Branch-and-Bound Style Resource
Constrained Scheduling using
Efficient Structure-Aware Pruning

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Outline

- Introduction
- RCS using Branch-and-Bound Approaches
  - Graph–based Notations
  - BULB Approach
- Our Structure-Aware Pruning Approach
  - Motivation
  - Level-Bound Pruning Heuristics
  - HLS Scheduling using Our Approach
- Experiments
- Conclusion
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SoC Design Cost Model

Big Savings by using ESL Methodology

Rising cost of IC design and effect of CAD tools
(Courtesy: Andrew Kahng, UCSD and SRC)
High Level Synthesis

- Convert ESL specification to RTL implementation, and satisfy the design constraints.
  - **Input:** Behavior specifications (C, SystemC, etc.), and design constraints (delay, power, area, etc.)
  - **Output:** RTL implementation (datapath, controller)

```c
int Sample(){
    var A,B,C,D,E,F,G : int;
    Read(A, B, C, D, E);
    F = E * (A + B);
    G = (A + B) * (C + D);
    ......
}
```
Resource Constrained Scheduling

- Various resource constraints (e.g., functional units, power, ...).
- Scheduling is a mapping of operations to control steps
  - Given a DFG and a set of resource constraints, RCS tries to find a (optimal) schedule with minimum overall control steps.

Constraints:
Delay(+) = 1, Delay(*) = 2,
functional units: 1+, 1*

RCS is NP-Complete. RCS should take care of
1) Operation precedence.  2) Resource sharing constraints.
Basic Solutions

- Non-optimal heuristics
  - Force Directed Scheduling
  - List scheduling
    - **Pros:** Fast to get near-optimal results
    - **Cons:** Schedules may not be tight

- Optimal approaches
  - Integer linear programming
    - **Pros:** Easy modeling
    - **Cons:** Scalability, cannot handle non-integer time
  - Branch-and-bound
    - **Pros:** Can prune the fruitless search space efficiently
    - **Cons:** Only investigate the bound length information,
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Graph-Based Notations

- [ASAP, ALAP] intervals indicate the earliest and latest start time of operations
- Input operations and output operations
- Level(op) indicates the longest length from some input operations to the current operation op

![Graph Diagram]

- Level 1: \([1,5]\) v1 * v2 + v3 *
- Level 2: v4 + [3,5]
- Level 3: v5 + [3,6]
Scheduling Using [ASAP, ALAP]

- **A schedule** is a binary relation of operations and corresponding dispatching control step
  - E.g., \{(v1, 1), (v2, 1), (v3, 3), (v4, 5), (v5, 5)\}

- Based on [ASAP, ALAP], naively enumerating all the possibilities can be extremely time consuming
  - The operations are enumerated in a specific order
  - Each operation are enumerated from ASAP to ALAP
Branch and Bound Style RCS (BULB)

- BULB tries to prune fruitless enumerations.
- B&B approach keeps two data structure regarding bound information.
  - $S_{bsf}$, best complete schedule searched so far
  - $S$, current incomplete schedule
Based on the bound information, no further pruning can be conducted for current B&B approaches when $\omega$ is in [lower, upper].
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Pruning based on the structural information of the best schedule (i.e., $S_{opt}$) searched so far.
A cut is an edge set which can separate a DFG into two parts, one part contains all input operations, the other one contains all output operations.

The $k_{th}$ complete level of a cut is a set node, which are adjacent input nodes of all the edges across $k_{th}$ level and $(k+1)_{th}$ level.

1st Complete level: $\{v1,v2,v3\}$

2nd Complete level: $\{v1,v4\}$
Let $OP_k$ be the operation set of $k^{th}$ complete level. The level-bound pruning can be enabled when the following conditions hold:

1. $\forall op_i, op_i \in OP_k \rightarrow S(op_i) > 0$;
2. $\forall op_i, op_i \in OP_k \rightarrow S_{bsf}(op_i) \leq S(op_i)$;
3. $\exists op_i, op_i \in OP_k \rightarrow S_{bsf}(op_i) < S(op_i)$.

\[ S_{bsf} \quad \text{Len}(S_{bsf}) \leq \text{Len}(S) \]
Basic Proof of Level-Bound Pruning

1. Enumeration of operations starts from ASAP to ALAP

2. When a level bound pruning condition holds, for $S_{bsf}$, all the combination of operation dispatching under the complete level has been fully explored.

3. $S_{bsf}$ is the best schedule founded in all combinations in 2.

4. Level bound pruning condition indicates that

$$\text{Len}(S_{bsf}) \leq \text{Len}(\text{best of all possible } S)$$

Therefore, the enumeration can be safely pruned.
Structure-Aware Pruning approach

\[ \text{Structure-awarePruning} (D, i, N, S_{bsf}, S, \omega) \{ \]

\[ \text{if } i \leq n \text{ then} \{ \]

\[ \text{for step } = \text{ASAP}(op_i) \text{ to ALAP}(op_i) \{ \]

\[ 1. \text{ if } \text{LevelBound}(S, S_{bsf}, op_i) \text{ return } (S_{bsf}, \omega); \]

\[ \text{if precedence}(op_i) \text{ and resAvailable}(step, \text{type}(op_i)) \{ \]

\[ 2. \text{ recalculate lower and upper; } \]

\[ \text{if upper } < \omega \{ \]

\[ 3. \omega = \text{upper}; \]

\[ 4. S_{bsf} = \text{ListScheduling}(op_i); \]

\[ 5. \text{if } \omega == \text{globalLow}(D) \text{ Terminate; } \]

\[ 6. \text{UpdateALAP(); } \}

\[ \text{if lower } < \omega \{ \]

\[ 7. S(op_i) = \text{step}; \]

\[ 8. \text{ResOccupy}(step, \text{type}(op_i), \text{delay}(op_i)); \]

\[ 9. \text{Structure-awarePruning} (D, i+1, N, S_{bsf}, S, \omega); \]

\[ 10. \text{ResRestore}(step, \text{type}(op_i), \text{delay}(op_i)); \} \]

\} \} \]
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Benchmarks & Settings

- Used benchmarks from MediaBench.
- BULB & our approach are implemented using C++.
- All the experiments were conducted on a Linux machine with Intel 2.0GHz CPU and 3G RAM.
- Setting of functional units:

<table>
<thead>
<tr>
<th>Functional Unit</th>
<th>Operation class</th>
<th>Delay (unit)</th>
<th>Power (unit)</th>
<th>Energy (unit)</th>
<th>Area (unit)</th>
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<tbody>
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<td>ADD/SUB</td>
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<td>1</td>
<td>10</td>
<td>10</td>
<td>5</td>
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<td>Others</td>
<td>...</td>
<td>1</td>
<td>10</td>
<td>10</td>
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</table>
### Results under Functional Constraints

<table>
<thead>
<tr>
<th>Design</th>
<th># of +, ×</th>
<th>lower</th>
<th>upper</th>
<th>c-step</th>
<th>CPLEX ILP (sec.)</th>
<th>BULB (sec.)</th>
<th>Ours (sec.)</th>
<th>Speedup</th>
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<tr>
<td>ARFilter</td>
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<td>22</td>
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<td>Timeout</td>
<td>234.76</td>
<td>&gt;15.33</td>
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<td>Timeout</td>
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<td>6.50</td>
</tr>
</tbody>
</table>

RCS efforts are significantly improved:
- BULB approach outperforms ILP approach
- Our approach can still get up to 15X speedup against BULB
Scheduling Using Area of 140 Units

When power is 60, up to 22x speedup.
When power is 40, up to 101x speedup.
Conclusions

- RCS is a major bottleneck in HLS
  - Branch-and-bound approaches are promising for optimal resource-constrained scheduling.

- Proposed a structure-aware pruning heuristic
  - Based on structural scheduling information of explored optimal schedule candidates
  - Synergy with state-of-the-art B&B methods

- Successfully applied on various benchmark with different resource constraints
  - Significant reduction in overall RCS efforts
Thank you !