Outline

- Introduction
- Signal, random variable, random process and spectra
- **Analog modulation**
- Analog to digital conversion
- Digital transmission through baseband channels
- Signal space representation
- Optimal receivers
- Digital modulation techniques
- Channel coding
- Synchronization
- Information theory

Analog modulation

AM/FM radio **TV broadcast**

Source → Modulator → Channel → Demodulator → Output

Analog modulation

- Amplitude modulation
- Angle modulation (phase/frequency)

Analog modulation

- What is modulation?
  - Transform a message into another signal to facilitate transmission over a communication channel
  - Generate a carrier signal at the transmitter
  - Modify some characteristics of the carrier with the information to be transmitted
  - Detect the modifications at the receiver

- Why modulation?
  - Frequency translation (antenna theory)
  - Frequency-division multiplexing (multiple users)
  - Noise performance improvement (quality)

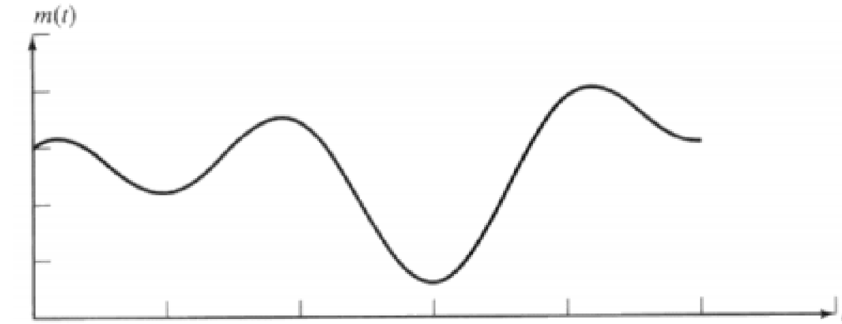
Analog modulation

- Characteristics that can be modified in sin carrier
  - Amplitude → **Amplitude modulation**
  - Frequency/phase → **Angle modulation**

Selected from Chapter 3, 4.1-4.4, 6.1-6.3

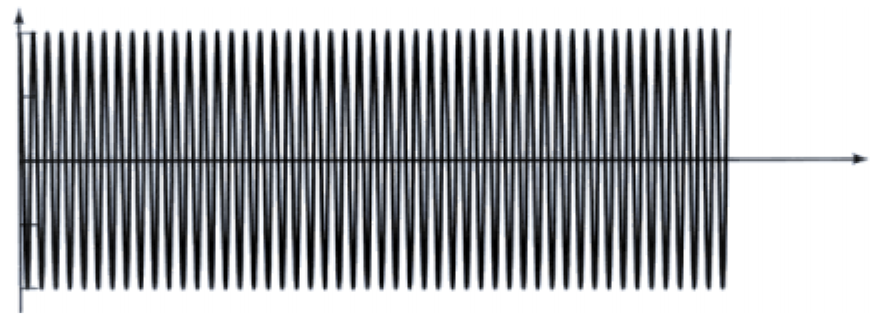
Amplitude modulation

- Double-sideband suppressed-carrier AM (DSB-SC)
  - ➢ Baseband signal (modulation wave) m(t)



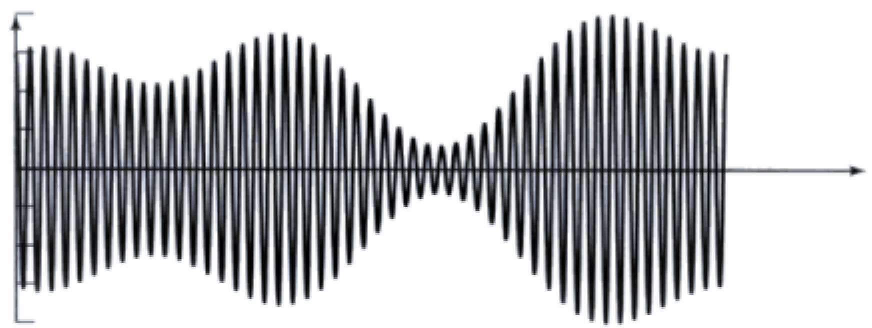
  - ➢ Carrier wave

$$c(t) = A_c \cos(w_c t + \theta_0)$$



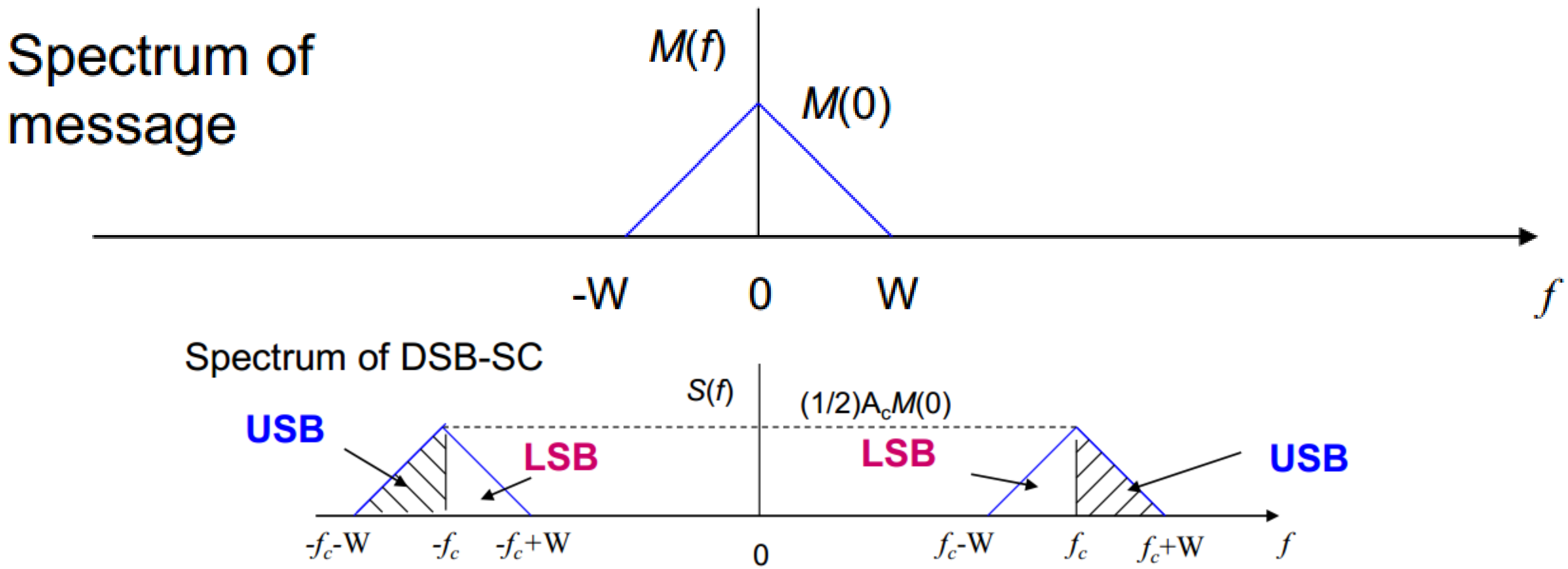
  - ➢ Modulated wave

$$s(t) = c(t)m(t) = A_c m(t)\cos\left(\omega_c t + \theta_0\right)$$



Amplitude modulation

- Double-sideband suppressed-carrier AM (DSB-SC)
  - Spectrum

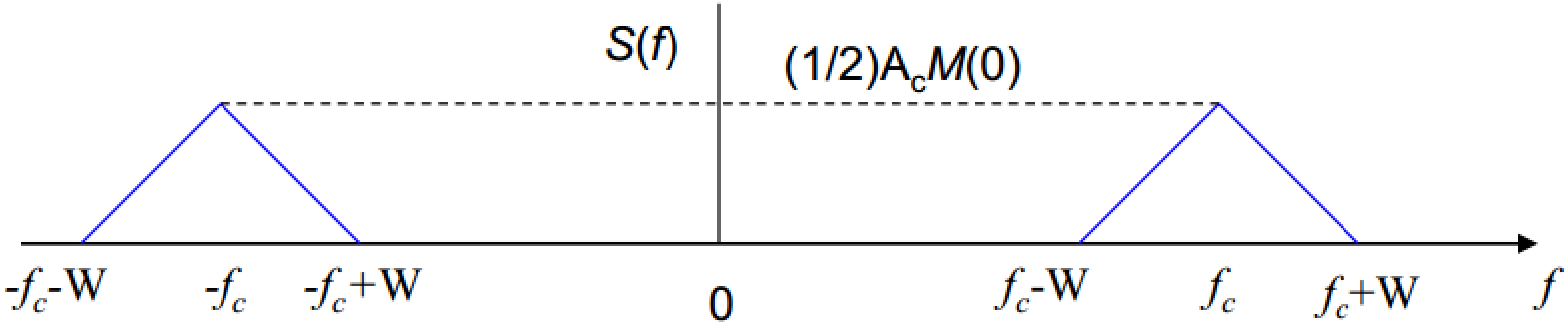


$$S(f)=\frac{1}{2}A_c[M(f-f_c)+M(f+f_c)]$$

Translation of the original message spectrum to carrier frequency

Amplitude modulation

- Double-sideband suppressed-carrier AM (DSB-SC)
  - Bandwidth and power efficiency



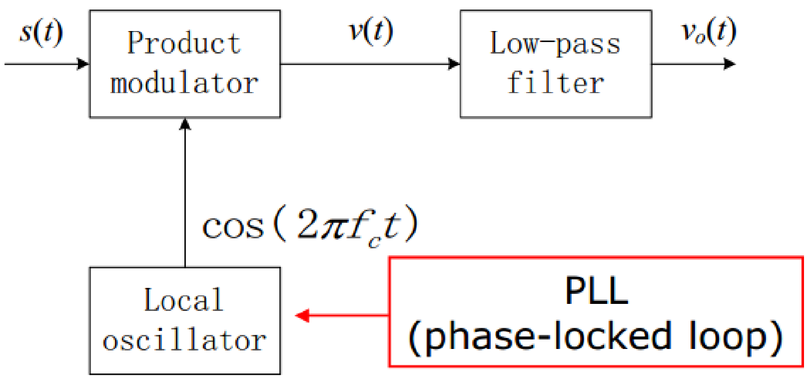
  - Required channel bandwidth B=2W
  - Required transmit power

$$P_s = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} s^2(t)dt = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} A_c^2 m^2(t)\cos^2(\omega_c t+\theta_0)dt$$

$$= \frac{A_c^2}{2}\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} m^2(t)\left[1+\cos(2\omega_c t+2\theta_0)\right]dt = \frac{A_c^2}{2}P_m$$

Amplitude modulation

- Double-sideband suppressed-carrier AM (DSB-SC)
  - ➢ Demodulation of DSB-SC



  - ➢ If there is a phase error ϕ , then

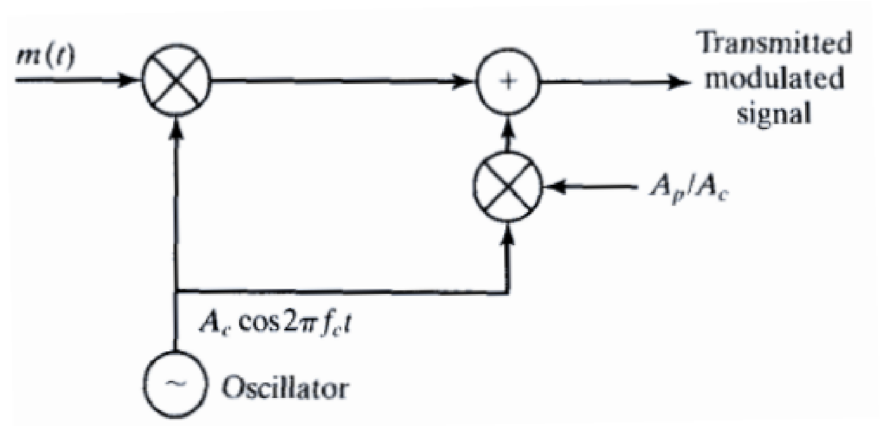
$$v(t) = s(t)\cos(2\pi f_c t + \phi) = A_c \cos(2\pi f_c t)\cos(2\pi f_c t + \phi) m(t)$$

$$= \underbrace{\frac{1}{2} A_c \cos(\phi) m(t)}_{\text{Scaled version of message signal}} + \underbrace{\frac{1}{2} A_c \cos(4\pi f_c t + \phi) m(t)}_{\text{Unwanted}}$$

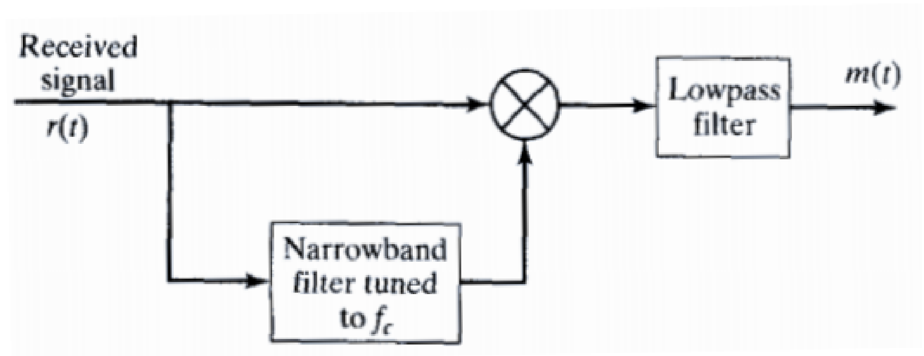
Amplitude modulation

- Double-sideband suppressed-carrier AM (DSB-SC)
  - ➢ Pilot-tone assisted demodulation

Add a pilot-tone into the transmitted signal



Filter out the pilot using a narrowband filter

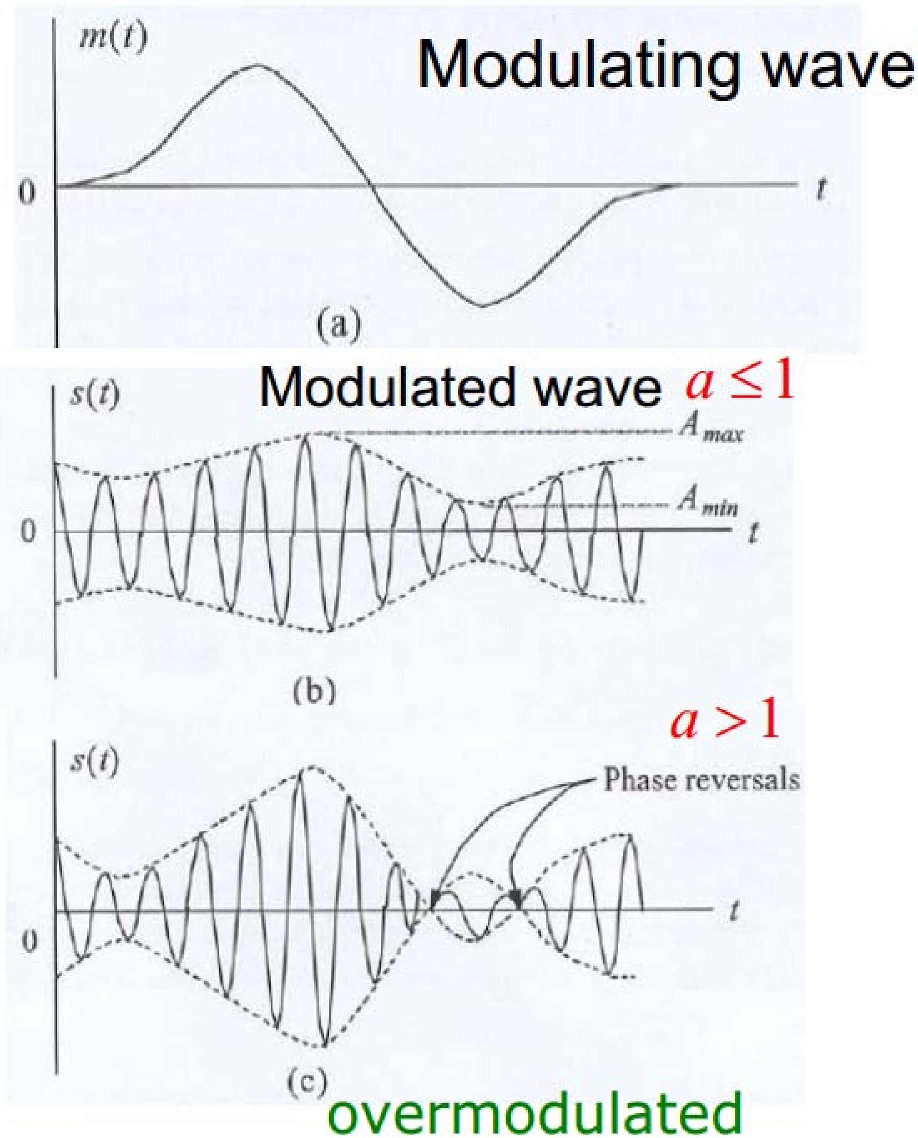


Amplitude modulation

- Conventional AM
  - Baseband signal (normalized) $m_n(t) = \dfrac{m(t)}{\max|m(t)|}$
  - Modulation index a
  - Carrier wave

    $$c(t) = A_c \cos(w_c t + \theta_0)$$
  - Modulated wave

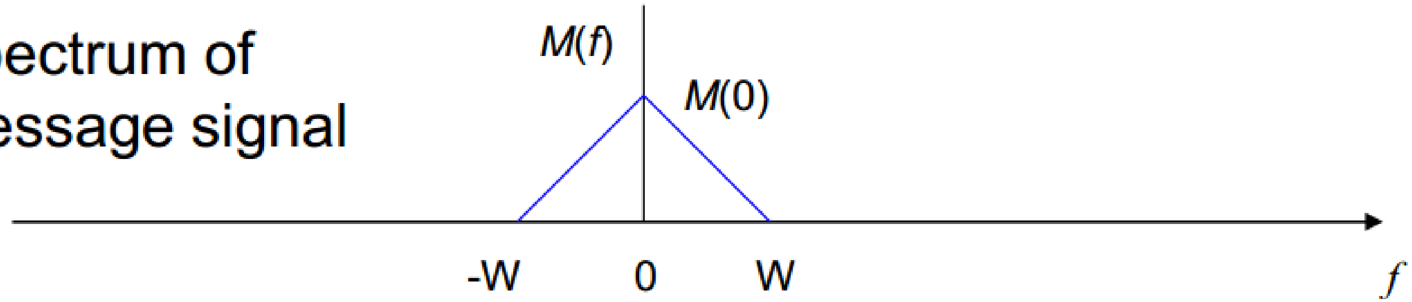
$$s(t) = A_c[1 + a m_n(t)] \cos(2\pi f_c t)$$
$$= A_c a m_n(t) \cos 2\pi f_c t)$$
$$+A_c \cos(2\pi f_c t)$$

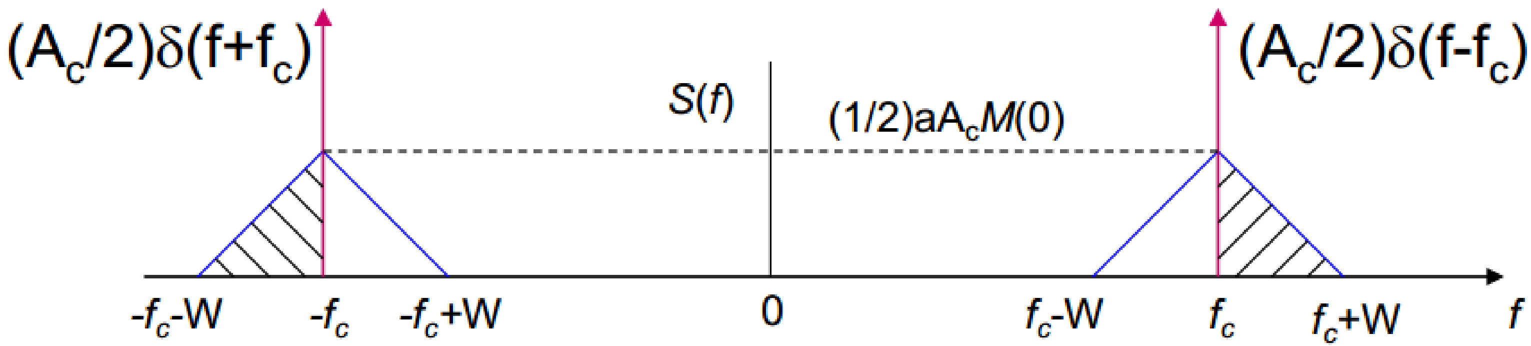


Amplitude modulation

- Conventional AM
  - Spectrum

Spectrum of message signal





$$S(f)=\frac{A_c}{2}\left[\delta(f-f_c)+\delta(f+f_c)\right]+\frac{A_c a}{2}\left[M(f-f_c)+M(f+f_c)\right]$$

Translation of the original message spectrum to carrier frequency and induction of carrier spectrum component

Amplitude modulation

- Conventional AM
  - Bandwidth and power efficiency
  - Required channel bandwidth B=2W
  - Required transmit power

$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} s^2(t)dt = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \left(A_c^2[1+am_n(t)]^2 \cos^2 \omega_c t\right)dt$$

$$= \underbrace{\frac{A_c^2}{2}}_{\text{carrier power}} + \underbrace{\frac{a^2 A_c^2}{2}P_m}_{\text{message power}}$$

  - Modulation efficiency

$$E = \frac{\text{power in sideband}}{\text{total power}} = \frac{\frac{a^2}{2}A_c^2 P_m}{\frac{A_c^2}{2} + \frac{a^2}{2}A_c^2 P_m} = \frac{a^2 P_m}{1+a^2 P_m}$$

Amplitude modulation

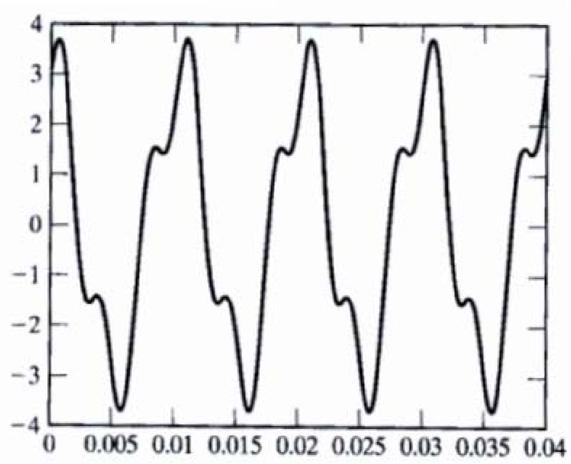
- Conventional AM
  - Consider for example message signal

$$m(t) = 3\cos(200\pi t) + \sin(600\pi t)$$

  - Carrier

$$c(t) = \cos(2\times 10^5 t)$$

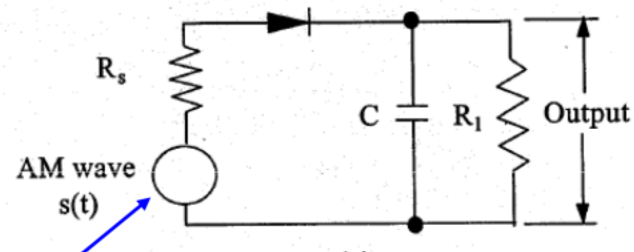
  - Modulation index a = 0.85
  - Determine the power in the carrier component and sideband components of the modulated signal



Amplitude modulation

- Conventional AM
  - Demodulation of conventional AM signals

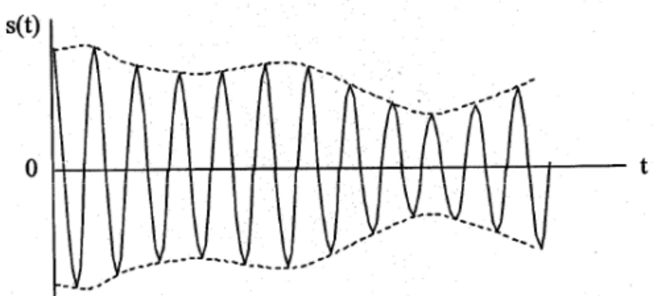
Envelope detector



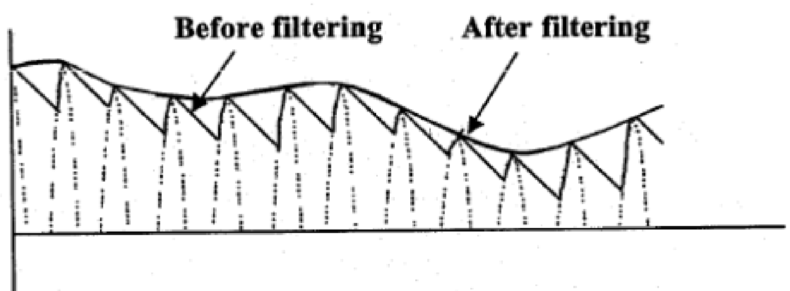
(a)

short charging time:
$R_s C << 1/f_c$

long discharging time:
$1/f_c << R_l C << 1/W$



(b)

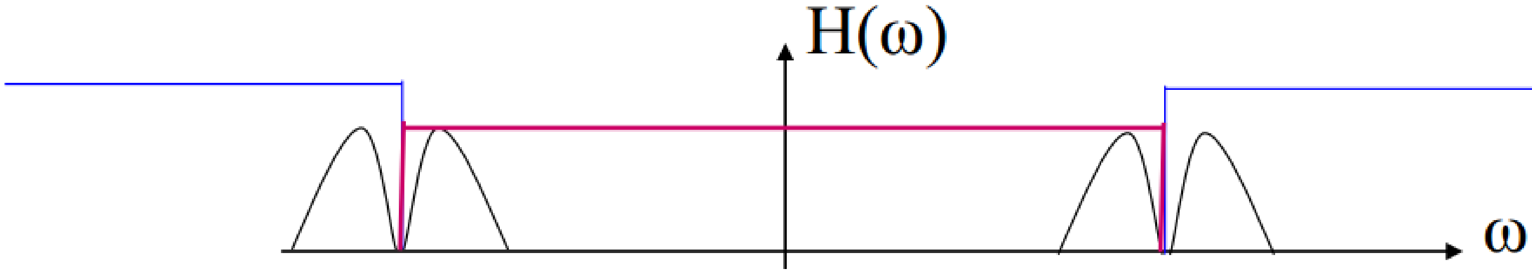


(c)

The simplicity of envelope detector has made the conventional AM a practical choice for AM-radio broadcasting.

Amplitude modulation

- Single Sideband (SSB) AM
  - Common problem in DSB is the **bandwidth wastage**
  - SSB is very bandwidth efficient



  - The baseband signal can be written as the sum of finite sinusoid signals

$$m(t) = \sum_{i=1}^{n} x_i \cos(2\pi f_i t + \theta_i), \; f_i \le f_c$$

  - Then the USB component is

$$m_c(t) = \frac{A_c}{2} \sum_{i=1}^{n} x_i \cos\left[2\pi(f_c + f_i)t + \theta_i)\right]$$

Amplitude modulation

- Single Sideband (SSB) AM
  - After manipulation

$$m_c(t)=\frac{A_c}{2}\left\{\left[\sum_{i=1}^{n}x_i\cos(2\pi f_i t+\theta_i)\right]\cos 2\pi f_c t-\underbrace{\left[\sum_{i=1}^{n}x_i\sin(2\pi f_i t+\theta_i)\right]}\sin 2\pi f_c t\right\}$$

$$=\frac{A_c}{2}m(t)\cos 2\pi f_c t-\frac{A_c}{2}\widehat{m}(t)\sin 2\pi f_c t$$

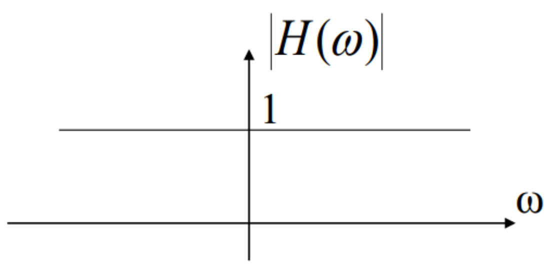
Hilbert transform of m(t)

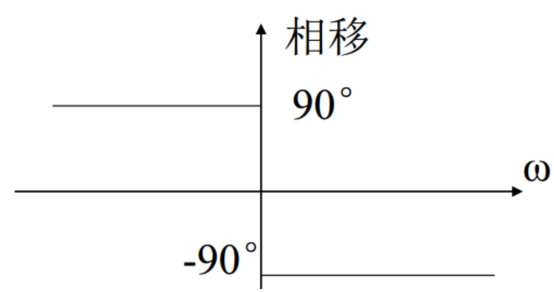
  - Hilbert transform

$$x(t)\Leftrightarrow\hat{x}(t)\quad\Rightarrow\quad\hat{x}(t)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x(\tau)}{t-\tau}d\tau=x(t)*\frac{1}{\pi t}$$

$$X(w)\leftrightarrow\hat{X}(w)\quad\Rightarrow\quad\hat{X}(w)=[-jsgn(w)]X(w)$$

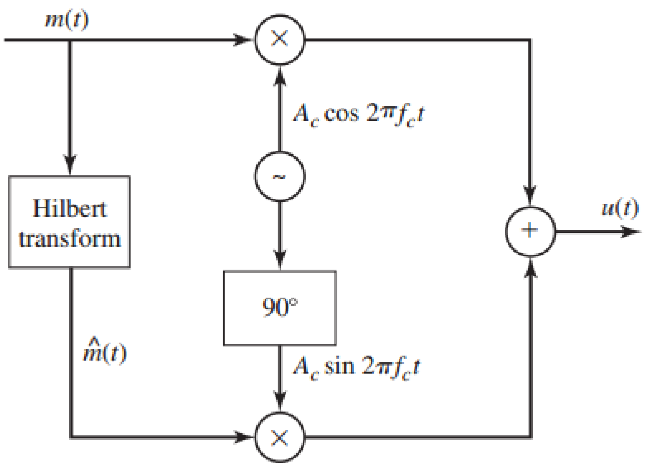
$$H(f)=\begin{cases}-j, & f>0\\ j, & f<0\\ 0, & f=0\end{cases}$$



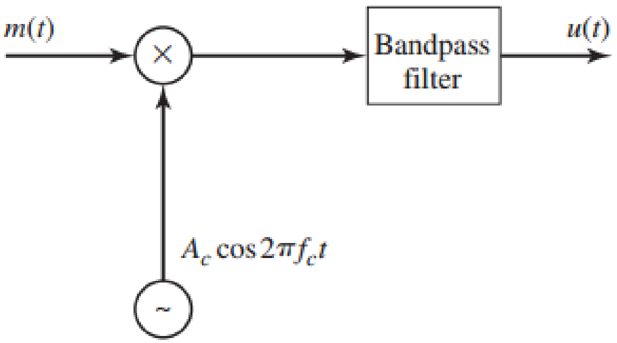


Amplitude modulation

- Single Sideband (SSB) AM
  - Generation of SSB-AM signal



Direct generation

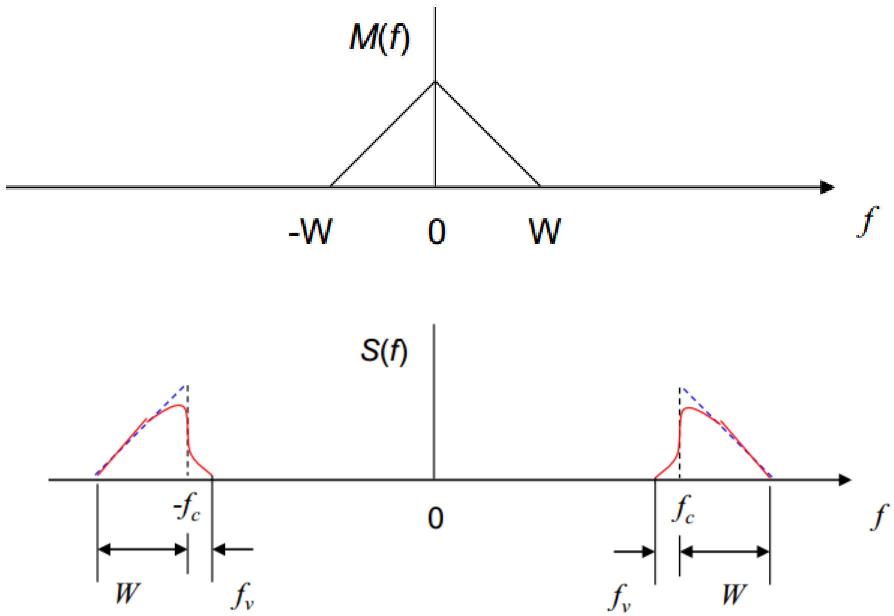


Filtering one of the sidebands of DSB-SC

  - The spectral efficiency of SSB makes it suitable for voice communication over telephone channels (line/cable)
  - Not suitable for signals with significant low frequency components due to the difficulty of implementing the filter.

Amplitude modulation

- Vestigial Sideband (VSB) AM
  - VSB is a compromise between SSB and DSB-SC



1. VSB signal bandwidth is $B=W+f_v$
2. VSB is used in TV broadcasting and similar signals where **low frequency components are significant**

Amplitude modulation

- Comparison of AM techniques

DSB-SC:

more power efficient. Seldom used

Conventional AM:

simple envelop detector. AM radio broadcast

SSB:

requires minimum transmitter power and bandwidth. Suitable for point-to-point and over long distances

VSB:

bandwidth requirement between SSB and DSBSC. TV transmission

Amplitude modulation

- Signal multiplexing is a technique where a number of independent signals are combined and transmitted in a common channel
- These signals are de-multiplexed at the receiver
- Two common methods for signal multiplexing
  - TDM (time-division multiplexing)
  - FDM(frequency-division multiplexing)

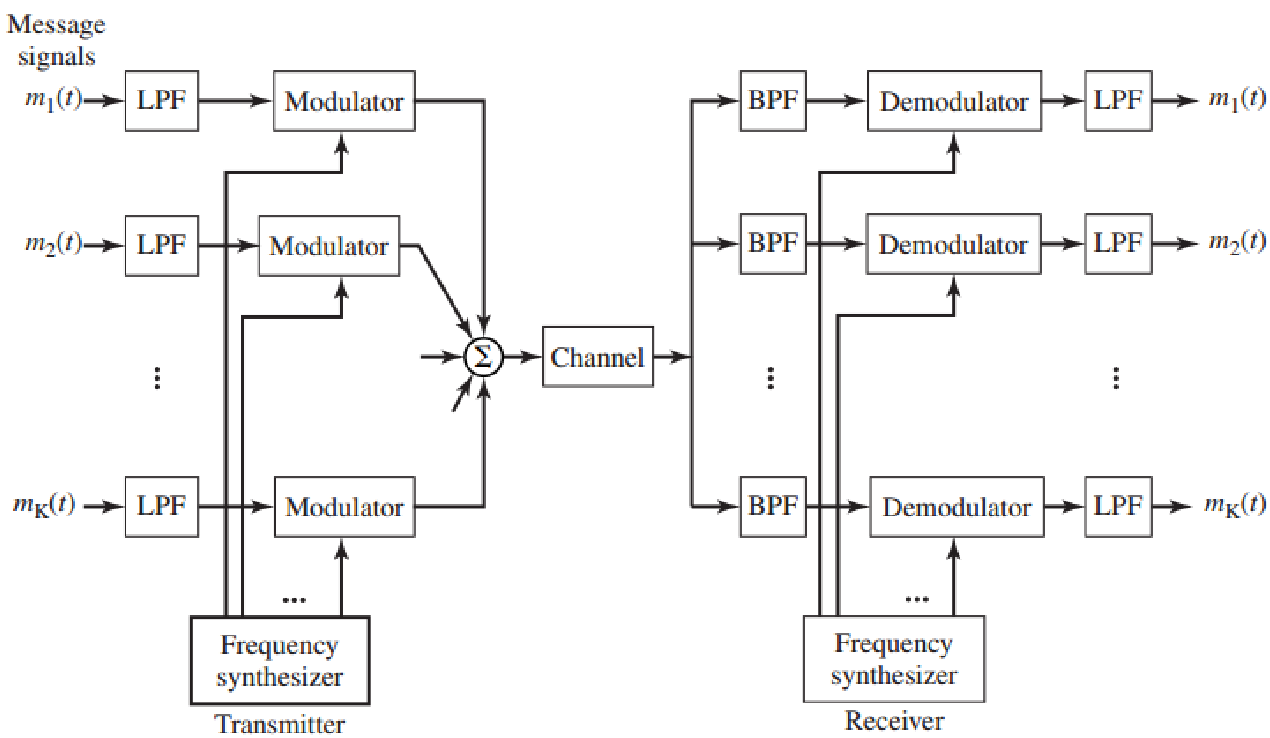
Amplitude modulation

- FDM

LPF: ensure signal bandwidth limited to W

MOD (modulator): shift message frequency range to mutually exclusive high frequency bands

BPF: restrict the band of each modulated wave to its prescribed range



Amplitude modulation

- FDM is widely used in radio and telephone communications
  - Voice signal: 300~3400Hz
  - Message is SSB modulated.
  - In 1st level FDM, 12 signals are stacked in frequency, with a freq. separation of 4 kHz between adjacent carriers.
  - A composite 48 kHz channel, called a **group channel**, transmits 12 voice-band signals simultaneously
  - In the next level of FDM, a number of group channels (typically 5 or 6) are stacked to form a **supergroup channel**
  - Higher-order FDM is obtained by combining several supergroup channels

An FDM hierarchy in telephone commun. Systems.

Amplitude modulation

- Quadarture-carrier multiplexing
  - Transmit two messages on the same carrier as

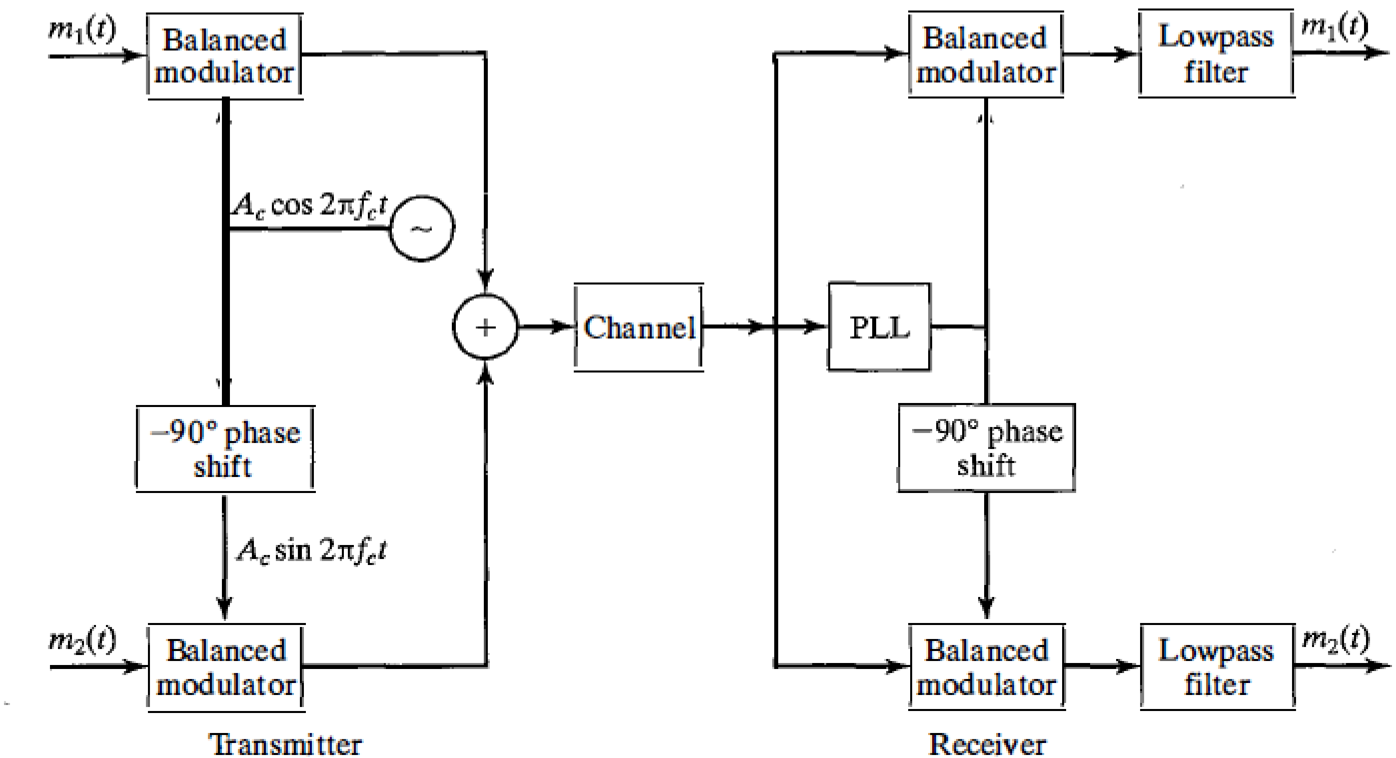
$$s(t) = A_c m_1(t)\cos\left(2\pi f_c t\right) + A_c m_2(t)\sin\left(2\pi f_c t\right)$$

  - cos() and sin() are two quadrature carriers
  - Each message signal is modulated by DSB-SC
  - Bandwidth-efficiency comparable to SSB AM
  - Synchronous demodulation of $m_1(t)$

$$\begin{aligned} s(t)\cos\left(2\pi f_c t\right) &= A_c m_1(t)\cos^2\left(2\pi f_c t\right) + A_c m_2(t)\sin\left(2\pi f_c t\right)\cos\left(2\pi f_c t\right) \\ &= \underbrace{\frac{A_c}{2} m_1(t)}_{\text{LPF}} + \frac{A_c}{2} m_1(t)\cos\left(4\pi f_c t\right) + \frac{A_c}{2} m_2(t)\sin\left(4\pi f_c t\right) \end{aligned}$$

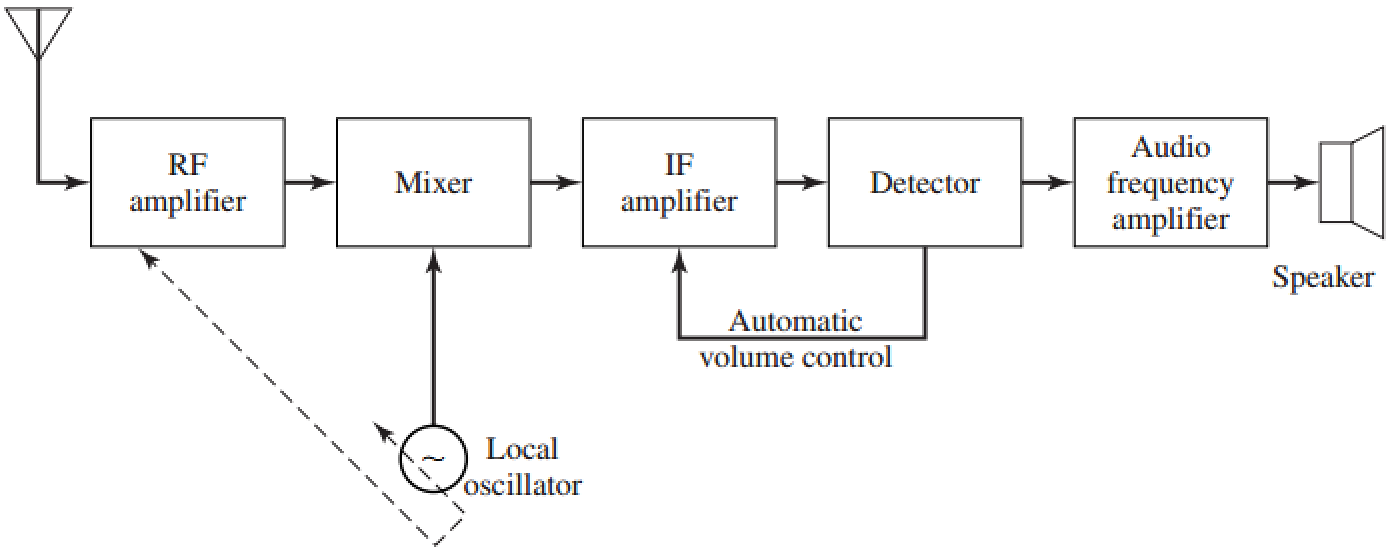
Amplitude modulation

- Quadarture-carrier multiplexing



Amplitude modulation

- AM radio broadcasting
  - Commercial AM radio uses conventional AM
  - Superheterodyne receiver



  - Every AM-radio signal is converted to a common IF frequency of 455 kHz, IF bandwidth 10 kHz, signal frequency range 535~1606 kHz

Angle modulation

- Either phase or frequency of the carrier is changed according to the message signal
- The general form

$$s(t) = A_c \cos\left[2\pi f_c t + \theta(t)\right]$$

$\theta(t)$: the time-varying phase

instantaneous frequency of s(t): $f_i(t) = f_c + \frac{1}{2\pi}\frac{d\theta(t)}{dt}$

- **Phase modulation (PM)**

$$\theta(t) = k_p m(t)$$

where k_p = phase deviation constant

- **Frequency modulation(FM)**

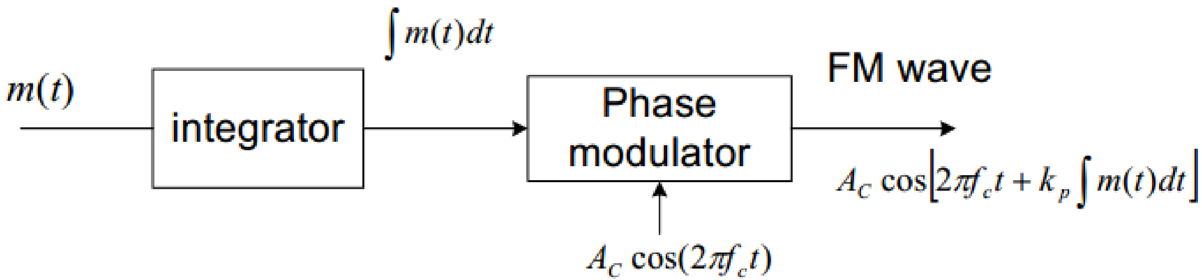
$$f_i(t) - f_c = k_f m(t) = \frac{1}{2\pi}\frac{d}{dt}\theta(t)$$

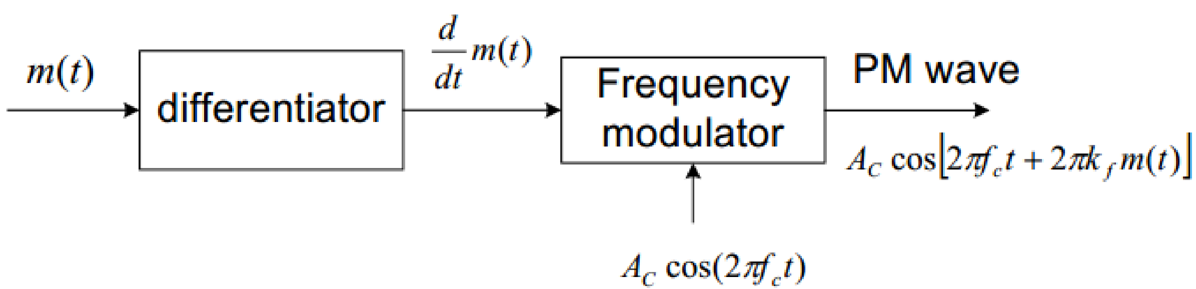
where k_f = frequency deviation constant/frequency sensitivity

The phase of FM is $\theta(t) = 2\pi k_f \int_0^t m(\tau)d\tau$

Angle modulation

- Constant envelope, i.e., amplitude of s(t) is constant
- Relationship between PM and FM



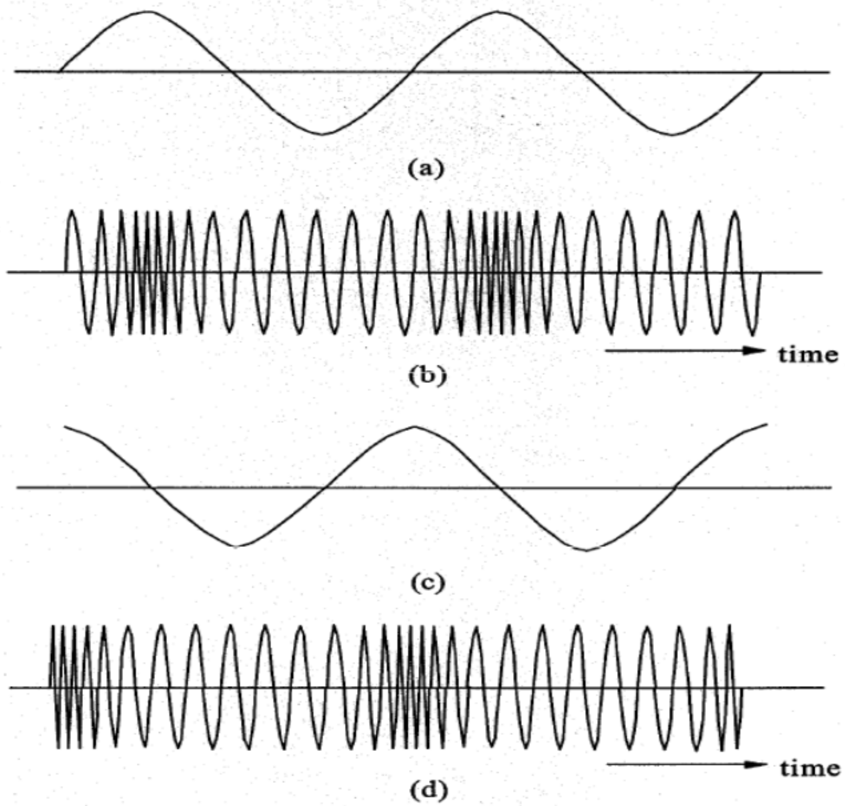


Will discuss the properties of FM only

Angle modulation

- Consider for example the **sinusoidal modulation**

Sinusoid modulating wave m(t)



FM wave

$$\frac{d}{dt}m(t)$$

PM wave

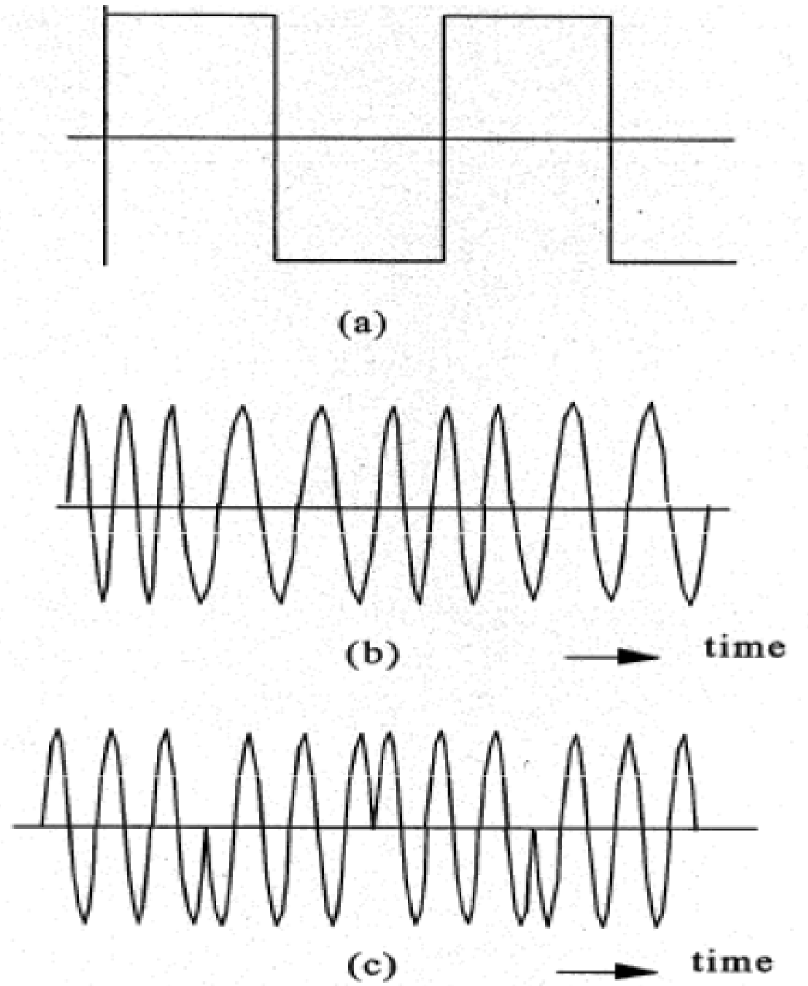
Angle modulation

- Consider for example the **square modulation**

Square modulating wave m(t)

FM wave

PM wave



Angle modulation

- FM by the **sinusoidal modulation**
  - Message $m(t) = A_m \cos(2\pi f_m t)$
  - Instantaneous frequency of resulting FM wave

$$f_i(t) = f_c + k_f A_m \cos(2\pi f_m t) = f_c + \Delta f \cos(2\pi f_m t)$$

Frequency deviation: $\Delta f = k_f A_m$

  - Carrier phase

$$\theta(t) = 2\pi \int_0^t \left(f_i(\tau) - f_c\right) d\tau = \frac{\Delta f}{f_m} \sin(2\pi f_m t)$$

$$= \beta \sin(2\pi f_m t)$$

Modulation index: $\beta = \Delta f / f_m$

Angle modulation

- FM by the **sinusoidal modulation**
  - Consider the following problem

Problem: a sinusoidal modulating wave of amplitude 5V and frequency 1kHz is applied to a frequency modulator. The frequency sensitivity is 40Hz/V. The carrier frequency is 100kHz.

Calculate (a) the frequency deviation

(b) the modulation index

Angle modulation

- FM by the **sinusoidal modulation**
  - Consider the following problem

<u>Problem</u>: a sinusoidal modulating wave of amplitude 5V and frequency 1kHz is applied to a frequency modulator. The frequency sensitivity is 40Hz/V. The carrier frequency is 100kHz.

Calculate (a) the frequency deviation

(b) the modulation index

Frequency deviation $\Delta f = k_f A_m = 40 \times 5 = 200 Hz$

Modulation index $\beta = \dfrac{\Delta f}{f_m} = \dfrac{200}{1000} = 0.2$

Angle modulation

- FM by the **sinusoidal modulation**
  - Spectrum analysis

$$s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]$$
$$= \underbrace{A_c \cos[\beta \sin(2\pi f_m t)]}_{\text{In-phase component}} \cos(2\pi f_c t) - \underbrace{A_c \sin[\beta \sin(2\pi f_m t)]}_{\text{Quadrature-phase component}} \sin(2\pi f_c t)$$

$$s_I(t) = A_c \cos[\beta sin(2\pi f_m t)] \qquad s_Q(t) = A_c \sin[\beta \sin(2\pi f_m t]$$

  - Define the complex envelope of FM wave

$$\tilde{s}(t) = s_I(t) + js_Q(t) = A_c e^{j\beta \sin(2\pi f_m t)}$$

  - $\tilde{s}(t)$ retains complete information about s(t)

$$s(t) = \mathrm{Re}\left\{A_c e^{j[2\pi f_c t + \beta \sin(2\pi f_m t)]}\right\} = \mathrm{Re}\left[\tilde{s}(t) e^{j2\pi f_c t}\right]$$

Angle modulation

- FM by the **sinusoidal modulation**
  - $\tilde{s}(t) = A_c e^{j\beta\sin(2\pi f_m t)}$ is periodic, expanded in Fourier series as

$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_m t}$$

with

$$c_n = f_m \int_{-1/(2f_m)}^{1/(2f_m)} \tilde{s}(t) e^{-j2\pi n f_m t} dt$$

$$= f_m A_c \int_{-1/(2f_m)}^{1/(2f_m)} e^{j[\beta\sin(2\pi f_m t) - 2\pi n f_m t]} dt$$

$$\overset{x = 2\pi f_m t}{=} \frac{A_c}{2\pi} \int_{-\pi}^{\pi} \exp\left[j(\beta\sin x - nx)\right] dx$$

  - n-th order Bessel function of the first kind

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp\left[j\left(\beta\sin x - nx\right)\right] dx$$

  - Hence $c_n = A_c J_n(\beta)$

Angle modulation

- FM by the **sinusoidal modulation**
  - Substituting $c_n = A_c J_n(\beta)$ into $\tilde{s}(t)$

$$\tilde{s}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \exp\left(j2\pi n f_m t\right)$$

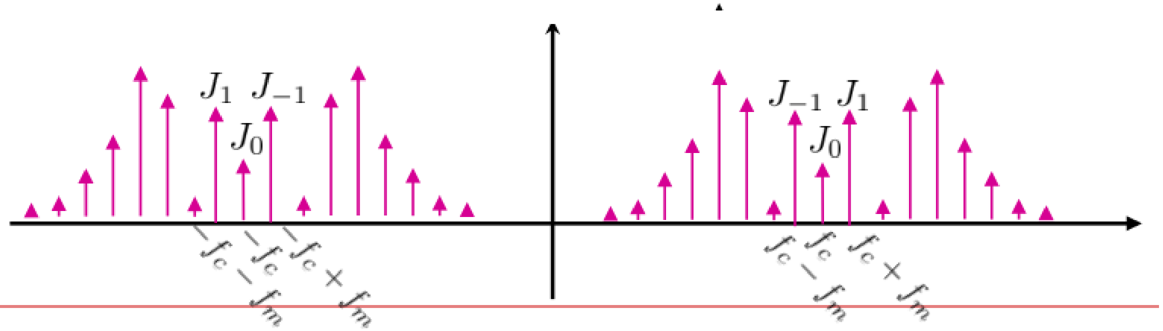
  - FM wave in time domain

$$s(t) = A_c Re\left\{ \sum_{n=-\infty}^{\infty} J_n(\beta) \exp[j2\pi(f_c + nf_m)t] \right\}$$

$$= A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos\left[2\pi(f_c + nf_m)t\right]$$

  - FM wave in frequency domain

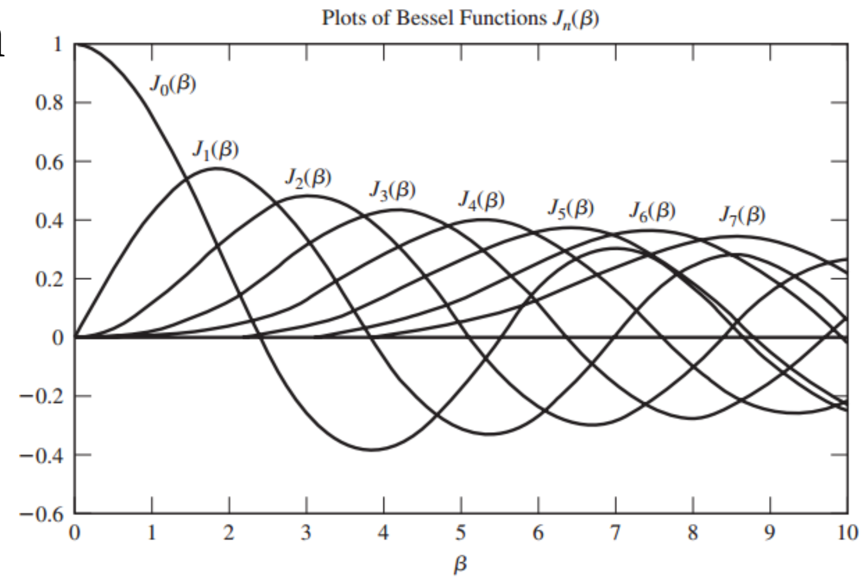
$$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta)\left[\delta(f - f_c - nf_m) + \delta(f + f_c + nf_m)\right]$$



Angle modulation

- FM by the **sinusoidal modulation**
  - Properties of Bessel function
  - For small $\beta \leq 0.3$, we have the approximations

$$J_0(\beta) \approx 1$$
$$J_1(\beta) \approx \beta/2$$
$$J_n(\beta) \approx 0, n > 1$$



$$|J_n(\beta)| \to 0 \text{ as } \beta \to \infty$$
$$J_{-n}(\beta) = \begin{cases} J_n(\beta), & n \text{ even} \\ -J_n(\beta), & n \text{ odd} \end{cases}$$

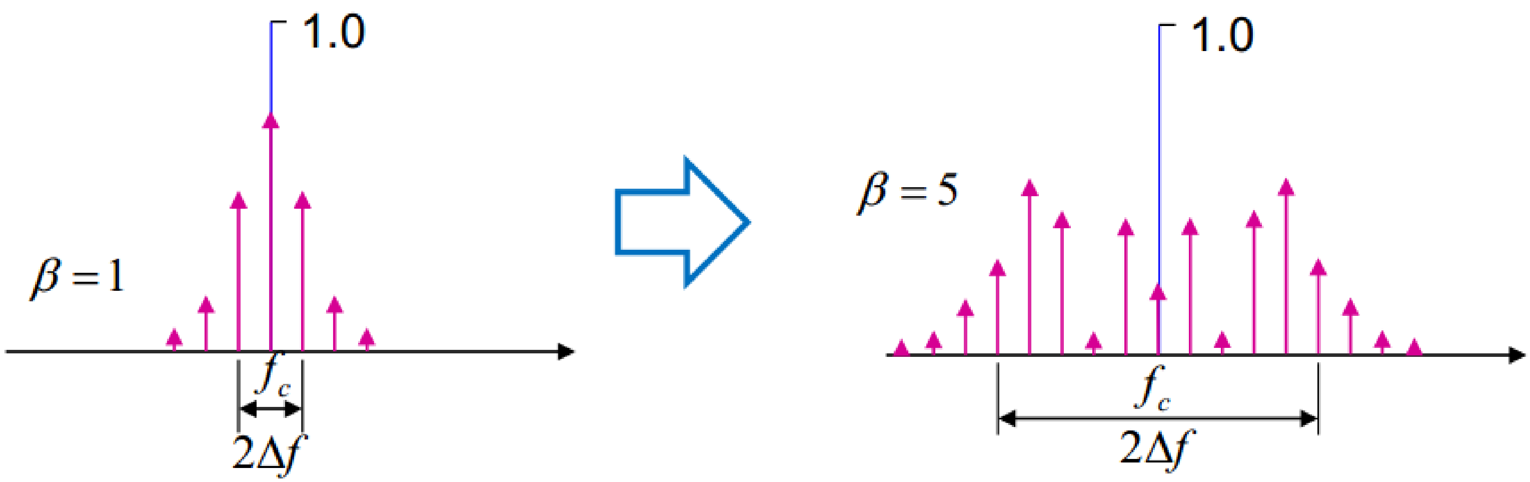
  - Then,

$$s(t) \approx A_c \cos(2\pi f_c t) + \frac{\beta A_c}{2}\cos[2\pi(f_c + f_m)t] - \frac{\beta A_c}{2}\cos[2\pi(f_c - f_m)t]$$

Approximate bandwidth = 2f_m

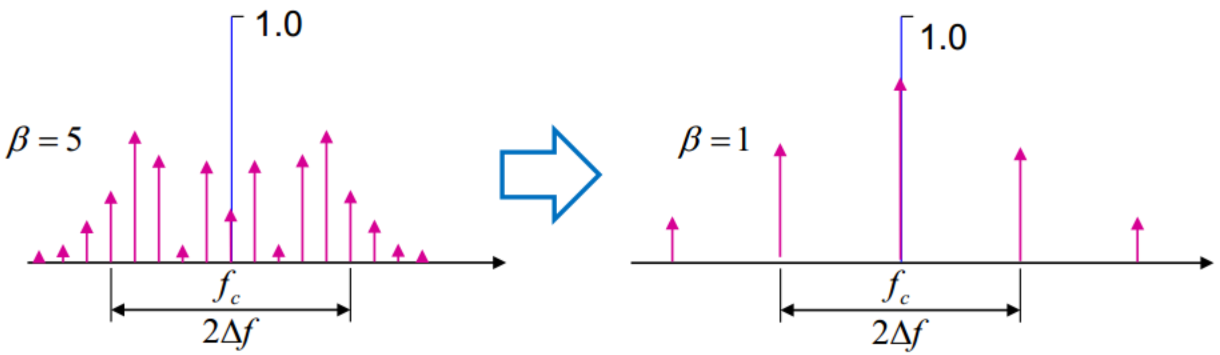
Angle modulation

- FM by the **sinusoidal modulation**
  - For the general case, we would like to see how A_m and f_m affect the spectrum
  - Fix f_m and vary A_m, then $\Delta f = k_f A_m$ and $\beta = \Delta f / f_m$ are varied



Angle modulation

- FM by the **sinusoidal modulation**
  - Fix A_m and vary f_m , then
    $\Delta f = k_f A_m$ is fixed, but $\beta = \Delta f / f_m$ is varied



Angle modulation

- FM by the **sinusoidal modulation**
  - Effective bandwidth of FM waves
  - For large β , B is only slightly greater than $2\Delta f$
  - For small β , the spectrum is limited to $[f_c - f_m, f_c + f_m]$
  - Carson's rule

$$B \approx 2\Delta f + 2f_m = 2(1+\beta)f_m$$

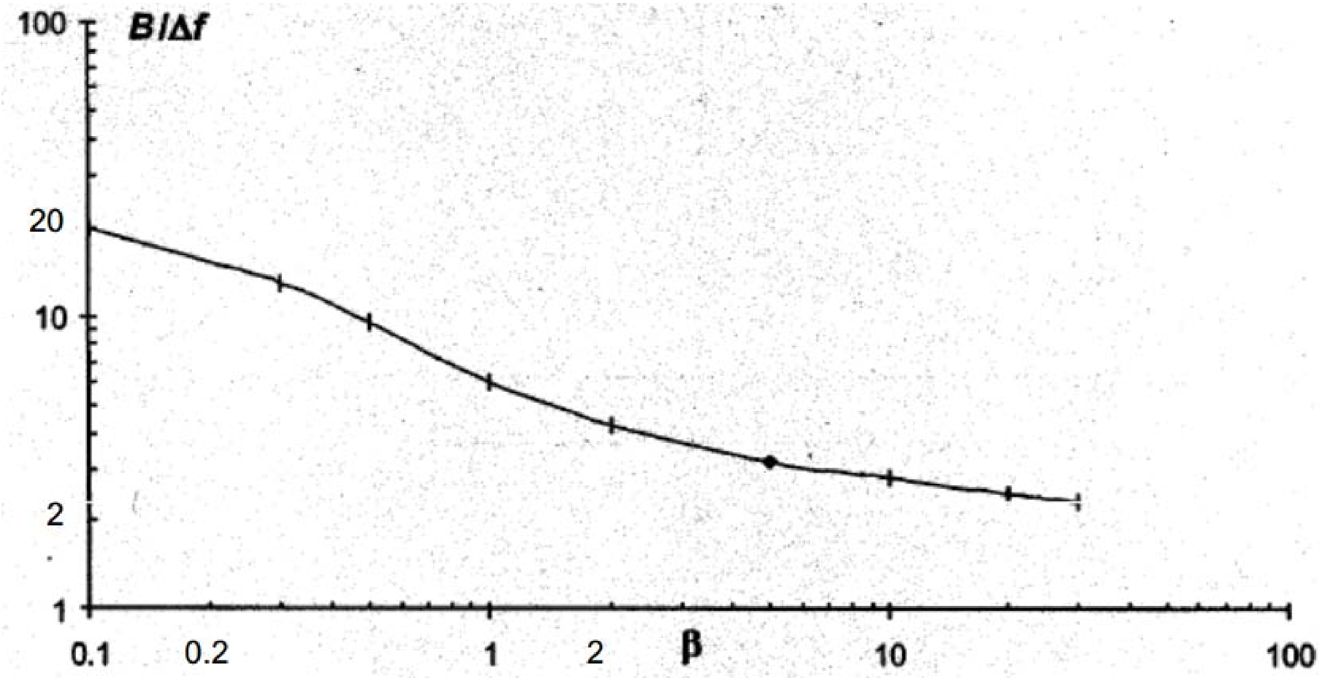
  - 99% bandwidth approximation specify the separation between the two frequencies beyond which none of the side-frequencies is greater than 1% of the unmodulated carrier amplitude, i.e.,

$B \approx 2n_{max} f_m$ where n_{max} is the max n that satisfies $|J_n(\beta)| > 0.01$

β	0.1	0.3	0.5	1.0	2.0	5.0	10	20	30
$2n_{max}$	2	4	4	6	8	16	28	50	70

Angle modulation

- FM by the **sinusoidal modulation**
  - ➢ A universal curve for evaluating the 99% bandwidth



As β increases, the bandwidth occupied by the significant side-frequencies drops toward that over which the carrier frequency actually deviates, i.e. B become less affected by β

Angle modulation

- FM by **an arbitrary message**
  - Consider an arbitrary m(t) with highest freq. component W
  - **Frequency deviation** $\Delta f = k_f \max|m(t)|$
  - **Modulation index**: $\beta = \dfrac{\Delta f}{W}$
  - Carson's rule applies as $B = 2(1+\beta)W$
  - Carson's rule underestimates the FM bandwidth requirement
  - Universal curve yields a conservative result

Angle modulation

- FM by **an arbitrary message**
  - Consider for example in north America, the maximum value of frequency deviation Δf is fixed at 75 kHz for commercial FM broadcasting by radio.
  - Take W=15 kHz, typically the maximum audio frequency of interest in FM transmission, the modulation index is

$$\beta = 75/15 = 5$$

  - Using Carson's rule,

$$B = 2(75 + 15) = 180KHz$$

  - Using universal curve,

$$B = 3.2\Delta f = 3.2 \times 75 = 240KHz$$

Angle modulation

- FM by **an arbitrary message**
  - Consider the following exercise.

    Assuming that $m(t) = 10\text{sinc}\left(10^4 t\right)$, determine the transmission bandwidth of an FM modulated signal with $k_f = 4000$

Angle modulation

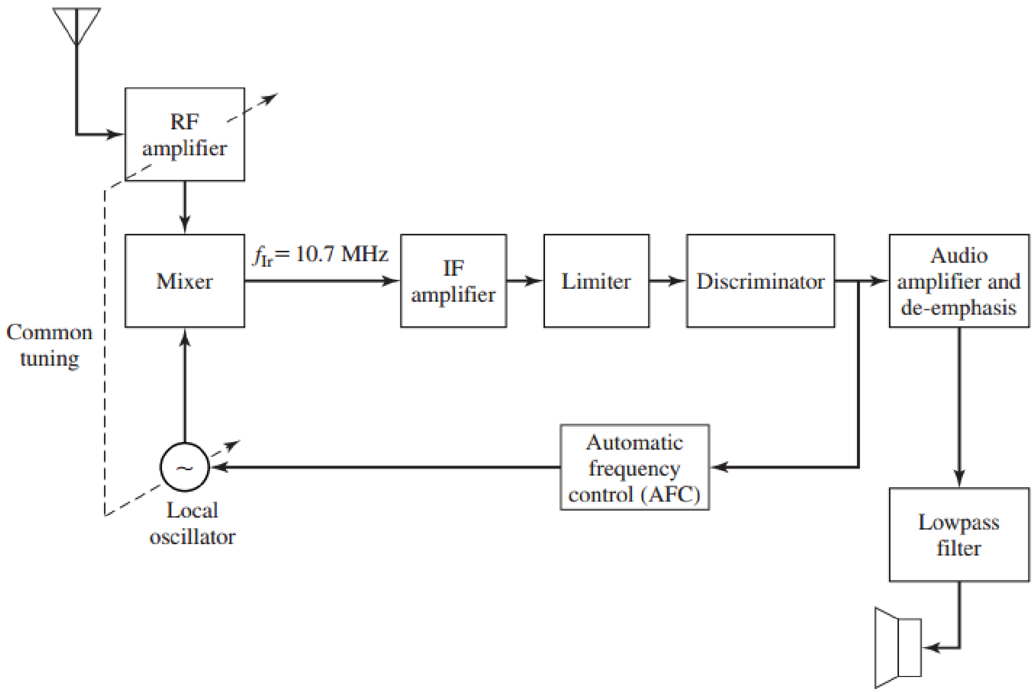
- FM by **an arbitrary message**
  - Consider the following exercise.

    Assuming that $m(t) = 10\mathrm{sinc}\left(10^4 t\right)$, determine the transmission bandwidth of an FM modulated signal with $k_f = 4000$

By Carson's rule: $B = 90KHz$

Angle modulation

- FM radio broadcasting
  - As with standard AM radio, most FM radio receivers are of super-heterodyne type



1. RF carrier range: **88~108 MHz**
2. Midband of IF: **10.7 MHz**
3. IF bandwidth: **200 kHz**
4. Peak freq. deviation: **75 kHz**

Angle modulation

- Generation of FM waves
  - **Direct approach**: design an oscillator whose frequency changes with the input voltage (voltage-controlled oscillator (VCO))
  - **Indirect approach**: first generate a narrowband FM signal and then change it to a wideband signal (due to the similarity with the conventional AM, the generation of narrowband FM signals is straightforward)

Angle modulation

- Generation of **narrowband** FM waves
  - Consider a narrow band FM wave

$$s_1(t) = A_1 \cos\left[2\pi f_1 t + \phi_1(t)\right]$$

with $\phi_1(t) = 2\pi k_1 \int_0^t m(\tau)\,d\tau$ $\quad f_1$ = carrier frequency

k_1 = frequency sensitivity

  - Given $\phi_1(t) \ll 1$ with $\beta \le 0.3$, we can use the approximations

$$\begin{cases} \cos\left[\phi_1(t)\right] \approx 1 \\ \sin\left[\phi_1(t)\right] \approx \phi_1(t) \end{cases}$$

  - Then, we can approximate $s_1(t)$ as

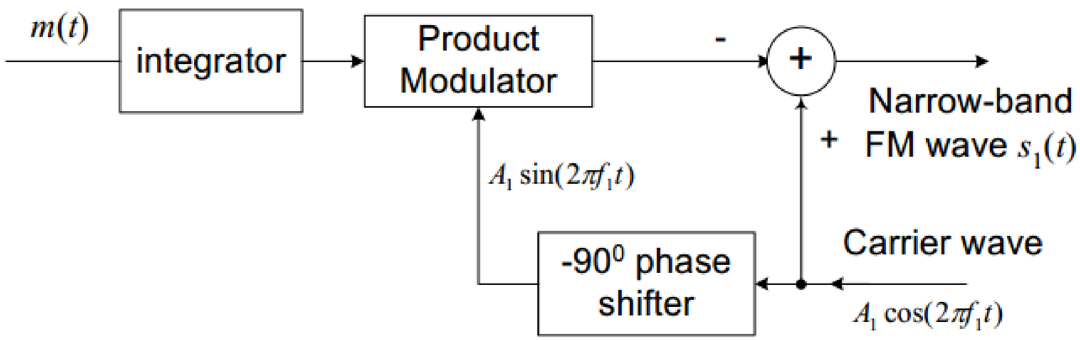
$$s_1(t) = A_1 \cos(2\pi f_1 t) - A_1 \sin(2\pi f_1 t)\phi_1(t)$$

$$= A_1 \cos(2\pi f_1 t) - 2\pi k_1 A_1 \sin(2\pi f_1 t)\int_0^t m(\tau)\,d\tau$$

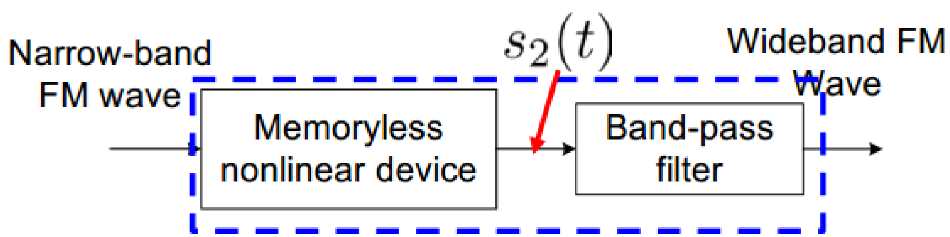
Narrowband FM wave

Angle modulation

- Generation of **narrowband** FM waves
  - Narrow-band frequency modulator



  - Next, pass $s_1(t)$ through a frequency multiplier



Angle modulation

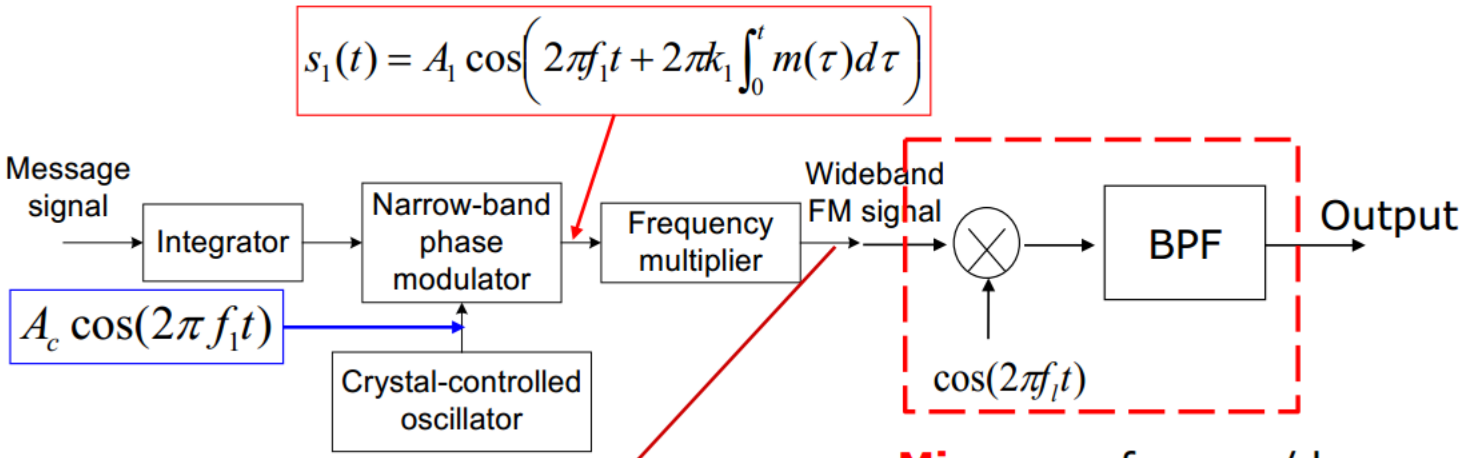
- Generation of **narrowband** FM waves
  - The input-output relationship of the non-linear device is
    $$s_2(t) = a_1 s_1(t) + a_2 s_1^2(t) + \ldots + a_n s_1^n(t)$$
  - The BPF is used to pass the FM wave centered at nf_1 and with deviation $n\Delta f_1$ and suppress all other FM spectra
  - Exam: Consider for example a square-law device based frequency multiplier $s_2(t) = a_1 s_1(t) + a_2 s_1^2(t)$ with
    $$s_1(t) = A_1 \cos\left(2\pi f_1 t + 2\pi k_1 \int_0^t m(\tau)d\tau\right)$$
  - Specify the **midband freq.** and **bandwidth** of BPF used in freq. multiplier for the resulting freq. deviation to be twice at the nonlinear device.

$$s_2(t) = a_1 A_1 \cos\left(2\pi f_1 t + 2\pi k_1 \int_0^t m(\tau)d\tau\right) + a_2 A_1^2 \cos^2\left(2\pi f_1 t + 2\pi k_1 \int_0^t m(\tau)d\tau\right)$$

$$= \underbrace{a_1 A_1 \cos\left(2\pi f_1 t + 2\pi k_1 \int_0^t m(\tau)d\tau\right) + \frac{a_2 A_1^2}{2}}_{\text{Removed by BPF with}} + \frac{a_2 A_1^2}{2} \cos\left(4\pi f_1 t + 4\pi k_1 \int_0^t m(\tau)d\tau\right)$$

$f_c = 2f_1$
BW $> 2\Delta f = 4\Delta f_1$

- Generation of **wideband** FM waves



$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right]$$

Mixer: perform up/down conversion to shift the signal to the desired center freq.

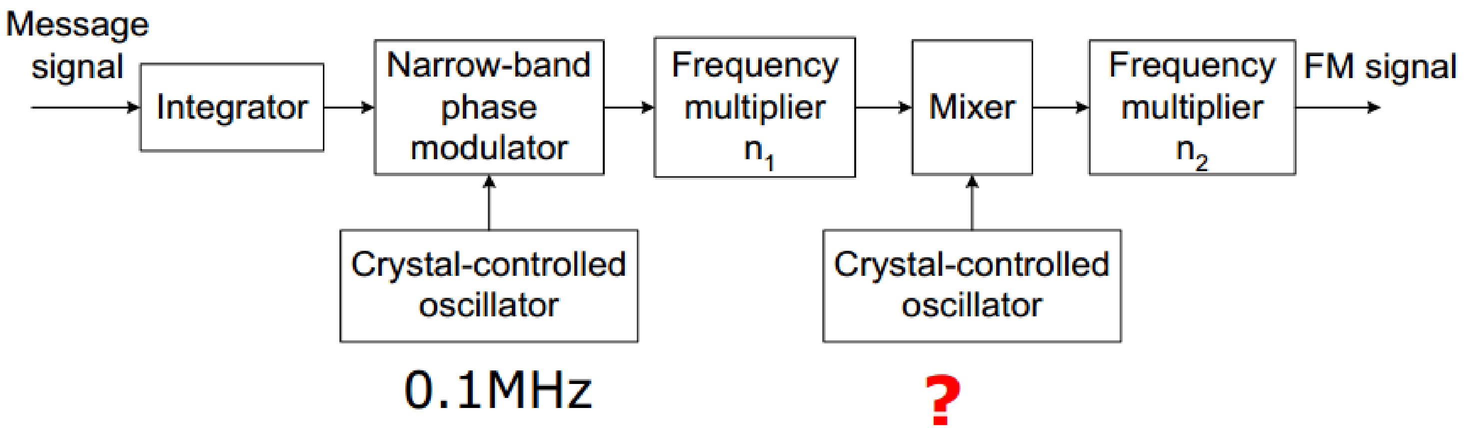
$f_c = nf_1$ → may not be the desired carrier frequency

$k_f = nk_1$

$\Delta f = n\Delta f_1$

Angle modulation

- Generation of **wideband** FM waves
  - Exam: Consider the following simplified diagram of a typical FM transmitter used to transmit audio signals containing frequencies in the range 100 Hz to 15 kHz
  - Desired FM wave: $f_c = 100\text{MHz}$, $\Delta f = 75\text{kHz}$
  - Set $\beta_1 = 0.2$ in the narrowband phase modulation to limit harmonic distortion.
  - Specify the two-stage frequency multiplier factors n_1 and n_2



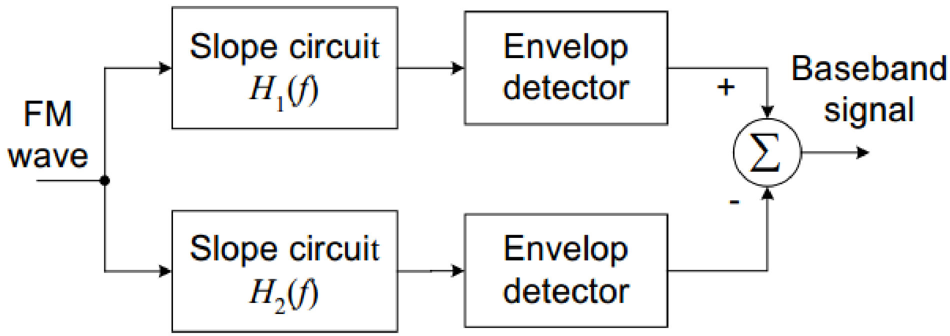
Angle modulation

- Demodulation of FM – Balanced Frequency Discriminator
  - Given FM wave $s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right)$

$$\frac{d}{dt}s(t) = -A_c\left[2\pi f_c + 2\pi k_f m(t)\right]\sin\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right]$$

Hybrid-modulated wave with AM and FM

  - **Differentiator** + **envelope detector** = FM demodulator
  - Frequency discriminator: a freq. to amplitude transform device



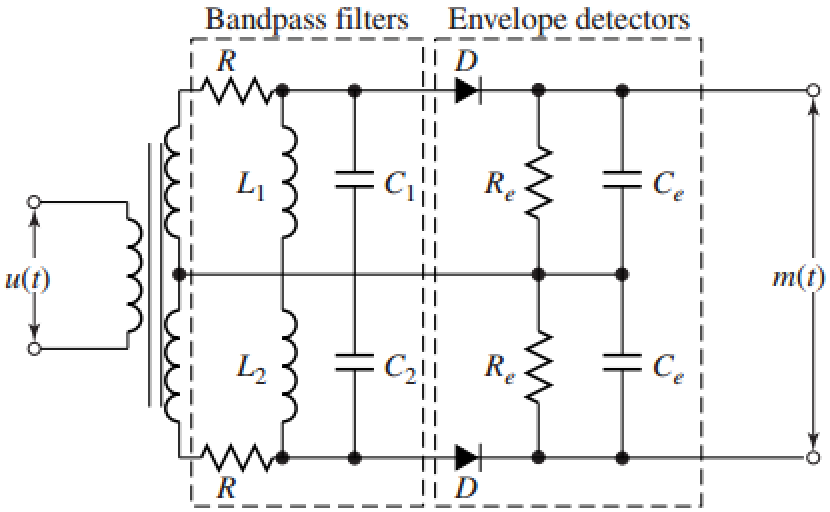
$$|H(f)| = 2\pi f$$

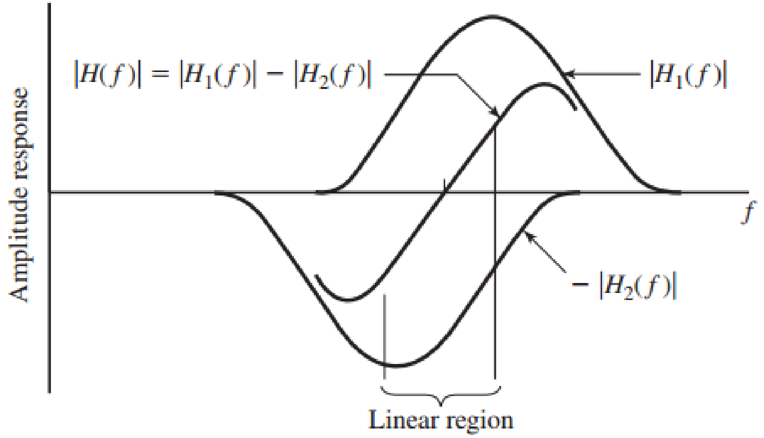
$$H_1(f) = \begin{cases} j2\pi a\left(f - f_c + B/2\right), & f_c - B/2 \le f \le f_c + B/2 \\ j2\pi a\left(f + f_c - B/2\right), & -f_c - B/2 \le f \le -f_c + B/2 \\ 0, & \text{elsewhere} \end{cases}$$

$$H_2(f) = H_1(-f)$$

Angle modulation

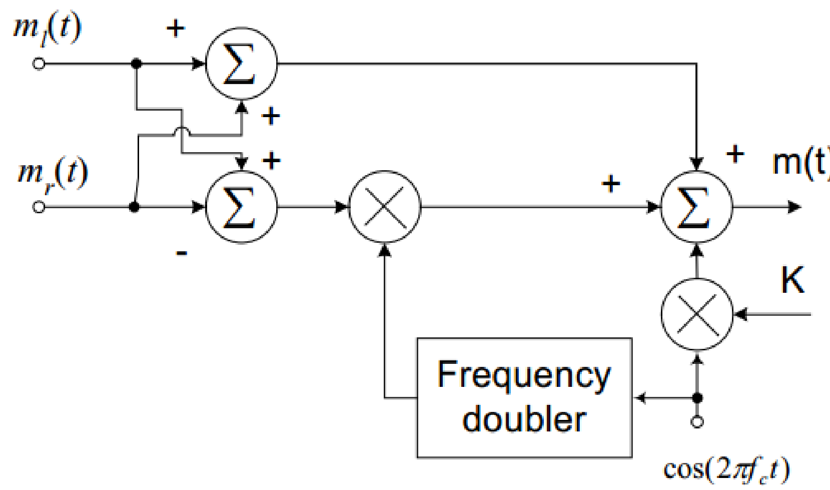
- Demodulation of FM – Balanced Frequency Discriminator
  - ➢ Circuit diagram and frequency response





Angle modulation

- FM radio stereo multiplexing
  - Stereo multiplexing is a form of FDM designed to transmit two separate signals via the same carrier.
  - Widely used in FM broadcasting to send two different elements of a program (e.g., vocalist and accompanist in an orchestra) to give a spatial dimension to its perception by a listener at the receiving end



$$m(t) = [m_l(t) + m_r(t)] + [m_l(t) - m_r(t)]\cos(4\pi f_c t) + K\cos(2\pi f_c t)$$

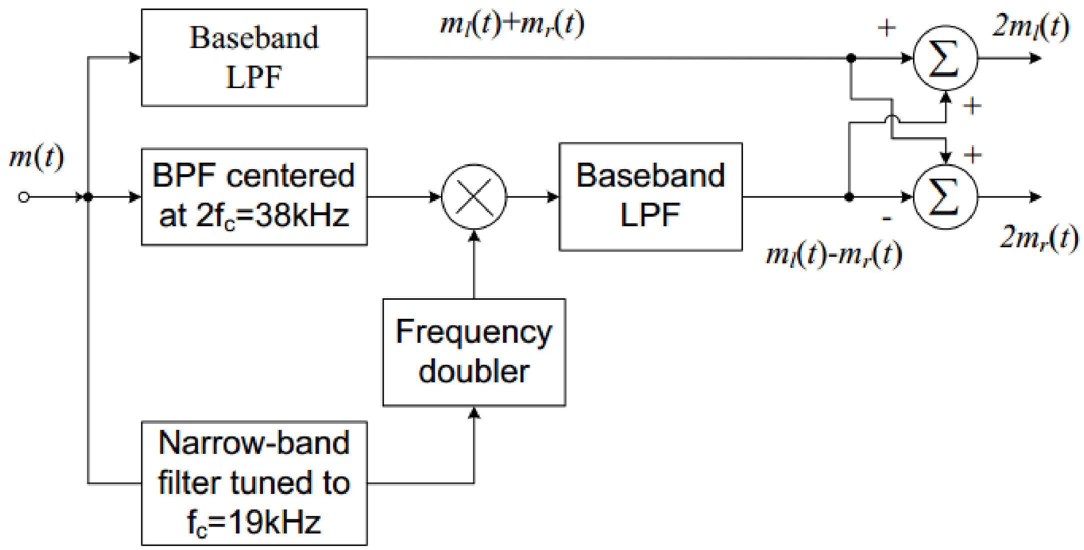
The sum signal is left unprocessed in its baseband form

The difference signal and a 38-kHz subcarrier produce a DSBSC wave

The 19-kHz pilot is included as a reference for coherent detection

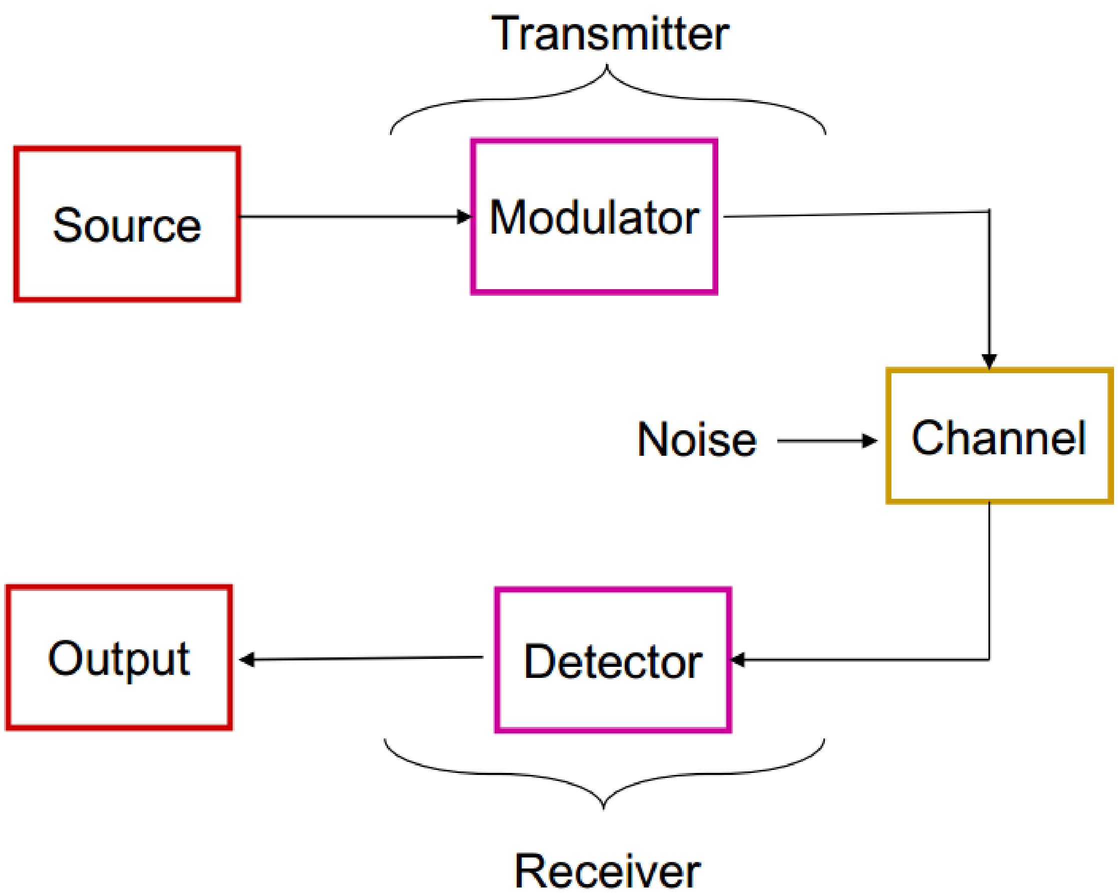
Angle modulation

- FM radio stereo multiplexing
  - FM-stereo receiver



To two loudspeakers

Analog Communication Systems



Performance: AM

- No carrier modulation
  - Ideal low-pass filter with bandwidth W.
  - With white noise, the noise power of the output

$$\begin{aligned} P_{no} &= \int_{-W}^{+W} \frac{N_0}{2}\, df \\ &= N_0 W. \end{aligned}$$

  - The baseband SNR is

$$\left(\frac{S}{N}\right)_b = \frac{P_R}{N_0 W}.$$

Performance: AM

- DSB-SC AM
  - Transmitted signal

$$u(t) = A_c m(t)\cos(2\pi f_c t);$$

  - The received signal with additive white noise is

$$\begin{aligned} r(t) &= u(t) + n(t) \\ &= A_c m(t)\cos(2\pi f_c t) + n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t). \end{aligned}$$

  - Suppose the demodulator multiplies $\cos(2\pi f_c t + \phi)$

$$\begin{aligned} r(t)\cos(2\pi f_c t + \phi) &= A_c m(t)\cos(2\pi f_c t)\cos(2\pi f_c t + \phi) + n(t)\cos(2\pi f_c t + \phi) \\ &= \frac{1}{2}A_c m(t)\cos(\phi) + \frac{1}{2}A_c m(t)\cos(4\pi f_c t + \phi) \\ &\quad + \frac{1}{2}[n_c(t)\cos(\phi) + n_s(t)\sin(\phi)] \\ &\quad + \frac{1}{2}[n_c(t)\cos(4\pi f_c t + \phi) - n_s(t)\sin(4\pi f_c t + \phi)]. \end{aligned}$$

  - After low-pass filter

$$y(t) = \frac{1}{2}A_c m(t)\cos(\phi) + \frac{1}{2}[n_c(t)\cos(\phi) + n_s(t)\sin(\phi)].$$

Performance: AM

- DSB-SC AM
  - Assume $\phi = 0$, $y(t) = \frac{1}{2}[A_c m(t) + n_c(t)]$
  - The message signal power is $P_o = \frac{A_c^2}{4} P_M$
  - The noise power is

$$P_{n_o} = \frac{1}{4} P_{n_c} = \frac{1}{4} P_n, \qquad S_n(f) = \begin{cases} \frac{N_0}{2} & |f - f_c| < W \\ 0 & \text{otherwise} \end{cases}.$$

  - Then, the output SNR is

$$\left(\frac{S}{N}\right)_o = \frac{P_o}{P_{n_o}} = \frac{\frac{A_c^2}{4} P_M}{\frac{1}{4} 2W N_0} = \frac{A_c^2 P_M}{2W N_0}.$$

Performance: AM

- DSB-SC AM
  - ➢ The received signal power

$$P_R = \frac{A_c^2 P_M}{2}$$

  - ➢ Then, we can express the output SNR as

$$\left(\frac{S}{N}\right)_{o_{\mathrm{DSB}}} = \frac{P_R}{N_0 W}$$

No SNR improvement for DSB-SC

Performance: AM

- SSB-SC AM
  - Transmitted signal

$$u(t) = A_c m(t)\cos(2\pi f_c t) \mp A_c \hat{m}(t)\sin(2\pi f_c t).$$

  - The received signal is

$$\begin{aligned} r(t) &= A_c m(t)\cos(2\pi f_c t) \mp A_c \hat{m}(t)\sin(2\pi f_c t) + n(t) \\ &= (A_c m(t) + n_c(t))\cos(2\pi f_c t) + (\mp A_c \hat{m}(t) - n_s(t))\sin(2\pi f_c t). \end{aligned}$$

  - With ideal phase, after the low-pass filter

$$y(t) = \frac{A_c}{2} m(t) + \frac{1}{2} n_c(t).$$

  - Similar to DSB, we have

$$P_o = \frac{A_c^2}{4} P_M \quad P_{n_o} = \frac{1}{4} P_{n_c} = \frac{1}{4} P_n \quad P_n = \int_{-\infty}^{\infty} S_n(f)\, df = \frac{N_0}{2} \times 2W = W N_0.$$

  - Therefore

$$\left(\frac{S}{N}\right)_o = \frac{P_o}{P_{n_o}} = \frac{A_c^2 P_M}{W N_0}.$$

Performance: AM

- SSB-SC AM
  - ➢ Now, we know that

$$P_R = P_U = A_c^2 P_M;$$

  - ➢ Hence,

$$\left(\frac{S}{N}\right)_{o\mathrm{SSB}} = \frac{P_R}{W N_0} = \left(\frac{S}{N}\right)_{\mathrm{b}}$$

No SNR improvement for SSB

Performance: AM

- Conventional AM
  - Transmitted signal

$$u(t) = A_c [1 + am(t)] \cos 2\pi f_c t.$$

  - The received signal is

$$r(t) = [A_c [1 + am_n(t)] + n_c(t)] \cos 2\pi f_c t - n_s(t) \sin 2\pi f_c t,$$

  - With ideal mixing and low-passing filter

$$y_1(t) = \frac{1}{2}[A_c [1 + am_n(t)] + n_c(t)]$$

  - DC component is removed by a DC block, so output

$$y(t) = \frac{1}{2} A_c a m_n(t) + \frac{n_c(t)}{2}.$$

  - Now, the received signal power

$$P_R = \frac{A_c^2}{2} [1 + a^2 P_{M_n}]$$

Performance: AM

- Conventional AM
  - The output SNR is

$$\begin{aligned}\left(\frac{S}{N}\right)_{o_{\mathrm{AM}}} &= \frac{\frac{1}{4}A_c^2 a^2 P_{M_n}}{\frac{1}{4}P_{n_c}} \\ &= \frac{A_c^2 a^2 P_{M_n}}{2N_0 W} \\ &= \frac{a^2 P_{M_n}}{1+a^2 P_{M_n}} \frac{\frac{A_c^2}{2}\left[1+a^2 P_{M_n}\right]}{N_0 W} \\ &= \frac{a^2 P_{M_n}}{1+a^2 P_{M_n}} \frac{P_R}{N_0 W} \\ &= \frac{a^2 P_{M_n}}{1+a^2 P_{M_n}} \left(\frac{S}{N}\right)_b \\ &= \eta \left(\frac{S}{N}\right)_b ,\end{aligned}$$

SNR loss for conventional AM

Performance: AM

- Exam
  - Consider for example a WSS random process M(t) with the autocorrelation function
$$R_M(\tau) = 16\,\mathrm{sinc}^2(10{,}000\tau).$$
  - Also, $\max|m(t)| = 6$
  - The channel attenuation is 50 dB and the PSD of AWGN is given by
$$S_n(f) = \frac{N_0}{2} = 10^{-12}\ \mathrm{W/Hz}$$
  - If we want the output SNR of the modulator to be at least 50 dB.
  - Required transmitter power and channel bandwidth for DSB, SSB and conventional AM.

Performance: AM

- Exam
  - The bandwidth of the message

$$S_M(f) = \mathscr{F}[R_M(\tau)] = \frac{16}{10{,}000}\Lambda\left(\frac{f}{10{,}000}\right)$$

  - $W = 10{,}000$ Hz.
  - So the baseband SNR

$$\left(\frac{S}{N}\right)_b = \frac{P_R}{N_0 W} = \frac{P_R}{2 \times 10^{-12} \times 10^4} = \frac{10^8 P_R}{2}$$

  - 50 dB attenuation $P_R = 10^{-5} P_T$

$$\left(\frac{S}{N}\right)_b = \frac{10^{-5} \times 10^8 P_T}{2} = \frac{10^3 P_T}{2}.$$

Performance: AM

- Exam
  - Bandwidth

    DSB-SC and conventional AM: 20 kHz

    SSB: 10kHz
  - Power

    DSB: $\frac{10^3 P_T}{2} = 10^5 \Longrightarrow P_T = 200 \text{ Watts}$

    SSB:

$$\left(\frac{S}{N}\right)_o = \left(\frac{S}{N}\right)_b = \frac{10^3 P_T}{2} = 10^5 \Longrightarrow P_T = 200 \text{ Watts}$$

Conventional AM:

$$\left(\frac{S}{N}\right)_o = \eta\left(\frac{S}{N}\right)_b = \eta\frac{10^3 P_T}{2}$$

$$\eta = \frac{a^2 P_{M_n}}{1 + a^2 P_{M_n}}.$$

$$P_{M_n} = \frac{P_M}{(\max|m(t)|)^2} = \frac{P_M}{36}$$

$$P_M = R_M(\tau)|_{\tau=0} = 16;$$

$$\eta = \frac{0.8^2 \times \frac{4}{9}}{1 + 0.8^2 \times \frac{4}{9}} \approx 0.22.$$

$$\left(\frac{S}{N}\right)_o \approx 0.22\frac{10^3 P_T}{2}$$

$$= 0.11 \times 10^3 P_T = 10^5$$

$$P_T \approx 909 \text{ Watts}.$$

Performance: FM

- In AM, message contained in the amplitude, and noise is directly added.
- In FM, the noise affects the zero crossings of the modulated signal.

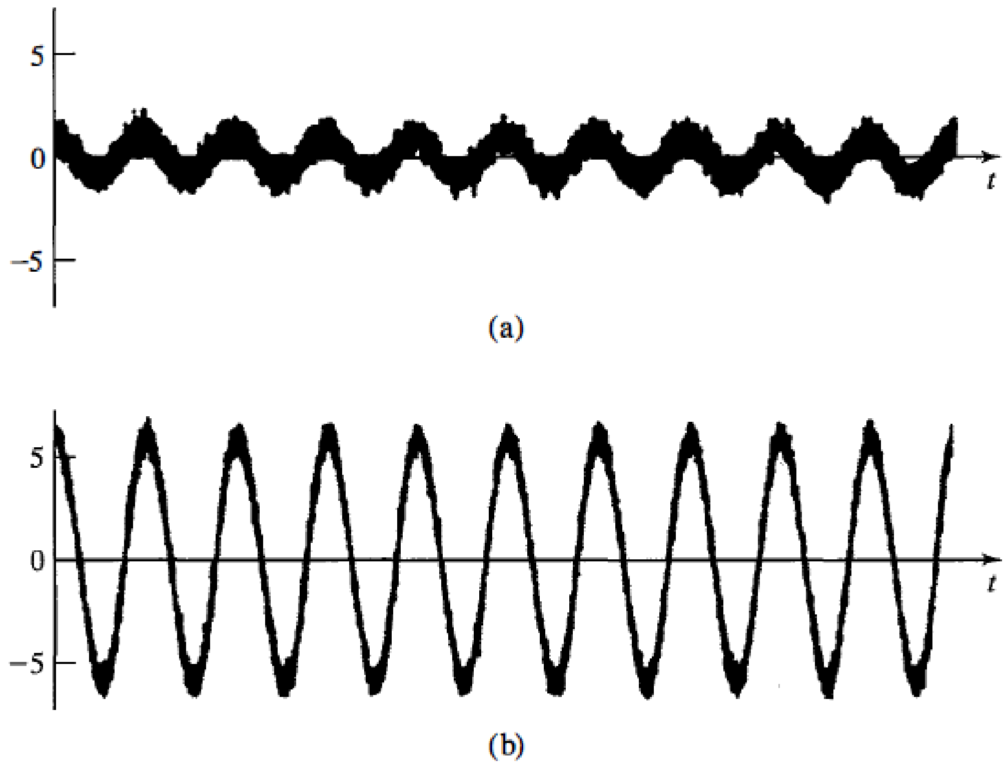


Figure 6.1 Effect of noise in frequency modulation.

Performance: FM

- Transmitted signal

$$u(t) = A_c \cos(2\pi f_c t + \phi(t))$$
$$= A_c \cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)\, d\tau\right)$$

- The block diagram for the receiver

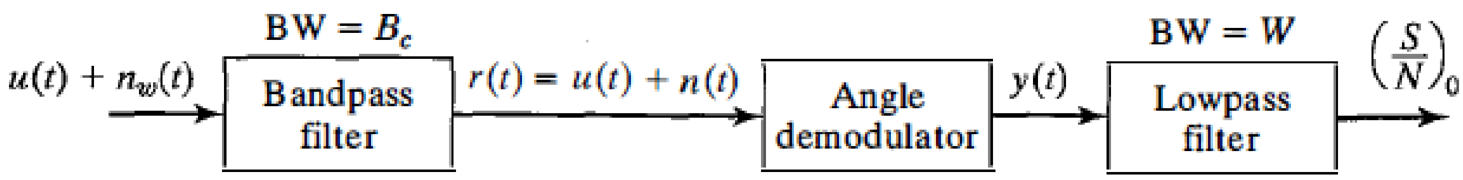


Figure 6.2 The block diagram of an angle demodulator.

- The received signal is

$$r(t) = u(t) + n(t)$$
$$= u(t) + n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t).$$

- The bandpass noise is

$$n(t) = \sqrt{n_c^2(t) + n_s^2(t)}\cos\left(2\pi f_c t + \arctan\frac{n_s(t)}{n_c(t)}\right)$$
$$= V_n(t)\cos(2\pi f_c t + \Phi_n(t)),$$

Performance: FM

- Assume that $P(V_n(t) \ll A_c) \approx 1.$

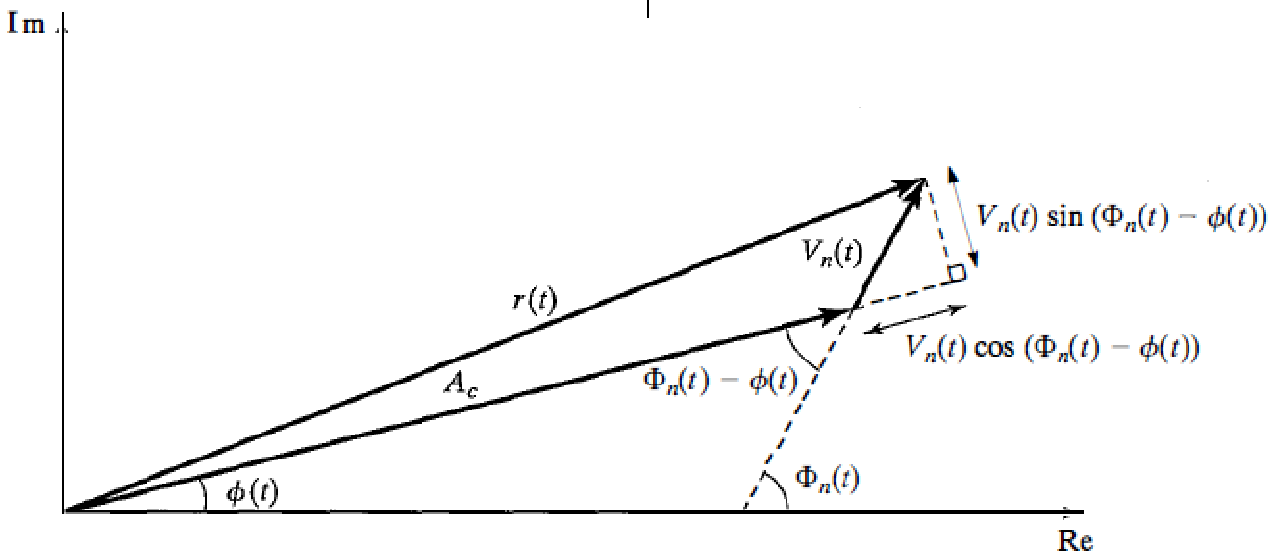


Figure 6.3 Phasor diagram of an angle-modulated signal when the signal is much stronger than the noise.

$$
\begin{aligned}
r(t) &\approx (A_c + V_n(t)\cos(\Phi_n(t)-\phi(t))) \\
&\quad \times \cos\left(2\pi f_c t + \phi(t) + \arctan\frac{V_n(t)\sin(\Phi_n(t)-\phi(t))}{A_c + V_n(t)\cos(\Phi_n(t)-\phi(t))}\right) \\
&\approx (A_c + V_n(t)\cos(\Phi_n(t)-\phi(t))) \\
&\quad \times \cos\left(2\pi f_c t + \phi(t) + \frac{V_n(t)}{A_c}\sin(\Phi_n(t)-\phi(t))\right).
\end{aligned}
$$

Performance: FM

- After angular demodulator,

$$y(t) = \frac{1}{2\pi}\frac{d}{dt}\left(\phi(t) + \frac{V_n(t)}{A_c}\sin(\Phi_n(t) - \phi(t))\right)$$

$$= k_f m(t) + \frac{1}{2\pi}\frac{d}{dt}\frac{V_n(t)}{A_c}\sin(\Phi_n(t) - \phi(t))$$

$$= k_f m(t) + \frac{1}{2\pi}\frac{d}{dt}Y_n(t)$$

where

$$Y_n(t) \stackrel{\text{def}}{=} \frac{V_n(t)}{A_c}\sin(\Phi_n(t) - \phi(t)) .$$

Higher signal level decreases the noise level, as a stark difference with AM.

Performance: FM

- Next,

$$Y_n(t) = \frac{V_n(t)}{A_c} \sin(\Phi_n(t) - \phi(t))$$
$$= \frac{1}{A_c}\left[V_n(t) \sin\Phi_n(t) \cos\phi(t) - V_n(t) \cos\Phi_n(t) \sin\phi(t)\right]$$
$$= \frac{1}{A_c}[n_s(t) \cos\phi(t) - n_c(t) \sin\phi(t)].$$

- Bandwidth of the noise (1/2 the modulated signal) is much larger than that of the message signal.
- Then,

$$Y_n(t) = \frac{1}{A_c}[n_s(t) \cos\phi - n_c(t) \sin\phi].$$

Filter response is symmetric at carrier → $S_{Y_n}(f) = (a^2 + b^2) S_{n_c}(f) = \frac{S_{n_c}(f)}{A_c^2}$

Performance: FM

- The PSD of Yn is

$$S_{Y_n}(f) = \begin{cases} \dfrac{N_0}{A_c^2} & |f| \le \dfrac{B_c}{2} \\ 0 & \text{otherwise} \end{cases}.$$

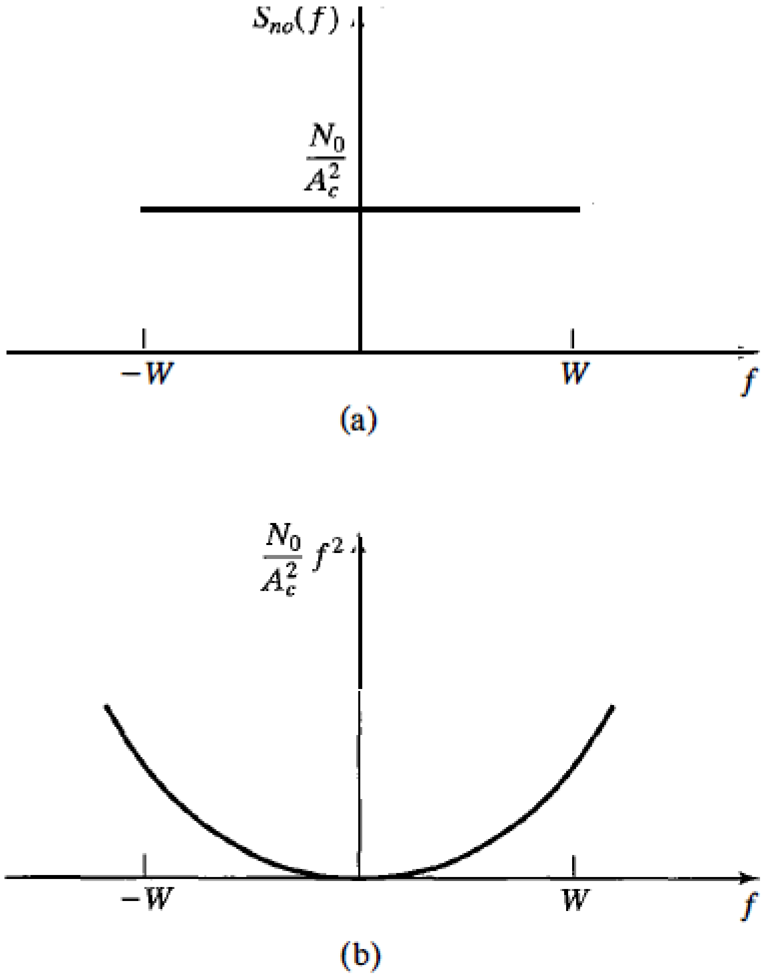
- Then the PSD of $\frac{1}{2\pi}\frac{d}{dt}Y_n(t)$ becomes

$$\frac{4\pi^2 f^2}{4\pi^2} S_{Y_n}(f) = f^2 S_{Y_n}(f) = \begin{cases} \dfrac{N_0}{A_c^2} f^2 & |f| \le \dfrac{B_c}{2} \\ 0 & \text{otherwise} \end{cases}.$$

- The noise PSD

$$S_{n_o}(f) = \begin{cases} \dfrac{N_0}{A_c^2} & \text{PM} \\ \dfrac{N_0}{A_c^2} f^2 & \text{FM} \end{cases}.$$

Performance: FM



After filtering

Figure 6.4 Noise power spectrum at demodulator output for $|f| < W$ in (a) PM and (b) FM.

Performance: FM

- The noise power is

$$P_{n_o} = \int_{-W}^{+W} S_{n_o}(f)\,df = \frac{2N_0 W^3}{3A_c^2}$$

- The output signal power is $P_{s_o} = k_f^2 P_M$

- The output SNR

$$\left(\frac{S}{N}\right)_o = \frac{3k_f^2 A_c^2}{2W^2}\frac{P_M}{N_0 W}$$

$$= 3P_R\left(\frac{\beta_f}{\max|m(t)|}\right)^2 \frac{P_M}{N_0 W}$$

$$\boxed{\beta_f = \frac{k_f \max|m(t|}{W}}$$

$$= 3\frac{P_M \beta_f^2}{(\max|m(t|)^2}\left(\frac{S}{N}\right)_b$$

Performance: FM

- The output SNR

$$\Omega \overset{\text{def}}{=} \frac{B_c}{W} = 2(\beta + 1)$$

$$\left(\frac{S}{N}\right)_o = 3\frac{P_M\beta_f^2}{(\max|m(t)|)^2}\left(\frac{S}{N}\right)_b = 3P_M\left(\frac{\frac{\Omega}{2}-1}{\max|m(t)|}\right)^2\left(\frac{S}{N}\right)_b$$

1. SNR is proportional to modulation index, albeit larger β results in larger bandwidth such that the assumption may not hold. **Threshold effect**
2. Increase in the received SNR is at the cost of bandwidth, where the quadrature tradeoff is far from optimal one, i.e., exponential.
3. Increase transmitter power decreases the receiver noise power, instead of message power (AM).
4. Noise is higher at higher frequencies.

Performance

- AM and FM Comparison
  - Compared with AM, FM requires a higher implementation complexity and a higher bandwidth occupancy. What is the advantage of FM then?
  - Why AM radio is mostly for news broadcasting while FM is mostly for music program?

Analog modulation

- Suggested reading
  - Chapter 3, Chapter 4.1-4.4, Chapter 6.1-6.3
  - HW2