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A low noise optical frequency synthesizer at 700–990 nm

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Optical frequency synthesizers can generate single-frequency laser light with high precision and accuracy at any desired wavelength over a wide optical region. Here, we demonstrate such an optical frequency synthesizer, which yields coherent light at any wavelength within 700–990 nm with more than 500 mW of power. The relative fractional frequency instability and uncertainty between the output light and the reference light of the optical frequency synthesizer are 6×10^{-19} at 1 s averaging time and 2×10^{-21}, respectively. This synthesis noise is two orders of magnitude better than the frequency stability and accuracy provided by optical clocks, supporting optical frequency synthesis from the most accurate optical clocks. When the optical frequency synthesizer is referenced to a cavity-stabilized laser at 1064 nm, the output of the optical frequency synthesizer is tested to have an average linewidth of 1 Hz and frequency instability of 1.5×10^{-15} at 1 s, limited by the reference laser. *Published by AIP Publishing.* [http://dx.doi.org/10.1063/1.4963690]

Frequency synthesizers from audio frequency to the microwave region have been widely used in technology and scientific research. However, in the optical region there is no such frequency synthesizer, which can generate single-frequency, coherent laser light at any desired wavelength over a wide optical region with high precision on demand. Optical atomic clocks can serve as the ultimate reference for an optical frequency synthesizer (OFS). Recently, the fractional frequency instability and uncertainty of optical clocks have been reduced to the level of 10^{-18}.[1–4] The unprecedented frequency stability and accuracy provided by optical clocks have been opening new applications, such as searching for possible variations of fundamental constants, tests of fundamental physics, precision spectroscopy, and redefinition of the international system of time.[5–9] It is necessary to use an OFS to convert the superior performance of the optical clocks at specific wavelengths to a broad optical and microwave region without degrading their frequency stability, accuracy, and coherence.

The invention of the optical frequency comb[10,11] paved the way to the OFS.[12,13] A comb can transfer the coherence and frequency accuracy of a high performance laser as a reference to all the comb teeth over a wide region with uncertainty at the 10^{-19} level.[14] However, all the comb teeth are typically in the same light beam and each tooth has very little light power (nW $\sim$ μW range). For this reason, combs cannot meet the requirements directly in many applications that demand single-frequency coherent light at a desired frequency with ample power. By employing an additional wavelength-tunable, single-frequency continuous-wave (c.w.) laser with the useful power to inherit the coherence and frequency accuracy of a reference light, this challenge can be overcome. This c.w. laser is the output of an OFS.

There are two ways to phase-lock the OFS output laser to a reference laser through a comb. First, by phase-locking a comb tooth to the reference laser tightly, the properties of the reference laser are transferred to all the comb teeth.[14–18] Then a c.w. laser is phase-locked to a frequency-stabilized comb tooth near its frequency. A relative linewidth of millihertz and frequency transfer instability of a few 10^{-16} at 1 s averaging time have been achieved.[18] In an alternative approach, the properties of a reference laser are directly transferred to a c.w. laser with a free-running comb as a transfer oscillator.[19,20] In this method, the error signal for phase-locking the c.w. laser is derived from the heterodyne signals by beating the reference laser and the c.w. laser against the comb, where the comb frequency noise is removed. Coherence transfer from 1064 nm to 1542 nm with relative frequency instability of 4×10^{-18} at 1 s has been demonstrated.[21] As a step toward the OFS, we have used the second approach to construct an accurate optical frequency divider (OFD). The OFD is demonstrated to have a division noise (the relative frequency fluctuation between the input and output light) of 6×10^{-19} at 1 s and division uncertainty at the 10^{-21} level.[22]

Here, an OFS has been constructed based on the OFD. The OFD accurately converts the frequency and transfers the coherence from a reference laser to a Ti:sapphire c.w. laser as the output of the OFS. Consequently, the OFS outputs coherent light at any desired wavelength within 700–990 nm with more than 500 mW of power. The frequency synthesis instability and uncertainty of OFSs are demonstrated to be 6×10^{-19} at 1 s and 2×10^{-21}, respectively.

Figure 1(a) shows the diagram of the OFS. A single-frequency, Ti:sapphire c.w. laser with more than 1 W of power over the tuning range of 700–1000 nm is used as the output of the OFS with frequency f_{out}. First, a computer coarsely sets f_{out} to a targeted frequency with an uncertainty of $\pm$30 MHz via network connection with a wave meter and the controller of the Ti:sapphire c.w. laser. The laser frequency is coarsely adjusted by tuning a birefringence filter inside the laser cavity.

In a second step, using an OFD,[22] the output light frequency is precisely set as $f_{out} = f_{ref}/R$, where R is a preset

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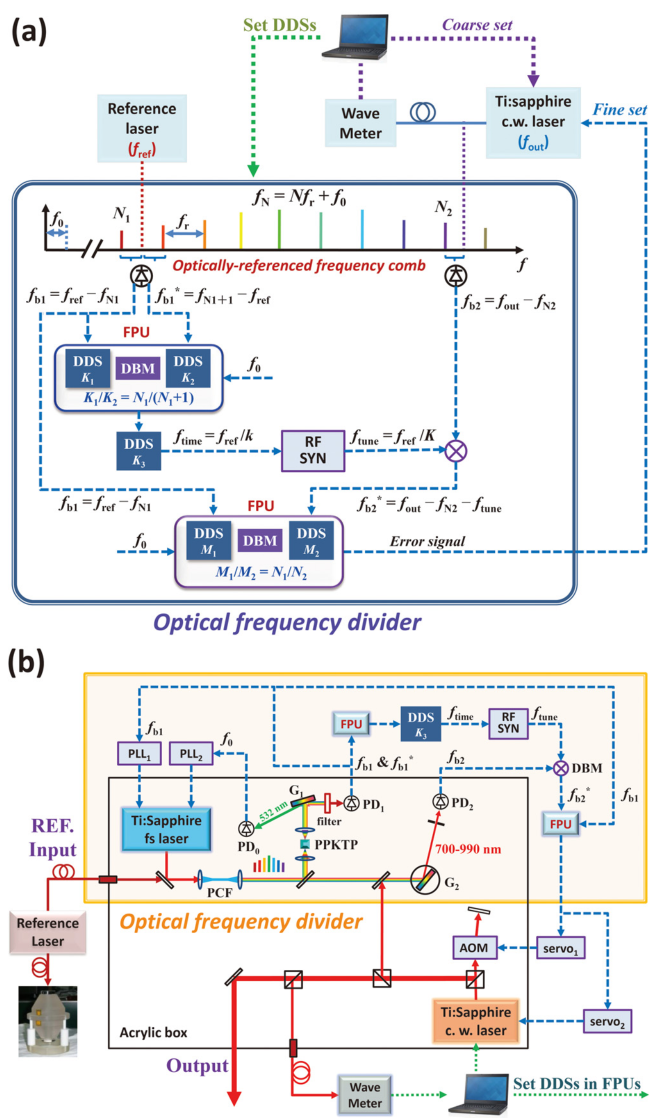

FIG. 1. (a) Schematic diagram and (b) experimental setup of an OFS. An optical frequency comb is stabilized to a reference laser with frequency of f_{ref}. The frequency of a Ti:sapphire c.w. laser f_{out}, as the output of the OFS, is coarsely set by a computer with the aid of a wave meter, and is accurately set to $f_{out} = f_{ref}/R$ by an OFD. f_{b1} (f_{b1}*) is the beat signal between f_{ref} and its nearby comb tooth f_{N1} (f_{N1+1}); f_{b2} is the beat signal between f_{out} and its nearby comb tooth f_{N2}; f_{time} is a self-referenced RF time base of $f_{time} = f_{ref}/k$; and f_{tune} is an RF signal synthesized as $f_{tune} = f_{ref}/K$ for precise tunability of f_{out}.

ratio and f_{ref} is the reference light frequency of the OFS. The OFD is based on an optically referenced comb as a transfer oscillator to bridge the frequency gap between f_{ref} and f_{out}. Here, the comb is based on a Ti:sapphire mode-locked femtosecond (fs) laser with a repetition rate (f_r) of 800 MHz. As shown in Fig. 1(b), the light from the fs laser is coupled into a piece of photonic crystal fiber (PCF) for spectral broadening. Light from the reference laser co-propagates with the comb light through the PCF for eliminating the light-path-fluctuation-induced frequency noise.[17] The beat note f_{b1} between f_{ref} and its nearby comb tooth (f_{N1}) is detected on a photo detector (PD$_1$) and is then phase-locked to a stable signal from a radio frequency (RF) synthesizer (SYN) with a phase lock loop (PLL) by controlling the cavity length of the fs laser and thus its frequency via two piezo transducers (PZT) attached to cavity mirrors (fast and slow servo). To fully stabilize the comb, a collinear self-referencing technique[23] is employed to detect the comb carrier-envelope offset frequency f_0 on PD$_0$ with a signal-to-noise ratio (SNR) of more than 50 dB in a resolution bandwidth (RBW) of 300 kHz. The light beams on the red and blue sides of the comb spectrum for generating the signal f_0 propagate in a common path through a PPKTP (periodically poled KTP) crystal to PD$_0$. The signal of f_0 is then phase-locked to a stable RF signal by controlling the pumping power of the fs laser. Consequently, both f_{b1} and f_0 fluctuate within ± 1 mHz measured on a frequency counter with a gate time of 1 s.

The beat note f_{b2} between f_{out} and its nearest comb tooth with frequency f_{N2} is detected on PD$_2$. If the desired output light frequency f_{out} coincides with a comb tooth f_{N2}, we can easily shift f_{N2} by 100 MHz via tuning the cavity length of the fs laser. The comb teeth close to f_{out} are filtered out on the first diffraction beam of a grating (G$_2$) and with an aperture before PD$_2$. The grating G$_2$ is mounted on a rotation stage to find the correct comb tooth when switching f_{out} to a new wavelength.

By mixing f_{b2} with a tunable RF signal f_{tune} on a double balanced mixer (DBM), f_{out} can be continuously and finely tuned via tuning f_{tune} and feeding back to the Ti:sapphire c.w. laser. In a frequency process unit (FPU), we use the beat notes f_{b1} and f_{b1}* between f_{ref} and the comb to synthesize an RF signal directly from f_{ref}. The signals f_0 and f_r are subtracted from f_{b1} and f_{b1}* using DBMs and direct digital synthesizers (DDS), whose divisors are set as $K_1/K_2 = N_1/(N_1+1)$. Then the resulting signal is sent to a third DDS with divisor K_3 to synthesize a self-referenced time base at nearly 10 MHz as

$$f_{time} = \left(\frac{f_{b1}+f_0}{K_1} + \frac{f_{b1}^* - f_0}{K_2}\right)\frac{1}{K_3} = \left(\frac{f_{ref}}{K_1} - \frac{f_{ref}}{K_2}\right)\frac{1}{K_3} = \frac{f_{ref}}{k}, \tag{1}$$

where $k = K_3/(1/K_1 - 1/K_2)$. Then an RF signal f_{tune} is synthesized as $f_{tune} = f_{ref}/K$ on an RF SYN with time base of f_{time}. Here, K is a number depending on the setting of the DDSs and RF SYN.

In a similar manner as described above, a second FPU generates an error signal

$$\Delta = \frac{f_{b1}+f_0}{M_1} - \frac{f_{b2}^* + f_0}{M_2} = \frac{f_{ref}}{M_1} - \frac{f_{out} - f_{tune}}{M_2}, \tag{2}$$

which is also free from comb frequency noise by setting the divisors of two DDSs to meet the requirement of $M_2/M_1 = N_2/N_1$ on the computer via RS 232 cables. The values of N_1, N_2, and f_{tune} are calculated in advance based on the values of f_r, f_0, f_{b1}, and f_{b2} measured on frequency counters and the values of f_{ref} and f_{out} measured on the wave meter. With this error signal Δ, f_{out} is phase-locked to f_{ref} by tuning the driving frequency of a double-passed acoustic-optic modulator (AOM, fast servo) at the output of the Ti:sapphire c.w. laser and the voltage applied on a PZT inside the laser cavity (slow servo). Finally, f_{out} is related to f_{ref} with an accurate ratio $R = 1/(M_2/M_1 + 1/K)$. Profiting from the techniques of the transfer oscillator scheme based on our optically referenced frequency comb, the self-referenced time base, and careful elimination of light path fluctuation, the division noise and uncertainty of the OFD are 6×10^{-19} at 1 s and 1.4×10^{-21}, respectively, by comparing the division ratio of the OFD against the frequency ratio between the fundamental and the second harmonic of a laser.[22]

The optics for the OFS are sealed in a box to reduce the light path fluctuation due to airflow turbulence. The laser light from the reference laser and that to the wave meter are transferred through optical fibers. After the output light from the Ti:sapphire c.w. laser double-passes the AOM, the power of the first order diffraction light of the AOM (OFS output light) is about 500 mW.

To switch f_{out} to a completely new wavelength, we need to follow the above procedure. By now, manual intervention includes grating rotation, determination of the divisors of the DDSs and f_{tune}, and laser phase locking. It is possible to foresee all the manual intervention can be replaced by using a motor-driven rotation stage for G_2 and digital phase lock loops.

Using an OFS, we are able to accurately synthesize optical signals at any wavelength within 700–990 nm from the best optical clocks. OFSs based on a single frequency comb can link several optical clocks for frequency comparisons. To simulate this, we used OFSs in series to connect optical frequencies, as shown in Fig. 2(a). For a two-step optical frequency synthesis, OFS_1 was employed to synthesize laser light with frequency f_{Ti} from a cavity-stabilized 1064 nm laser[24] ($f_{1064\text{–}1}$) with a preset division ratio R_1. Therefore, $f_{Ti} = f_{1064\text{–}1}/R_1$. Then OFS_2 used f_{Ti} as the reference to synthesize light from a second independent 1064 nm laser ($f_{1064\text{–}2}$) as $f_{1064\text{–}2} = f_{Ti}/R_2$. The beat frequency f_b between $f_{1064\text{–}2}$ and $f_{1064\text{–}1}$ is measured on a counter with the time base of $f_{time} = f_{1064\text{–}1}/k$. The product of the frequency ratios, $R = R_1 \times R_2$, can be determined according to $f_b = f_{1064\text{–}1} - f_{1064\text{–}1}/(R_1 \times R_2) = A \times f_{1064\text{–}1}/(k \times 10^7)$. Here, A is the reading number in hertz on the counter.

As shown in Fig. 2(b), the fractional instability of R (blue squares) follows $6 \times 10^{-19}/\sqrt{\tau}$ (τ is the averaging time). The noise is from servo systems, DDSs, and residual light path fluctuation. It is close to the noise floor of $4 \times 10^{-19}/\sqrt{\tau}$ when $f_{1064\text{–}2}$ are phase-locked to $f_{1064\text{–}1}$ without using any OFS (green circles), where the noise comes from a servo and light path fluctuation. The measured instability of R implies that during optical frequency synthesis the relative frequency instability between the OFS output and input light is two orders of magnitude better than that of the most stable lasers,[25–29] adding a negligible noise onto the output of the OFS. The linewidth of f_b was measured on a fast Fourier transform (FFT) spectrum analyzer, as shown in the inset of Fig. 2(b). It is a RBW-limited linewidth of 1 mHz, indicating that the coherence has been faithfully transferred from $f_{1064\text{–}1}$ to $f_{1064\text{–}2}$ via the OFSs. In the three-step frequency synthesis, OFS_3 was appended. The relative fractional instability corresponding to $R = R_1 \times R_2 \times R_3$ is slightly higher, as shown with red triangles in Fig. 2(b).

The product of the ratios, R, has been measured in sixteen different days over five months. Figure 2(c) shows the deviation of R from the setting value. During the measurement, the wavelength of the Ti:sapphire c.w. laser varied from 700 to 990 nm, and R_1 and R_2 varied accordingly. Each data point results from averaging over a roughly 5000 s measurement time. Blue squares (red triangles) denote the measurement in the two-step (three-step) frequency synthesis. Using standard statistical methods, we combine 22 sets of data (total measurement time of 10^5 s) to calculate the fractional deviation of R to be $(1.5 \pm 2.0) \times 10^{-21}$ (99% confidence level).[30] The synthesis uncertainty of the OFSs here is three orders of magnitude better than the most accurate optical clocks,[3,4,31] limited mainly by the OFD.[21]

When the OFS is referenced to a 1064 nm laser frequency-stabilized to a 7.75-cm-long cavity (f_{1064}),[24] the performance of the OFS output light at 778.6 nm is measured

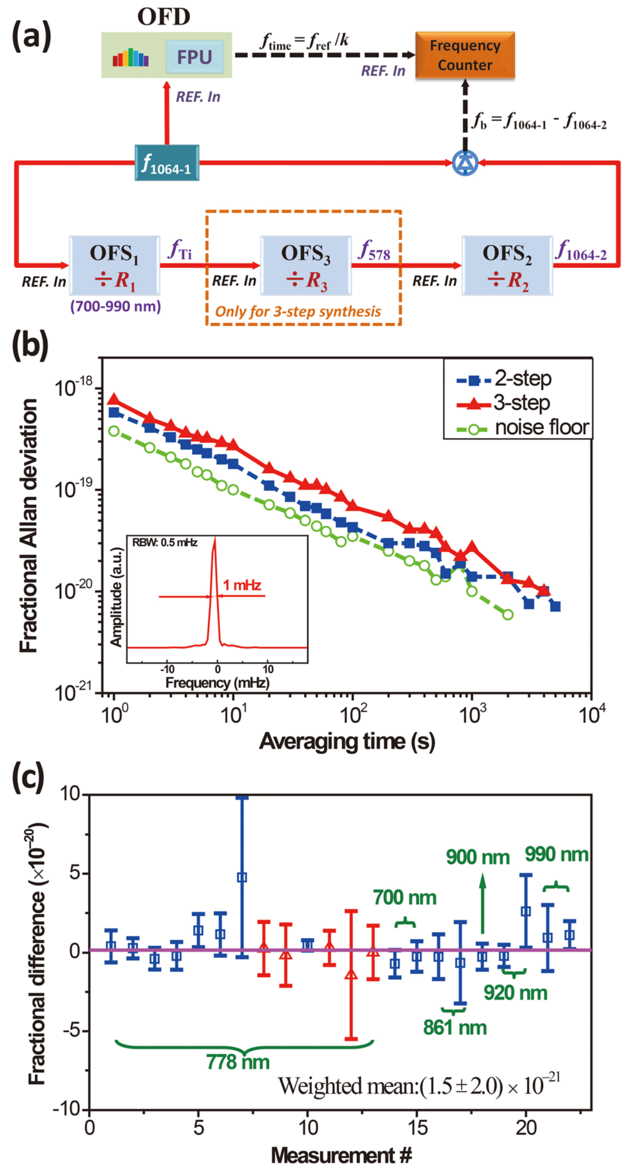

FIG. 2. Performance measurement of multiple OFSs based on a single comb. (a) Diagram of the experimental setup. (b) Fractional instability of R during the two-step (blue squares) and three-step (red triangles) optical frequency synthesis. The green circles show the instability when $f_{1064\text{-}2}$ is phase-locked to $f_{1064\text{-}1}$ without using OFSs. (c) Frequency synthesis uncertainty measurement. The wavelength of the Ti:sapphire c.w. laser varied from 700 nm to 990 nm as marked in green. The product of the ratios of the OFSs, R, is derived by measuring f_b on a counter referenced to $f_{time} = f_{1064\text{-}1}/k$. Blue squares (red triangles) denote the measured R deviated from the set value in the two-step (three-step) frequency synthesis. The error bar for each data point is the standard deviation of five mean values. Each mean value is averaged over 1000 s measurement time.

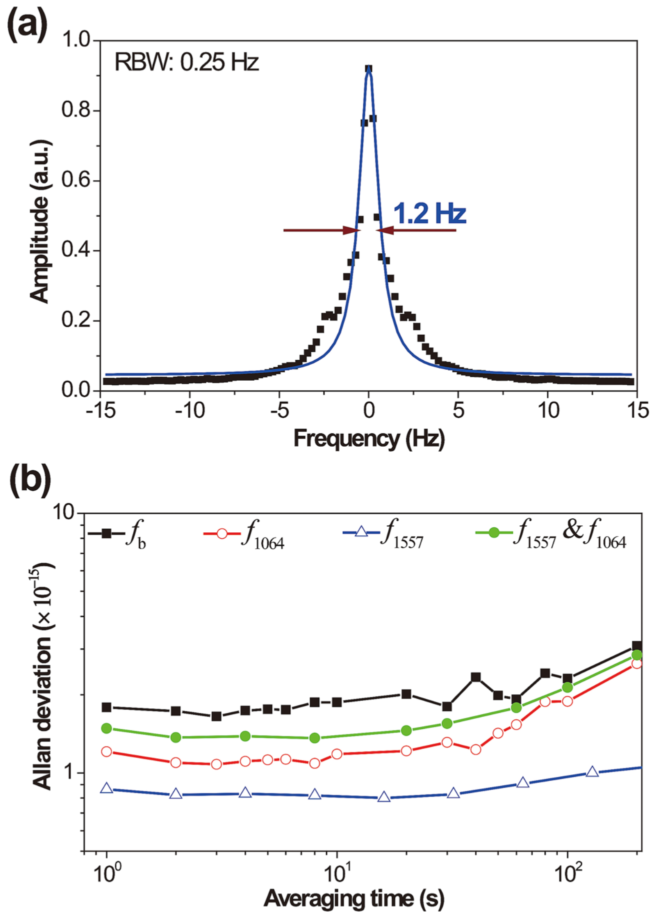

FIG. 3. Performance measurement of an OFS when referenced to a cavity-stabilized 1064 nm laser. The output of the OFS at 778.6 nm is compared against the SHG of a cavity-stabilized 1557 nm laser. (a) Averaged spectrum of a total 850 measurements of the beat note with center overlapped. Each measurement is observed on an FFT spectrum analyzer with a RBW of 0.25 Hz and measurement time of 4 s. (b) Fractional frequency instability of the beat note f_b between the OFS output and the SHG of f_{1557} (black squares), f_{1064} (red circles), f_{1557} (blue triangles), and that contributed by f_{1557} and f_{1064} (calculated, green dots).

against the second-harmonic generation (SHG) of an independent cavity-stabilized laser at 1557 nm (f_{1557}).[32] The average linewidth and frequency instability of f_{1064} (f_{1557}) are 0.7 Hz (0.35 Hz) and 1×10^{-15} (8×10^{-16}) at 1 s, accordingly. The beat signal between f_{out} and the SHG of f_{1557} was analyzed on an FFT spectrum analyzer with a RBW of 0.25 Hz. A total of 850 groups of spectra were recorded automatically by a computer without any compensation of the laser frequency drift. Figure 3(a) shows the average linewidth of all the beat notes with center overlapped, where the laser frequency drift between neighboring measured spectra is removed. Using a Lorentzian fit (blue line), the linewidth of the averaged beat note is 1.2 Hz. After subtracting the frequency noise contribution of f_{1557}, we deduce that the average linewidth of the OFS output light at 778.6 nm is 1.0 Hz. The pedestals shown on the spectrum arise from f_{1557}, which is modulated by vibration at low frequencies.

Since the power of the SHG of f_{1557} is not high enough, an erbium-doped fiber amplifier (EDFA) was employed to boost the power of f_{1557} to 90 mW, resulting in 15 μW SHG light power. The beat signal with frequency f_b between f_{out} and the SHG of f_{1557} with a SNR of 40 dB in 300 kHz RBW was measured on a counter. As shown in Fig. 3(b), the fractional frequency instability is 1.7×10^{-15} at 1 s, limited by f_{1064} and extra noise from the EDFA and a piece of noise-uncompensated optical fiber connected between the EDFA and the 1557 nm laser. After subtracting the contribution of f_{1557}, the frequency instability of the OFS is 1.5×10^{-15} at 1 s.

In conclusion, an OFS has been constructed. It outputs coherent light at any desired wavelength within 700–990 nm with more than 500 mW of power. Benefiting from the high output power of the OFS, it is possible to broaden the output wavelength range with nonlinear optical processes. The relative frequency instability and uncertainty during optical frequency synthesis are demonstrated to be 6×10^{-19} at 1 s and 2×10^{-21}, respectively. It supports the optical frequency synthesis from the most accurate optical clocks. When the OFS is referenced to a cavity-stabilized laser at 1064 nm, the output of the OFS is tested to have an average linewidth of 1 Hz and frequency instability of 1.5×10^{-15} at 1 s, mainly limited by the reference laser. We expect such a frequency synthesizer in the optical region will be instrumental in precision spectroscopy-based applications.

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[1]N. Hinkley, J. A. Sherman, N. B. Phillips, M. Schioppo, N. D. Lemke, K. Beloy, M. Pizzocaro, C. W. Oates, and A. D. Ludlow, Science **341**, 1215 (2013).
[2]B. J. Bloom, T. L. Nicholson, J. R. Williams, S. L. Campbell, M. Bishof, X. Zhang, W. Zhang, S. L. Bromley, and J. Ye, Nature **506**, 71 (2014).
[3]I. Ushijima, M. Takamoto, M. Das, T. Ohkubo, and H. Katori, Nat. Photonics **9**, 185 (2015).
[4]T. L. Nicholson, S. L. Campbell, R. B. Hutson, G. E. Marti, B. J. Bloom, R. L. McNally, W. Zhang, M. D. Barrett, M. S. Safronova, G. F. Strouse, W. L. Tew, and J. Ye, Nat. Commun. **6**, 6896 (2015).
[5]A. D. Ludlow, M. M. Boyd, J. Ye, E. Peik, and P. O. Schmidt, Rev. Mod. Phys. **87**, 637 (2015).
[6]C. W. Chou, D. B. Hume, T. Rosenband, and D. J. Wineland, Science **329**, 1630–1633 (2010).
[7]M. Fischer, N. Kolachevsky, M. Zimmermann, R. Holzwarth, Th. Udem, T. W. Hänsch, M. Abgrall, J. Grünert, I. Maksimovic, S. Bize, H. Marion, F. Pereira Dos Santos, P. Lemonde, G. Santarelli, P. Laurent, A. Clairon, C. Salomon, M. Haas, U. D. Jentschura, and C. H. Keitel, Phys. Rev. Lett. **92**, 230802 (2004).
[8]R. M. Godun, P. B. R. Nisbet-Jones, J. M. Jones, S. A. King, L. A. M. Johnson, H. S. Margolis, K. Szymaniec, S. N. Lea, K. Bongs, and P. Gill, Phys. Rev. Lett. **113**, 210801 (2014).
[9]P. Gill, Philos. Trans. R. Soc. A **369**, 4109 (2011).
[10]J. L. Hall, Rev. Mod. Phys. **78**, 1279 (2006).
[11]T. W. Hänsch, Rev. Mod. Phys. **78**, 1297 (2006).
[12]J. D. Jost, J. L. Hall, and J. Ye, Opt. Express **10**, 515 (2002).
[13]T. R. Schibli, K. Minoshima, F.-L. Hong, H. Inaba, Y. Bitou, A. Onae, and H. Matsumoto, Opt. Lett. **30**, 2323 (2005).
[14]L.-S. Ma, Z. Bi, A. Bartels, L. Robertsson, M. Zucco, R. S. Windeler, G. Wilpers, C. Oates, L. Hollberg, and S. A. Diddams, Science **303**, 1843 (2004).
[15]A. D. Ludlow, T. Zelevinsky, G. K. Campbell, S. Blatt, M. M. Boyd, M. H. G. de Miranda, M. J. Martin, J. W. Thomsen, S. M. Foreman, J. Ye, T. M. Fortier, J. E. Stalnaker, S. A. Diddams, Y. Le Coq, Z. W. Barber, N. Poli, N. D. Lemke, K. M. Beck, and C. W. Oates, Science **319**, 1805 (2008).

[16]I. Coddington, W. C. Swann, L. Lorini, J. C. Bergquist, Y. Le Coq, C. W. Oates, Q. Quraishi, K. S. Feder, J. W. Nicholson, P. S. Westbrook, S. A. Diddams, and N. R. Newbury, Nat. Photonics **1**, 283 (2007).
[17]S. Fang, H. Chen, T. Wang, Y. Jiang, Z. Bi, and L. Ma, Appl. Phys. Lett. **102**, 231118 (2013).
[18]T. R. Schibli, I. Hartl, D. C. Yost, M. J. Martin, A. Marcinkevičius, M. E. Fermann, and J. Ye, Nat. Photonics **2**, 355 (2008).
[19]H. R. Telle, B. Lipphardt, and J. Stenger, Appl. Phys. B **74**, 1 (2002).
[20]J. Stenger, H. Schnatz, C. Tamm, and H. R. Telle, Phys. Rev. Lett. **88**, 073601 (2002).
[21]D. Nicolodi, B. Argence, W. Zhang, R. Le Targat, G. Santarelli, and Y. Le Coq, Nat. Photonics **8**, 219 (2014).
[22]Y. Yao, Y. Jiang, H. Yu, Z. Bi, and L. Ma, "Optical frequency divider with division uncertainty at the 10^{-21} level," e-print arXiv:1608.03690.
[23]Y. Y. Jiang, Z. Y. Bi, L. Robertsson, and L. S. Ma, Metrologia **42**, 304 (2005).
[24]H. Chen, Y. Jiang, S. Fang, Z. Bi, and L. Ma, J. Opt. Soc. Am. B **30**, 1546 (2013).
[25]S. Häfner, S. Falke, C. Grebing, S. Vogt, T. Legero, M. Merimaa, C. Lisdat, and U. Sterr, Opt. Lett. **40**, 2112 (2015).
[26]Y. Y. Jiang, A. D. Ludlow, N. D. Lemke, R. W. Fox, J. A. Sherman, L. S. Ma, and C. W. Oates, Nat. Photonics **5**, 158 (2011).
[27]T. L. Nicholson, M. J. Martin, J. R. Williams, B. J. Bloom, M. Bishof, M. D. Swallows, S. L. Campbell, and J. Ye, Phys. Rev. Lett. **109**, 230801 (2012).
[28]T. Kessler, C. Hagemann, C. Grebing, T. Legero, U. Sterr, F. Riehle, M. J. Martin, L. Chen, and J. Ye, Nat. Photonics **6**, 687 (2012).
[29]M. J. Thorpe, L. Rippe, T. M. Fortier, M. S. Kirchner, and T. Rosenband, Nat. Photonics **5**, 688 (2011).
[30]I. Lira, *Evaluating the Measurement Uncertainty* (Institute of Physics, Bristol, UK, 2002).
[31]C. W. Chou, D. B. Hume, J. C. J. Koelemeij, D. J. Wineland, and T. Rosenband, Phys. Rev. Lett. **104**, 070802 (2010).
[32]L. Wu, Y. Jiang, C. Ma, W. Qi, H. Yu, Z. Bi, and L. Ma, Sci. Rep. **6**, 24969 (2016).