# Adaptive Function Projective Synchronization of Discrete-Time Chaotic Systems *

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*By backstepping control law and active control method, adaptive function projective synchronization of 2D and 3D discrete-time chaotic systems with uncertain parameters are investigated. To illustrate the effectiveness of new scheme, some numerical examples are given.*

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Since the synchronization of chaotic system is discovered,[1] the synchronization problem in chaotic systems has been intensively and extensively studied in recent decades. Up to now, there exist many types of chaos synchronization schemes in dynamical systems.[2−15] Parameters adaptive control,[2] active control,[3] feedback approach,[4] backstepping design,[5] and so on have been successfully applied to chaos synchronization. Backstepping design[6] has become a systematic and powerful method for the construction of both feedback controllers and associated Lyapunov functions.

Over the last decade, some articles have been reported to extend the backstepping design to deduce some proper controllers to investigate chaos control and synchronization.[7−9] However, they are based on the exactly knowing of the system parameters. In real situation, some or all of the parameters are unknown. In this Letter, we study global chaos synchronization of discrete-time chaotic system with uncertain parameters base on Chen and Li[11] and Lü's scheme.[15,16] First, we give a definition of adaptive function projective synchronization (AFPS). Then a synchronization scheme is applied to investigate AFPS between two identical 2D and 3D discrete-time chaotic systems based on backstepping design. Moreover, we provide numerical examples to demonstrate the effectiveness of proposed method.

The discrete-time chaotic system (called the drive system) in the form

$$x(k+1) = F(x(k)), \tag{1}$$

and the response system of (1) is defined by

$$y(k+1) = G(y(k)) + U, \tag{2}$$

where $F : R^m \to R^m$, $G : R^m \to R^m$ are the vector-valued functions, $x(k) = [x_1(k), \cdots, x_m(k)]^T \in R^m$, $y(k) = [y_1(k), \cdots, y_m(k)]^T \in R^m$ are the state vectors, and $U = [u_1(x_1(k), y_1(k)), \cdots, u_m(x_m(k), y_m(k))] \in R^m$ is an unknown controller vector. Let the error state be

$$\begin{aligned} e(k) =& \big(e_1(k), e_2(k), \cdots, e_m(k)\big) \\ =& \big(x_1(k+\tau) - f_1(x(k+\tau))y_1(k), x_2(k+\tau) \\ &- f_2(x(k+\tau))y_2(k), \cdots, x_m(k+\tau) \\ &- f_m(x(k+\tau))y_m(k)\big), \end{aligned} \tag{3}$$

where $f_i (i = 1 \cdots, m)$ are the scaling function, $\tau \in Z/Z^-$ is a constant. It is said that Eqs. (1) and (2) are globally AFPS when $\tau \in N$ ($\tau$ is called the synchronization anticipation), if there exists proper controllers $U = (u_1, u_2, \cdots, u_m)^T$ such that $\lim_{k\to\infty}(e(k)) = 0$, we can say that there exist AFPS between the systems (1) and (2).

*Remark.* When we choose $\tau = 0$, (i) $f_1 = f_2 = \cdots, = f_n = 1$, (ii) $f_1 = f_2 = \cdots, = f_n = \alpha$, (iii) $f_1 = \alpha_1$, $f_2 = \alpha_2$, $\cdots$, $f_n = \alpha_n$, (iv) $f_1 = f_1(x)$, $f_2 = f_2(x)$, $\cdots$, $f_n = f_3(x)$, and we will obtain CS[7], PS[13], MPS[14] and FPS[11], respectively.

The Kawakami map[6]

$$\begin{aligned} x_1(k+1) =& \alpha x_1(k) + x_2(k), \\ x_2(k+1) =& -\beta + x_2^2(k). \end{aligned} \tag{4}$$

as the drive system, and the response system[14] reads

$$\begin{aligned} y_1(k+1) =& \alpha_1(k)y_1(k) + y_2(k) + u_1(x,y), \\ y_2(k+1) =& -\beta_1(k) + y_2^2(k) + u_2(x,y), \end{aligned} \tag{5}$$

where $\alpha_1(k)$ and $\beta_1(k)$ are uncertain parameters which estimate parameters of $\alpha$ and $\beta$. Here $u_1$ and $u_2$ are the controllers such that two chaotic systems can be synchronized in the sense of AFPS.

In the following we would like to realize Eqs. (4) and (5) by backstepping design method.

Let the error states be $e_1(k) = x_1(k+\tau) - 3y_1(k)$, $e_2(k) = x_2(k+\tau) - (1 + \tanh(x_2(k+\tau)^2))y_2(k)$, $e_3(k) = \alpha_1(k) - \alpha$, $e_4(k) = \beta_1(k) - \beta$. Then from

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Eqs. (4) and (5), we have the discrete-time error system

$$\begin{aligned} e_1(k+1) =& x_1(\xi) - 3\alpha_1 y_1 - 3y_2 - 3u_1(x,y), \\ e_2(k+1) =& x_2(\xi) - (1+\tanh(x_2(\xi)^2) \\ &\cdot(-\beta_1 + y_2^2 + u_2(x,y)). \end{aligned} \quad (6)$$

while $\xi = k+1+\tau, \eta = k+\tau$, $\alpha_1 = \alpha_1(k)$, $\beta_1 = \beta_1(k)$, $x_1 = x_1(k)$, $x_2 = x_2(k)$, $y_1 = y_1(k)$, $y_2 = y_2(k)$. Based on the backstepping design and the improved ideas of Refs. [11,15,16], we give a systematic and constructive algorithm to derive the controllers $u(x,y)$ step by step such that systems Eqs. (4) and (5) are synchronized together.

*Step 1.* The first error variable $E_1(k) = e_1(k)$. Let the first partial Lyapunov function be $L_1(k) = |E_1(k)| = |e_1(k)|$ and the second error variable be

$$e_2(k) = e_1(k+1) - \delta_{11} e_1(k), \quad (7)$$

where $\delta_{11} \in R$. We have the derivative of $L_1(k)$

$$\begin{aligned} \Delta L_1(k) &= |e_1(k+1)| - |e_1(k)| \\ &\le (|\delta_{11}| - 1)|e_1(k)| + |e_2(k)|. \end{aligned} \quad (8)$$

*Step 2.* Let the second partial Lyapunov function candidate be $L_2(k) = L_1(k) + c_1|e_2(k)|$ and the third error variable be

$$e_3(k) = e_2(k+1) - \delta_{21} e_1(k) - \delta_{22} e_2(k), \quad (9)$$

where $c_1 > 1, \delta_{21}$, $\delta_{22} \in R$. Therefore, from Eqs. (7) and (9) we have the derivative $L_2(k)$

$$\begin{aligned} \Delta L_2(k) &= L_2(k+1) - L_2(k) \\ &\le (c_1|\delta_{21}| + |\delta_{11}| - 1)|e_1(k)| + (c_1|\delta_{22}| \\ &\quad + 1 - c_1)|e_2(k)| + c_1|e_3(k)|. \end{aligned} \quad (10)$$

*Step 3.* Let the third partial Lyapunov function candidate be $L_3(k) = L_2(k) + c_2|e_3(k)|$ and the fourth error state be

$$e_4(k) = e_3(k+1) - \delta_{31} e_1(k) - \delta_{32} e_2(k) - \delta_{33} e_3(k), \quad (11)$$

where $c_2 > c_1 > 1$, $\delta_{31}$, $\delta_{32}$, $\delta_{33} \in R$. Therefore, from Eqs. (9) and (11) we have the derivative $L_3(k)$

$$\begin{aligned} &\Delta L_3(k) = L_3(k+1) - L_3(k) \\ &\le (c_2|\delta_{31}| + c_1|\delta_{21}| + |\delta_{11}| - 1)|e_1(k)| \\ &\quad + (c_2|\delta_{32}| + c_1(|\delta_{22}| - 1) + 1)|e_2(k)| \\ &\quad + (c_2|\delta_{33}| + c_1 - c_2)|e_3(k)| + c_2|e_4(k)|. \end{aligned} \quad (12)$$

*Step 4.* Let the fourth partial Lyapunov function candidate be $L_4(k) = L_3(k) + c_3|e_4(k)|$ and the fourth error state be

$$\begin{aligned} &e_4(k+1) - \delta_{41} e_1(k) - \delta_{42} e_2(k) \\ &- \delta_{43} e_3(k) - \delta_{44} e_4(k) = 0, \end{aligned} \quad (13)$$

where $c_3 > c_2 > c_1 > 1$, $\delta_{41}, \delta_{42}, \delta_{43}, \delta_{44} \in R$. Therefore, from (11) and (13) we have the derivative $L_4(k)$

$$\begin{aligned} &\Delta L_4(k) = L_4(k+1) - L_4(k)| \\ &\le (c_3|\delta_{41}| + c_2|\delta_{31}| + c_1|\delta_{21}| + |\delta_{11}| - 1)|e_1(k)| \\ &\quad + (c_3|\delta_{42}| + c_2|\delta_{32}| + c_1(|\delta_{22}| - 1) + 1)|e_2(k)| \\ &\quad + (c_3|\delta_{43}| + c_2|\delta_{33}| + c_1 - c_2)|e_3(k)| \\ &\quad + (c_3|\delta_{44}| + c_2 - c_3)|e_4(k)|. \end{aligned} \quad (14)$$

From Eq. (14), we know that the right-hand side of Eq. (14) is negative and infinite, if the parameters $c_i (i = 1, 2, 3, 4)$ and $\delta_{ij} (1 \le j \le i \le 4)$ satisfy

$$\begin{aligned} c_1|\delta_{21}| + c_2|\delta_{31}| + c_3|\delta_{41}| + |\delta_{11}| &< 1, \\ c_1|\delta_{22}| + c_2|\delta_{32}| + c_3|\delta_{42}| &< c_1 - 1, \\ c_2|\delta_{33}| + c_3|\delta_{43}| < c_2 - c_1,\ |\delta_{44}| &< \frac{c_3 - c_2}{c_3}. \end{aligned} \quad (15)$$

Here $\Delta L(k)$ is negative and infinite. From Eqs. (7), (9), (11) and (13) we obtain the controllers

$$\begin{aligned} u_1(x,y) =& \frac{1}{3}x_1(\xi) - \alpha_1 y_1 - \frac{2}{3}y_2 - \frac{1}{3}\delta_{11}x_1(\eta) \\ &+ \delta_{11}y_1 - \frac{1}{3}x_2(\eta) + \frac{1}{3}\tanh(x_2(\eta))^2 y_2, \\ u_2(x,y) =& \frac{1}{1+\tanh(x_2(\xi))^2}((-x_2(\xi) + y_2^2 - \beta_1 \\ &- \tanh(x_2(\xi))^2 y_2^2 - \tanh(x_2(\xi))^2 \beta_1 \\ &+ \delta_{21}x_1(\eta) - 3\delta_{21}y_1 + \delta_{22}x_2(\eta) \\ &- \delta_{22}y_2) - \delta_{22}y_2)\tanh(x_2(\eta))^2, \end{aligned} \quad (16)$$

$$\begin{aligned} \alpha_1(k+1) =& \alpha + \delta_{31}x_1(\eta) - 3\delta_{31}y_1 + \delta_{32}x_2(\eta) \\ &- \delta_{32}y_2 - \delta_{32}\tanh(x_2(\eta)y_2 \\ &+ \delta_{33}\alpha_1 - \delta_{33}\alpha + \beta_1 - \beta, \\ \beta_1(k+1) =& \beta + \delta_{41}x_1(\eta) - 3\delta_{41}y_1 + \delta_{42}x_2(\eta) \\ &- \delta_{42}y_2) - \delta_{42}\tanh(x_2(\eta))y_2) + \delta_{43}\alpha_1 \\ &- \delta_{43}\alpha + \delta_{44}\beta_1 - \delta_{44}\beta, \end{aligned} \quad (17)$$

$\xi = k+1+\tau$, $\eta = k+\tau$, $\alpha_1 = \alpha_1(k)$, $\beta_1 = \beta_1(k)$, $x_1 = x_1(k)$, $y_1 = y_1(k)$.

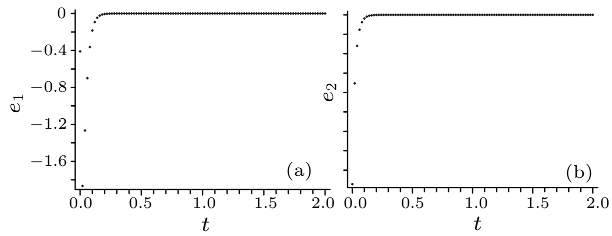


**Fig. 1.** Orbits of the error states: (a) $e_1(k) = x_1(k+\tau) - 3y_1(k)$, $\tau = 1$, (b) $e_2(k) = x_2(k+\tau) - [1 + \tanh(x_2(k+\tau))^2]y_2(k)$, $\tau = 1$.

We use numerical simulations to verify the effectiveness of the above-mentioned controllers. The parameters are chosen as $\alpha = -0.1$, $\beta = 1.6$, $\delta_{11} = 0.3$, $\delta_{21} = 0.02$, $\delta_{22} = 0.4$, $\delta_{31} = 0.05$, $\delta_{32} = 0.1$, $\delta_{33} = -0.2$, $\delta_{41} = 0.01$, $\delta_{42} = 0.02$, $\delta_{43} = 0.03$,

$\delta_{44} = 0.04$, $c_1 = 2$, $c_2 = 3$, $c_3 = 5$ and the initial values $[x_1(0) = 0.1, x_2(0) = 0.2]$, $[y_1(0) = 0.2, y_2(0) = 0.1]$, and $\alpha_1(0) = 0.1$, $\beta_1(0) = 0.1$, and the figures of synchronization errors are displayed in Fig. 1(a)–1(b), and simulations of the two parameters $\alpha_1(k), \beta_1(k)$ are displayed in Fig. 2(a) and 2(b). Finally the attractors after being synchronized with controllers are displayed in Fig. 3.

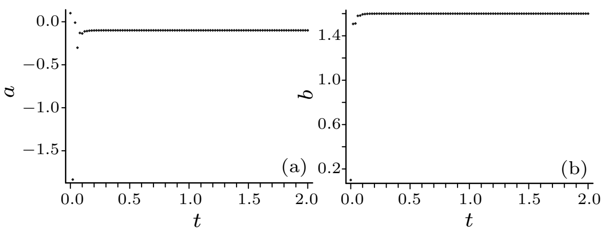


**Fig. 2.** Orbits of uncertain parameters.

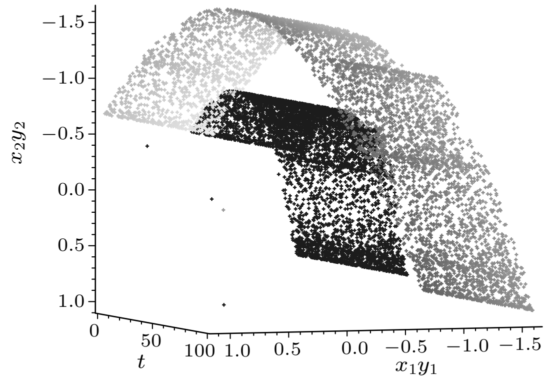


**Fig. 3.** Two attractors after synchronized with $\tau = 1$, the dark one is the response system with the controllers, and the other is the drive system.

The generalized Hénon map[12]

$$\begin{aligned} x_1(k+1) &= -\beta x_2(k), \\ x_2(k+1) &= 1 + \beta x_3(k) - \alpha x_1^2(k), \\ x_3(k+1) &= \beta x_2(k) + x_1(k). \end{aligned} \tag{18}$$

as the drive system, and the response system is as follows:

$$\begin{aligned} y_1(k+1) &= -\beta_1(k) y_2(k) + u_1(x,y), \\ y_2(k+1) &= 1 + \beta_1(k) y_3(k) - \alpha_1(k) y_1^2(k) + u_2(x,y), \\ y_3(k+1) &= \beta_1(k) y_2(k) + y_1(k) + u_3(x,y), \end{aligned} \tag{19}$$

where $\alpha_1(k)$ and $\beta_1(k)$ are uncertain parameters which estimate the parameters of $\alpha$ and $\beta$, while $u_1$, $u_2$ and $u_3$ are the controllers such that two chaotic systems can be synchronized in the sense of AFPS. Then we would like to realize AFPS of Eqs. (18) and (19).

Let the error states be $e_1(k) = x_1(k+\tau) - 3y_1(k)$, $e_2(k) = x_2(k+\tau) + y_2(k)$, $e_3(k) = x_3(k+\tau) - (1 + \coth(x_3(k+\tau)))^2 y_3(k)$, $e_4(k) = \alpha_1(k) - \alpha$, $e_5(k) = \beta_1(k) - \beta$. Then from Eqs. (18) and (19), we have the discrete-time error dynamical system

$$\begin{aligned} e_1(k+1) &= x_1(\xi) + 3\beta_1 y_2 - 3u_1(x,y), \\ e_2(k+1) &= 1 + y_3 + x_2(\xi) - \alpha_1 y_2^2 + u_2(x,y), \\ e_3(k+1) &= x_3(\xi) - (1 + \coth(x_3(\xi))^2) \\ &\quad \cdot (\beta_1 y_2 + y_1 + u_3(x,y)). \end{aligned} \tag{20}$$

while $\xi = k+1+\tau$, $\eta = k+\tau$, $\alpha_1 = \alpha_1(k)$, $\beta_1 = \beta_1(k)$, $x_1 = x_1(k)$, $x_2 = x_2(k)$, $x_3 = x_3(k)$, $y_1 = y_1(k)$, $y_2 = y_2(k)$, $y_3 = y_3(k)$. In the following we consider AFPS between Eqs. (18) and (19) via the following scheme.

*Step 5.* Let the fourth partial Lyapunov function candidate be $L_4(k) = L_3(k) + c_3|e_4(k)|$ and the fourth error state be

$$\begin{aligned} e_5(k) &= e_4(k+1) - \delta_{41}e_1(k) - \delta_{42}e_2(k) \\ &\quad - \delta_{43}e_3(k) - \delta_{44}e_4(k), \end{aligned} \tag{21}$$

where $c_3 > c_2 > c_1 > 1$, $\delta_{41}, \delta_{42}, \delta_{43}, \delta_{44} \in R$. Therefore, from Eqs. (11) and (21) we have the derivative $L_4(k)$

$$\begin{aligned} &\Delta L_4(k) = L_4(k+1) - L_4(k) \\ &\le (c_3|\delta_{41}| + c_2|\delta_{31}| + c_1|\delta_{21}| + |\delta_1 1| - 1)|e_1(k)| \\ &\quad + (c_3|\delta_{42}| + c_2|\delta_{32}| + c_1(|\delta_{22}| - 1) + 1)|e_2(k)| \\ &\quad + (c_3|\delta_{43}| + c_2|\delta_{33}| + c_1 - c_2)|e_3(k)| \\ &\quad + (c_3|\delta_{44}| + c_2 - c_3)|e_4(k)| + c_3|e_5(k)|. \end{aligned} \tag{22}$$

*Step 6.* Let the Lyapunov function be $L(k) = L_4(k) + c_4|e_5(k)|$. From the above steps we have

$$\begin{aligned} &e_5(k+1) - \delta_{51}e_1(k) - \delta_{52}e_2(k) - \delta_{53}e_3(k) \\ &\quad - \delta_{54}e_4(k) + \delta_{55}e_5(k) = 0, \end{aligned} \tag{23}$$

where $c_4 > c_3 > c_2 > c_1 > 1$, $\delta_{51}, \delta_{52}, \delta_{53}, \delta_{54}, \delta_{55} \in R$. Then from Eqs. (21),(22), and (23), we obtain the derivative of the Lyapunov function $L(k)$

$$\begin{aligned} &\Delta L(k) = L(k+1) - L(k) \le \big(c_4|\delta_{51}| + c_3|\delta_{41}| \\ &\quad + c_2|\delta_{31}| + c_1|\delta_{21}| + |\delta_1 1| - 1\big)|e_1(k)| + \big(c_4|\delta_{52}| \\ &\quad + c_3|\delta_{42}| + d_2|c_{32}| + c_1(|\delta_{22}| - 1) + 1\big)|e_2(k)| \\ &\quad + \big(c_4|\delta_{53}| + c_3|\delta_{43}| + c_2|\delta_{33}| + c_1 - c_2\big)|e_3(k)| \\ &\quad + \big(c_4|\delta_{54}| + c_3|\delta_{44}| + c_2 - c_3\big)|e_4(k)| \\ &\quad + \big(c_3 - c_4 + c_4|\delta_{55}|\big)|e_5(k)|. \end{aligned} \tag{24}$$

From Eq. (24), we know that the right-hand side of Eq. (24) is negative and infinite, if the parameters $c_i (i = 1, 2, 3, 4)$ and $\delta_{ij} (1 \le j \le i \le 4)$ satisfy

$$\begin{aligned} &c_1|\delta_{21}| + c_2|\delta_{31}| + c_3|\delta_{41}| + c_4|\delta_{51}| + |\delta_{11}| < 1, \\ &c_1|\delta_{22}| + c_2|\delta_{32}| + c_3|\delta_{42}| + c_4|\delta_{52}| < c_1 - 1, \\ &c_2|\delta_{33}| + c_3|\delta_{43}| + c_4|\delta_{53}| < c_2 - c_1, \\ &c_3|\delta_{44}| + c_4|\delta_{54}| < c_3 - c_2, \ |\delta_{55}| < \frac{c_4 - c_3}{c_4}, \end{aligned} \tag{25}$$

then $\Delta L(k)$ is negative and infinite. From Eqs. (7), (9), (11), (21) and (23), we can determine the scalar controllers of $u(x,y)$ in the form

$$u_1(x,y) = \beta_1 y_2 + \frac{1}{3}x_1(\xi) + \delta_{11}y_1 - \frac{1}{3}\delta_{11}x_1(\eta)$$

$$
\begin{aligned}
& - \frac{1}{3}y_2 - \frac{1}{3}x_2(\eta), \\
u_2(x,y) = & -1 + \alpha_1 y_2^2 - x_2(\xi) - 2y_3 - 3\delta_{21}y_1 \\
& + \delta_{21}x_1(\eta) + \delta_{22}y_2 \\
& + \delta_{22}x_2(\eta) + x_3(\eta) - y_3\coth(x_3(\eta))^2, \\
u_3(x,y) = & \frac{1}{1+\coth(x_3(\xi))^2}(-\beta_1 y_2 - y_1 + x_3(\xi) \\
& + \beta_1 y_2 \coth(x_3(\xi)^2 - y_1\coth(x_3(\xi)^2 \\
& + 3\delta_{31}y_1 - \delta_{31}x_1(\eta) - \delta_{32}y_2 - \delta_{32}x_2(\eta) \\
& + \delta_{33}y_3 - \delta_{33}x_3(\eta) \\
& + \delta_{33}y_3\coth(x_3(\eta))^2 - \alpha_1 + \alpha), \qquad (26) \\
\alpha_1(k+1) = & \alpha - 3\delta_{41}y_1 + \delta_{41}x_1(\eta) + \delta_{42}y_2 + \delta_{42}x_2(\eta) \\
& + \delta_{43}x_3(\eta) - \delta_{43}y_3\coth(x_3(\eta))^2 + \delta_{44}\alpha_1 \\
& - \delta_{44}\alpha + \beta_1 - \beta, \\
\beta_1(k+1) = & \beta - 3\delta_{51}y_1 + \delta_{51}x_1(\eta) + \delta_{52}y_2 + \delta_{52}x_2(\eta) \\
& - \delta_{53}y_3 + \delta_{53}x_3(\eta) - \delta_{53}y_3\coth(x_3(\eta))^2 \\
& + \delta_{54}\alpha_1 - \delta_{54}\alpha + \delta_{55}\beta_1 - \delta_{55}\beta, \qquad (27)
\end{aligned}
$$

while $\xi = k+1+\tau$, $\eta = k+\tau$, $\alpha_1 = \alpha_1(k)$, $\beta_1 = \beta_1(k)$, $x_1 = x_1(k)$, $x_2 = x_2(k)$, $x_3 = x_3(k)$, $y_1 = y_1(k)$, $y_2 = y_2(k)$, $y_3 = y_3(k)$. Then we use numerical simulations to verify the effectiveness of the obtained controllers $u(x,y)$. Here take $\alpha = 1.07$, $\beta = 0.3$, $\delta_{11} = 0.3$, $\delta_{21} = 0.02$, $\delta_{22} = 0.4$, $\delta_{31} = 0.05$, $\delta_{32} = 0.1$, $\delta_{33} = -0.2$, $\delta_{41} = 0.01$, $\delta_{42} = 0.02$, $\delta_{43} = 0.03$, $\delta_{44} = 0.04$, $\delta_{51} = 0.01$, $\delta_{52} = 0.02$, $\delta_{53} = 0.03$, $\delta_{54} = 0.04$, $\delta_{55} = 0.05$, $c_1 = 2$, $c_2 = 3$, $c_3 = 5$, $c_4 = 6$ and the initial values $[x_1(0) = 0.2, x_2(0) = 0.7, x_3(0) = 0.06]$, $[y_1(0) = 0.06, y_2(0) = 0.7, y_3(0) = 0.2]$, and $\alpha_1(0) = 0.1$, $\beta_1(0) = 0.1$, respectively.

In the case $\tau > 0$, without loss of generality, we set $\tau = 1$, Thus the figures of AFPS errors are displayed in Figs. 4(a)–4(c), and simulations of the two parameters $\alpha_1(k), \beta_1(k)$ are displayed in Figs. 5(a) and 5(b). Finally the attractors after synchronized are displayed in Fig. 6.

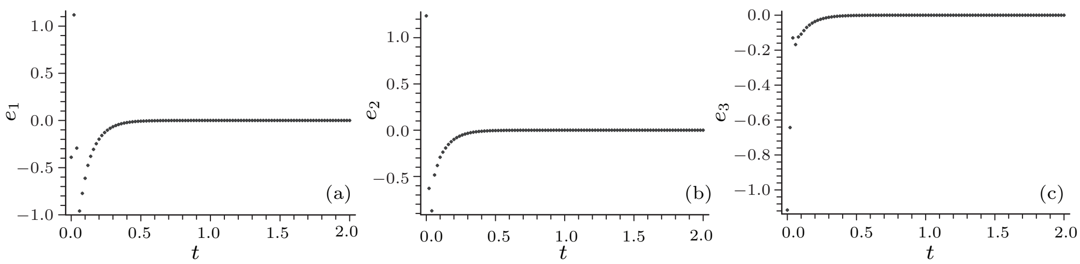


**Fig. 4.** Orbits of the error states: (a) $e_1 = x_1(k+\tau) - 3y_1(k)$, $\tau = 1$. (b) $e_2 = x_2(k+\tau) + y_2(k)$, $\tau = 1$, (c) $e_3 = x_3(k+\tau) - (1+\coth(x_3(k+\tau))^2)y_3(k)$, $\tau = 1$.

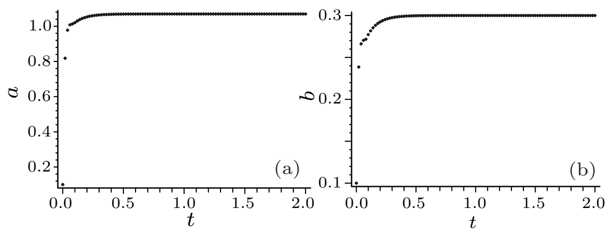


**Fig. 5.** The orbits of uncertain parameters.

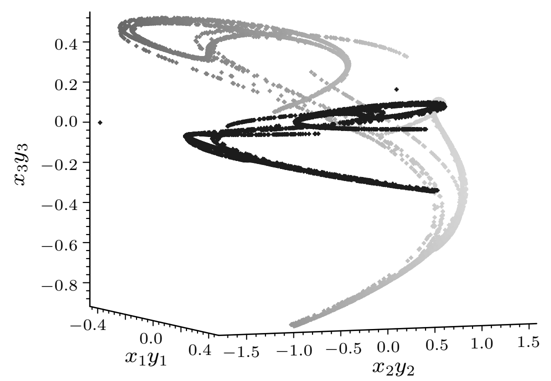


**Fig. 6.** Two attractors after synchronized with $\tau = 1$, the dark one is the response system with the contollers, and the other is the drive system.

In summary, we have presented a synchronization scheme to study AFPS in discrete-time chaotic systems. The scheme is applied to investigate AFPS between two identical 2D and 3D discrete-time chaotic systems with uncertain parameters. Numeric simulations are used to verify the effectiveness of our scheme.

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