those ultra-stable laser systems, the laser frequency drifts linearly at tens of mHz/s to several Hz/s on a relatively short timescale, and drifts nonlinearly on hundreds of seconds' timescale which is hardly compensated by applying a feed forward frequency correction. The aging of the cavity material and optical contact between cavity mirrors and spacers leads to a fractional length change on the order of 10^{-17}/s to 10^{-16}/s [21]. Therefore, temperature fluctuation of reference cavities is the key cause of laser frequency drift. Well-designed temperature controllers can achieve sub-mK stability over several days [10,22]. When a reference cavity made of ULE glass is temperature-stabilized at the zero-crossing thermal expansion temperature ($T_{CTE=0}$) with accuracy of 0.1 °C, it has a coefficient of thermal expansion (CTE) of less than 2×10^{-10} /K [18]. To achieve laser frequency instability of 10^{-16} or even lower, careful considerations of passive thermal shielding and supporting configuration for reference cavities should be made. For systems that can not be temperature stabilized close to $T_{CTE=0}$ or that have a lower temperature stability, it is also possible to realize high length stability of reference cavities by carefully designing their thermal shields [23].

This paper gives two models on how reference cavities respond to (1) a step environmental temperature change and (2) a periodic temperature fluctuation through heat transfer of thermal conduction and thermal radiation separately. The analysis shows that passive thermal shielding configurations of reference cavities with a larger thermal time constant will be less sensitive to environmental temperature fluctuations. All the thermal shields discussed in this paper are passive ones without active temperature control. With additional numerical simulation results based on finite element analysis, we will discuss the details on the design of cavity thermal shielding configurations to make it insensitive to environmental temperature perturbations, including material, layer numbers and aperture size of thermal shields. The number of thermal shields has a great effect on the time constant of reference cavities. A two-layer thermal shield will help to reduce the temperature fluctuation-induced length instability of a reference cavity below 1×10^{-15} on a day timescale when the cavity has a CTE of 1×10^{-10} /K and is enclosed in a vacuum chamber with temperature fluctuation amplitude of 1 mK and period of 24 hours.

2. Time constant

2.1. *Thermal conduction*

Consider body M_1 that has a length of L_1, cross-section area of A_1 and heat conductivity of k_1, as shown in Fig. 1(a). Another body M_2 with a mass of m_2 and heat capacity of C_2 is on the top of M_1. Assume that M_2 is in good thermal contact with M_1, and both are initially in thermal equilibrium.

At time $t=0$, the temperature at the bottom of M_1 changes from T_i to T_f ($\Delta T_1 = T_f - T_i$). If ignoring the heat absorbed by M_1 itself when M_1 has a much smaller mass and thermal capacity compared with those of M_2, the heat flow rate transferred between M_1 and M_2 is given by

$$\frac{dQ}{dt} = \frac{k_1 A_1}{L_1}(T_f - T_2), \tag{1}$$

where Q is transferred heat and T_2 is the temperature of M_2. M_2 absorbs the heat that flows from M_1, resulting in a temperature change

$$dT_2 = \frac{dQ}{C_2 m_2}. \tag{2}$$

By solving the above equations, we obtain the temperature change of M_2 over time

$$T_2(t) = T_f - \Delta T_1 \cdot e^{-\frac{k_1 A_1}{C_2 m_2 L_1}t} = T_f - \Delta T_1 \cdot e^{-\frac{t}{\tau_{c1}}}. \tag{3}$$

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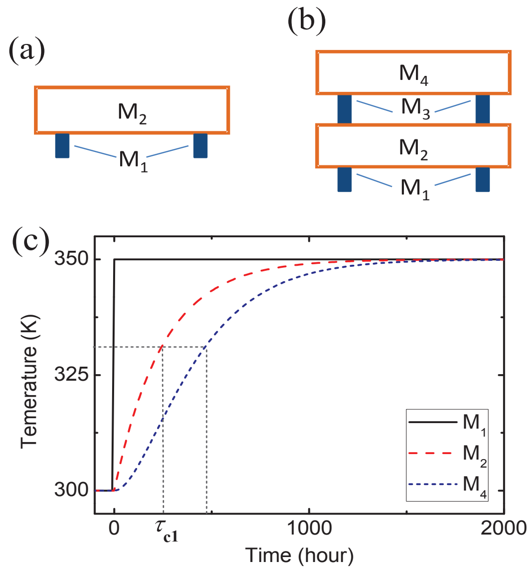

Fig. 1. Thermal conduction. (a) One-layer thermal conduction. (b) Two-layer thermal conduction. (c) The temperature variation of different layers over time when the temperature of M_1 changes from $T_i = 300$ K to $T_f = 350$ K at $t = 0$.

It takes M_2 time of $\tau_{c1} = \frac{C_2 m_2 L_1}{k_1 A_1}$ (time constant of thermal conduction) to change its temperature by $(1 - \frac{1}{e})\Delta T_1$.

For the case of thermal conduction in a typical supporting configuration of reference cavities, M_1 could be supporting rods of reference cavities or supporting rods of thermal shields. To enlarge the time constant, those supporting rods might have small thermal conductivity and cross-section area, while thermal shields on the top of the supporting rods have a large thermal capacity and mass. The red dashed line in Fig. 1(c) shows an illustration of the temperature change of M_2 (made of copper with $m_2 = 20$ kg) assuming the temperature of M_1 (Teflon cylinders with $L_1 = 3$ cm and total cross-section area $A_1 = 10^{-3}$ m^2) changes from 300 K to 350 K. Here the time constant of thermal conduction is $\tau_{c1} \approx 256.7$ h. Parameters for materials used throughout this paper can be found in Table 1.

Table 1. Parameters of materials.

Material	ρ (kg/m^3)	k (W/(m·K))	C (J/(kg·K))	ε^*
ULE glass	2210	1.31	767	0.85
Copper	8930	398	385	0.1
Aluminum	2700	210	900	0.2
Teflon	2200	0.25	1400	0.85
Gold	19320	301	128	0.07

*Emissivity relies on the surface finish. Details please refer to Ref. [24].

Suppose there are other layers M_3 and M_4 on the top of M_2, as shown in Fig. 1(b). The temperature change of M_4 is determined by

$$\frac{dT_4}{dt} = \frac{k_3 A_3}{C_4 m_4 L_3} \left(T_f - \Delta T_1 \cdot e^{-\frac{t}{\tau_{c1}}} - T_4 \right). \tag{4}$$

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Thus the temperature change of M_4 which responds to the temperature variation of M_1 is

$$T_4(t) = T_f - \frac{\Delta T_1}{\tau_{c1} - \tau_{c2}} \left(\tau_{c1} \cdot e^{-\frac{t}{\tau_{c1}}} - \tau_{c2} \cdot e^{-\frac{t}{\tau_{c2}}} \right), \tag{5}$$

where $\tau_{c1} = \frac{C_2 m_2 L_1}{k_1 A_1}$ and $\tau_{c2} = \frac{C_4 m_4 L_3}{k_3 A_3}$. If M_3 is also Teflon rods with $L_3 = 3$ cm and $A_3 = 10^{-3}$ m^2, and M_4 is a piece of thermal shield made of copper with $m_4 = 15$ kg, the time constants are $\tau_{c1} \approx 256.7$ h and $\tau_{c2} \approx 192.5$ h. The blue short dashed line in Fig. 1(c) shows how M_4 responds to the step temperature change of M_1. It takes M_4 about 480.0 h to change its temperature by $(1 - \frac{1}{e})\Delta T_1$. As it shows, in the two-layer thermal conduction the time constant is almost twice of that in the single-layer thermal conduction.

2.2. Thermal radiation

Thermal radiation is a dominant way of heat transfer for optical reference cavities since those cavities are usually put in vacuum chambers. Suppose there is an outer layer P_1 and an inner layer P_2, as shown in Fig. 2(a). Layer P_1 could be a vacuum chamber, and P_2 could be a layer of thermal shield. Initially, both of them are in thermal equilibrium. At $t = 0$, the temperature of P_1 changes from T_i to T_f ($\Delta T_1 = T_f - T_i$). According to ref. [25], when ignoring the geometry of radiation bodies and distances between radiation surfaces, and supposing the view factor of radiation surfaces is unity (Indeed, the value of view factor is less than 1, which is related to radiation angle and relative spacing of radiation surfaces.), the rate of radiated energy absorbed by P_2 is

$$\frac{dQ}{dt} = \frac{\sigma(T_f^4 - T_2^4)}{\frac{1}{A_1} \cdot \left(\frac{1}{\varepsilon_1} - 1 \right) + \frac{1}{A_2} \cdot \frac{1}{\varepsilon_2}}, \tag{6}$$

where A_1 and A_2 are inner surface area of P_1 and outer surface area of P_2 respectively, and $\sigma = 5.67 \times 10^{-8}$ W/($m^2 \cdot K^4$) is the Stefan-Boltzmann constant. ε_1 and ε_2 are the emissivity of the corresponding surfaces of P_1 and P_2 respectively. Since the emissivity of a material varies as a function of temperature and surface finish, the exact emissivity of a material should be determined with absolute measurements. All the emissivity values concerning in this paper refer to Table 1, which are used just for reference. The temperature of P_2 changes by

$$\frac{dT_2}{dt} = \frac{\sigma(T_f^4 - T_2^4)}{C_2 m_2 \left[\frac{1}{A_1} \cdot \left(\frac{1}{\varepsilon_1} - 1 \right) + \frac{1}{A_2} \cdot \frac{1}{\varepsilon_2} \right]} = \beta_{12}(T_f^4 - T_2^4). \tag{7}$$

Using the first order approximation when ΔT_1 is much smaller than T_f and T_i, the temperature variation of P_2 over time is given by

$$T_2(t) \approx T_f - \Delta T_1 \cdot e^{-4\beta_{12} T_f^3 \cdot t} = T_f - \Delta T_1 \cdot e^{-\frac{t}{\tau_{r1}}}, \tag{8}$$

where $\tau_{r1} = \frac{1}{4\beta_{12} T_f^3}$ is the time constant of thermal radiation for P_2 to respond to the temperature change of P_1. Note that in Eq. (8) τ_{r1} depends on T_f instead of T_i due to the approximation($\Delta T_1 \ll T_i, T_f$), while in fact τ_{r1} has a weak dependence on T_i. This approximation gives an error of less than 5% for τ_{r1} when $\Delta T_1 < 10$ K, T_f and $T_i \approx 300$ K. The red dashed line in Fig. 2(c) shows an illustration of the temperature change of P_2 over time when the temperature of P_1 changes from 300 K to 310 K, assuming P_1 is made of aluminum with $\varepsilon_1 = 0.2$ and $A_1 = 1$ m^2, and P_2 is made of copper with $m_2 = 20$ kg, $A_2 = 0.6$ m^2 and $\varepsilon_2 = 0.1$. The time constant is $\tau_{r1} \approx 6.5$ h.

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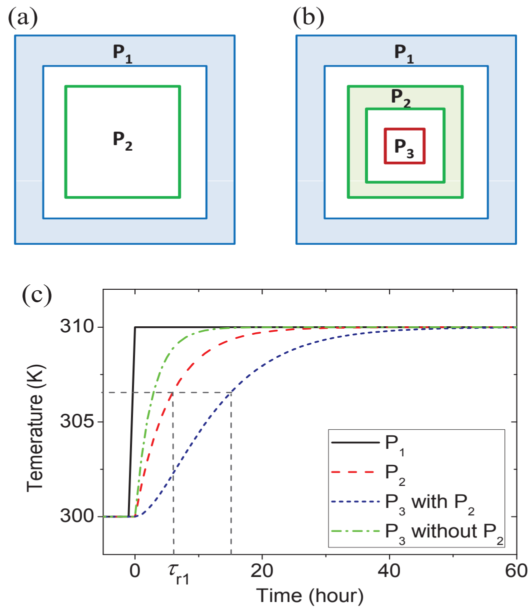

Fig. 2. Thermal radiation. (a) One-layer thermal radiation. (b) Two-layer thermal radiation. (c) The temperature change of different layers over time when the temperature of P_1 changes from $T_i = 300$ K to $T_f = 310$ K at $t = 0$.

If there is another layer P_3, which could be a cavity, inside P_2, as shown in Fig. 2(b), the temperature change of P_3 is determined by

$$\frac{dT_3}{dt} = \beta_{23}\left[\left(T_f - \Delta T_1 \cdot e^{-\frac{t}{\tau_{r1}}}\right)^4 - T_3^4\right], \tag{9}$$

where $\beta_{23} = \frac{\sigma}{C_3 m_3\left[\frac{1}{A_2}\left(\frac{1}{\varepsilon_2}-1\right)+\frac{1}{A_3}\frac{1}{\varepsilon_3}\right]}$. By solving Eq. (9) to the first order approximation, we have the temperature change of P_3, which is

$$T_3(t) \approx T_f - \frac{\Delta T_1}{\tau_{r1} - \tau_{r2}} \cdot \left[\tau_{r1} \cdot e^{-\frac{t}{\tau_{r1}}} - \tau_{r2} \cdot e^{-\frac{t}{\tau_{r2}}}\right], \tag{10}$$

where $\tau_{r1} = \frac{1}{4\beta_{12}T_f^3}$ and $\tau_{r2} = \frac{1}{4\beta_{23}T_f^3}$. If P_3 is made of ULE glass with $A_3 = 0.2\ \mathrm{m}^2$, $m_3 = 10$ kg and $\varepsilon_3 = 0.85$, $\tau_{r2} \approx 6.6$ h. The blue short dashed line in Fig. 2(c) shows the response of P_3 to the step temperature change of P_1 of 10 K. It takes P_3 about 14.2 h to change its temperature by $\left(1 - \frac{1}{e}\right)\Delta T_1$. The green dash-dot line shows the response of P_3 to the step temperature change of P_1 if the intermediate layer P_2 is missing ($\tau_r \approx 3.1$ h). As shown, the intermediate layer P_2 helps to enlarge the time constant of P_3 by more than four times.

3. Thermal sensitivity

Next we will discuss the situation in which the environmental temperature fluctuates periodically, we will see how reference cavities respond to the temperature variations.

3.1. *Thermal conduction*

In Fig. 1(a), suppose the temperature at the bottom of M_1 fluctuates as $T_1(t) = \bar{T} + \Delta T_1 \cdot \sin\left(\frac{2\pi t}{\xi}\right)$, in which $\bar{T}$ is the mean temperature, ΔT_1 and ξ are the amplitude and period of

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the temperature fluctuation respectively. Then the temperature of M_2 changes periodically due to the existence of the heat transfer between M_1 and M_2 via thermal conduction. We use similar equations as Eqs. (1) and (2), and obtain the temperature of M_2 as

$$\frac{dT_2}{dt} = \frac{1}{\tau_{c1}} \left[\bar{T} + \Delta T_1 \cdot \sin\left(\frac{2\pi t}{\xi} \right) - T_2 \right]. \tag{11}$$

By solving the above equation, we have

$$T_2(t) = \bar{T} + \frac{\xi}{\sqrt{\xi^2 + 4\pi^2 \tau_{c1}^2}} \cdot \Delta T_1 \cdot \sin\left(\frac{2\pi t}{\xi} - \phi \right) + \frac{2\pi \xi \tau_{c1}}{\xi^2 + 4\pi^2 \tau_{c1}^2} \cdot \Delta T_1 \cdot e^{-\frac{t}{\tau_{c1}}}. \tag{12}$$

Figure 3(a) shows the temperature change of M_2 when the temperature at the bottom of M_1 fluctuates as $T_1(t) = 300 + 10 \cdot \sin(\frac{2\pi t}{5\times10^5})$. The large fluctuation period ($\xi = 5 \times 10^5$ s) used here is to make the temperature fluctuation of M_2 large enough to be clearly seen in the figure. As shown, the temperature of M_2 fluctuates with the same period as that of M_1, but it has a phase lag $\phi = \arcsin\left(\frac{2\pi\tau_{c1}}{\sqrt{\xi^2+4\pi^2\tau_{c1}^2}} \right)$. And the mean value of $T_2(t)$ does not equal to $\bar{T}$ due to the last term of Eq. (12). The temperature fluctuation amplitude of M_2 is $\Delta T_2 = \frac{\xi \Delta T_1}{\sqrt{\xi^2+4\pi^2\tau_{c1}^2}}$.

The normalized sensitivity of M_2 to the temperature fluctuation of M_1 can be written

$$S = \frac{\Delta T_2}{\Delta T_1} = \frac{\xi}{\sqrt{\xi^2 + 4\pi^2 \tau_{c1}^2}}. \tag{13}$$

As shown in Eq. (13), if the period of the temperature fluctuation is much smaller than the time constant ($\xi \ll \tau_{c1}$), $S \approx \frac{\xi}{2\pi\tau_{c1}}$. In the other limit, if $\xi \gg \tau_{c1}$, $S \approx 1$.

If there are more layers on the top of M_2, then the thermal sensitivity of M_n is

$$S_n = \prod_{i=1}^{n} \frac{\xi}{\sqrt{\xi^2 + 4\pi^2 \tau_{c_i}^2}}. \tag{14}$$

Figure 3(b) shows the thermal sensitivity of conduction when using $\tau_{c1} \approx 256.7$ h for S (the blue solid line) and $\tau_{c1} \approx 256.7$ h and $\tau_{c2} \approx 192.5$ h for two-layer sensitivity S_2 (the red dashed line). As we can see in the figure, multiple-layer structure helps to make reference cavities less sensitive to temperature fluctuations with short fluctuation periods. For example, as seen in Fig. 3(b), when the temperature perturbations have fluctuation periods of 1 s to 10^6 s , the two-layer thermal sensitivity is $S_2 \approx S^2$.

3.2. *Thermal radiation*

For the model shown in Fig. 2(a), if the temperature of P_1 fluctuates as $T_1(t) = \bar{T} + \Delta T_1 \cdot \sin\left(\frac{2\pi t}{\xi}\right)$, due to thermal radiation the temperature of the inner layer P_2 varies as

$$\frac{dT_2}{dt} = \beta_{12} \left[\left(\bar{T} + \Delta T_1 \cdot \sin\left(\frac{2\pi t}{\xi} \right) \right)^4 - T_2^4 \right]. \tag{15}$$

When ΔT_1 is small, the temperature variation of P_2 is obtained to the first order approximation as

$$T_2(t) \approx \bar{T} + \frac{\xi}{\sqrt{\xi^2 + 4\pi^2 \tau_{r1}^2}} \cdot \Delta T_1 \cdot \sin(\frac{2\pi t}{\xi} - \psi) + \frac{2\pi \xi \tau_{r1}}{\xi^2 + 4\pi^2 \tau_{r1}^2} \cdot \Delta T_1 \cdot e^{-\frac{t}{\tau_{r1}}}, \tag{16}$$

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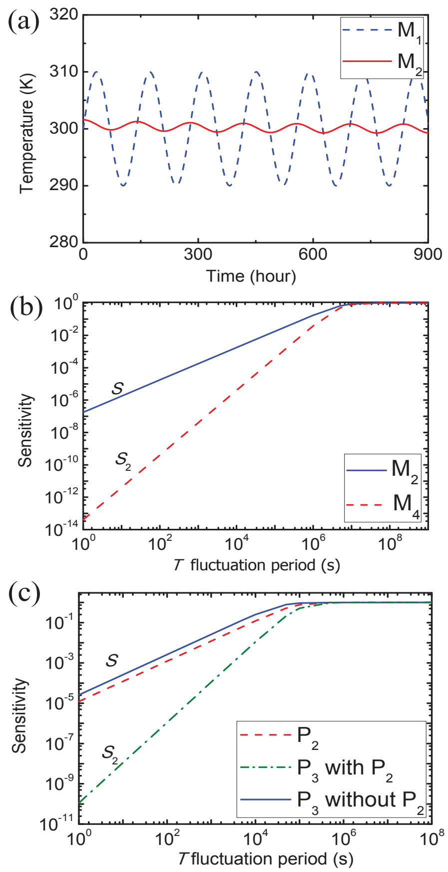

Fig. 3. Thermal sensitivity. (a) The temperature change of M_2 when the temperature of M_1 fluctuates as $T_1(t) = 300 + 10 \cdot \sin[\frac{2\pi t}{5\times10^5}]$. (b) The sensitivity of thermal conduction as a function of temperature fluctuation period. S (S_2) is the sensitivity of one (two)-layer thermal conduction. (c) The sensitivity of thermal radiation as a function of temperature fluctuation period. S (S_2) is the sensitivity of one (two)-layer thermal radiation.

where $\psi = \arcsin\left(\frac{2\pi\tau_{r1}}{\sqrt{\xi^2+4\pi^2\tau_{r1}^2}}\right)$. As we can see from the above equation, the temperature of P_2 also fluctuates with the same period as that of P_1. And the normalized sensitivity of P_2 to the temperature fluctuation of P_1 is given by

$$S = \frac{\xi}{\sqrt{\xi^2 + 4\pi^2\tau_{r1}^2}}. \tag{17}$$

When the outer temperature fluctuation period ξ is much smaller compared to the thermal time constant of radiation τ_{r1}, $S \approx \frac{\xi}{2\pi\tau_{r1}}$. In the opposite limit, if $\xi \gg \tau_{r1}$, $S \approx 1$.

If there are other layers inside P_2, the thermal sensitivity of the most inside layer P_n to the temperature fluctuation of P_1 is given by

$$S_n = \prod_{i=1}^{n} \frac{\xi}{\sqrt{\xi^2 + 4\pi^2\tau_{r_i}^2}}. \tag{18}$$

Figure 3(c) shows the thermal sensitivity of P_2 when using $\tau_{r1} \approx 6.5$ h (the blue solid line) and the thermal sensitivity of P_3 with P_2 as an intermediate layer using $\tau_{r1} \approx 6.5$ h and $\tau_{r2} \approx 6.6$ h (the red dashed line). The green dash-dot line is the thermal sensitivity of P_3 without P_2 using $\tau_{r1} \approx 3.1$ h. As we can see that multiple-layer thermal radiation also helps to make reference cavities less sensitive to temperature fluctuations with short fluctuation period.

4. Simulation results for less thermal sensitivity

In the above analysis, whenever heat transfers via thermal conduction or thermal radiation, cavity thermal shielding configurations with a large time constant are insensitive to temperature fluctuations. Thereby, it is significant to enlarge the time constant of reference cavities.

Considering thermal conduction in an apparatus of a reference cavity, its thermal time constant (τ_c) is proportional to thermal capacity and mass of thermal shields and reference cavities, and inversely proportional to thermal conductivity and cross section area of supporting rods. To enlarge τ_c, material with small thermal conductivity should be chosen for supporting rods, such as vacuum compatible ceramic or Teflon. The geometry of those supporting rods is usually designed to be small in radius and large in length as long as it is stable enough to support.

For thermal radiation in an apparatus of a reference cavity, its thermal time constant τ_r is also proportional to thermal capacity and mass of thermal shields and reference cavities. For those reasons, the thermal shields for reference cavities are usually have a large mass. In the case of limited weight, aluminum is a good choice since its density is nearly one-third of that of copper while its heat capacity is three times bigger if without considering emissivity. To reduce thermal radiation between thermal shields and reference cavities, the emissivity of thermal shields should be small. Even for a specified material, the emissivity varies according to its surface treatment. Usually highly polished surfaces have lower emissivity, though it is technically difficult [24].

From the estimations of time constant listed above, the time constant of thermal radiation is more than an order of magnitude smaller than that of thermal conduction. Therefore thermal conduction could be neglected when calculating the combined time constant of reference cavities. It is also true when reference cavities are designed to be even longer in length for lower thermal noise [18, 19, 26], which makes the geometry of thermal shields even bigger. Since the mass as well as the radiation areas of thermal shields increase, the time constant of thermal conduction increases more than that of thermal radiation.

In the above analysis, both time constant and thermal sensitivity of thermal radiation are given to the first order approximation. Moreover, the geometry of radiation bodies and the distance between radiation surfaces are not taken into consideration, which affect the view factor of radiation and thus time constant of reference cavities as well. Therefore, in the following section, finite element analysis is performed in order to obtain a clearer and more accurate answer on the design of cavity shielding configurations, aiming at making reference cavities less sensitive to environmental temperature perturbations.

4.1. Material for thermal shields

We use ANSYS software to numerically simulate the temperature change of a reference cavity over time. The simulation model is shown in Fig. 4. A reference cavity and thermal shields are in the shape of cylinder. The cavity is made of ULE glass, and the outer vacuum chamber is made of aluminum. Only one layer of thermal shield, layer B as shown in the figure, is used to study the effect of different materials of thermal shield on the thermal time constant. The initial temperature of the whole setup is 25 °C. At $t = 0$, the temperature of the outer vacuum chamber raises to 30 °C.

The simulation results are shown in Fig. 5(a) and listed in Table 2. The thermal time constant

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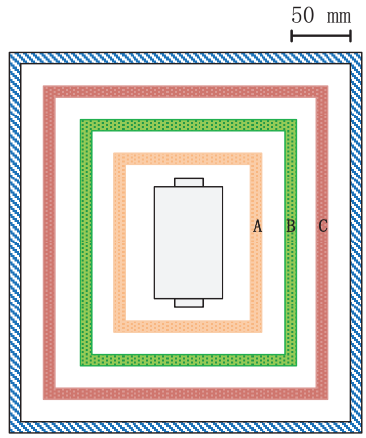

Fig. 4. The simulation model. The outer layer is a vacuum chamber made of aluminum, while the most inner layer is a reference cavity made of ULE glass. There are three layers of thermal shields inserted between the vacuum chamber and the reference cavity, labeled as A, B and C. All are in the shape of cylinder.

of the reference cavity is 26.3 h, 23.8 h and 11.0 h when the thermal shield with the same size is made of gold, copper (roughly polished) and aluminum (roughly polished), respectively. For aluminum thermal shield, if radiation surfaces are highly polished, the emissivity could be smaller, e.g. 0.07, resulting in a thermal time constant of 26.0 h for the reference cavity. For the copper thermal shield, if it is plated by gold, the time constant increases to 32.5 h.

When using Eq. (10), the calculated time constants are also listed in the table, which are slightly smaller than the results based on finite element analysis since finite element analysis takes the geometry of each part, radiation angle and relative spacing of radiation surfaces into consideration. While in the calculation the thermal radiation is more efficient since the view factor is assumed to be unity. However, the calculation gives an estimation.

Table 2. The thermal time constants when the material of thermal shield varies.

Material of thermal shield	Time constant (hour)	
	Simulation	Calculation
Gold	26.3	19.9 ($\tau_{r1} = 15.8$ h and $\tau_{r2} = 3.6$ h)
Copper	23.8	19.2 ($\tau_{r1} = 16.1$ h and $\tau_{r2} = 2.8$ h)
Aluminum	11.0	8.7 ($\tau_{r1} = 6.5$ h and $\tau_{r2} = 1.9$ h)

If there are three supporting rods made of Teflon with a diameter of 20 mm and length of 50 mm inserted between layers, a similar simulation is performed when considering both thermal radiation and thermal conduction. The combined thermal time constant of the reference cavity is 31.9 h when the thermal shield is made of gold-plated copper, as shown in Fig. 5(a). Comparing with the time constant of 32.5 h in the case that without supporting rods, it indicates that thermal conductivity plays a less significant role.

4.2. Layers of thermal shield

To study the effect of multiple-layer thermal shields, we numerically simulated the temperature variation of the reference cavity when the temperature of the outer vacuum chamber changes from 25 °C to 30 °C using the same model with different layer numbers of thermal shield made

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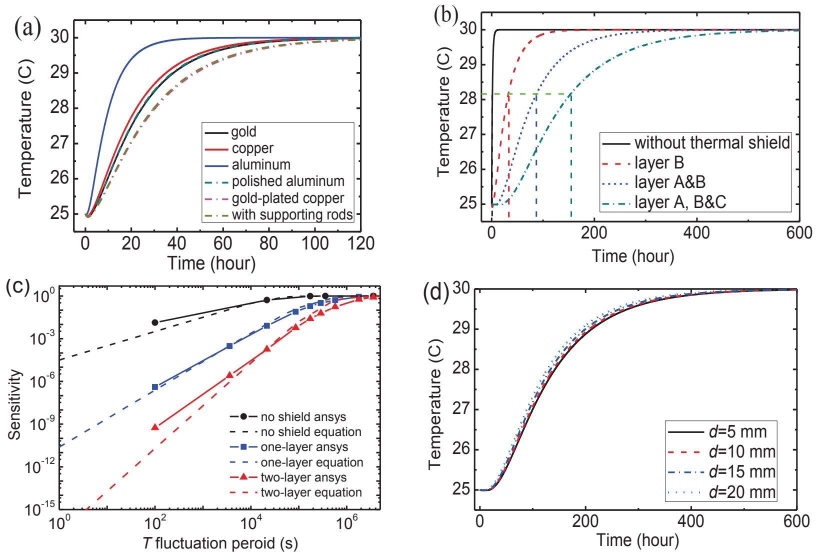

Fig. 5. The simulation results of temperature change for the reference cavity over time when the temperature of outer vacuum chamber is changed from 25 °C to 30 °C if (a) the material of one-layer thermal shield varies and (b) the layer number of thermal shields varies. (c) The simulated and calculated thermal sensitivity of a reference cavity to outer temperature fluctuation when it is enclosed in zero to two layers thermal shields. (d) The simulated time constant of a reference cavity when the aperture size of the three-layer thermal shield varies.

of gold-plated copper. As shown in Fig. 5(b), the time constant of the reference cavity is 1.5 h, 32.5 h, 87.2 h and 153.8 h if there are zero to three layers of thermal shields, respectively. This indicates that when adding one more layer, the thermal time constant of the reference cavity increases significantly.

The thermal sensitivities of the reference cavity as a function of temperature fluctuation period are also simulated when it is enclosed in different layers of thermal shields, as shown in Fig. 5(c). For each data in the figure, the temperature of the outer vacuum chamber fluctuates at a certain period with different amplitude ΔT_{out}. By simulation, the temperature change of the inner reference cavity ΔT_{cav} is obtained separately and thus the thermal sensitivity $S = \Delta T_{cav}/\Delta T_{out}$ is obtained. In the figure, the simulation results of the sensitivities for reference cavities with no thermal shield, one-layer thermal shield and two-layer thermal shield are shown with dots, squares and triangles respectively, while the corresponding calculations results based on Eq. (10) are shown with dashed lines, which agree with the simulation results quite well.

From the above analysis, we can see that when using multiple-layer thermal shield the thermal time constant of reference cavities gets larger and reference cavities become less sensitive to environmental temperature fluctuations. However, it is not necessary to use multiple-layer thermal shields in each case. In Table 3, the temperature fluctuation-induced fractional length instability of reference cavities ($\Delta L/L$) are estimated according to achievable outer temperature instability and the CTE of reference cavities. For example, if an outer vacuum chamber is temperature-stabilized with instability within 1 mK (24 hour fluctuation period) near $T_{CTE=0}$, which might have a CTE of 1×10^{-10}/K, then the reference cavity can have a length instability at the 10^{-16} level on hundreds of seconds' timescale even without any thermal shield, assuming

Table 3. Estimation of temperature fluctuation-induced fractional length instability based on outer temperature variation rate, the CTE and the sensitivity of a reference cavity.

$\Delta T_{out}/\xi$	$\mathrm{d}T_{out}/\mathrm{d}t$ (K/s)	CTE (1/K)	$S = \frac{\Delta T_{cav}}{\Delta T_{out}}$	$\Delta L/L$		
				1 s	100 s	1 day
1 mK/24 hour	7.3×10^{-8}	1×10^{-10}	0.8 [a]	6×10^{-18}	6×10^{-16}	8×10^{-14}
			0.07 [b]	5×10^{-19}	5×10^{-17}	7×10^{-15}
			0.006 [c]	4×10^{-20}	4×10^{-18}	6×10^{-16}
1 mK/24 hour	7.3×10^{-8}	5×10^{-9}	0.8 [a]	3×10^{-16}	3×10^{-14}	4×10^{-12}
			0.07 [b]	3×10^{-17}	3×10^{-15}	4×10^{-13}
			0.006 [c]	2×10^{-18}	2×10^{-16}	3×10^{-14}
1 mK/1 hour	1.7×10^{-6}	1×10^{-10}	6×10^{-2} [a]	1×10^{-17}	1×10^{-15}	6×10^{-15}
			3×10^{-4} [b]	5×10^{-20}	5×10^{-18}	3×10^{-17}
			3×10^{-6} [c]	5×10^{-22}	5×10^{-20}	3×10^{-19}
1 mK/1 hour	1.7×10^{-6}	5×10^{-9}	6×10^{-2} [a]	5×10^{-16}	5×10^{-14}	3×10^{-13}
			3×10^{-4} [b]	3×10^{-18}	3×10^{-16}	2×10^{-15}
			3×10^{-6} [c]	3×10^{-20}	3×10^{-18}	2×10^{-17}

[a] Without thermal shield.
[b] One-layer thermal shield.
[c] Two-layer thermal shield.

the thermal noise limit of the reference cavity is below 1×10^{-16}. If a reference cavity can not be conveniently controlled near $T_{CTE=0}$, for example, which has a CTE of 5×10^{-9}/K (for ULE glass, about 30 °C above $T_{CTE=0}$), in order to achieve 10^{-16} length instability in an averaging time above 100 s, it should be enclosed in thermal shields. If the temperature variation rate of the outer vacuum chamber is large, the need for multiple-layer thermal shield becomes significant. Moreover, multiple-layer thermal shield is also important to achieve a high frequency stability on the timescale over a day. For example, as shown in Table 3, if the environmental temperature fluctuates with amplitude of 1 mK and period of 24 hours, a reference cavity with a CTE of 1×10^{-10}/K is enclosed in a two-layer thermal shield, the temperature-induced length instability of the reference cavity can be at the 10^{-16} level over one day.

4.3. Aperture size of thermal shield

In the previous simulations, the thermal shields do not have any holes to let laser light access the reference cavities. However, all designs are going to need an aperture. To study the effect of the aperture size in the thermal shield on the thermal time constant, we numerically analyze the thermal time constants of reference cavities when the thermal shields having optical apertures with different diameters. In the case of one-layer thermal shield, layer B, the time constant decreases from 31.6 h to 30.0 h (changes by 5%) if the diameter of aperture in the thermal shield increases from 5 mm to 20 mm. If the reference cavity is enclosed in three-layer gold-plated copper thermal shields that have optical apertures d of 5 mm to 20 mm, the time constant decreases from 151 h to 135 h (changes by 12%), as shown in Fig. 5(d). This implies that the aperture size of thermal shields has a relatively weak effect on the thermal time constant compared to the material and layer numbers of thermal shields.

5. Conclusion

In this paper, thermal time constant and temperature sensitivity of reference cavities are studied in order to describe how reference cavities respond to environmental temperature fluctuations via heat transfer of thermal conduction and thermal radiation. The thermal sensitivity of reference cavities is proportional to fluctuation periods of temperature perturbations, and inversely proportional to the time constant of reference cavities. Both calculations based on equations and numerical simulations indicate that thermal radiation plays a significant role in enlarging the thermal time constant of reference cavities. To make reference cavities less sensitive to temperature perturbations, they might be enclosed in multiple layers of thermal shields with large mass, high thermal capacity and small emissivity. One choice for the material of thermal shields is copper with low emissivity. The aperture size of thermal shields has a weak effect on the thermal time constant of reference cavities. The layer number of thermal shields can be chosen according to experimentally achievable temperature stability and the CTE of reference cavities. By using multiple-layer thermal shield, the temperature fluctuation-induced fractional length instability at the 10^{-16} level on a day timescale could be achieved.

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