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Vibration insensitive optical ring cavity*

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The mounting configuration of an optical ring cavity is optimized for vibration insensitivity by finite element analysis. A minimum response to vertical accelerations is found by simulations made for different supporting positions.

Keywords: F-P cavity, laser frequency stabilization, vibration, optical clock
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1. Introduction

Highly stable and ultra-narrow-linewidth lasers are essential in the development of optical clocks that will serve as future optical frequency standards.[1−8] They also have important applications in many other fields such as high-precision laser spectroscopy and fundamental physics tests.[9,10] Usually, a highly stable and ultra-narrow-linewidth laser is achieved by servo-locking the frequency of the laser to the resonance of a Fabry–Perot cavity.[11,12] Then the remaining instability of the locked laser results from a combination of the instability of the reference cavity length and the defects of the servo-locking system. Length fluctuation of the Fabry–Perot cavity acts as a broadband noise source which modulates the laser carrier frequency, resulting in a broadened linewidth. The most severe fluctuations of the length are caused by low frequency seismic and acoustic accelerations. Those vibrations couple through the mounting to the cavity and induce quasi-static deformations of the cavity which result in the change of its length. Besides improving the isolation to such perturbations, it is for this reason important to find geometries and mounting configurations of reference cavities which have a minimal sensitivity to vibrations.

Fabry–Perot cavities which are widely used typically comprise two highly reflective mirrors. To achieve vibration insensitivity, one has to ensure that the cavity length remains essentially invariant when the cavity is perturbed by vibrations. A commonly used implementation is to hold the cavity in its symmetrical plane with its axis aligned with gravity, thus leading to a reduced acceleration sensitivity of 10 kHz/ms^{-2} and a laser linewidth of 1 Hz.[13] Furthermore, through optimization of the support positions of a cavity horizontally mounted with its optical axis, the distance between the centers of the mirrors can be made invariant, even though the cavity still deforms on the application of a vertical force. A vertical acceleration sensitivity of 1.5 kHz/ms^{-2} has been demonstrated in this case.[14] Similar results are achieved by removing material from the underside of the cavity and supporting the cutout cavity by four points, a vertical acceleration sensitivity of as low as 0.1 kHz/ms^{-2} has been achieved.[15] All the Fabry–Perot cavities numerically analyzed with respect to their vibration sensitivity mentioned above are linear cavities. By applying such linear cavities, lasers with Hertz level or even sub-Hertz level linewidth can be obtained.

Optical ring cavities have the characteristic of avoiding optical feedback effects and purifying the linear polarization state of the resonant light. They have already been applied successfully in gravitational wave detection experiments.[16] The optical feedback from a linear cavity is dominated by direct reflection from the input mirror. While for an optical ring cavity, the light does not strike any of the mirrors at normal incidence to give such a feedback-reflected beam. The

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optical feedback results instead from the backscattering which has distribution function that is strongly reduced at low angles of incidence. The optical feedback from the optical ring cavity is therefore much smaller than that from a linear cavity.[17] If an optical ring cavity could be made insensitive to vibrations, it would have great attraction in laser stabilization to obtain an ultra-narrow linewidth. In this paper, we give numerical results of vibration sensitivity analysis based on finite element methods of the ring cavity configuration for different mounting conditions. A minimum response to vertical accelerations has been simulated related to the supporting position.

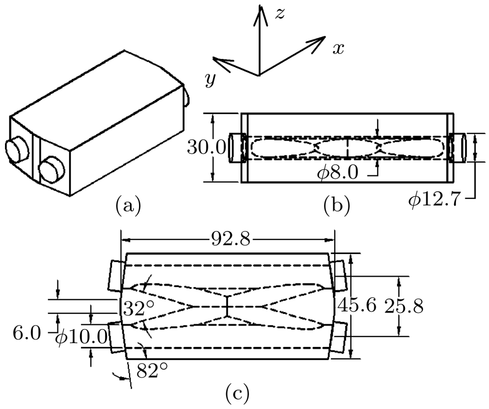


Fig.1. An optical ring cavity with four mirrors. (a) Three-dimensional view of the optical ring cavity; (b) and (c) show dimensions of the optical ring cavity from side view and top view (unit/mm) respectively.

2. Geometrical structure of the optical ring cavity

For a linear cavity supported vertically with its axis aligned with gravity, the fractional change of its length L is $\Delta L/L = -\rho a L/2E$, where ρ is the density of the cavity spacer material, E is Young's modulus, and a is the perturbing acceleration in the direction of gravity, so shorter cavities can be less sensitive to accelerations. On the other hand, for shorter cavities, frequency fluctuations caused by the birefringence of mirror coatings, cavity thermal noise[18] and cavity resonance linewidth become proportionally larger. In order to achieve vibration immunity while enjoying the benefits of a relatively long cavity, suitable cavity geometry and supporting methods are required.

In this paper the geometry of the optical ring cavity adopted is $92.8\times30.0\times45.6$ mm^3 as shown in Fig.1. The angle of incidence of each mirror in this model is as small as 8° to eliminate astigmatism, and the mirrors are 12.7 mm in diameter and 6 mm thick. While the optical path length of the cavity $L_{\rm op}$ is 367.3 mm and the free spectral range (FSR) is 0.82 GHz.

To detect the total displacement of the mirror surfaces, we place four probe points A, B, C and D at the centers of the four mirror surfaces, as shown in Fig.2. A Cartesian coordinate system is used in the simulation with its origin in the geometrical center of the optical ring cavity. From the finite element analysis (FEA), we obtain the coordinates (p_x, p_y and p_z) and displacements (u_x, u_y and u_z) of the four probe points in x, y and z axes. Without deformation, the displacements (u_x, u_y and u_z) of the four points are equal to zero, and the total optical path length of the cavity can be described by $L_{\rm op} = L_{\rm AB} + L_{\rm CD} + L_{\rm BD} + L_{\rm AC}$. When a force is applied, the cavity deforms, and the change of the optical path length can be determined from the displacements of the four probe points.

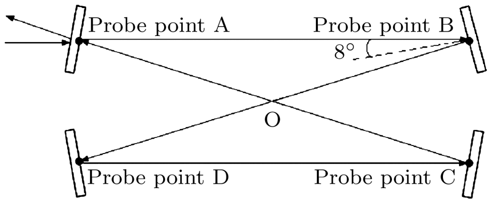


Fig.2. The probe points (solid dots) on the mirror surfaces of the optical ring cavity in an axial cross section.

3. Simulation results and discussion

We use FEA to quantitatively investigate the elastic deformation of the optical ring cavity in order to reduce the acceleration sensitivity of its optical path length. Three mounting configurations of the cavity have been analyzed and the results are given in this section.

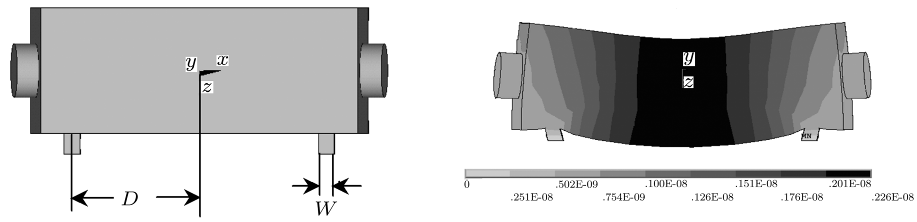
The cavity spacer and the four mirrors are made of ultra-low expansion (ULE) glass. The Young's modulus E, Poisson's ratio υ and mass density ρ of ULE glass are $\sim 6.7\times10^{10}$ N/m^2, 0.17 and 2.21×10^3 kg/m^3 respectively. Finite element simulations have been carried out for this design using the ANSYS program. For making direct comparisons among different mounting configurations, we use an upward acceleration $a = 9.8\ \mathrm{m/s^2}$ along the gravity throughout the paper as the deformation is linearly dependent on the magnitude of the acceleration. The supporting surface of the cavity is restrained to prevent the mechanical structure from undergoing translational and rotational movements. One may refer to the work of Chen *et al* for a detailed description of FEA of cavity deformation with a number of cavity geometries and mounting configurations under the influence of vibrations.[19,20]

The following section presents numerical results for the optical ring cavity mounted vertically and horizontally under the influence of an upward acceleration of $a = 9.8\ \mathrm{m/s^2}$. Since the displacements (u_x, u_y and u_z) of all the points on the mirror surfaces in the simulation are less than 10^{-9} m, the beam trajectory can be considered unchanged.

3.1. Horizontal mounting

Horizontal mounting is a common experimental configuration in laser frequency stabilization. However, bending of the optical ring cavity spacer can affect its optical path. Figure 3(a) gives the optical ring cavity with two supporting beams at its bottom. This method has been mentioned in Chen's work. The light trajectory in the optical ring cavity is in the same horizontal plane, thus making it convenient for the experimental setup. The mounting configuration is simulated by fixing the bottom surfaces of the supporting beams. Figure 3(b) plots the deformation of the optical ring cavity under an upward acceleration of $a = 9.8\ \mathrm{m/s^2}$. The height of the beam is fixed at 5 mm. In order to find the positions of the supporting beams D and the beam width W where the change of cavity's optical path is minimized, the values of D and W are adjusted.

The fractional change of the optical path length as a function of the beam position D for various beam width W is plotted in Fig.4. We add a dashed line in the figure as a mark for $\Delta L_{\rm op}/L_{\rm op} = 0$. All the curves cross the dashed line, which demonstrates that the optical path can remain unchanged at a certain supporting position D for each beam width W. It offers an experimental parameter that can be adjusted to obtain vibration immunity.



(a) (b)

Fig.3. Optical ring cavity horizontally mounted on two beams (case A). (a) Side view of the ring cavity. (b) Calculated deformation of the optical ring cavity. Total displacement is shown under an upward acceleration of $a = 9.8\ \mathrm{m/s^2}$. The plot of the deformation has been magnified by a factor of 3×10^6.

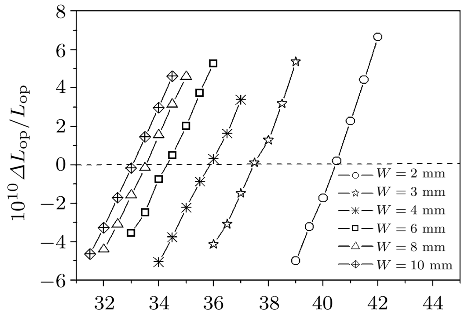


Fig.4. Fractional change of the optical path as a function of the beam position D for various beam width W.

Figure 5 shows the slopes of the curves at $\Delta L_{\rm op}/L_{\rm op} = 0$ plotted in Fig.4 for various W. If the slope is smaller, $\Delta L_{\rm op}/L_{\rm op}$ will be less sensitive to beam position D. Take $W = 4$ mm for instance, we can find, according to Fig.4, that $\Delta L_{\rm op}/L_{\rm op}$ will be less than 2.4×10^{-11} if the accuracy of the supporting position reaches 0.1 mm, corresponding to a calculated sensitivity to vertical accelerations of less than $0.7\ \mathrm{kHz/ms^{-2}}$ at 1064 nm.

If we change the supporting beams to the side of the optical ring cavity, the cavity can still be horizontally mounted, as shown in Fig.6(a). The mounting configuration is simulated by fixing the bottom surfaces of the supporting beams. Figure 6(b) plots the deformation of the optical ring cavity under the same upward acceleration.

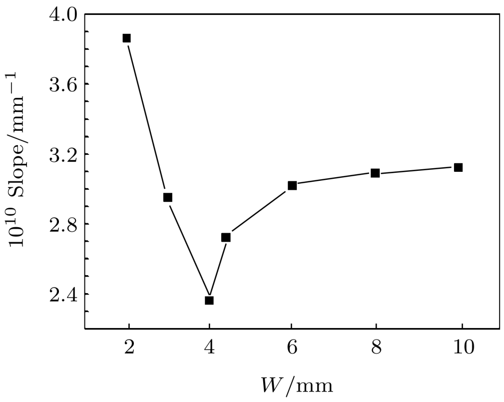


Fig.5. Slopes of the curves at $\Delta L_{\mathrm{op}}/L_{\mathrm{op}} = 0$ plotted in Fig.4 as a function of D for various W.

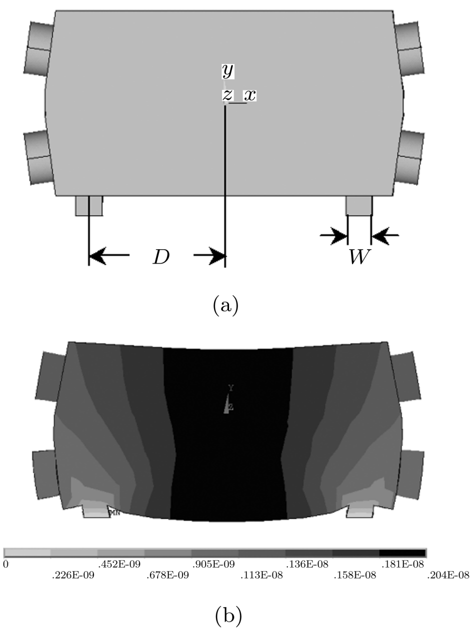


Fig.6. Optical ring cavity horizontally mounted on two beams (case B). (a) Side view of the ring cavity. (b) Calculated deformation of the optical ring cavity. Total displacement is shown under an upward acceleration of $a = 9.8$ m/s^2. The plot of the deformation has been magnified by a factor of 3×10^6.

Figure 7 shows the fractional change of the optical path as a function of the beam position D for $W = 8$ mm. We can see that the curve also crosses the dashed line where $\Delta L_{\mathrm{op}}/L_{\mathrm{op}}$ is insensitive to vertical accelerations for a certain position of the beams. As we have demonstrated in case A, one can make $\Delta L_{\mathrm{op}}/L_{\mathrm{op}}$ reach zero by adjusting the beam position for various beam width W with the mounting configuration of case B as shown in Fig.6.

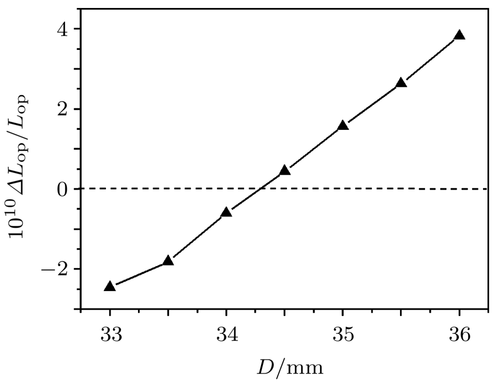


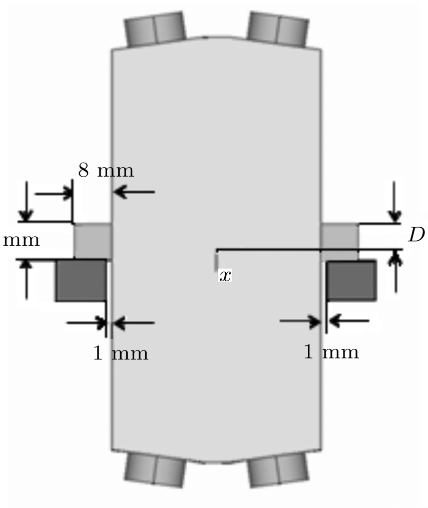
Fig.7. Fractional change of the optical path as a function of the beam position D for $W = 8$ mm.

3.2. Vertical mounting

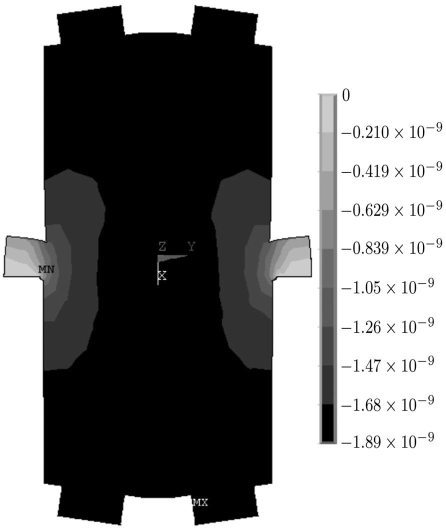
Since vertical mounting has successful applications in linear cavities, in this section, we numerically model a vertically supported ring cavity for comparisons. Figure 8(a) shows the optical ring cavity mounted vertically with two supporting beams placed near its midplane in order to avoid effects of vertical acceleration. Theoretically, both the top half and the bottom half of the cavity will move downward by the same amount under the influence of gravity, resulting in a cavity length insensitive to vertical acceleration. The position of the two supporting beams ($8 \times 8 \times 30$ mm^3) is adjusted and optimized based on the FEA calculations. The supporting surfaces are 1 mm away from the cavity body as shown in Fig.8(a).

We simulate this mounting configuration by fixing the bottom surfaces of the supporting beams and adjusting the location D of the beam from its top surface to the cavity's midplane. Figure 8(b) shows the deformation of the optical ring cavity.

Figure 9 shows the fractional change of the optical path as a function of the beam position D. One can see from the figure that the curve crosses the zero line again, which means that if the beam position is adjusted precisely, the optical path can be insensitive to vertical accelerations when the cavity is vertically mounted.



(a)



(b)

Fig.8. Optical ring cavity vertically mounted near its mid-plane. (a) Side view of the ring cavity. (b) Calculated deformation of the optical ring cavity. Total displacement is shown under an upward acceleration of $a = 9.8$ m/s^2. The plot of the deformation has been magnified by a factor of 3×10^6.

The slopes of the curves plotted in Figs.7 and 9 at $\Delta L_{\rm op}/L_{\rm op} = 0$ are 2.1×10^{-10} mm^{-1} and 3.2×10^{-10} mm^{-1} respectively, corresponding to a calculated sensitivity to vertical accelerations of 0.6 and 0.9 kHz/ms^{-2} at 1064 nm when the accuracy of the supporting position reaches 0.1 mm. If the accuracy of the position can be controlled within 0.01 mm or less, the optical path will be extremely insensitive to vibrations.

The results of numerical analysis show that all three mounting configurations of the ring cavity mentioned above can reach vibration insensitivity through finely adjusting supporting position. However, the horizontal mounting shown in Fig.3 is preferred for easy construction in experiments.

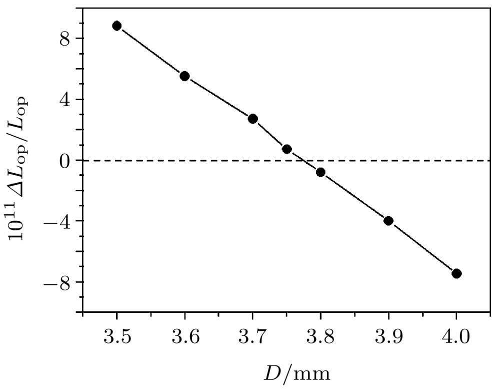


Fig.9. Fractional change of the optical path as a function of the beam position D.

4. Conclusion

We have presented applicable mounting methods of the ring cavity which has the advantage of intrinsically low optical-feedback and can be used in laser stabilization to obtain ultra-narrow linewidth. With the high resolution provided by FEA calculations, the sensitivity to vertical vibrations of the optical ring cavity with three introduced mounting configurations can be analyzed prior to experiments. It has been shown that the application of optical ring cavities in the field of laser frequency stabilization holds promising potential.

Acknowledgments

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