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The generalized Q-S synchronization between the generalized Lorenz canonical form and the Rössler system

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Abstract

In this paper, we investigate the generalized Q-S synchronization between the generalized Lorenz canonical form and the Rössler system. Firstly, we transform an arbitrary generalized Lorenz system to the generalized Lorenz canonical form, and the relation between the parameter of the generalized Lorenz system and the parameter of the generalized Lorenz canonical form are shown. Secondly, we extend the scheme present by [Yan ZY. Chaos 2005;15:023902] to study the generalized Q-S synchronization between the generalized Lorenz canonical form and the Rössler system, the more general controller is obtained. By choosing different parameter in the generalized controller obtained here, without much extra effort, we can get the controller of synchronization between the Chen system and the Rössler system, the Lü system and the Rössler system, the classic Lorenz system and the Rössler system, the Hyperbolic Lorenz system and the Rössler system, respectively. Finally, numerical simulations are used to perform such synchronization and verify the effectiveness of the controller.

1. Introduction

Chaos is an interesting complex dynamical phenomenon which plays important role in the field of non-linear science. Recently, the traditional trend of analyzing and understanding chaos has evolved to a new phase. Research in this field moved to chaos control, synchronization and modeling including not only suppressing chaos when it is harmful, but also chaotification, i.e. generating chaos intentionally when it is useful. Controlling and utilizing chaos has been extensively studied within the scientific, engineering and mathematical communities for more than three decades [1–11,13–25]. In Particular, chaos synchronization has received a significant attention in the last few years [1–11,21–25], since Pecora and Carroll [2] presented the chaos synchronization method to synchronize two identical chaotic systems with different initial values in 1990. More recently, based on a backstepping design with one controller, Yan [9] presented a systematic and constructive scheme to investigate the Q-S lag or anticipated synchronization between the continuous-time drive

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system and the response system with a strict-feedback form. With the aid of symbolic computation, the scheme can be performed automatically in the computer.

In this paper, we extend the scheme [9] to study the generalized Q-S synchronization between the canonical form and the Rössler system, the more generalized controller is obtained. As is known, Lorenz system in a simple three-dimensional autonomous system is the earliest system to be found [12]. Recently, Celikovsky and Chen [16] present a new generalized Lorenz canonical form, such a canonical representation enables subtle of the Lorenz system and chaos tuning, which is a useful tool for chaos analysis. In fact, the classical Lorenz system, the Chen system and the Lü system all belong to generalized Lorenz canonical family, therefore, they can be transformed into the canonical form. In addition, the Hyperbolic generalized Lorenz can also be transformed the same form [18]. We will list the response relations between the parameters of the generalized Lorenz system and the parameters of the canonical form. Based on the symbolic computation system *Maple*, the synchronization of canonical form and Rössler system is studied, and the common controller are obtained, from which we can obtain the controllers of synchronization between the classical Lorenz system (the Chen system, the Lü system, the Hyperbolic generalized Lorenz system and so on) and the Rössler system by choosing the some corresponding parameters without much extra effort. Moreover, the above scheme is effective for the Hyperbolic generalized Lorenz system. Numerical simulations are used to perform such synchronization and verify the effectiveness of the controller.

The paper is organized as follows: in Section 2, we introduce the generalized Lorenz canonical form, and give the relations of the parameters. In Section 3, we investigate the generalized Q-S synchronization of the canonical form and Rössler system. Finally, conclusions are presented.

2. The introduction of the generalized Lorenz canonical form

The non-linear system of ordinary differential equations in the three-dimension real space in the following form are called the generalized Lorenz system.

$$\dot{x} = \begin{pmatrix} A & 0 \\ 0 & \lambda_3 \end{pmatrix} x + x_1 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} x, \tag{2.1}$$

in which $x = (x_1, x_2, x_3)^T$, $\lambda_3 \in R$, A is a (2×2) real matrix.

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \tag{2.2}$$

with eigenvalues λ_1, $\lambda_2 \in R$, such that

$$-\lambda_2 > \lambda_1 > -\lambda_3 > 0 \tag{2.3}$$

Theory 1. For the non-singular generalized Lorenz system (2.1)–(2.3), there is a non-singular linear coordinate transformation $z = Tx$ via which (2.1) can be transformed the following generalized Lorenz canonical form:

$$\dot{z} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} z + (1, -1, 0) z \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & -1 \\ 1 & \tau & 0 \end{pmatrix} z \tag{2.4}$$

In fact, for an arbitrary three-dimension system, if A has two linear independent real eigenvalue, it must exist a non-singular linear coordinate transformation $z = Tx$, via which the system (2.1) can be transformed into the other five standard forms. Because the solutions of the other five stand forms approach zero or infinity or limit circle, it is to say, only the system (2.4) is the one we are interested in. For the other five forms have no role in this paper, we omit them.

In the canonical forms, the classical Lorenz system response to the $\tau > 0$, the Chen system response to $-1 < \tau < 0$, the Lü system response to $\tau = 0$, the Hyperbolic generalized Lorenz system response to $\tau < -1$. The graphs are shown in Figs. 1–3.

The scheme that transform a generalized Lorenz system into a canonical form can be seen in Ref. [6]. We design a *Maple* program, if inputting the a_{11}, a_{21}, a_{12}, a_{22}, λ_1, τ, the output is λ_1, λ_2, λ_3, τ. For the canonical forms are determined by λ_1, λ_2, λ_3, τ, only need to compute the value of λ_1, λ_2, λ_3, τ, the canonical form is solved.

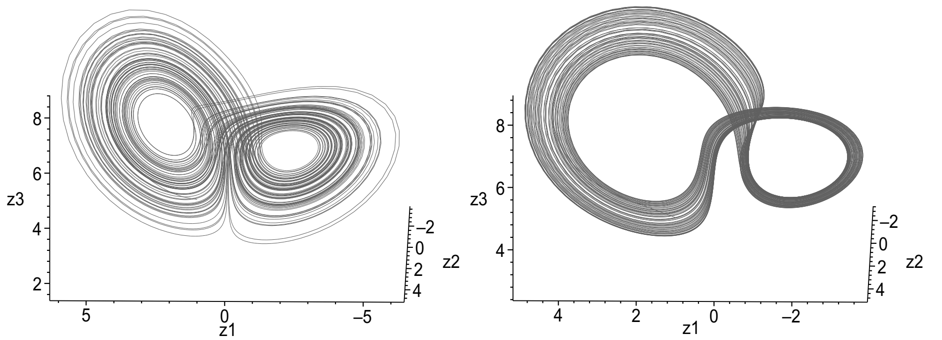

Fig. 1. The classical Lorenz system and the Lü system with conditions: $\lambda_1 = 8$, $\lambda_2 = -16$, $\lambda_3 = -1$, $\tau = 0.6$ and $\lambda_1 = 8$, $\lambda_2 = -16$, $\lambda_3 = -1$, $\tau = 0$.

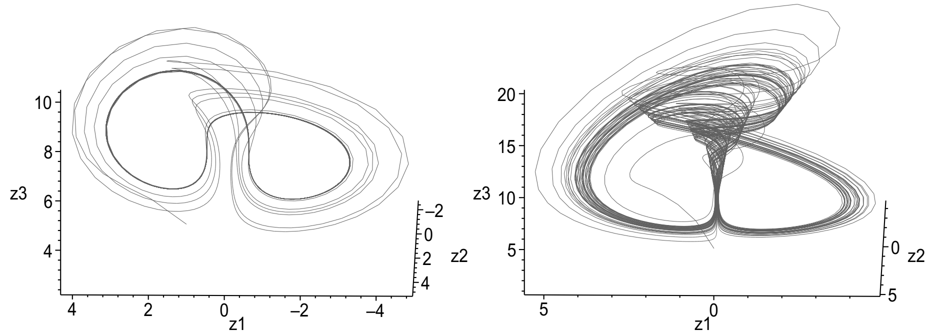

Fig. 2. The Chen system and the Hyperbolic generalized Lorenz system with conditions: $\lambda_1 = 8$, $\lambda_2 = -16$, $\lambda_3 = -1$, $\tau = -0.9$ and $\lambda_1 = 8$, $\lambda_2 = -16$, $\lambda_3 = -1$, $\tau = -5$.

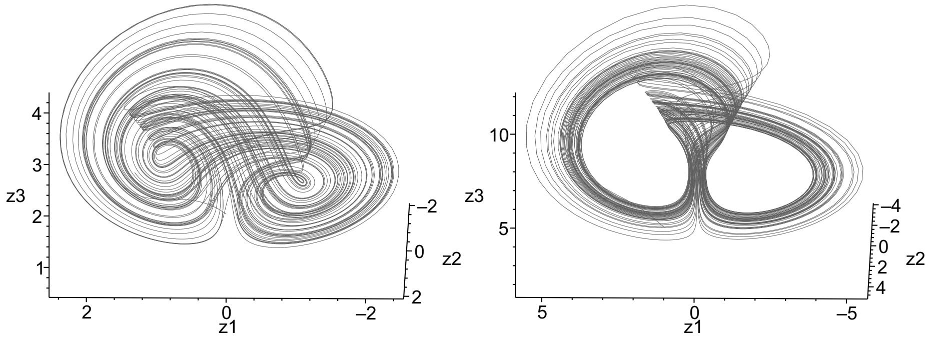

Fig. 3. The Chen system and the condition between the generalized Lorenz system and the Hyperbolic Lorenz system with conditions: $\lambda_1 = 3$, $\lambda_2 = -5$, $\lambda_3 = -1$, $\tau = -0.8$ and $\lambda_1 = 8$, $\lambda_2 = -16$, $\lambda_3 = -1$, $\tau = -1$.

Through our calculation, the relations between a_{11}, a_{21}, a_{12}, a_{22}, λ_1, τ and λ_1, λ_2, λ_3, τ, are as following:

$$\lambda_1 = \frac{a_{11} + a_{22} + \sqrt{(a_{11} - a_{22})^2 + 4a_{11}a_{22}}}{2}, \quad \lambda_2 = \frac{a_{11} + a_{22} - \sqrt{(a_{11} - a_{22})^2 + 4a_{11}a_{22}}}{2}, \quad \lambda_3 = \lambda_3, \quad \tau = \gamma\delta$$

in which

$$\gamma=\frac{b_{23}}{b_{13}\sqrt{b_{13}}},\quad \delta=\frac{b_{32}}{\sqrt{b_{31}}},\quad b_{13}=\frac{\sqrt{2}(\sin(\phi_1)\sqrt{\tilde{a}_{21}/\tilde{a}_{12}}+\cos(\phi_1))}{2},$$

$$b_{23}=\frac{\sqrt{2}(\sin(\phi_1)\sqrt{\tilde{a}_{21}/\tilde{a}_{12}}-\cos(\phi_1))}{2},\quad b_{31}=\frac{\sqrt{2}(\sin(\phi_1)\sqrt{\tilde{a}_{21}/\tilde{a}_{12}}+\cos(\phi_1))}{2},$$

$$b_{32}=-\frac{\sqrt{2}(\sin(\phi_1)\sqrt{\tilde{a}_{21}/\tilde{a}_{12}}-\cos(\phi_1))}{2},\quad \tilde{a}_{11}=\tilde{a}_{22}=\frac{a_{11}+a_{22}}{2},$$

$$\tilde{a}_{12}=\frac{a_{12}-a_{21}\pm\sqrt{(a_{12}+a_{21})^2+(a_{11}-a_{22})^2}}{2},\quad \tilde{a}_{12}=\frac{a_{21}-a_{12}\pm\sqrt{(a_{12}+a_{21})^2+(a_{11}-a_{22})^2}}{2}$$

$$\sin(2\phi_1)=\pm\frac{a_{22}-a_{11}}{\sqrt{\sqrt{(a_{12}+a_{21})^2+(a_{11}-a_{22})^2}}^2},\quad \cos(2\phi_1)=\pm\frac{a_{12}-a_{21}}{\sqrt{\sqrt{(a_{12}+a_{21})^2+(a_{11}-a_{22})^2}}^2}$$

in last two equations the sign of $\pm$ relies on the condition that $\phi_1\in[0,\frac{\pi}{2}]$.

Taking the new unified chaotic system for example, we use our *Maple* program to obtain the value of λ_1, λ_2, λ_3, τ The unified chaotic system as follows:

$$\begin{cases}\dot{x}_1=(25\alpha+10)(x_2-x_1)\\ \dot{x}_2=(28-35\alpha)x_1-x_1x_3+(29\alpha-1)x_2\\ \dot{x}_3=x_1x_2-\frac{8+\alpha}{3}x_3\end{cases}\tag{2.5}$$

where $\alpha\in[=0.016,1.15]$. Let $x=(x_1,x_2,x_3)^T$, system (2.5) can be written in the forms of (2.1), in which

$$A=\begin{pmatrix}-(25\alpha+10) & (25\alpha+10)\\ (28-35\alpha) & (29\alpha-1)\end{pmatrix},\quad \lambda_3=-\frac{8+\alpha}{3}$$

When $-0.016\leqslant\alpha\leqslant 0.8$, the system (2.5) belongs to the classical Lorenz system; when $\alpha=0.8$ the system (2.5) belongs to the Lü system; when $0.8<\alpha\leqslant 1.15$ the system (2.5) belongs to the Chen system. Now, we take $\alpha=0$, 0.8, 1, respectively to compute the λ_1, λ_2, λ_3, τ.

When $\alpha=0$, the input is $a_{11}=-(25\alpha+10)=-10$, $a_{12}=(25\alpha+10)=10$, $a_{21}=(28-35\alpha)=28$, $a_{22}=(29\alpha-1)=-1$, $\lambda_3=-\frac{8+\alpha}{3}=-\frac{8}{3}$, through our *Maple* program the output is $\lambda_1=11.8277$, $\lambda_2=-22.8277$, $\lambda_3=-\frac{8}{3}$, $\tau=0.5877$, it belongs to the classical Lorenz system. Similarly, when $\alpha=1$, the input is $(a_{11},a_{12},a_{21},a_{22},\lambda_3)=(-35,35,-7,28,-3)$, the output is $(\lambda_1,\lambda_2,\lambda_3,\tau)=(23.8359,-30.8359,-3,-0.7078)$, it belongs to the Chen system. when $\alpha=0.8$, the input is $(a_{11},a_{12},a_{21},a_{22},\lambda_3)=(-30,30,0,22.2,-8.8/3)$, the output is $(\lambda_1,\lambda_2,\lambda_3,\tau)=(22.2,-30,-2.93333,-0.5757\times 10^{-10})$, it belongs to the Lü system. It can be seen that the result according with the analysis above.

3. The generalized Q-S synchronization in the generalized Lorenz canonical form and the Rössler system

Recently, a kind of generalized-type synchronization called Q-S synchronization between two dynamical systems was defined by Yang [21]. More recently, Yan [9] investigated the Q-S synchronization between the Rössler system and the new unified chaotic system in [19].

For two dynamical systems

$$\dot{x}=F(x,y),\quad \dot{y}=G(x,y),\ \ (x,y)\in R^{n\times n}$$

let $Q_1(x)$, $Q_2(x),\ldots,Q_h(x)$ and $S_1(x)$, $S_2(x),\ldots,S_h(x)$ be observable of the above two systems respectively. The above two systems are said to be synchronizable with respect to $(Q_1(x), Q_2(x),\ldots,Q_h(x))$ and $(S_1(x), S_2(x),\ldots,S_h(x))$, if $\lim_{t\to\infty}[Q_i(x(t))-S_i(y(t))]=0$, $i=1, 2,\cdots,h$. For convenience, we call this synchronization the Q-S synchronization.

In this paper, we will study the generalized Q-S synchronization between the more general chaotic system: the generalized Lorenz canonical form and the Rössler system.

Taking the canonical form as the driven system

$$\begin{cases}\dot{x}_1=\lambda_1x_1-z_1(x_1-y_1)\\ \dot{y}_1=\lambda_2y_1-z_1(x_1-y_1)\\ \dot{z}_1=\lambda_3z_1+(x_1+\tau y_1)(x_1-y_1)\end{cases}\tag{3.1}$$

and the Rössler system is the response system

$$\begin{cases} \dot{x}_2 = -y_2 - z_2 \\ \dot{y}_2 = x_2 + \alpha y_2 \\ \dot{z}_2 = b + x_2 z_2 - c z_2 + u \end{cases} \tag{3.2}$$

where u is the controller.

Let the lyapunov function be $L(t) = \frac{1}{2}\left(E_1^2 + E_2^2 + E_3^2\right)$, $Q_1(t) = y_2$, $S_1(t) = y_1$, $\frac{\partial Q_1}{\partial y_2} = 1 \neq 0$. With the help of symbolic computation, by using the scheme [9], we can obtain E_1, E_2, E_3 as following:

$$E_1(t) = Q_1(t) - S_1(t) = y_2 - y_1 \tag{3.3}$$

$$E_2(t) = Q_2(t) - S_2(t) = c_1 E_1(t) + \dot{E}_1(t) \tag{3.4}$$

$$E_3(t) = Q_3(t) - S_3(t) = E_1(t) + c_2 E_2(t) + \dot{E}_2(t) \tag{3.5}$$

where $c_i \in R^+ (i = 1, 2, 3)$.

Finally, through solving the equation $E_2(t) + c_3 E_3(t) + \dot{E}_3(t) = 0$, we can get the controller u as following.

$$\begin{aligned} u = {} & c_2c_1x_2(t) - c_2\lambda_2^2 y_1(t) + c_3c_2x_2(t) - b - \tau(x_1(t))^3 z_1(t) - c_3\lambda_2^2 y_1(t) + c_1 a x_2(t) + (y_1(t))^2\lambda_1 x_1(t) + (z_1(t))^2\lambda_1 y_1(t) \\ & + c_1a^2y_2(t)^2 - 2\lambda_2 y_1(t) - x_2(t)z_2(t) + cz_2(t) + x_2(t) - c_1y_1(t) + 2z_1(t)x_1(t) - 2z_1(t)y_1(t) + c_3\tau(y_1(t))^3 \\ & + c_3a^2y_2(t) + c_3ax_2(t) + c_3x_1(t)(y_1(t))^2 + c_3c_1x_2(t) - 2c_3(x_1(t))^2y_1(t) + c_3c_1z_1(t)x_1(t) - 2c_3\lambda_2 z_1(t)y_1(t) \\ & - c_3c_2c_1y_1(t) + c_3c_2z_1(t)x_1(t) - c_3c_2\lambda_2y_1(t) + c_3c_2ay_2(t) - c_3c_2z_1(t)y_1(t) + c_3c_2c_1y_2(t) + c_3z_1(t)\lambda_1x_1(t) \\ & - c_3c_1\lambda_2y_1(t) - c_3c_1z_1(t)y_1(t) + c_3\lambda_2z_1(t)x_1(t) + c_3\tau y_1(t)(x_1(t))^2 + c_3x_1(t)\lambda_3z_1(t) - 2c_3x_1(t)\tau(y1(t))^2 \\ & - c_3y_1(t)\lambda_3z_1(t) + c_3c_1ay_2(t) - c_3y_1(t) + c_3(x_1(t))^3 - c_3z_2(t) - 3(y_1(t))^2z1(t)x1(t) + 3(x_1(t))^2z_1(t)y_1(t) \\ & + z_1(t)\lambda_1^2x_1(t) - (z_1(t))^2\lambda_1x_1(t) + \lambda_2(z_1(t))^2x_1(t) - \lambda_2(z_1(t))^2y_1(t) - 5(x_1(t))^2y_1(t)\lambda_1 + \tau(y_1(t))^3z_1(t) \\ & + 4(x_1(t))^3\lambda_1 - x_1((t))^3z_1(t) + 3\tau y_1(t)(x_1(t))^2\lambda_1 + 3\tau y_1(t)(x_1(t))^2z_1(t) - 3\tau(y_1(t))^2x_1(t)z_1(t) + c_2z_1(t)\lambda_1x_1(t) \\ & + c_1z_1(t)\lambda_1x_1(t) + \lambda_2z_1(t)\lambda_1x_1(t) + 2\lambda_3z_1(t)\lambda_1x_1(t) - 3\tau(y_1(t))^2\lambda_1x_1(t) + (y_1(t))^3z_1(t) - \lambda_2^3y_1(t) - 5(x_1(t))^2\lambda_2y_1(t) \\ & - 3\lambda_2^2z_1(t)y_1(t) + 4x_1(t)(y_1(t))^2\lambda_2 + \lambda_2^2z_1(t)x_1(t) + 5\tau(y_1(t))^3\lambda_2 - c1\lambda_2^2y_1(t) - c_2c_1\lambda_2y_1(t) + c_2c_1z_1(t)x_1(t) \\ & - c_2c_1z_1(t)y_1(t) - 2c_1z_1(t)\lambda_2y_1(t) + c_2\lambda_2z_1(t)x_1(t) - 2c_2\lambda_2z_1(t)y_1(t) - 7\tau(y_1(t))^2x_1(t)\lambda_2 + 2\tau(x_1(t))^2\lambda_2y_1(t) \\ & - 3\lambda_3z_1(t)\lambda_2y_1(t) + c_1\lambda_2z_1(t)x_1(t) + x_1(t)\lambda_3^2z_1(t) - 2c_1(x_1(t))^2y_1(t) + c_1(x_1(t))^3 + c_2(x_1(t))^3 + \lambda_2(x_1(t))^3 \\ & + (x_1(t))^3\lambda_3 - c_2y_1(t)\lambda_3z_1(t) - 2c_2(y_1(t))^2\tau x_1(t) + c_1x_1(t)\lambda_3z_1(t) + c_1(x_1(t))^2\tau y_1(t) - 2c_1x_1(t)\tau(y_1(t))^2 \\ & - 2(y_1(t))^2\lambda_3\tau x_1(t) + c_2x_1(t)\lambda_3z_1(t) + c_2(x_1(t))^2\tau y_1(t) + \lambda_2x_1(t)\lambda_3z_1(t) + (x_1(t))^2\lambda_3\tau y_1(t) - c_1y_1(t)\lambda_3z_1(t) \\ & - y_1(t)\lambda_3^2z_1(t) - 2c_2y_1(t)(x_1(t))^2 + c_2(y_1(t))^2x_1(t) + c_2(y_1(t))^3\tau - 2y_1(t)\lambda_3(x_1(t))^2 + (y_1(t))^2\lambda_3x_1(t) + (y_1(t))^3\lambda_3\tau \\ & + c_1(y_1(t))^2x_1(t) + c_1(y_1(t))^3\tau - c_2y_2(t) - c_2z_2(t) - c_1z_2(t) - az_2(t) + c_2c_1ay_2(t) + c_2a^2y_2(t) + c_2ax_2(t) \\ & + a^2x_2(t) + a^3y_2(t) \end{aligned}$$

Taking the given value of λ_1, λ_2, λ_3, τ from u can make synchronization between the different chaos systems belonging to the generalized Lorenz canonical family and the Rössler system.

In the following, we would like to use numerical simulation to verify the effectiveness of the obtained controller u. We take the initial values of system (3.2) and (3.1) as $a = 0.2$, $b = 0.2$, $c = 5.7$, $h_1 = 1$, $h_2 = 2$, $h_3 = 3$, $[x_2(0) = 0.2, y_2(0) = 0.3, z_2(0) = 0.5]$ and $[x_1(0) = 1, y_1(0) = 2, z_1(0) = 3]$.

Case 1. Synchronization of the classical Lorenz system and the Rössler system, taking $\lambda_1 = 8$, $\lambda_2 = -16$, $\lambda_3 = -1$, $\tau = 0.6$ from the general controller u, the canonical form belongs to the classical Lorenz system, so we can obtain the synchronization between the classical Lorenz system and the Rössler system, the initial values of the error dynamical system (3.3)–(3.5) is $E_1(0) = -1.700$, $E_2(0) = 27.560$, $E_3(0) = -256.868$. Numerical simulations of the synchronization of the driven system (3.1) and response system (3.2) are shown in the Fig. 4.

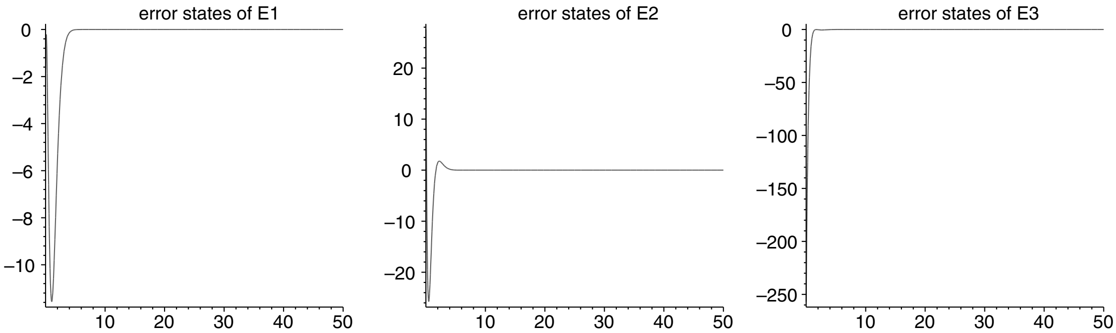

Fig. 4. The classical Lorenz synchronization errors with conditions: $\lambda_1 = 8$, $\lambda_2 = -16$, $\lambda_3 = -1$, $\tau = 0.6$.

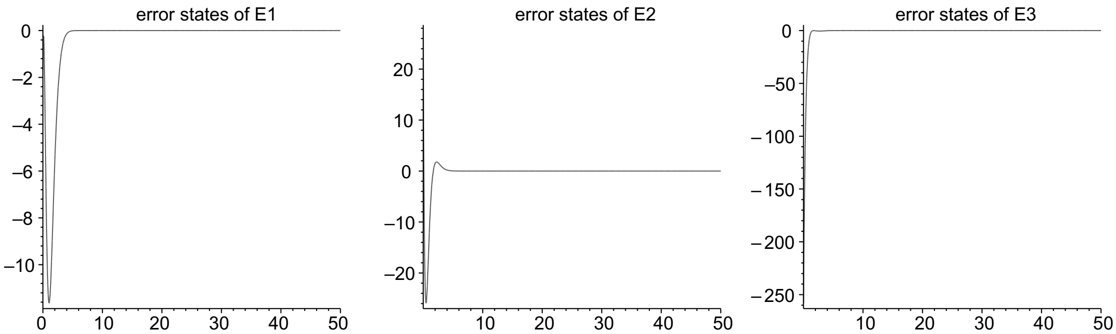

Fig. 5. The Lü synchronization errors with conditions: $\lambda_1 = 8$, $\lambda_2 = -16$, $\lambda_3 = -1$, $\tau = 0$.

Case 2. Synchronization of the Lü synchronization and the Rössler system, taking $\lambda_1 = 8$, $\lambda_2 = -16$, $\lambda_3 = -1$, $\tau = 0$ from the general controller u, the canonical form belongs to the Lü system, so we can obtain the synchronization between Lü system and the Rössler system, the initial values of the error dynamical system (3.3)–(3.5) is $E_1(0) = -1.700$, $E_2(0) = 27.560$, $E_3(0) = -256.068$. Numerical simulations of the synchronization of the driven system (3.1) and response system (3.2) are shown in Fig. 5.

Case 3. Synchronization of the Chen synchronization and the Rössler system, taking $\lambda_1 = 8$, $\lambda_2 = -16$, $\lambda_3 = -1$, $\tau = -0.2$ from the general controller u, the canonical form belongs to the Chen system, so we can obtain the synchronization between the Chen system and the Rössler system, the initial values of the error dynamical system (3.3)–(3.5) is $E_1(0) = -1.700$, $E_2(0) = 27.560$, $E_3(0) = 258.468$. Numerical simulations of the synchronization of the driven system (3.1) and response system (3.2) are shown in Fig. 6.

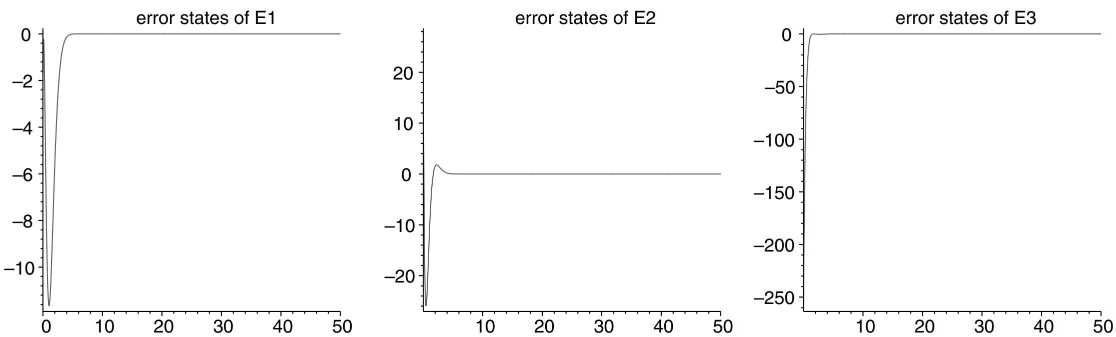

Fig. 6. The Chen synchronization errors with conditions: $\lambda_1 = 8$, $\lambda_2 = -16$, $\lambda_3 = -1$, $\tau = -0.2$.

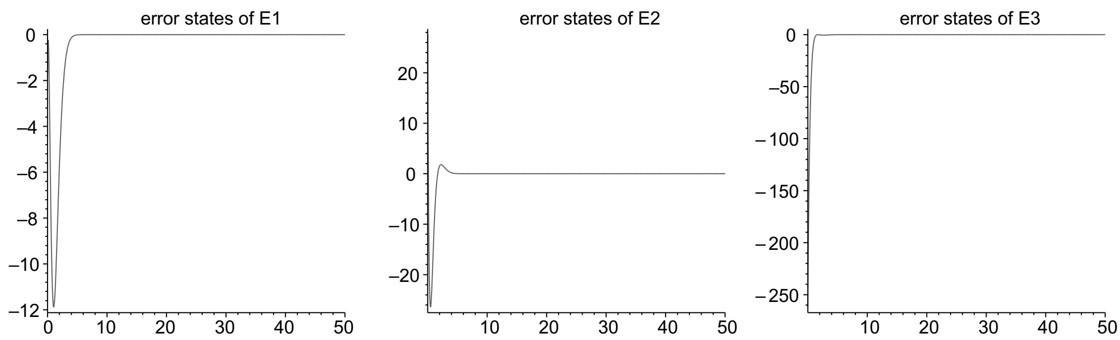

Fig. 7. The Hyperbolic generalized Lorenz synchronization errors with conditions: $\lambda_1 = 8$, $\lambda_2 = -16$, $\lambda_3 = -1$, $\tau = -2$.

Case 4. Synchronization of the Hyperbolic generalized Lorenz system and the Rössler system. Taking $\lambda_1 = 8$, $\lambda_2 = -16$, $\lambda_3 = -1$, $\tau = -2$ from the general controller u, the canonical form belongs to the Hyperbolic generalized Lorenz system, so we can obtain the synchronization between the Hyperbolic generalized Lorenz system and the Rössler system, the initial values of the error dynamical system (3.3)–(3.5) is $E_1(0) = -1.700$, $E_1(0) = 27.560$, $E_3(0) = -262.068$. Numerical simulations of the synchronization of the driven system (3.1) and response system (3.2) are shown in Fig. 7.

In fact, take any arbitrary λ_1, λ_2, λ_3, τ, we can obtain the synchronization between the other Lorenz chaotic system that belongs to canonical form and the Rössler system.

4. Summary and conclusions

Based on a systematic and constructive scheme to investigate synchronization and by means of *Maple*, we have studied synchronization between the continuous-time drive system: the generalized Lorenz canonical form and the response system with a strict-feedback form: the Rössler system. Due to the generalized Lorenz canonical form that covers a broader class of chaotic systems, we only need to choose different parameter in the generalized controller found in this paper, we can obtain some different controller for synchronization between the Chen system and the Rössler system, the Lü system and the Rössler system, the classic Lorenz system and the Rössler system, the Hyperbolic Lorenz system and the Rössler system, respectively. Numerical simulations are used to perform such synchronization and verify the effectiveness of the controller.

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