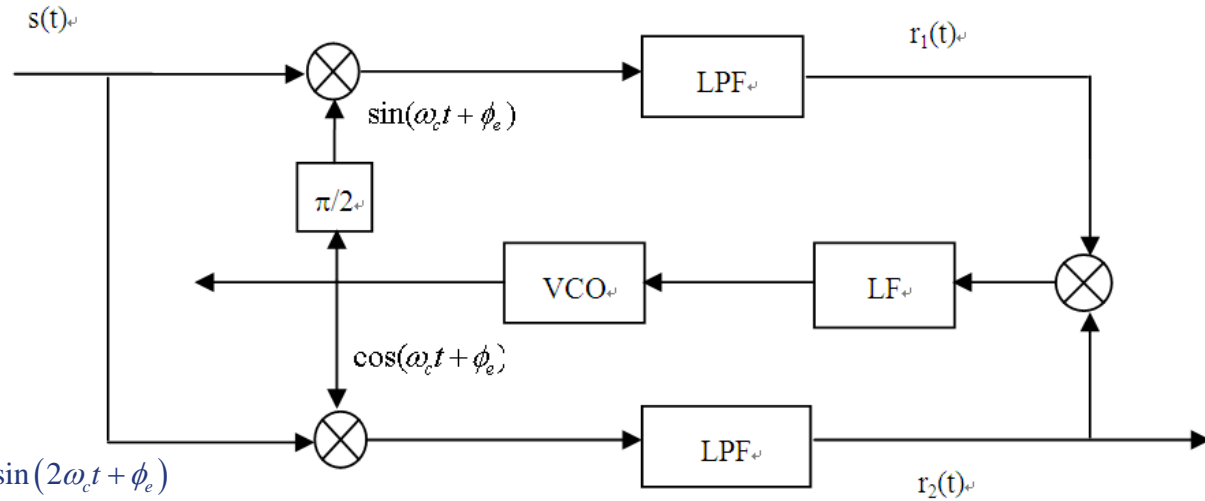
Since the carrier is extracted by frequency division, its phase may shift by 180° . Besides, modulation noise may cause random phase jitter.





Carrier synchronization

- **Nonlinear transformation based method:**
 - In-phase orthogonal loop (**Costas loop**)



Let $s(t) = f(t) \cos \omega_c t$

(1) upper branch

$$f(t) \cos \omega_c t \cdot \sin(\omega_c t + \phi_e) = \frac{1}{2} f(t) \sin \phi_e + \frac{1}{2} f(t) \sin(2\omega_c t + \phi_e)$$

ϕ_e is the phase difference between generated carrier and the original carrier

After LPF $r_1(t) = \frac{1}{2} f(t) \sin \phi_e$

When ϕ_e is small, $r_1(t) = \frac{1}{2} f(t) \phi_e$

(2) lower branch

$$r_2(t) = \frac{1}{2} f(t) \cos \phi_e \rightarrow \frac{1}{2} f(t)$$

(3) $r_1(t) \cdot r_2(t) \rightarrow \frac{1}{4} f^2(t) \phi_e = v_d(t)$

Contains in-phase branch and orthogonal branch. All parts except LF and VCO are similar with a “phase detector”.



Carrier synchronization

- **Nonlinear transformation based method:**
 - In-phase orthogonal loop (Costas loop)
Advantages of Costas loop:
 1. Costas loop works on f_c instead of $2f_c$ so when f_c is large Costas loop is easier to realize
 2. The output of in-phase loop $r_2(t)$ is the signal $f(t)$
- Performance of carrier synchronization technique
 - 1) Phase error: steady-state phase error, random phase error
 - 2) Synchronization build time and hold time



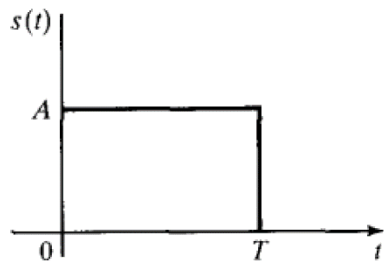
Symbol synchronization

- In a digital communication system, the output of the receiving filter must be sampled periodically at the symbol rate and at the precise sampling time instance.
- To perform this periodic sampling, we need a **clock signal** at the receiver
- The process of extracting such a clock signal is called **symbol synchronization** or **timing recovery**
- One method is for the transmitter to simultaneously transmit the clock frequency along with the information signal. The receiver can simply employ a narrowband filter or PLL to extract it. This method requires extra power and bandwidth and hence, but frequently used in telephone transmission systems.
- Another method is to extract the clock signal from the received data signal by using some kind of **non-linear transformation**.

Symbol synchronization

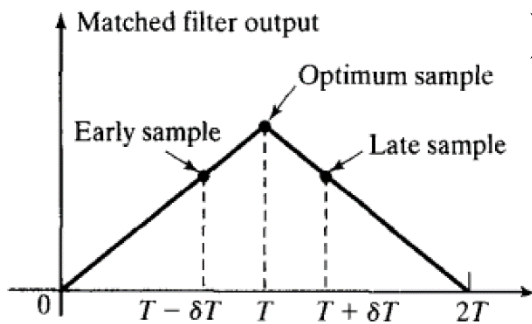
- **Early-late gate synchronization**

- Basic Idea: exploit the **symmetry** properties of the output signal of **matched filter** or correlator



(a)

- Due to the symmetry, the values of the correlation function at the early samples $t = T - \delta T$ and the late samples $t = T + \delta T$ are equal.



(b)

Figure 8.48 (a) Rectangular signal pulse and (b) its matched filter output.

- Thus, the proper sampling time is the midpoint between $t = T - \delta T$ and $t = T + \delta T$



Symbol synchronization

- Early-late gate synchronization
 - Block diagram.

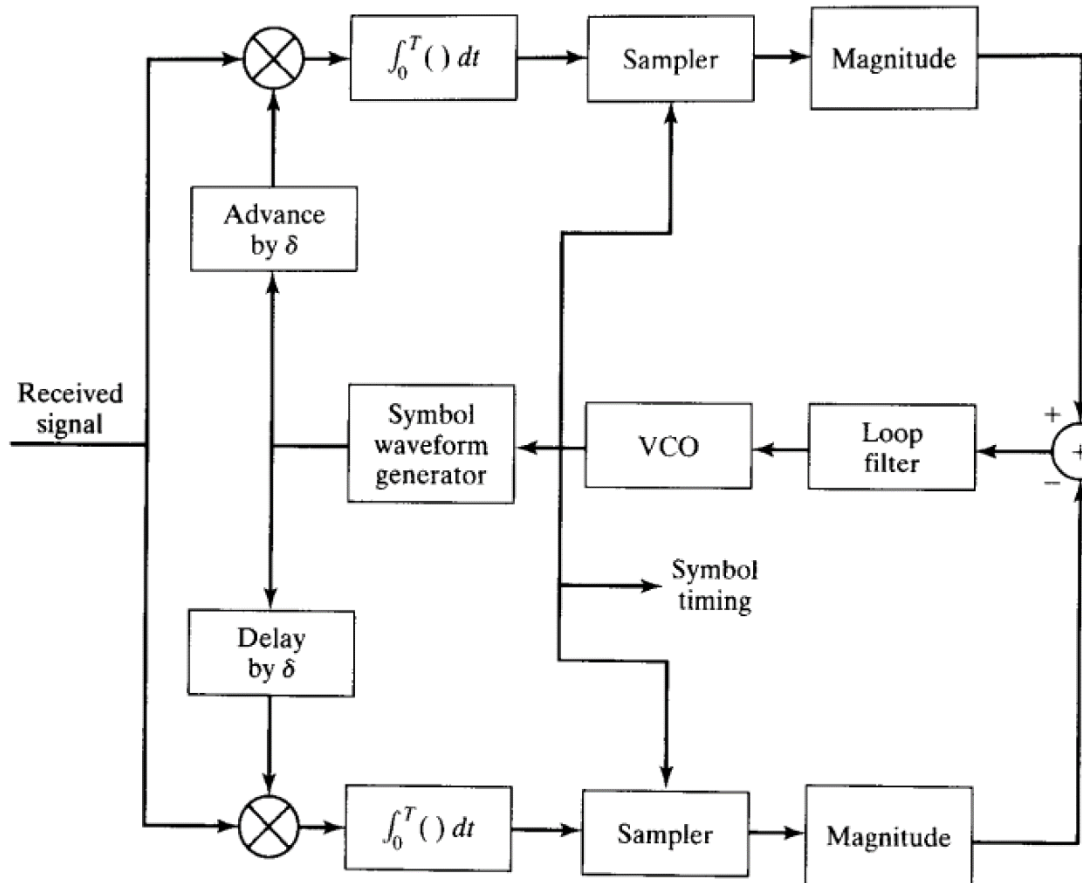


Figure 8.49 Block diagram of early-late gate synchronizer.



Symbol synchronization

- Nonlinear transformation based synchronization



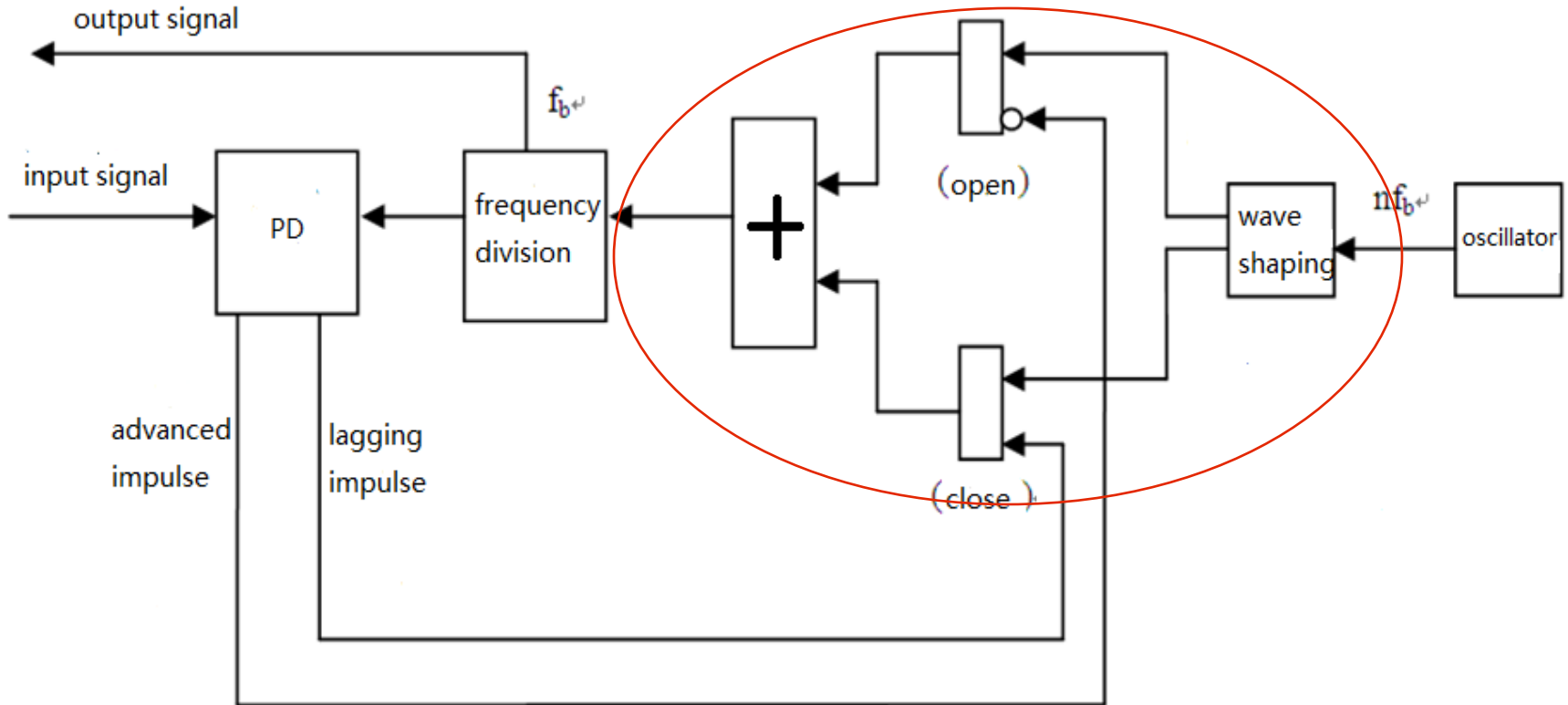
➤ Some transformations can add synchronous signal with $f=1/T$ to the original signal. For example, we can transform the signal to return-to-zero waveform. After narrowband filtering and phase shifting, we can generate the clock signal used for synchronization.

$$P_s(f) = f_s P(1-P) |G_1(f) - G_2(f)|^2 + f_s^2 \sum_{m=-\infty}^{\infty} |PG_1(mf_s) + (1-P)G_2(mf_s)|^2 \delta(f - mf_s)$$



Symbol synchronization

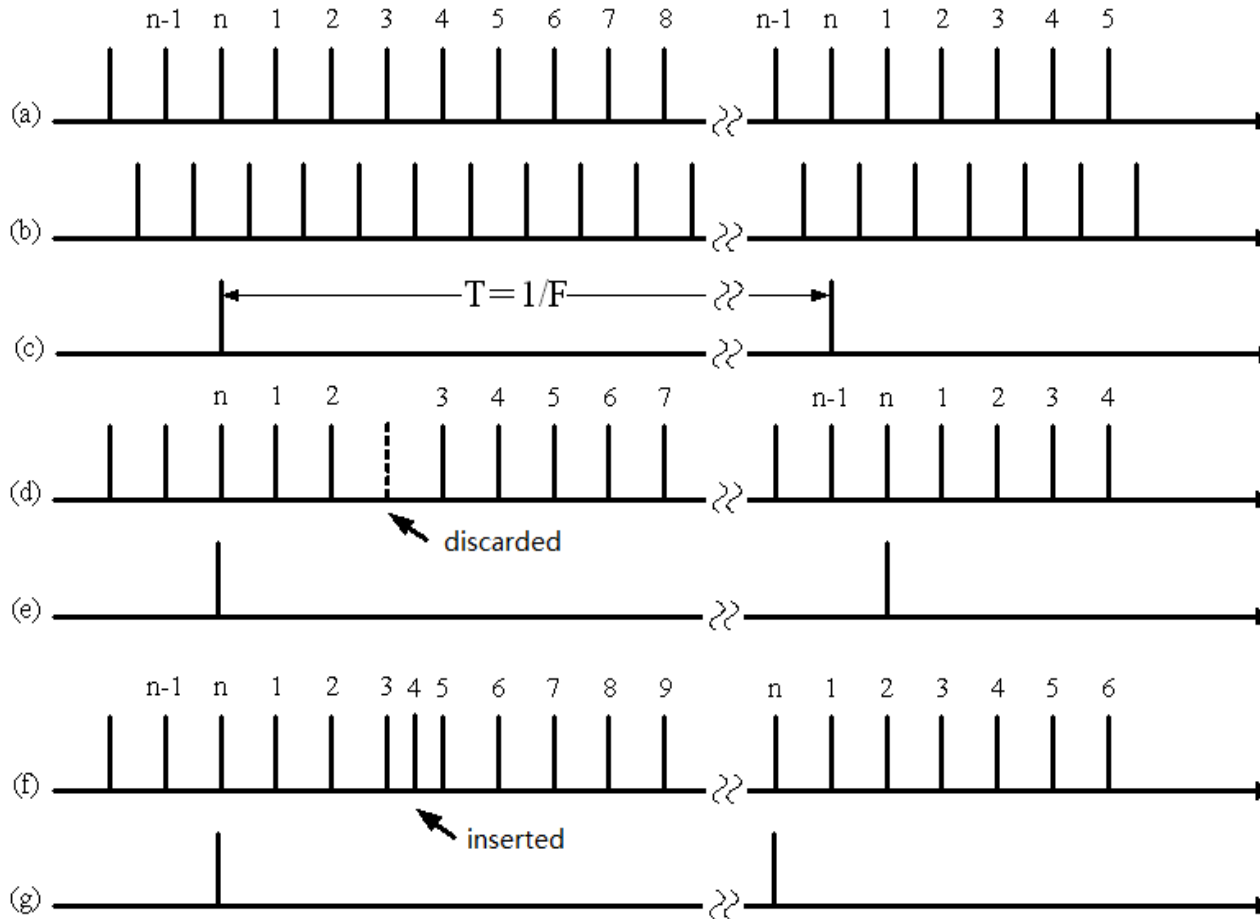
- DPLL





Symbol synchronization

- DPLL





Symbol synchronization

- DPLL
 - Performance.
 - 1). Phase error
 - 2). Synchronization build time
 - 3). Synchronization hold time
 - 4). Synchronous bandwidth



Frame synchronization

- Recall that carrier and symbol synchronization needs to estimate the **phase of synchronous signal** which can be realized by using a PLL.
- Frame synchronization is to insert frame **alignment** signal (distinctive bit sequence) and then detect the alignment symbol.
- Besides adding frame alignment bits, some code such as **self-synchronizing code** can be synchronized without adding extra bits.
- Here, we only focus on the first method ——inserting frame alignment signal.



Frame synchronization

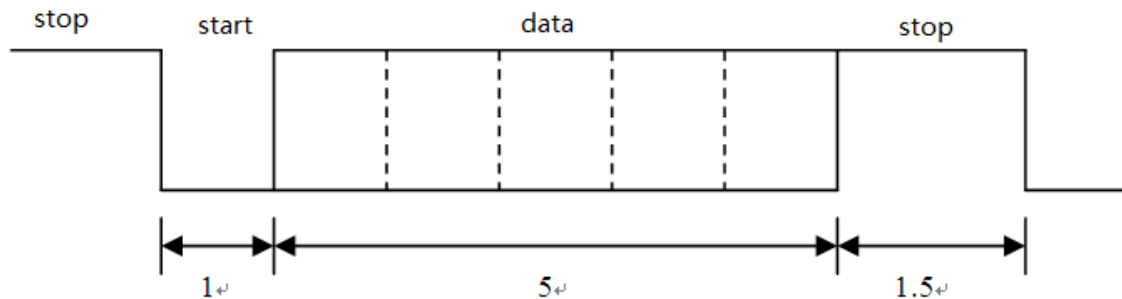
- Recall that carrier and symbol synchronization needs to estimate the **phase of synchronous signal** which can be realized by using a PLL.
- Frame synchronization is to insert frame **alignment** signal (distinctive bit sequence) and then detect the alignment symbol.
- Besides adding frame alignment bits, some code such as **self-synchronizing code** can be synchronized without adding extra bits.
- Here, we only focus on the first method ——inserting frame alignment signal.
 - Start-stop method
 - Bunched frame alignment signal
 - Distributed frame alignment signal



Frame synchronization

- **Start-stop method**

- It is widely used in teleprinter. Each symbol contains 5-8 data bits, a start bit and a stop bit.



start bit: "0": width T_b

stop bit: "1", width $\geq T_b$

System will keep sending stop bit when it is idle. When "1" \rightarrow "0", the receiver will start to receive a data symbol.

Low transmission efficiency and low timing accuracy



Frame synchronization

- **Bunched frame alignment signal**
- This method inserts **synchronous code** at a particular place in each frame. The code should have a **sharp self-correlation function**. The detector should be simple to implement.
- Frame synchronous code includes
 1. Barker code
 2. Optimal synchronous code
 3. Pseudo-random code



Frame synchronization

- **Bunched frame alignment signal**

- **Barker code**

(1) Barker code:

A n bits barker code $\{x_1, x_2, x_3 \cdots x_n\}$, $x_i = +1$ or -1 . its self-correlation function satisfies:

$$R_x(j) = \sum_{i=1}^{n-j} x_i x_{i+j} = \begin{cases} n & j = 0 \\ 0 \text{ or } \pm 1 & 0 < j < n \\ 0 & j \geq n \end{cases}$$

Barker code is not a periodic sequence. It is proved that when $n < 12100$, we can only find barker code with $n = 2, 3, 4, 5, 7, 11, 13$.



Frame synchronization

- **Bunched frame alignment signal**
 - Barker code

n	barker code
2	++
3	++-
4	+++-, ++-+
5	+++ - +
7	+++ - - + -
11	+++ - - - + - - + -
13	++++ - - + + - + - +



Frame synchronization

- **Bunched frame alignment signal**

- **Barker code**

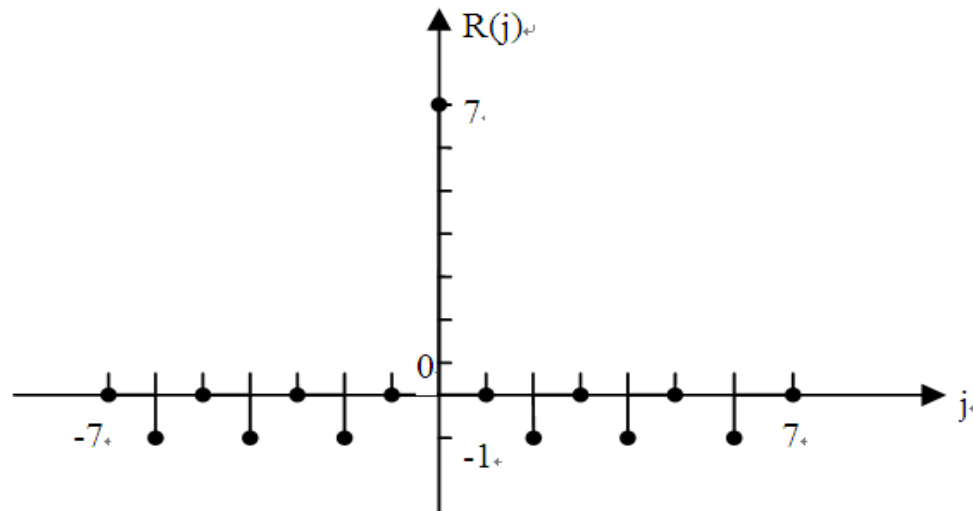
Example: A barker code with $n = 7$ find its self-correlation function

$$j = 0: R_x(0) = x_1x_1 + x_2x_2 + \dots + x_7x_7 = 7$$

$$j = 1: R_x(1) = x_1x_2 + x_2x_3 + \dots = 0$$

Similarly, we can determine $R_x(j)$.

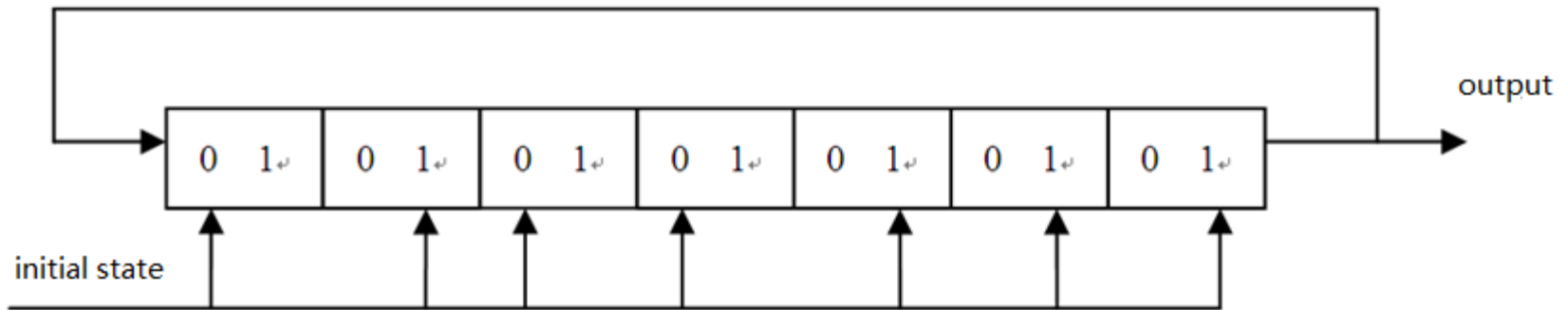
The result is shown below, we can see it has a sharp peak when $j = 0$.





Frame synchronization

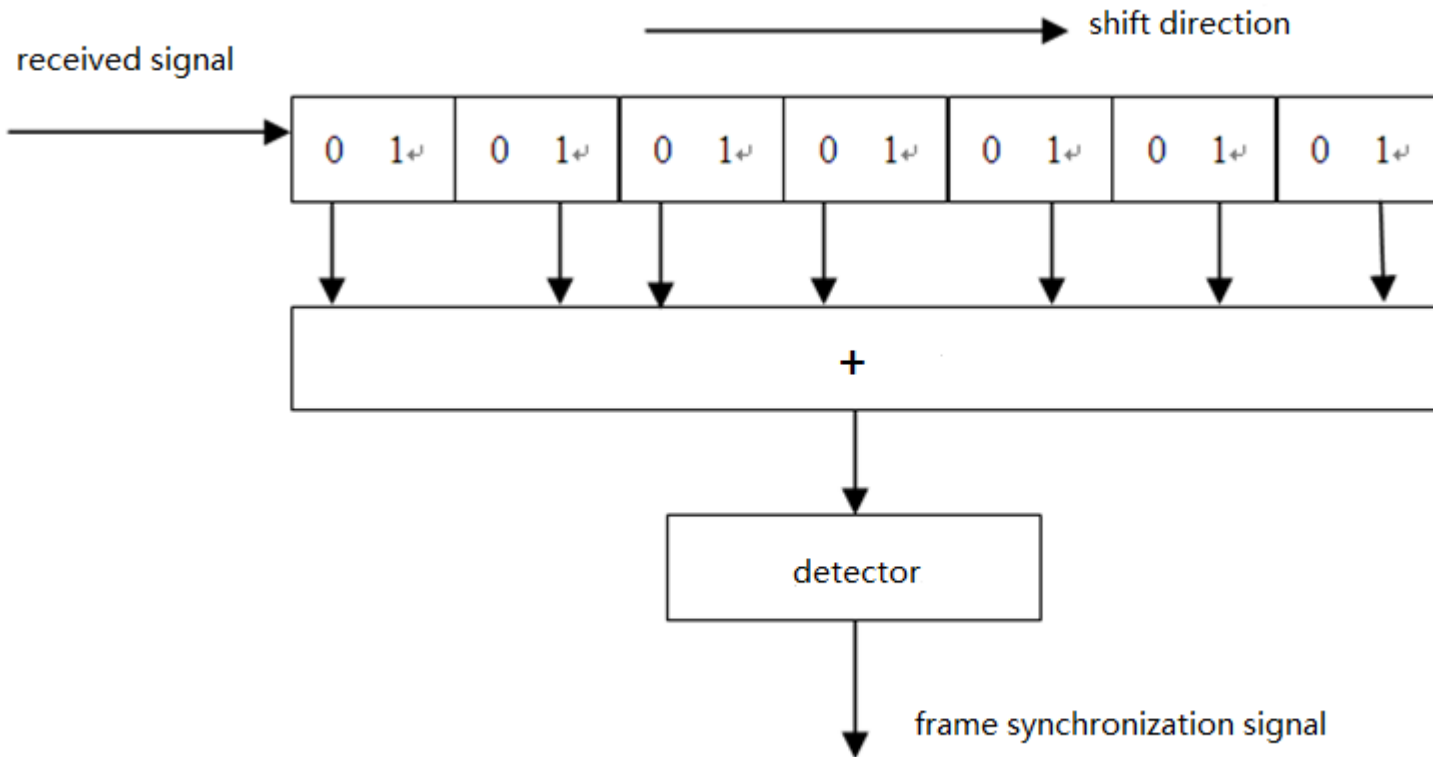
- **Bunched frame alignment signal**
 - Barker code generator – shift register
 - Example: when $n=7$, a 7 bits shift register. The initial state is a barker code.





Frame synchronization

- **Bunched frame alignment signal**
- Barker code detector





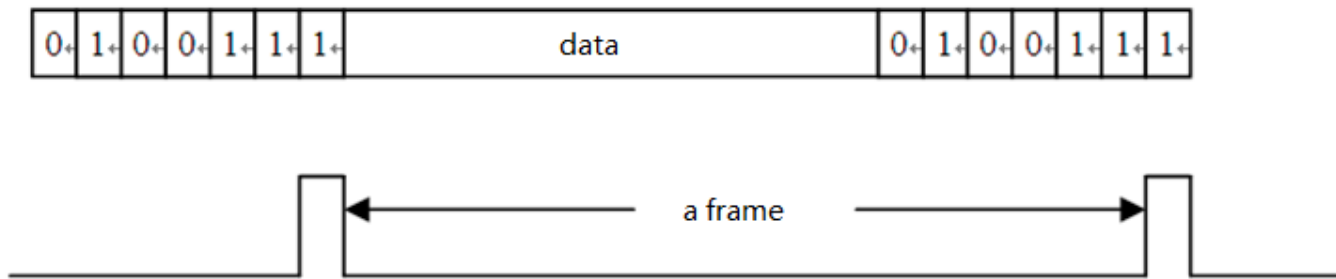
Frame synchronization

- **Bunched frame alignment signal**
- **Barker code detector**

The barker code detector follows:

$$\begin{aligned} \text{input: "1"} & \begin{cases} \text{output "1": "?"} + 1 \\ \text{output "0": "?"} - 1 \end{cases} \\ \text{input: "0"} & \begin{cases} \text{output "1": "?"} - 1 \\ \text{output "0": "?"} + 1 \end{cases} \end{aligned}$$

If the output connection of the shift register is the same with a barker code, then when the input is a barker code, the output of the shift register is "1111111". The detector will send a synchronous impulse.





Frame synchronization

- **Distributed frame alignment signal**
- The synchronous code is distributed in the data signal. That means between each n bits, a synchronous bit is inserted.
- Design criteria of **synchronous code**:
 1. Easy to detect. For example: “11111111” or “10101010”
 2. Easy to separate synchronous code from data code. For example: In some digital telephone system, all “0” stands for ring, so synchronous code can only use “10101010”



Frame synchronization

- Performance of Bunched frame alignment signal
- **Probability of missing synchronization P_L**
 1. Affected by noise, the detector may not be able to detect the synchronous code. The probability of this situation is called probability of missing synchronization P_L .
 2. Assume the length of synchronous code is n , bit error rate is P_e . The detector will not be able to detect if more than m bit errors happen, then:

$$P_L = 1 - \sum_{x=0}^m C_n^x P_e^x (1 - P_e)^{n-x}$$



Frame synchronization

- Performance of Bunched frame alignment signal

- **Probability of false synchronization P_F**

Since data code can be arbitrary, it may be the same with synchronous code. The probability of this situation is called probability of false synchronization P_F . P_F equals to the probability of appearance of synchronous code in the data code.

- a. In a binary code, assume 0 and 1 appears with the same probability. There are 2^n combinations of a n bit code.
- b. Assume when there are more than m bit errors, the data code will also be detected as synchronous code.



Frame synchronization

- Performance of Bunched frame alignment signal
- **Probability of false synchronization P_F**

When $m = 0$, only $1(C_n^0)$ code will be detected as synchronous code

When $m = 1$, there are C_n^1 codes will be detected as synchronous code;

.....

Therefore, the probability of false synchronization is:

$$P_F = \frac{\sum_{x=0}^m C_n^x}{2^n} = \left(\frac{1}{2}\right)^n \sum_{x=0}^m C_n^x$$



Frame synchronization

- Performance of Bunched frame alignment signal

➤ Performance

P_L and P_F depends on the length of synchronous code n and the maximum bit error m .

When $n \uparrow$, $P_F \downarrow$, $P_L \uparrow$ when $m \uparrow$, $P_L \downarrow$ $P_F \uparrow$

3. Average build time t_s

Assume both P_L and P_F will not happen, the worst case is we need one frame to build frame synchronization. Assume each frame contains N bits, each bit has a width T_b , then one frame costs NT_b .

Now assume a missing synchronization or a false synchronization also needs NT_b to rebuild the synchronization, then:

$$t_s^1 = NT_b (1 + P_L + P_F)$$

Besides, the average build time of using the distributed frame alignment signal is:

$$t_s^2 = N^2 T_b (N \gg 1)$$

Apparently, $t_s^1 < t_s^2$, so the previous method is more widely used.