SCIENCE CHINA
Technological Sciences

July 2013 Vol.56 No.7: 1589–1596
doi: 10.1007/s11431-013-5192-7

Progress and trend of narrow-linewidth lasers

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Received January 7, 2013; accepted February 21, 2013; published online April 15, 2013

High frequency stability, narrow-linewidth lasers have been long dreamed of since the invention of the laser. They have recently developed dramatically due to the advent of optical clocks. State-of-the-art narrow-linewidth lasers have been constructed by using the Pound-Drever-Hall (PDH) technique to lock the laser frequencies to the resonance of ultra-stable external optical cavities with high finesse. This paper introduces the developments of narrow-linewidth lasers, with a focus on the improvements of length stability of optical reference cavities, including optical cavity designs of vibration insensitivity and low thermal noise. Future trends and alternative methods for narrow-linewidth lasers are also discussed.

narrow-linewidth laser, optical clock, the Pound-Drever-Hall technique, Fabry-Perot optical cavity, thermal noise

Citation: Chen H Q, Jiang Y Y, Bi Z Y, et al. Progress and trend of narrow-linewidth lasers. Sci China Tech Sci, 2013, 56: 1589–1596, doi: 10.1007/s11431-013-5192-7

1 Introduction

A time standard is to provide the most stable and the most accurate time/frequency interval to form the base of the definition of the second. Time is a fundamental quantity in the International System of Units (SI). The second is the most accurately realized unit and has been used widely. Among the seven fundamental quantities of the SI, three units (meter, candela, and ampere) are defined in terms of the second. For the past few decades, cesium (Cs) clocks have been the primary time standard and the base of the definition of the second in SI, in which a second is defined as the duration of 9192631770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 (^{133}Cs) atom [1]. This definition, as well as other frequency standards, is based on unperturbed atomic transitions. With remarkable progress in techniques of laser cooling and trapping [2–4], significant improvements have been made for Cs clocks on solving the problem of frequency shifts and spectrum broadening by slowing down atoms. Nowadays, the best Cs fountain clock has approached a relative inaccuracy of ~3×10^{-16} [5].

The precision of an atomic clock is derived from its frequency stability. The fractional frequency instability for an atomic clock can be described as [6]

$$\sigma(\tau) \propto \frac{\Delta\nu}{\nu\sqrt{N}}\frac{1}{\sqrt{\tau}}, \qquad (1)$$

where $\Delta\nu$ and ν are the spectral linewidth and center frequency of quantum transition, respectively. N is the number of atoms and τ is the averaging time. Since frequencies in the optical region are 5 orders of magnitude larger than those in the microwave region, an atomic clock based on an optical transition with the same spectral linewidth $\Delta\nu$ can have a better frequency stability and a potential inaccuracy at the 10^{-18} level by systematic evaluation in relatively short averaging times [6, 7].

Although optical clocks based on optical oscillations have distinct advantages in frequency stability and accuracy, there is no device which can directly count such fast oscillations in the optical region. The initial work on optical fre-

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*Corresponding author (email: yyjiang@phy.ecnu.edu.cn; lsma@phy.ecnu.edu.cn) quency measurement by using frequency chains [8] was complex and had poor precision. Fortunately, optical femtosecond frequency comb [9, 10] solved this problem by down-converting such fast oscillations to that in the microwave region, which can be counted with radio frequency (RF) counters. Since then, several national laboratories around the world have been constructing optical atomic clocks based on different species, such as Hg^+, Al^+, Ca^+, Yb^+, Sr, Yb and Hg [6, 11–18]. Nowadays, direct comparisons between optical atomic clocks show that the uncertainties of some optical clocks [11, 13, 19] have already surpassed that of Cs clocks and they have become strong candidates for the redefinition of the second. For example, a fractional frequency uncertainty of 8.6×10^{-18} for an Al^+ optical clock has been reported [19]. The International Committee for Weights and Measures (Comité International des Poids et Mesures, CIPM) recommended four optical clock systems in 2006 as secondary representations of the second [20], the first time of establishing secondary representations of a unit in the SI history. Later in 2012, three other optical clocks were recommended [21]. All the seven optical clocks for secondary representations of the second are shown in Table 1 [20, 21], making preparations for the redefinition of the second. The fractional frequency uncertainties of these optical clocks are also given when comparing against Cs clocks, which are mostly limited by the inaccuracy of the Cs clocks and the measurement itself.

Figure 1 shows the schematic diagram of an optical clock. A narrow-linewidth laser, acting as a local oscillator (LO, also called clock laser), is used to probe and servo-locked to a narrow transition between two energy levels (E_0 and E_1) of a single trapped ion or neutral atoms in optical lattice sites [22]. The narrow-linewidth laser usually has a linewidth, Δf, of a few Hz or below (FWHM: full width at half maximum of the frequency spectrum). The narrow transition between the two energy levels (also called the clock transition) has a linewidth, $\Delta\nu$, at the Hz level or below and is extremely insensitive to environmental perturbations such as variations of magnetic field and electric field. By tuning the frequency of the pre-stabilized laser f around the atomic frequency transition to determine the center frequency ν, we can servo-lock the LO frequency to the resonance of the clock transition. Then, the frequency-determined laser light is delivered to a femtosecond frequency comb for optical to microwave down-conversion for counting. Additionally, optical fibers are usually used to conveniently transfer laser light. A scheme for fiber noise cancellation (FNC) [23] to maintain the coherence of the narrow-linewidth laser light is often employed in an optical clock.

Table 1 Optical clocks for secondary representations of the second [20, 21]

Atom/ion	λ (nm)	Clock transition frequency (Hz)	Frequency uncertainty (compared against Cs clocks)
$^{199}Hg^+$	282	1 064 721 609 899 145.3	1.9×10^{-15}
$^{88}Sr^+$	674	444 779 044 095 485. 3	4.0×10^{-15}
$^{171}Yb^+$	436	688 358 979 309 307. 1	3.0×10^{-15}
$^{171}Yb^+$	467	642 121 496 772 645. 6	1.3×10^{-15}
$^{27}Al^+$	268	1 121 015 393 207 857. 3	1.9×10^{-15}
^{87}Sr	698	429 228 004 229 873. 4	1.0×10^{-15}
^{171}Yb	578	518 295 836 590 865. 0	2.7×10^{-15}

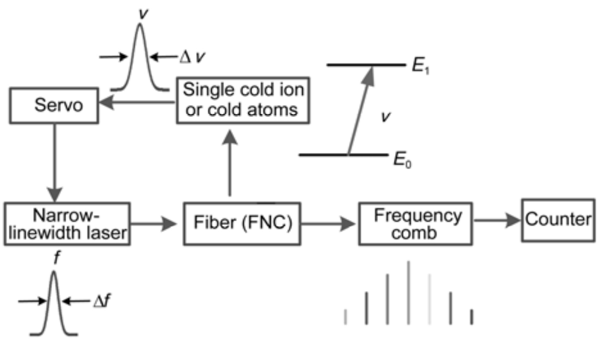

Figure 1 Schematic diagram of an optical clock. FNC: fiber noise cancellation.

As described above, in an optical clock, a local oscillator is used to resolve a narrow-linewidth clock transition. Thereby the local oscillator should have a linewidth comparable to that of the clock transition. However, free-running lasers, usually with linewidth from several kHz to MHz, would fail to meet the requirements in optical atomic clocks. Even by some active frequency stabilization techniques, such as those based on saturation absorption spectroscopy [24] and modulation transfer spectroscopy [25, 26], the laser linewidth could not be reduced to the Hz level. Benefiting from frequency modulation [27] and heterodyne techniques, the Pound-Drever-Hall (PDH) technique [28] can largely reduce the frequency noise of lasers and narrow laser linewidth to the shot noise limit. With the PDH technique, several groups have realized narrow linewidth lasers at the Hz [29–32] or even sub-Hz [33–41] level by frequency-locking the laser frequency to the resonance of an external optical reference cavity.

Narrow-linewidth lasers have spectrally high resolution, ultra-low phase noise and high frequency stability at short averaging time. They play a significant role in many other fields such as high-resolution laser spectroscopy, low-phase-noise microwave signals generation [42, 43], measurements of fundamental constants [13, 44], fundamental tests of physics [45], gravitational wave detection [46] and quantum computation [47].

In this paper, we will focus our efforts on the research and development of narrow-linewidth lasers from the following aspects: introduction of the PDH technique, the developments of narrow linewidth lasers and future trends.

2 The PDH technique

Generally speaking, in the PDH technique, a laser frequency is stabilized to the resonance of an external Fabry-Perot (FP) optical cavity.

The schematic of the PDH technique is shown in Figure 2. A laser light, whose field is expressed as $E=E_0\exp(i\omega t)$, is phase-modulated by an electro-optic modulator (EOM) with a modulation angular frequency of Ω at RF range and a depth of β. After the modulation, the optical field becomes $E'=E_0\exp\{i[\omega t+\beta\sin(\Omega t)]\}$. When $\beta<1$, one can simplify the electric field with Bessel functions as

$$E' \approx E_0[J_0(\beta)e^{i\omega t}+J_1(\beta)e^{i(\omega+\Omega)t} - J_1(\beta)e^{i(\omega-\Omega)t}]. \qquad (2)$$

From eq. (2), we can see that in the frequency domain the laser light contains three frequency components: a carrier (ω) and two sidebands ($\omega\pm\Omega$) with the same amplitude and opposite phase. After that, the modulated light is incident into an FP optical cavity and the optical cavity reflection undergoes absorption and dispersion which can be described as [48, 49]

$$f(\omega) = -\frac{(\sqrt{R_1}-\sqrt{R_2}e^{i(2nL\omega/c)})}{1-\sqrt{R_1R_2}e^{i(2nL\omega/c)}}, \qquad (3)$$

where n is the refractive index, L is the length of the optical cavity, c is the speed of light, R_1 and R_2 are the reflectivity of the mirrors, respectively.

Then, the reflected beam from the FP optical cavity is steered onto a fast photodiode by using a polarization beam splitter (PBS) and a quarter wave plate ($\lambda/4$). The output signal of the detector is sent to a mixer for phase demodulation against the signal that is applied to the EOM with a shifted phase ϕ. By choosing the modulation frequency properly, we can get rid of most technical noise and reduce the amplitude noise close to the shot noise. After passing through a low-pass filter, the frequency discrimination signal (the PDH signal) is obtained as

$$\varepsilon \propto 2E_0^2 J_0(\beta)J_1(\beta)\,\mathrm{Im}[f(\omega)f^*(\omega+\Omega) - f(\omega-\Omega)f^*(\omega)]. \qquad (4)$$

The PDH signal is also shown in Figure 2. When the laser frequency is exactly on the resonance of the optical cavity, the two sidebands will undergo the same absorption and dispersion, and the heterodyne beating of the reflection of the optical cavity on the detector between the carrier and two sidebands of the laser equals zero. However, if there is a small laser frequency deviation from the optical cavity resonance, the two sidebands will suffer slightly different absorption and dispersion. The heterodyne beating, scaled with both sidebands and carrier as shown in eq. (4), will generate an error signal to give a fast response to correct the laser frequency. The sign of the error signal near the optical cavity resonance depends on whether the laser frequency is higher or lower than the optical cavity resonance frequency. Moreover, in the PDH technique, high finesse optical cavities (cavity mirrors with reflectivity of >99.99%) are often used for larger slope of the error signal, e.g., 0.1 mV/Hz, for sensitive frequency discrimination [48, 49].

The PDH technique for laser frequency stabilization takes advantages of frequency modulation spectroscopy and heterodyne detection. By choosing modulation frequency properly, the signal-to-noise ratio (SNR) of the discrimination signal can achieve the shot-noise-limited performance. The shot noise, due to the quantum nature of light, sets a fundamental limit for laser frequency stabilization. Suppose that a laser light at 1064 nm with laser power of 10 μW is incident on an FP optical cavity with a linewidth $\Delta\nu_c$~10 kHz and finesse F~300000. According to the theoretical estimation [49], the shot-noise-limited frequency noise is approximately 0.35 mHz/Hz, corresponding to a linewidth of 0.4 μHz [50].

However, in the PDH technique for laser frequency stabilization, the effect of residual amplitude modulation (RAM) [51, 52] in EOMs may limit the performance of the PDH technique from shot-noise-limited performance. Therefore, efforts should also be made to reduce the effect of RAM [53–55].

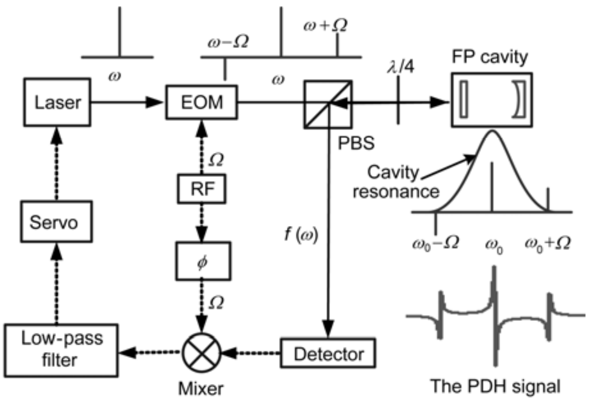

Figure 2 Schematic of the PDH technique for laser frequency stabilization. PBS: polarization beam splitter; RF: radio frequency; EOM: electro-optic modulator; FP: Fabry-Perot; PDH: Pound-Drever-Hall; $\lambda/4$: quarter wave plate; ϕ: phase shifter; $f(\omega)$: reflection function.

3 The developments of narrow-linewidth lasers

The PDH technique has been used for laser frequency stabilization since the 1980s. In 1983, [28] stabilized a dye laser and a gas laser independently to adjacent resonant modes of the same optical reference cavity by the PDH technique for the first time and obtained a beat note of these lasers with a linewidth less than 100 Hz. In this system, some technical issues had to be solved for better locking performance. Later in 1988, a frequency beating with approximately 50 mHz linewidth between two lasers, which were independently locked to adjacent modes of one optical cavity, was realized [56]. Theoretical calculations also demonstrated that the linewidth of some lasers could be reduced to the mHz level when they were locked to optical reference cavities with high finesse optimally [56]. Thus, the idea of constructing sub-Hz linewidth lasers became possible.

In the limit of good SNR of the error signal and tight lock, optical cavity length fluctuation, ΔL, mainly accounts for the resulting laser frequency instability, Δf, which can be described as

$$\frac{\Delta f}{f} \sim -\frac{\Delta L}{L}. \tag{5}$$

Therefore, optical reference cavities should be well isolated from perturbations to keep the length constant. Temperature fluctuation, seismic and acoustic vibration and change of atmospheric pressure are the main causes for length fluctuation of optical reference cavities. Low expansion materials are often chosen for optical cavities to reduce optical cavity length fluctuation caused by temperature variation. Ultra-low expansion glass (ULE) from Corning Inc., is widely used since it has a zero thermal expansion coefficient (CTE) near room temperature ($|\mathrm{CTE}|<5\times10^{-8}$ /K). Typically, optical reference cavities are temperature controlled near this temperature to reduce thermal expansion. To reduce shift of optical cavity resonance frequency due to the variation of refraction index of the air, optical cavities are often enclosed in vacuum chambers. Vibration isolation systems are usually used to reduce the environmental vibration coupling to optical reference cavities. In addition, scientists have used various types of supporting structures to reduce vibration, such as suspension systems [57, 58]. The linewidth of some lasers have been reduced to below 100 Hz [57, 58].

3.1 First sub-Hz narrow-linewidth laser

The first sub-Hz linewidth laser was realized in 1999 at National Institute of Standards and Technology (NIST) [35]. The scientists there took the lead in narrowing the linewidth of a dye laser by the PDH technique to 0.6 Hz, a record which stood for many years. The performance of the previous work on narrow-linewidth lasers was limited by length instability of optical reference cavities due to imperfect vibration isolation. In ref. [35], they isolated the optical reference cavities from vibration noise by mounting them independently on elaborate vibration isolation platforms. One of the platforms was suspended by rubber bands stretched to approximately 3 meters long from the ceiling of the laboratory. The resonance for the basic mode of the stretch as well as the pendulum mode was about 0.3 Hz. The suspension system reduced vibrations at frequencies higher than the resonance. For example, at frequencies higher than 3 Hz, it provided a reduction of vibration noise by more than a factor of 50. However, the vibration noise below the resonance frequency continued to modulate the optical cavity length. The method for vibration reduction is complex and hard to realize. Nowadays, in most other experiments, an alternative choice for vibration isolation is to apply commercial vibration isolation platforms (VIP) with resonant frequencies at about 1 Hz [29, 36–38]. Vibration isolation of more than 30 dB at frequencies higher than resonant frequencies can be achieved with some of these platforms.

3.2 Vibration-insensitive optical cavity designs

Vibration isolation systems are of great help for vibration noise reduction. However, some excess noise still exists and continues acting on the optical reference cavities, especially vibration in the low frequency region. Ideas of reducing the sensitivity of optical reference cavities to vibration noise were naturally proposed. In general, the vertical acceleration sensitivity of an optical reference cavity, e.g., a 10 cm long cylinder optical cavity on a V-shape block, is ~100 kHz/(m s^{-2}), corresponding to a fractional optical cavity length change $\Delta L/L$~1.8×10^{-10} under one unit of acceleration for a 532 nm laser [59]. When it is placed on a VIP, where vibration noise is at the 10^{-5} m s^{-2} $\mathrm{Hz}^{-1/2}$ level (in case of white noise), the linewidth of a laser is ~3 Hz when frequency is stabilized to this optical cavity and limited by the vibration noise [50]. However, if the acceleration sensitivity of an optical cavity with vibration-insensitive design is reduced to 20 kHz/(m s^{-2}), the vibration-limited laser linewidth will be remarkably decreased to 0.1 Hz.

3.2.1 Vertical optical cavity

The first vibration-insensitive optical cavity was introduced by Notcutt et al. at JILA in 2005. It is a vertically-oriented cylindrical optical reference cavity and vertically mounted on its horizontal mid-plane symmetrically [31, 38].

The structure of a vertical vibration-insensitive optical cavity is shown in Figure 3 [38]. This design takes the advantage of potential constant length distance between the two mirrors at the ends by setting the optical axis along the direction of gravity. The acceleration along the direction of

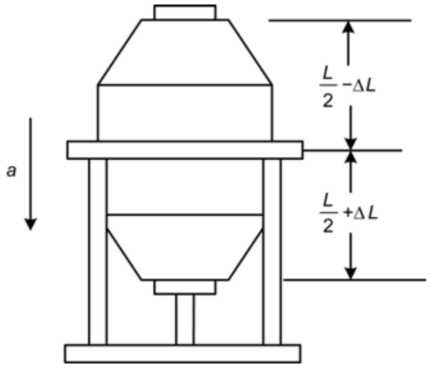

Figure 3 Structure of a vertical vibration-insensitive optical cavity [38]. *a*: acceleration along the gravity; *L*: optical cavity length; ΔL: length change.

gravity causes the compression of the upper part accompanied by stretch of the under part, which results in an unchanged optical cavity length [31, 38]. For this reason, such an optical cavity can be insensitive to vertical accelerations. The vertical acceleration sensitivity of the optical reference cavity at JILA had been reduced to 30 kHz/(m s^{-2}) [38]. This optical cavity design would be more suitable for optical cavities with shorter length because the length instability scales with the optical cavity length as $\Delta L/L=0.5\rho La/E$ [31], where ρ is the density of the material of the optical cavity spacer, E is Young's modulus, and a is an acceleration of the perturbation along the gravity.

The vertically-mounted optical reference cavities are now commercially available from Advanced Thin Films. Lasers with linewidth of Hz or sub-Hz level and frequency instability at the 10^{-15} level have been obtained by using those optical cavities [29, 37, 38]. Our group in East China Normal University (ECNU) has obtained 1-Hz-linewidth lasers at 1064 nm by stabilizing them to this kind of optical cavities with the PDH technique. More details can be found in ref. [29]. An example of the beat note between two laser systems and its Lorentzian fitting is shown in the inset of Figure 4 with the resolution bandwidth (RBW) of 0.5 Hz. The linewidth distribution of the beat note measured over 11 days is also shown in Figure 4 with dots. As shown with the Gauss fitting of the linewidth distribution, the most probable linewidth of the beat note is about 1.5 Hz. Thereby the linewidth of each laser has approached 1 Hz by assuming that each laser contributes equally. With improvements on the two laser systems, lasers with sub-Hz linewidth have been obtained recently.

3.2.2 Horizontal optical cavity

Optical cavities mounted horizontally with their optical axis in the horizontal plane were commonly used before the development of vertically-mounted vibration-insensitive optical cavities. At that time, a horizontal optical cavity in the shape of a cylinder was usually supported by a V-block [35] or two U-shaped brackets [59]. After the design of vertically-mounted optical cavities for vibration insensitivity, horizontally-oriented vibration-insensitive optical cavities were also designed similarly through numerically simulations by finite-element analysis [60–63]. Although the optical cavity mounted horizontally will be deformed under the acceleration along the direction of gravity, the distance between the centers of the mirrors can remain constant by carefully seeking for the optimal supporting positions. Figure 5 shows the mounting configuration of a horizontal optical cavity for vibration sensitivity reduction. By adjusting the positions of four support points along the X and Z directions, the optical path between two mirrors can remain constant under accelerations. A vertical acceleration sensitivity of 0.1 kHz/(m s^{-2}) was achieved by using such a kind of horizontal optical cavity in experiment [61].

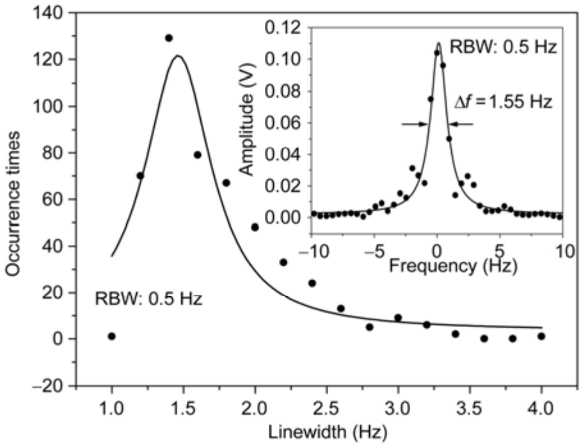

Figure 4 Linewidth distribution of the beat note of two 1 Hz level lasers at 1064 nm in ECNU shown with dots and their Gauss fitting with solid line with the RBW of 0.5 Hz [29]. The inset shows an example of the beat note between two laser systems (dots) and its Lorentzian fitting (solid line) with the RBW of 0.5 Hz.

3.3 Low thermal noise optical cavities

With the significant improvements on the optical cavity length stability, many narrow-linewidth lasers have approached a fundamental limit: the length stability of the optical reference cavity is limited by Brownian motion of the molecules of optical cavity materials, called thermal noise [64]. Numata et al. have estimated the thermal noise from optical cavity spacer and mirrors by using the fluctuation dissipation theorem (FDT) [65]. Calculation shows the thermal noise estimation is in agreement with the experimental result in ref. [35].

For the most-often used ULE optical cavities, the dominant thermal noise source of the optical cavity is from two mirrors (including substrate and coating), which limits the laser frequency instability as [39, 64]

$$\sigma_{\text{therm}}=\sqrt{\ln 2\,\frac{8k_{\text{B}}T}{\pi^{3/2}}\frac{1-\sigma^2}{E\omega_0L^2}\left(\phi_{\text{sub}}+\phi_{\text{coat}}\frac{2}{\sqrt{\pi}}\frac{1-2\sigma}{1-\sigma}\frac{d}{\omega_0}\right)},\quad(6)$$

where k_{B} is the Boltzmann constant, T is mirror temperature, σ is Poisson's ratio, E is Young's modulus, ω_0 is beam radius on the mirror, L is optical cavity length, ϕ_{sub} and ϕ_{coat} are the mechanical losses for the mirror substrate and coat-

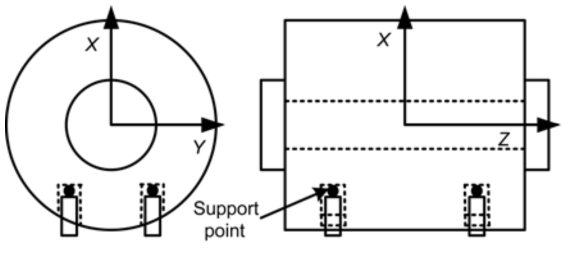

Figure 5 A horizontal vibration-insensitive optical cavity from the front and side views.

ing, respectively, and d is the thickness of the thin film coating.

From eq. (6), we can see several steps can be taken to reduce the thermal noise of optical reference cavities, such as substrate material with low loss, long optical cavity length, large beam radius, low optical cavity temperature and coating with low mechanical loss.

Recently, new optical cavities with low thermal noise have been redesigned, aiming to obtain the thermal-noise-limited frequency instability of lasers at the 10^{-16} level or lower [39–41, 63].

3.3.1 *Substrate material with low mechanical loss*

Fused silica (FS) has a lower mechanical loss (ϕ_{sub}=1×10^{-6}) than that of ULE (ϕ_{sub}=1.6×10^{-5}). By replacing ULE mirrors with FS mirrors, the thermal noise of optical cavities will be reduced by a small amount [39, 41, 63, 66, 67]. For example, for a 10 cm long optical cavity with beam size of ω_0=130 μm on the mirrors, the thermal-noise-limited frequency instability can be reduced by a factor of ~$\sqrt{2}$. However, the FS material has a larger CTE (~5×10^{-7} /K) than that of ULE (within ±5×10^{-8} /K) at room temperature. Because of different thermal expansions of ULE spacer and FS mirrors, the composite CTE zero-crossing temperature of an optical cavity with an ULE spacer and FS mirror substrates will be largely decreased [67]. This problem can be solved by contacting ULE rings to the back surface of the FS mirrors without adding excess thermal noise [41, 67] or using an ULE spacer with negative CTE at room temperature [39].

3.3.2 *Long optical cavity*

A longer optical cavity length will help to reduce the thermal-noise-limited frequency instability of a laser since σ_{therm} scales with $1/L$. In ref. [35], they obtained the sub-Hz-linewidth laser with a fractional instability of 3×10^{-16} at 1 s averaging time by locking a laser to a 24 cm long optical cavity, while later attempts to achieve that goal with short optical cavities (<10 cm) failed [29, 37, 38]. As illuminated by the thermal noise analysis by Numata et al., 29 cm long optical cavities using FS mirror substrates with curvature R=1 m were constructed in 2011. The thermal-noise-limited fractional frequency instability was reduced to 1.4×10^{-16} [39]. When locked to these low-thermal-noise optical cavities, 578 nm lasers with a linewidth of less than 250 mHz (RBW=85 mHz) and a fractional frequency instability of 3×10^{-16} at 1 s averaging time were achieved [39]. Some groups used even longer optical cavities, such as a 39.4 cm long optical cavity with a thermal noise floor of 1×10^{-16} at JILA. A laser stabilized to this cavity has achieved a fractional frequency instability of 1×10^{-16} at the averaging time of 1 to 1000 s [41, 68].

3.3.3 *Cooling*

Since thermal noise arises from the Brownian motion, it is natural to reduce it by cooling optical cavities to cryogenic temperature, also seen from eq. (6). Mono-crystalline silicon is a good choice of optical cavity material at low temperature because it has many excellent properties such as low mechanical loss (<10^{-7}) and low CTE (within ±5×10^{-9} /K) at about 120 K [40, 69]. Recently, a 21-cm-long optical cavity with spacer and substrates made of silicon has been designed by Physikalisch-Technische Bundesanstalt (PTB) and JILA and cooled to 124 K [40]. The thermal-noise-limited frequency instability was estimated to be 7×10^{-17}. With proper vibration isolation and support for minimum vibration sensitivity for the optical cavity, a laser at 1.5 μm was locked to the optical cavity, resulting in a linewidth less than 40 mHz (RBW=37.5 mHz) and a fractional frequency instability of ~1×10^{-16} at short averaging time.

4 Future trends

With efforts described above, today the linewidth of lasers has been reduced to the level of sub-Hz or even less and the frequency instability has approached 1×10^{-16}. With the performance of the ultra-stable lasers described above, optical clocks have been improved in different ways. Firstly, the coherence of lasers resolved narrower atomic spectra of clock transition by increasing the probe time. Secondly, it reduced the fractional instability of the optical clock to new records, e.g., 5×$10^{-16}/\sqrt{\tau}$ [39] and 3×$10^{-16}/\sqrt{\tau}$ [68]. The frequency instability of optical clocks will continue to get smaller by further improving the performance of narrow-linewidth lasers.

In addition to optical clocks, some other applications, such as low-phase-noise microwave generation, gravitational wave detection and quantum computation, also set high demand on the spectral purity of lasers. For example, in gravitational wave detection, with further improvements on lasers, two arms of interferometers can get even longer, thereby largely increasing the sensitivity of the detectors.

To meet those applications, for optical cavity-stabilized lasers, the Brownian motion of their optical reference cavities needs to be further reduced. In both long optical cavities and cryogenic silicon optical cavities described above, the thermal noise of the mirror coating (usually using SiO_2 and Ta_2O_5) dominantes (>90% contribution), due to its relatively large mechanical loss (ϕ_{coat}~5×10^{-4}). Mechanical structures with high optical reflectivity as well as low mechanical loss are of great interest, such as $Al_xGa_{1-x}As$ heterostructures [70, 71]. High-quality Bragg reflectors can have an optical reflectivity over 99.98% and mechanical loss at 10^{-5} level and are thus strong candidates for optical mirrors with ultralow thermal noise [70, 71]. Instead of reducing the thermal noise of these optical cavity-stabilized lasers, some alternative methods may also be possible, such as spectral-hole burning and superradiant lasing. In laser frequency stabilization based on spectral-hole burning, the ground state population of a rare-earth-ion-doped crystal is excited and a hole is created which plays a role as the resonance of an FP optical cavity. A recent result has shown the frequency instability of such kind of lasers may approach 6×10^{-16} [72]. Superradiant lasers reported recently are another powerful candidate for ultra-stable narrow-linewidth lasers. The key to this technique is that the atomic linewidth of laser gain medium is much less than the optical cavity linewidth [73, 74]. And the emitted light is less sensitive to the fluctuation of optical cavity length (technical noise and thermal noise).

With applications of narrow-linewidth lasers becoming much wider and more profound, the future trends for ultra-stable narrow-linewidth lasers are no longer limited at a particular single frequency or in a particular laboratory. By using a femtosecond frequency comb, the coherence of a narrow-linewidth laser can be transferred to other optical frequencies or microwave region [75, 76]. And when using optical fiber with fiber noise cancellation, narrow-linewidth laser light can also be transferred to other places for a variety of other important applications [77, 78]. Endeavors will also be made for the constructions of transportable narrow-linewidth lasers for space applications. Some initial works have already tried to solve the problem of moving these elegant and fragile optical cavities while enjoying vibration insensitivity [79, 80].

This work was supported by the National Basic Research Program of China ("973" Project) (Grant Nos. 2010CB922903, 2012CB821302) and the National Natural Science Foundation of China (Grant Nos. 11104077, 11127405).

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