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ORIGINAL PAPER

Mixed interactions of localized waves in the three-component coupled derivative nonlinear Schrödinger equations

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Abstract The Darboux transformation of the three-component coupled derivative nonlinear Schrödinger equations is constructed. Based on the special vector solution generated from the corresponding Lax pair, various interactions of localized waves are derived. Here, we focus on the higher-order interactional solutions among higher-order rogue waves, multi-solitons, and multi-breathers. It is defined as the identical type of interactional solution that the same combination appears among these three components q_1, q_2, and q_3, without considering different arrangements among them. According to our method and definition, these interactional solutions are completely classified as six types, among which there are four mixed interactions of localized waves in these three different components. In particular, the free parameters μ and ν play the important roles in dynamics structures of the interactional solutions. For example, different nonlinear localized waves merge with each other by increasing the absolute values of these two parameters. Additionally, these results demonstrate that more abundant and novel localized waves may exist in the multi-component coupled systems than in the uncoupled ones.

Keywords Interactions of localized waves · Rogue wave · Soliton · Breather · Three-component coupled derivative nonlinear Schrödinger equations · Darboux transformation

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1 Introduction

Recently, there have been a variety of researches about nonlinear localized waves in the field of nonlinear science, such as bright [1–4] or dark soliton [5–7], breather [8,9], and rogue wave [10–14]. In most cases, soliton (bright or dark soliton) keeps its amplitude and speed unchanged during propagating. Based on the instability of small-amplitude perturbations, breathers may be generated, and they localize in space or (and) time. They are greatly different from soliton solutions, such as their periodic properties. Up to the present, there are mainly two special kinds of breathers such as Akhmediev breathers (ABs) [15,16] and Kuznetsov-Ma breathers (KMBs) [17]. The former localizes in time and propagates periodically in space, whereas the latter propagate periodically in time and localizes in space. A simple rational solution—the Peregrine soliton, which is the rogue wave prototype—was first derived by Peregrine [18]. Besides, the Peregrine soliton can be generated through taking limiting process about ABs or KMBs. Localizing in both space and time, rogue wave (RW) always appears from nowhere and disappears without a trace, and its peak amplitude is usually more than twice of the plane background's height [19]. There have been many experimental observations [20,21] and theoretical researches [22–26] on RWs in different nonlinear models.

In recent years, different interactions of localized waves have been reported in many single-component systems. The hybrid solutions between RWs and cnoidal periodic waves were constructed in the focusing nonlinear Schrödinger (NLS) equation through Darboux transformation (DT) method [27]. Some interactional solutions between solitons and several other types of nonlinear waves were obtained in a lot of nonlinear systems by the consistent Riccati expansion and consistent tanh expansion methods [28,29]. It is found that RWs interacted with solitons and breathers at the same time in the Boussinesq equation by Hirota bilinear methods [30]. Besides, some novel semi-rational solutions were constructed in several nonlocal nonlinear integrable models [31,32]; for example, lump solitons interacted separately with RWs, solitons, and breathers, and lump solitons interacted with breathers and periodic line waves at the same time and so on. RW was triggered by the interaction between lump soliton and a pair of resonance kink stripe solitons through Hirota bilinear method [33,34].

Additionally, the studies for interactions of localized waves have been extended to multi-component coupled systems. It was reported that dark-dark and bright-dark solitons existed in various coupled nonlinear systems by KP reduction technique [6,35,36]. Bright-dark-rogue wave solutions were constructed in both two-component NLS [37] equations and Hirota [38] equations by DT. Interactional solutions including breather and dark soliton, breather and anti-dark soliton, dark soliton, and anti-dark soliton were all constructed in multi-component coupled nonlocal NLS equations [39,40]. It indicates that more novel and abundant nonlinear localized waves may be constructed in the coupled systems [41] than the ones in single-component systems. In this paper, we focus on constructing some novel interactional solutions in the following three-component coupled derivative nonlinear Schrödinger (DNLS) equations [42,43]

$$\begin{cases} iq_{1t} + q_{1xx} - \frac{2i}{3}\epsilon[(|q_1|^2 + |q_2|^2 + |q_3|^2)q_1]_x = 0, \\ iq_{2t} + q_{2xx} - \frac{2i}{3}\epsilon[(|q_1|^2 + |q_2|^2 + |q_3|^2)q_2]_x = 0, \\ iq_{3t} + q_{3xx} - \frac{2i}{3}\epsilon[(|q_1|^2 + |q_2|^2 + |q_3|^2)q_3]_x = 0. \end{cases} \tag{1}$$

Here, q_1, q_2, and q_3 are the complex envelops of three fields along the coordinate x, and t is the time. Besides, the subscripted variables x and t in Eq. (1) denote for the corresponding partial differentiation and $\epsilon = \pm 1$. The DNLS system is important in plasma physics and the ultra-short pulse field [42,44–48]. In a low-β plasma (the ratio of kinetic to magnetic pressure), the DNLS equations (1) can describe the evolution of small-amplitude Alfvén waves, which propagate at a small angle [44,45] or parallel [46] to the ambient magnetic field. Additionally, it is shown that the DNLS system (1) can be also used to describe the behavior of large-amplitude magnetohydrodynamic waves in a high-β plasma propagating at an arbitrary angle [47] to the ambient magnetic field. The standard NLS models cannot be used in the ultra-short pulses field, because the spectrum of these ultra-short pulses is approximately of the order $10^{15}\ \mathrm{s}^{-1}$ and the width of optical pulse is in the order of femtosecond [43,48]. However, the DNLS system can describe the propagation of the ultra-short pulses well.

Higher-order RWs and the hierarchy of higher-order solutions of the single-component DNLS equation were constructed in [48–51] by DT, respectively. Besides, some higher-order semi-rational solutions of the single-component DNLS equation, which included higher-order RWs and higher-order breathers, was constructed in [52]. There have been many other results about single-component DNLS equation, such as soliton solutions [53], stationary solutions [54] and breather solutions [55]. The two-component coupled DNLS system was derived by Morris and Dodd [42]. N-soliton solutions of the two-component case of the coupled system (1) was constructed by DT [43]. In [56], Baronio et al. constructed the first-order interactional solutions of the two-component coupled NLS equations, which included first-order RW, first-order RW interacting with one-bright (dark) soliton or one-breather. Compared to the first-order RW, the higher-order RWs are the nonlinear superposition of some first-order ones and can describe the realistic phenomena well [57]. It is greatly necessary to investigate the interactional solutions between higher-order RWs and other nonlinear localized waves in the DNLS system (1), such as multi-soliton and multi-breather.

Compared to the two-component systems [58–60], there may exist some novel and interesting mixed interactions of localized waves in the three-component ones [61,62]. Here, we extend the two-component coupled DNLS equations in [42,43] to three-component system (1) and successfully construct the corresponding Lax pair. It is very complicated to construct the interactional solutions among RWs, breathers and solitons in multi-component DNLS system. The integrability and N-soliton in two-component DNLS system (even) are discussed in [42,43], respectively. Here, we focus on investigating the interactions of localized waves in three-component case (odd). Considering both two- (even) and three-component (odd) DNLS systems, we can understand the interactional solutions between RWs and other two states (breathers and solitons) in multi-component case well.

In this paper, hybrid (interactional) solution is defined as RW interacting with other two states (soliton or breather) in each component. Besides, it is named mixed hybrid (interactional) solution that different hybrid (interactional) solutions combine together in the three components q_j $(j = 1, 2, 3)$ at the same time. Through defining the same combination as the same type solution among the three components q_1, q_2, and q_3, these interactional solutions are completely classified as six cases, among which there are four mixed types.

This article is organized as follows. In Sect. 2, the DT of the three-component coupled DNLS equations is constructed. In Sect. 3, higher-order mixed interactional solutions are obtained and some dynamics are discussed. The last section contains several conclusions and discussion.

2 Darboux transformation for the three-component DNLS equations

For convenience, the parameter ϵ in the coupled system (1) is chosen as $\epsilon = -1$ in the following contents. Here, the two-component coupled DNLS equations [42,43] is extended to three-component case (1), and the corresponding Lax pair can be constructed as

$$\Psi_x = U\Psi = \lambda^2 U_1 + \lambda U_2, \tag{2}$$

$$\Psi_t = V\Psi = \lambda^4 V_1 + \lambda^3 V_2 + \lambda^2 V_3 + \lambda V_4, \tag{3}$$

with

$$U_1 = \mathrm{diag}(-2i, i, i, i), \quad U_2 = \begin{pmatrix} 0 & Q^\dagger \\ -Q & 0_{3\times 3} \end{pmatrix},$$
$$V_1 = \mathrm{diag}(-9i, 0, 0, 0),$$
$$V_2 = 3U_2, \quad V_3 = -i\sigma U_2^2,$$
$$V_4 = i\sigma U_{2x} - \frac{2}{3}(|q_1|^2 + |q_2|^2 + |q_3|^2)U_2,$$
$$Q = (q_1^*, q_2^*, q_3^*)^T, \quad \sigma = \mathrm{diag}(1, -1, -1, -1),$$

where $\Psi(x, t) = (\psi_1, \psi_2, \psi_3, \psi_4)^T$ and λ is the spectral parameter. Besides, T denotes the transpose of the vector and $\dagger$ represents the Hermitian conjugation. Additionally, the three-component coupled DNLS system (1) can be easily generated from the compatibility condition $U_t - V_x + [U, V] = 0$.

In [43], the DT of the two-component coupled DNLS equations was constructed, and it was also generalized to the multi-component case. Based on the DT in [43], we can construct the elementary DT of the coupled system (1) as

$$\Psi[1] = T\Psi_{10}, \quad T = M_0\lambda^2 + M_1\lambda - I, \tag{4}$$

$$q_k[1] = q_k - \frac{\lambda_1^* - \lambda_1^2}{|\lambda_1|^2}\left(\frac{\psi_{10}\psi_{(k+1)0}^*}{\Omega_{10}}\right)_x \ (k = 1, 2, 3), \tag{5}$$

with

$$I = \mathrm{diag}(1, 1, 1, 1),$$
$$M_0 = \frac{1}{\lambda_1^{*2}} I + \frac{\lambda_1^{*2} - \lambda_1^2}{\lambda_1^{*2}\lambda_1} \begin{pmatrix} \frac{\psi_{10}\psi_{10}^*}{\Omega_{10}} & 0_{1\times 3} \\ 0_{3\times 1} & \frac{P_1 P_1^\dagger}{\Omega_{10}^*} \end{pmatrix},$$
$$M_1 = \frac{\lambda_1^* - \lambda_1^2}{\lambda_1\lambda_1^*} \begin{pmatrix} 0 & \frac{\psi_{10}P_1^\dagger}{\Omega_{10}} \\ \frac{P_1\psi_{10}^*}{\Omega_{10}^*} & 0_{3\times 3} \end{pmatrix}, \quad \Omega_{10} = \lambda_1|\psi_{10}|^2 + \lambda_1^*(|\psi_{20}|^2 + |\psi_{30}|^2 + |\psi_{40}|^2),$$
$$P_1 = (\psi_{20}, \psi_{30}, \psi_{40})^T;$$

here, the subscript x in (5) denotes for partial differentiation and $\Psi_{10}(x, t) = (\psi_{10}, \psi_{20}, \psi_{30}, \psi_{40})^T$ is the solution of the Lax pair (2–3) under the condition that $\lambda = \lambda_1$.

We choose $q_1 = q_1[0], q_2 = q_2[0], q_3 = q_3[0]$ as the seed solution of the three-component coupled DNLS system (1) with $\lambda = \lambda_1 + \delta$. Then, a special vector eigenfunction of the Lax pair (2–3) can be derived as

$\Psi_1(x,t)(\lambda_1+\delta) = (\psi_{11}, \psi_{21}, \psi_{31}, \psi_{41})^T$. Expanding the column vector $\Psi_1(\lambda_1+\delta)$ at $\delta = 0$, we have

$$\Psi_1 = \sum_{j=0}^{\infty} \Psi_1^{[j]}\delta^j, \ \Psi_1^{[j]} = \left(\psi_1^{[j]}, \psi_2^{[j]}, \psi_3^{[j]}, \psi_4^{[j]}\right)^T,$$

$$\Psi_1^{[j]} = \frac{1}{j!}\frac{\partial^l \Psi_1}{\partial \delta^j}|_{\delta=0} \quad (j = 0, 1, 2, 3, \ldots).$$

Iterating the generalized DT N times, the N-fold generalized DT of the three-component coupled DNLS equations (1) can be derived as follows

$$\Psi[N] = T[N]T[N-1]\ldots T[2]T[1]\Psi,$$
$$T[j] = M_0[j]\lambda^2 + M_1[j]\lambda - I \quad (j = 1, 2, \ldots, N), \tag{6}$$

$$q_k[N] = q_k[N-1] - \frac{\lambda_1^{*2} - \lambda_1^2}{|\lambda_1|^2}\left(\frac{\psi_1[N-1]\psi_{(k+1)}[N-1]^*}{\Omega_N}\right)_x, \quad (k = 1, 2, 3), \tag{7}$$

where

$$\Psi_1[N-1] = (\psi_1[N-1], \psi_2[N-1], \psi_3[N-1], \psi_4[N-1])^T = \lim_{\delta\to 0}\frac{\Psi[N-1]|_{\lambda=\lambda_1+\delta}}{\delta^{N-1}}, \tag{8}$$

$$\Omega_N = \lambda_1|\psi_1[N-1]|^2 + \lambda_1^*(|\psi_2[N-1]|^2 + |\psi_3[N-1]|^2 + |\psi_4[N-1]|^2), \tag{9}$$

$$M_0[j] = \frac{1}{\lambda_1^{*2}}I + \frac{\lambda_1^{*2} - \lambda_1^2}{\lambda_1^{*2}\lambda_1}\begin{pmatrix} \frac{\psi_1[j-1]\psi_1[j-1]^*}{\Omega_j} 0_{1\times 3} \\ 0_{3\times 1} \frac{P[j]P[j]^\dagger}{\Omega_j^*}\end{pmatrix}, \tag{10}$$

$$M_1[j] = \frac{\lambda_1^{*2} - \lambda_1^2}{\lambda_1\lambda_1^*}\begin{pmatrix} 0 \frac{P[j]^\dagger\psi_1[j-1]}{\Omega_j} \\ \frac{P[j]\psi_1[j-1]^*}{\Omega_j^*} 0_{3\times 3}\end{pmatrix},$$

$$P[j] = (\psi_2[j], \psi_3[j], \psi_4[j])^T. \tag{11}$$

3 Higher-order mixed interactions of localized waves

In order to construct various interactional solutions of the three-component DNLS equations (1), we choose a general nontrivial seed solution as

$$q_1[0] = c_1 e^{-\frac{2}{3}i\theta}, \quad q_2[0] = c_2 e^{-\frac{2}{3}i\theta},$$
$$q_3[0] = c_3 e^{-\frac{2}{3}i\theta}, \tag{12}$$

where $\theta = (c_1^2 + c_2^2 + c_3^2)x$ and c_j $(j = 1, 2, 3)$ are all real constants. For convenience, the above seed solution is chosen periodically in space variable x without depending on time variable t. Starting with the above seed solution $q_1 = q_1[0]$, $q_2 = q_2[0]$, $q_3 = q_3[0]$ with the spectral parameter λ, the special vector solution of the Lax pair (2)–(3) can be elaborately constructed as follows

$$\Phi_1 = \begin{pmatrix} (l_1 e^{\eta_1+\eta_2} - l_2 e^{\eta_1-\eta_2})e^{-\frac{i}{3}\theta} \\ r_1(l_1 e^{\eta_1-\eta_2} - l_2 e^{\eta_1+\eta_2})e^{\frac{i}{3}\theta} - (\mu c_2 + \nu c_3)e^{ix\lambda^2} \\ r_2(l_1 e^{\eta_1-\eta_2} - l_2 e^{\eta_1+\eta_2})e^{\frac{i}{3}\theta} + \mu c_1 e^{ix\lambda^2} \\ r_3(l_1 e^{\eta_1-\eta_2} - l_2 e^{\eta_1+\eta_2})e^{\frac{i}{3}\theta} + \nu c_1 e^{ix\lambda^2} \end{pmatrix}, \tag{13}$$

where

$$l_1 = \frac{i(9\lambda^2 - 2\tau - \sqrt{81\lambda^4 + 4\rho^2})^{\frac{1}{2}}}{\sqrt{81\lambda^4 + 4\rho^2}},$$

$$l_2 = \frac{i(9\lambda^2 - 2\rho + \sqrt{81\lambda^4 + 4\rho^2})^{\frac{1}{2}}}{\sqrt{81\lambda^4 + 4\rho^2}},$$

$$\eta_1 = -\frac{i}{2}\lambda^2(x + 9\lambda^2 t),$$

$$\eta_2 = \frac{i}{6}\sqrt{81\lambda^4 + 4\rho^2}\left(x + 3\lambda^2 t + \sum_{k=1}^{N} s_k\delta^{2k}\right),$$

$$r_1 = \frac{c_1}{\sqrt{\rho}}, \quad r_2 = \frac{c_2}{\sqrt{\rho}}, \quad r_3 = \frac{c_3}{\sqrt{\rho}},$$

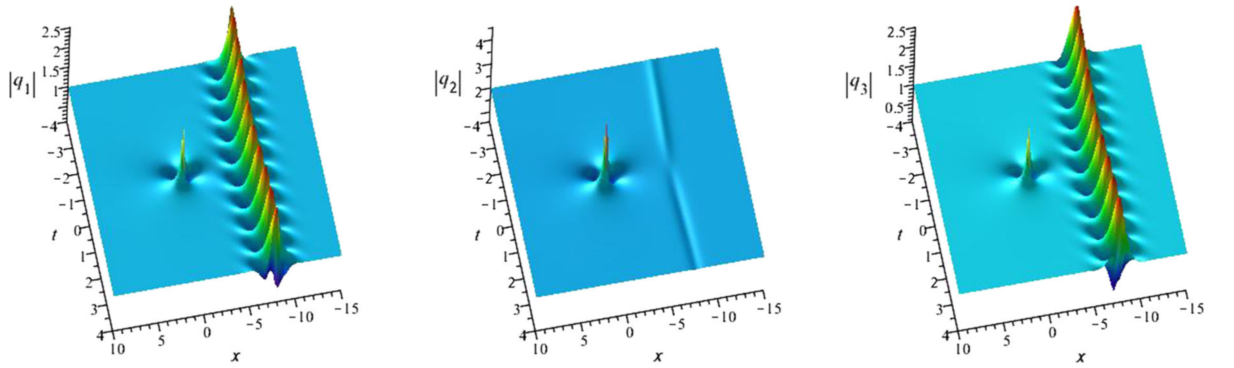
$$s_k = m_k + in_k, \quad \rho = c_1^2 + c_2^2 + c_3^2,$$

the parameters m_k, n_k $(k = 1, 2, \ldots, N)$, μ and ν are all real free constants. The structures of high-order RWs in the hybrid solutions is controlled by the parameters $s_k = m_k + in_k$; for example, the fundamental second-order RW can split into three first-order ones if $s_1 \neq 0$.

Here, we choose the spectral parameter $\lambda = \frac{\sqrt{\rho}}{3}(1 + i + \delta^2)$ with arbitrary small parameter δ and expand the Taylor expansion of the special vector function Φ_1 in (13) at $\delta = 0$ as follows:

$$\Phi_1 = \sum_{j=0}^{\infty} \Phi_1^{[j]}\delta^{2j}, \quad \Phi_1^{[j]} = \left(\psi_1^{[j]}, \psi_2^{[j]}, \psi_3^{[j]}, \psi_4^{[j]}\right)^T$$
$$= \frac{1}{(2j)!}\frac{\partial^j \Phi_1}{\partial\delta^j}|_{\delta=0} \quad (j = 0, 1, 2, 3, \ldots).$$

The concrete expressions of $\Phi_1^{[j]}$ can be easily calculated through Maple software. Besides, these expressions are very tedious and complicated, and we omit them.



(a) **(b)** **(c)**

Fig. 1 (Color online) Evolution plot of the first-order interactional solution in case (2) with the parameters chosen by $c_1 = 1, c_2 = -2, c_3 = -1, \mu = 0, \nu = \frac{1}{200000}$

Through formulae (7), the general expressions of the first-order interactions of localized waves for the three-component coupled DNLS equations (1) can be derived as

$$q_1[1] = c_1 \mathrm{e}^{-\frac{2}{3}i\theta} + 2i\left(\frac{6c_1\rho^{\frac{3}{2}}K_1\mathrm{e}^{-\frac{2}{3}i\theta} + 54(1+i)(\mu c_2 + \nu c_3)\rho^{\frac{3}{2}}K_2\mathrm{e}^{\xi_1}}{-(1+i)\sqrt{2}L_1 + 324\rho^2(1-i)L_2\mathrm{e}^{\xi_2}}\right)_x, \tag{14}$$

$$q_2[1] = c_2 \mathrm{e}^{-\frac{2}{3}i\theta} + 2i\left(\frac{6c_2\rho^{\frac{3}{2}}K_1\mathrm{e}^{-\frac{2}{3}i\theta} - 54(1+i)c_1\mu\rho^{\frac{3}{2}}K_2\mathrm{e}^{\xi_1}}{-(1+i)\sqrt{2}L_1 + 324\rho^2(1-i)L_2\mathrm{e}^{\xi_2}}\right)_x, \tag{15}$$

$$q_3[1] = c_3 \mathrm{e}^{-\frac{2}{3}i\theta} + 2i\left(\frac{6c_3\rho^{\frac{3}{2}}K_1\mathrm{e}^{-\frac{2}{3}i\theta} - 54(1+i)c_1\nu\rho^{\frac{3}{2}}K_2\mathrm{e}^{\xi_1}}{-(1+i)\sqrt{2}L_1 + 324\rho^2(1-i)L_2\mathrm{e}^{\xi_2}}\right)_x, \tag{16}$$

where

$$\xi_1 = \frac{\rho}{9}[(-3+3i)x + 2i\rho], \quad \xi_2 = -\frac{2}{3}\rho x,$$

$$K_1 = \frac{\left(12\,ix\rho + 8\,\rho^2 t + 9 - 9\,i\right)\left(12\,ix\rho - 8\,\rho^2 t + 9 + 9\,i\right)}{\sqrt{(-2-2\,i)\,\rho}\rho\,\sqrt{-2+2\,i}},$$

$$K_2 = \frac{12\,ix\rho - 8\,\rho^2 t + 9 + 9\,i}{\sqrt{(-2+2\,i)\,\rho}},$$

$$\begin{aligned} L_1 = {} & \left(72\,i\rho^2{c_1}^2 + 72\,i\rho^2{c_2}^2 + 72\,i\rho^2{c_3}^2 - 72\,\rho^3\right)x^2 \\ & - 108\,\rho\,\left(i{c_1}^2 + i{c_2}^2 + i{c_3}^2 + \rho\right)x \\ & + 32\,t^2\rho^4\left(i{c_1}^2 + i{c_2}^2 + i{c_3}^2 - \rho\right) \\ & + 72\,t\rho^2\left(i{c_1}^2 + i{c_2}^2 + i{c_3}^2 + \rho\right) \\ & + 81(i{c_1}^2 + i{c_2}^2 + i{c_3}^2 - \rho), \end{aligned}$$

$$L_2 = \mu^2{c_1}^2 + \mu^2{c_2}^2 + 2\,\mu\,\nu\,c_2c_3 + \nu^2{c_1}^2 + \nu^2{c_3}^2.$$

(1) When $\mu = \nu = 0$, the first-order interactional solutions degenerate to the first-order RWs.
(2) If one of the two parameters μ and ν is zero, and the backgrounds are all non-vanished ($\mu = 0, \nu \neq 0, c_j \neq 0\ (j = 1, 2, 3)$), we can find that a first-order RW interacts with a breather in q_1 and q_3 components, and a first-order RW interacts with an amplitude-varying soliton in q_2 component in Fig. 1. Choosing the case that $\mu = 0, \nu \neq 0$ and $c_j \neq 0$, we can get a different various arrangement of three components q_1, q_2 and q_3. However, the combination is the same, namely one component is RW and amplitude-varying soliton, two components are RW and breather. Here, we define the same combination as the same kind of interactional solution.

The density plot of q_2 in Fig. 2a shows that the soliton in q_2 component is anti-dark soliton if $t < 0$ and becomes dark soliton if $t > 0$. Besides, this kind of amplitude-varying soliton annihilates if $t = 0$. When $t < 0$, the amplitude of anti-dark soliton in q_2 component becomes big with t increasing, see Fig. 2b; otherwise, if $t > 0$, the amplitude of dark soliton in q_2 component becomes small with t increasing, see Fig. 2d. Moreover, it demonstrates that the soliton annihilates and only a first-order RW exist at $t = 0$ in q_2 component from Fig. 2c. By increasing the absolute value of ν, the first-order RW can merge with one-breather or one-amplitude-varying soliton.

(3) When $\mu = 0, \nu \neq 0, c_1 \neq 0, c_2 \neq 0$, and $c_3 = 0$, it demonstrates that hybrid solution between a

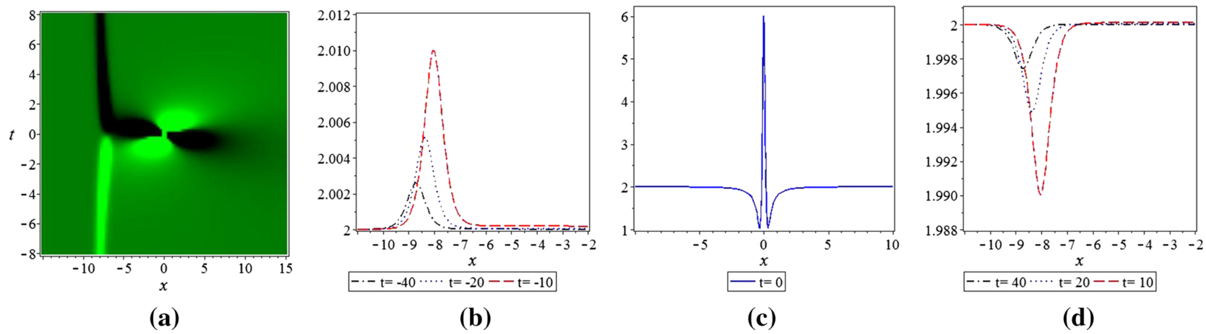


Fig. 2 (Color online) **a** Evolution density plot of q_2 component in Fig. 1b. Plane evolution plot of the interactional process between the first-order RW and the amplitude-varying soliton of q_2 component in Fig. 1b at different moments: **b** $t < 0$; **c** $t = 0$; **d** $t > 0$

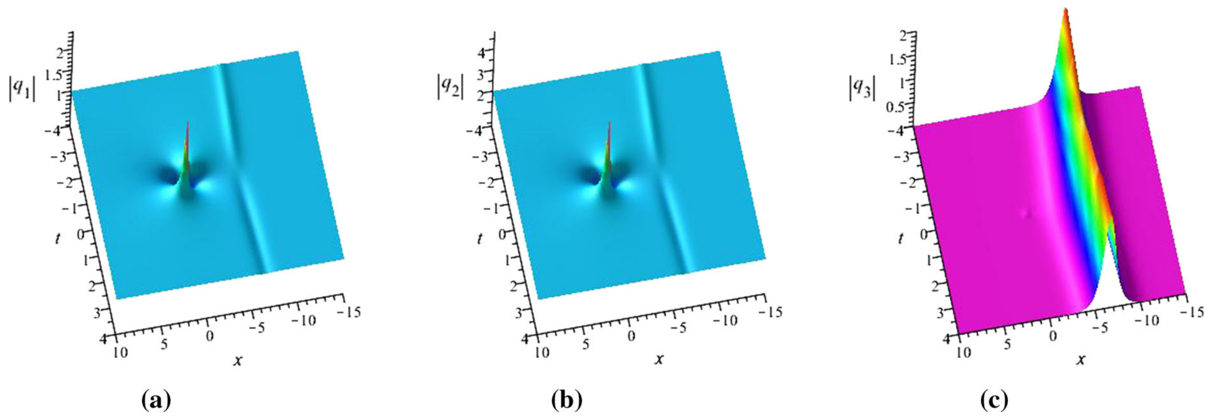


Fig. 3 (Color online) Evolution plot of the first-order interactional solution in case (3) with the parameters chosen by $c_1 = 1, c_2 = -2, c_3 = 0, \mu = 0, \nu = \frac{1}{2000}$

first-order RW and an amplitude-varying soliton exists in q_1 and q_2 components, and hybrid solution between a first-order RW and a bright soliton exists in q_3 component in Fig. 3. In Fig. 3c, the first-order RW in q_3 component cannot be easily observed since it emerges from zero plane background. By increasing the absolute value ν, the first-order RW can merge with amplitude-varying soliton and bright soliton, respectively.

(4) When $\mu \neq 0, \nu \neq 0$, and one of the three backgrounds is vanished ($c_1 \neq 0, c_2 \neq 0$ and $c_3 = 0$), we can find that a first-order RW merges with a breather in q_1 and q_2 components, and a first-order RW merges with a bright solitons in q_3 component from Fig. 4. Similar with case (3), the nonlinear localized waves can separate in the three components q_1, q_2, and q_3 by decreasing the absolute values of μ and ν. Here, we omit these figures.

(5) Similar with case (4), setting $\mu \neq 0, \nu \neq 0$, and two of the three backgrounds be vanished ($c_1 \neq 0, c_2 = c_3 = 0$), we can find that a first-order RW merges with a bright soliton in q_2 and q_3 components, and a first-order RW merges with an amplitude-varying soliton in q_1 component from Fig. 5.

(6) When these five free parameters μ, ν and c_j $(j = 1, 2, 3)$ are all not zero, the interactional solutions can be constructed that a first-order interacts with a breather in three components q_1, q_2 and q_3, see Fig. 6. Furthermore, it is interesting that the breather in q_2 component is different from ones in other two components. Similarly, the first-order RW can merge with the breather through increasing the absolute values of μ and ν.

Without considering different arrangements among the three components q_1, q_2 and q_3, the same combina-

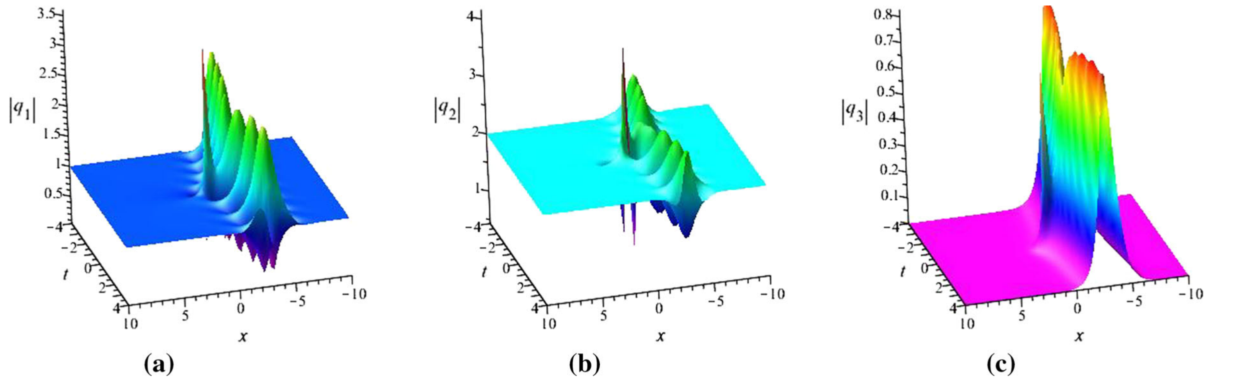


Fig. 4 (Color online) Evolution plot of the first-order interactional solution in case (4) with the parameters chosen by $c_1 = 1, c_2 = -2, c_3 = 0, \mu = \frac{1}{20}, \nu = -\frac{1}{20}$.

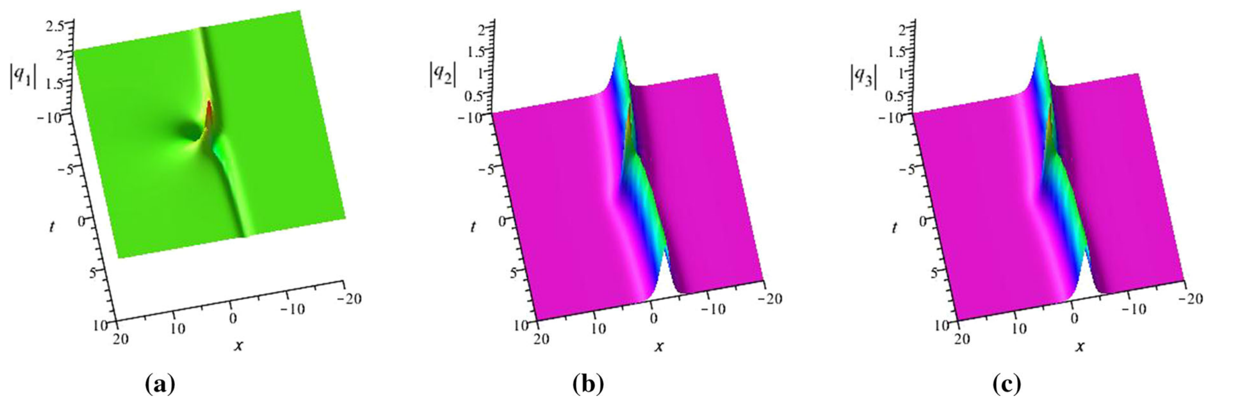


Fig. 5 (Color online) Evolution plot of the first-order interactional solution in case (5) with the parameters chosen by $c_1 = 2, c_2 = 0, c_3 = 0, \mu = \frac{1}{2}, \nu = -\frac{1}{2}$

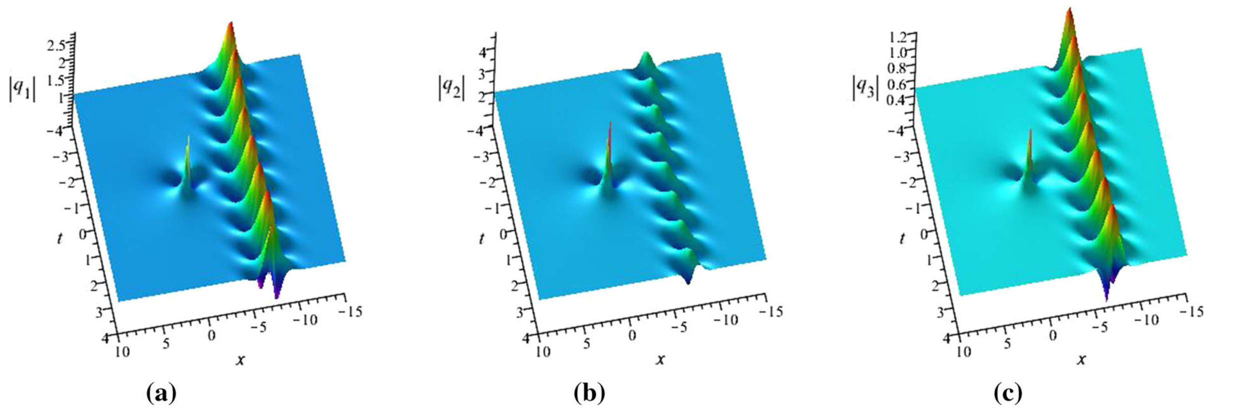


Fig. 6 (Color online) Evolution plot of the first-order interactional solution in case (6) with the parameters chosen by $c_1 = 1, c_2 = -2, c_3 = \frac{1}{2}, \mu = \frac{1}{20000}, \nu = -\frac{1}{20000}$

tion is defined as the same type of hybrid solution. The first-order interactions of localized waves for the three-component coupled system (1) can be completely classified as six types according to our method and definition. It is shown that cases (2)–(5) are four mixed interactions of localized waves. Iterating DT of the three-component coupled DNLS equations (1), the higher-order interactions of localized waves can be obtained through the formulae (7). Similar to the first-order case, the higher-order interactional solutions can be also classified as six types. Among the above six types of interactional solutions, there are also four mixed cases.

4 Conclusion

Utilizing both a limiting process and the peculiar vector solution (13) of the Lax pair (2)–(3), various novel and interesting higher-order interactional solutions in the three-component coupled DNLS equations are constructed through DT technique. Considering both disturbing terms (μ and ν) and the backgrounds (c_j, $j = 1, 2, 3$), the interactional solutions of Eq. (1) are completely classified as six types, among which there are four mixed cases. These results provide evidence of some obvious interactions between higher-order RWs and multi-soliton or multi-breather. When the components of the DNLS system are more than 3, the matrices in the corresponding Lax pair (2)–(3) are more than 4×4. At this point, the degree of the characteristic equation of transformed matrix U_0 (U in Eq. (2) with exponential functions is transformed to constant coefficient matrix U_0) is equal or greater than 5. The equation with one unknown quantity whose degree is equal or greater than 5 has not radical solutions and this is an open problem. Based on the above facts, the special vector solution of the corresponding Lax pair (2)–(3) of the DNLS system whose components are more than 3 cannot be directly constructed. For the 4-coupled (or more than 4) DNLS system, the special vector solution of the Lax pair is expected to be constructed by numerical simulation method in our future work.

In this work, we generalize Baronio's results [56] to the higher-order case in the three-component DNLS equations (1) and obtain four mixed interactions of localized waves. Besides, we constructed mixed interactions of localized waves in three-component NLS equations [61] and Hirota equations [62] through generalized DT. However, these mixed interactions cannot be obtained in single- and two-component systems by DT technique. Based on the above facts, a conclusion can be drawn that these kinds of mixed interactions of localized waves can be only obtained by DT in the nonlinear systems, whose components are more than 3 with the corresponding Lax pair including the matrices larger than 3×3. In [56], Baronio et al. gave the experimental conditions for observing the first-order interactional solutions among RW, one-bright (dark) soliton and one-breather in two-component coupled NLS equations. Motivated by Baronio's experimental conditions in [56], we expect that these interactions of localized waves obtained in this article will be verified and observed in the physical experiments in the future.

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Compliance with ethical standards

Conflicts of interest The authors declare that there is no conflict of interests regarding the publication of this paper.

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