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Adomian Decomposition Method and Padé Approximants for Nonlinear Differential-Difference Equations*

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Abstract *Combining Adomian decomposition method (ADM) with Padé approximants, we solve two differential-difference equations (DDEs): the relativistic Toda lattice equation and the modified Volterra lattice equation. With the help of symbolic computation Maple, the results obtained by ADM-Padé technique are compared with those obtained by using ADM alone. The numerical results demonstrate that ADM-Padé technique give the approximate solution with faster convergence rate and higher accuracy and relative in larger domain of convergence than using ADM.*

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Key words: Adomian decomposition method, Padé approximants, relativistic Toda lattice equation, modified Volterra lattice equation

1 Introduction

Since the work of Fermi *et al.* in the 1950s,[1] differential-difference equations (DDEs) have been the focus of many nonlinear studies. There is a vast body of work on it.[2−8] The DDEs play an important role in modeling complicated physical phenomena such as particle vibrations in lattices, currents flow in electrical networks, and pulses in biological chains. Unlike difference equations, which are fully discrete, DDEs are semi-discrete with some (or all) of their spacial variables discrete while time is usually kept continuous.

The Adomian decomposition method (ADM)[9−11] is powerful to obtain approximate solution or even closed form analytical solution of differential equation[12−18] without linearization or perturbation and provides an efficient numerical solution with minimal calculation. However, in some cases, the convergence interval of the ADM series solution is very small and outside it high error is obtained. To overcome the drawback, the Padé approximants, which often show superior performance over series approximation, are employed to the series solution of ADM to improve the accuracy and enlarge the convergence domain. The nature idea is that we first use ADM to obtain series solution of the DDEs and PDEs and employ Padé approximants to improve the accuracy and enlarge the convergence domain. Recently, combing ADM with Padé approximants, Mehdi Dehghan *et al.*[19] Wazwaz[20] investigated the continuous solitary system. More recently, Yang *et al.*[21] and Wang *et al.*[22] studied the differential-difference equations. Numerical and graphical illustrations show that it is a promising tool for solving nonlinear problem.

In this paper, we combine ADM with Padé approximants to solve the relativistic Toda lattice equation,[23,24]

$$\begin{aligned} u_t(n,t) &= (1+\alpha u(n,t))(v(n,t)-v(n-1,t))\,, \\ v_t(n,t) &= v(n,t)(u(n+1,t)-u(n,t) \\ &\quad + \alpha v(n+1,t)-\alpha v(n-1,t))\,, \end{aligned} \tag{1}$$

and the modified Volterra lattice equation,[25,26]

$$u_t(n,t) = (u(n,t)^2-1)(u(n+1,t)-u(n-1,t))\,, \tag{2}$$

where the subscript n in Eqs. (1) and (2) represents the n-th lattice.

It is arranged as follows. In Sec. 2, ADM–Padé technique for solving the discrete differential-difference equations is outlined. In Sec. 3, the relativistic Toda lattice equation is studied. And the modified Volterra lattice equation is investigated in Sec. 4. Finally, conclusions are followed.

2 Description of ADM-Padé Technique

2.1 *Description of ADM for Solving DDEs*

For the purposes of the illustration of the decomposition method, we consider a system of nonlinear differential-difference equation as follows:

$$\begin{aligned} L(u(n,t)) = g(n,t) &+ R(u_i(n,t),u_i(n-1,t), \\ & u_i(n+1,t),\ldots) + N(u_i(n,t), \\ & u_i(n-1,t),u_i(n+1,t),\ldots)\,, \end{aligned} \tag{3}$$

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where $u_i(n,t)$, $u_i(n-1,t)$, and $u_i(n+1,t)$ is the unknown function with respect to the discrete spatial variable n and the temporal variable t; L is the highest-order derivative, which is assumed to be invertible; R is the remaind linear operator and N is the nonlinear operator and g is the source term.

Applying the inverse operator L^{-1} on both sides of Eq. (3) gives

$$L^{-1}L(u(n,t)) = L^{-1}g(n,t) + L^{-1}R(u_i(n,t), u_i(n-1,t), u_i(n+1,t),\ldots) + L^{-1}N(u_i(n,t), u_i(n-1,t), u_i(n+1,t),\ldots)\,. \tag{4}$$

Using the initial conditions, we get

$$u(n,t) = f(n,t) + L^{-1}R(u_i(n,t), u_i(n-1,t), u_i(n+1,t),\ldots) + L^{-1}N(u_i(n,t), u_i(n-1,t), u_i(n+1,t),\ldots)\,, \tag{5}$$

where the function f represents the term arising from integrating the source term g and the given initial or boundary conditions. According to the ADM,[9−11] we assume that a series solution of the unknown function $u(n,t)$ can be expressed by an infinite series of the form:

$$u(n,t) = \sum_{m=0}^{\infty} u_m(n,t)\,. \tag{6}$$

The nonlinear term N will be decomposed by the infinite series of the Adomian polynomials,

$$N(u_i(n,t), u_i(n-1,t), u_i(n+1,t),\ldots) = \sum_{m=0}^{\infty} A_m\,,$$
$$A_m = A_m(u_i(n,t), u_i(n-1,t), u_i(n+1,t),\ldots)\,, \tag{7}$$

A_m are the so-called Adomian polynomials. In order to determine the Adomian polynomials, we introduce a parameter λ and equation (7) becomes

$$N\Big(\sum_{m=0}^{\infty} u_{i,m}(n,t)\lambda^m, \sum_{m=0}^{\infty} u_{i,m}(n-1,t)\lambda^m, \sum_{m=0}^{\infty} u_{i,m}(n+1,t)\lambda^m,\ldots\Big) = \sum_{m=0}^{\infty} A_m\lambda^m\,. \tag{8}$$

Let $u_{i,\lambda}(u,t) = \sum_{m=0}^{\infty} u_{i,m}(n,t)\lambda^m$, then

$$\begin{aligned} A_m &= \frac{1}{m!}\Big[\frac{\mathrm{d}^m}{\mathrm{d}\lambda^m} N(u_{i,\lambda}(u,t), u_{i,\lambda}(u-1,t), u_{i,\lambda}(u+1,t),\ldots)\Big]_{\lambda=0} \\ &= \frac{1}{m!}\Big[\frac{\mathrm{d}^m}{\mathrm{d}\lambda^m} N\Big(\sum_{m=0}^{\infty} u_{i,m}(n,t)\lambda^m, \sum_{m=0}^{\infty} u_{i,m}(n-1,t)\lambda^m, \sum_{m=0}^{\infty} u_{i,m}(n+1,t)\lambda^m,\ldots\Big)\Big]_{\lambda=0}\,. \end{aligned} \tag{9}$$

To determine the components $u_m(n,t)$, $m \geq 0$, we employ the recursive relation

$$u_0(n,t) = f(n,t)\,, \quad u_{m+1}(n,t) = L^{-1}R(u_m(n,t), u_m(n-1,t), u_m(n+1,t),\ldots) + L^{-1}A_m\,, \quad m \geq 0\,. \tag{10}$$

The expression

$$\phi_r = \sum_{m=0}^{r} u_m(n,t)\,, \tag{11}$$

denotes the r-term approximation to $u(n,t)$.

2.2 *Padé Approximants on Series Solution*

When we obtain the truncated series solution $u(n,t)$ of order at least $(L+M)$ in t by ADM and use it, we can obtain Padé $[L/M](n,t)$ approximants solution for $u(n,t)$. The procedure is to seek a rational function for the series. Given a known function $f(z)$ expanded in a Maclaurin series

$$f(z) = \sum_{n=0}^{\infty} c_n z^n\,, \tag{12}$$

we can use the coefficients of the series to represent the function by a ratio of two polynomials

$$\frac{A^{[L/M](Z)}}{B^{[L/M]}(z)} = \frac{a_0 + a_1 z + \cdots + a_L z^L}{b_0 + b_1 z + \cdots + b_M z^M}\,, \tag{13}$$

symbolized by $[L/M]$ and called the Padé approximants. The basic idea is to match the series coefficients as far as possible. Even though the series has a finite region of convergence, we can obtain the limit of the function as $z \to \infty$ if $L = M$. We note that there are $L+1$ independent coefficients in the numerator and $M+1$ coefficients in the denominator. To make the system determinable, let $b_0 = 1$. We then have M independent coefficients in the denominator and $L+M+1$ independent coefficients in all. Now the $[L/M]$ approximants can fit the power series through orders $1, z, z^2, \ldots, z^{L+M}$ with an error of $O(z^{L+M+1})$. Consequently,

$$a_0 + a_1 z + \cdots + a_L z^L = (b_0 + b_1 z + \cdots + b_M z^M) \times (c_0 + c_1 z + c_2 z^2 + \ldots)\,. \tag{14}$$

Balancing the each coefficients of $z^{L+1}, z^{L+2}, \ldots, z^{L+M}$, we can get

$$\begin{aligned} &b_M c_{L-M+1} + b_{M-1} c_{L-M+2} + \cdots + b_0 c_{L+1} = 0\,, \\ &b_M c_{L-M+2} + b_{M-1} c_{L-M+3} + \cdots + b_0 c_{L+2} = 0\,, \\ &\vdots \\ &b_M c_L + b_{M-1} c_{L+1} + \cdots + b_0 c_{L+M} = 0\,, \end{aligned} \tag{15}$$

Since $c_0, c_1, \ldots,$ are known, when taking $b_0 = 1$ into

Eq. (15) and solving these linear equations, we can obtain b_i ($i = 1, \ldots, M$). At the same time, we can get $a_0, a_1, \ldots, a_L$ by equating the coefficients of $1, z, z^2, \ldots, z^L$ of both sides in Eq. (14). Here

$$\begin{aligned} a_0 &= c_0\,,\\ a_1 &= c_1 + b_1 c_0\,,\\ a_2 &= c_2 + b_1 c_1 + b_2 c_0\,,\\ &\vdots\\ a_L &= c_L + c_{L-1} b_1 + \cdots + c_0 b_L\,, \end{aligned} \tag{16}$$

Therefore, substituting a_i and b_i into Eq. (13), we can calculate the diagonal approximants like [2/2], [3/3], ...

In the following, we will give two examples to illustrate the applications of ADM-Padé techniques in details.

3 Soliton Solution of Relativistic Toda Lattice Equation

Considering the relativistic Toda lattice equation (1) as follows:

$$\begin{aligned} u_t(n,t) &= (1+\alpha u(n,t))(v(n,t)-v(n-1,t))\,,\\ v_t(n,t) &= v(n,t)(u(n+1,t)-u(n,t)\\ &\quad + \alpha v(n+1,t) - \alpha v(n-1,t))\,, \end{aligned}$$

assuming the initial conditions as

$$u(n,0) = f_1(n)\,,\quad v(n,0) = f_2(n)\,, \tag{17}$$

where

$$\begin{aligned} f_1(n) &= -\frac{1}{\alpha} - c\coth(k) + c\tanh(kn+c_0)\,,\\ f_2(n) &= \frac{c}{\alpha}\coth(k) - \frac{c}{\alpha}\tanh(kn+c_0)\,, \end{aligned} \tag{18}$$

we rewrite Eq. (1) in operator form:

$$\begin{aligned} L_t u(n,t) &= v(n,t) - v(n-1,t) + \alpha u(n,t)v(n,t)\\ &\quad - \alpha u(n,t)v(n-1,t)\,,\\ L_t v(n,t) &= v(n,t)u(n+1,t) - v(n,t)u(n,t)\\ &\quad + \alpha v(n,t)v(n+1,t) - \alpha v(n,t)v(n-1,t)\,, \end{aligned} \tag{19}$$

where L_t is a first order differential operator and L_t^{-1} is an integral operator defined by

$$L_t^{-1} = \int_0^t (\cdot)\,\mathrm{d}t\,. \tag{20}$$

For convenience, we set the nonlinear terms as:

$$\begin{aligned} P(u(n,t),v(n,t)) &= u(n,t)v(n,t)\,,\\ Q(u(n,t),v(n-1,t)) &= u(n,t)v(n-1,t)\,,\\ S(v(n,t),u(n+1,t)) &= v(n,t)u(n+1,t)\,,\\ T(v(n,t),v(n+1,t)) &= v(n,t)v(n+1,t)\,,\\ K(v(n,t),v(n-1,t)) &= v(n,t)v(n-1,t)\,. \end{aligned} \tag{21}$$

Operating L_t^{-1} on both sides of Eq. (19) and using the initial conditions, we obtain

$$\begin{aligned} u(n,t) &= f_1(n) + L_t^{-1}(v(n,t) - v(n-1,t))\\ &\quad + \alpha L_t^{-1}(P(u(n,t),v(n,t))\\ &\quad - Q(u(n,t),v(n-1,t)))\,,\\ v(n,t) &= f_2(n) + L_t^{-1}(S(v(n,t),u(n+1,t))\\ &\quad - P(u(n,t),v(n,t)))\\ &\quad + \alpha L_t^{-1}(T(v(n,t),v(n+1,t))\\ &\quad - K(v(n,t),v(n-1,t)))\,. \end{aligned} \tag{22}$$

We assume the expressions of $u(n,t)$, $v(n,t)$, in the decomposition forms:

$$u(n,t) = \sum_{m=0}^{\infty} u_m(n,t)\,,\quad v(n,t) = \sum_{m=0}^{\infty} v_m(n,t)\,. \tag{23}$$

According to Eqs. (7) ∼ (9), the nonlinear terms (21) can be expressed in terms of Adomian polynomial as follows:

$$\begin{aligned} P(u(n,t),v(n,t)) &= \sum_{m=0}^{\infty} A_m\,,\\ Q(u(n,t),v(n-1,t)) &= \sum_{m=0}^{\infty} B_m\,,\\ S(v(n,t),u(n+1,t)) &= \sum_{m=0}^{\infty} C_m\,,\\ T(v(n,t),v(n+1,t)) &= \sum_{m=0}^{\infty} M_m\,,\\ K(v(n,t),v(n-1,t)) &= \sum_{m=0}^{\infty} N_m\,. \end{aligned} \tag{24}$$

For exmaple, we can get the first components of Adomian polynomial as follows:

$$\begin{aligned} A_0 &= u_0(n,t)v_0(n,t)\,,\quad B_0 = u_0(n,t)v_0(n-1,t)\,,\\ C_0 &= v_0(n,t)u_0(n+1,t)\,,\quad M_0 = v_0(n,t)v_0(n+1,t)\,,\\ N_0 &= v_0(n,t)v_0(n-1,t)\,,\\ A_1 &= u_0(n,t)v_1(n,t) + u_1(n,t)v_0(n,t)\,,\\ B_1 &= u_0(n,t)v_1(n-1,t) + u_1(n,t)v_0(n-1,t)\\ C_1 &= v_0(n,t)u_1(n+1,t) + v_1(n,t)u_0(n+1,t)\,,\\ M_1 &= v_0(n,t)v_1(n+1,t) + v_1(n,t)v_0(n+1,t)\,,\\ N_1 &= v_0(n,t)v_1(n-1,t) + v_1(n,t)v_0(n-1,t)\,,\\ A_2 &= u_0(n,t)v_2(n,t) + u_1(n,t)v_1(n,t) + u_2(n,t)v_0(n,t)\,,\\ B_2 &= u_0(n,t)v_2(n-1,t) + u_1(n,t)v_1(n-1,t)\\ &\quad + u_2(n,t)v_0(n-1,t)\,,\\ C_2 &= v_0(n,t)u_2(n+1,t) + v_1(n,t)u_1(n+1,t)\\ &\quad + v_2(n,t)u_0(n+1,t)\,,\\ M_2 &= v_0(n,t)v_2(n+1,t) + v_1(n,t)v_1(n+1,t)\\ &\quad + v_2(n,t)v_0(n+1,t)\,,\\ N_2 &= v_0(n,t)v_2(n-1,t) + v_1(n,t)v_1(n-1,t)\\ &\quad + v_2(n,t)v_0(n-1,t)\,. \end{aligned} \tag{25}$$

The components $u_m(n,t)$ and $v_m(n,t)$ can be determined by using recursive relations given by

$$\begin{aligned} u_0(n,t) &= f_1(n)\,, \\ u_m(n,t) &= L_t^{-1}(v_{m-1}(n,t) - v_{m-1}(n-1,t)) \\ &\quad + \alpha L_t^{-1}(A_{m-1} - B_{m-1})\,, \\ v_0(n,t) &= f_2(n)\,, \\ v_m(n,t) &= L_t^{-1}(C_{m-1} - A_{m-1}) \\ &\quad + \alpha L_t^{-1}(M_{m-1} - N_{m-1})\,. \end{aligned} \tag{26}$$

The r-term approximate solution is evaluated as

$$\phi_r = \sum_{m=0}^{r} u_m(n,t)\,, \quad \psi_r = \sum_{m=0}^{r} v_m(n,t)\,. \tag{27}$$

With the aid of *Maple*, we can obtain fourth-order approximation

$$\phi_4 = \sum_{m=0}^{4} u_m(n,t)\,, \quad \psi_4 = \sum_{m=0}^{4} v_m(n,t)\,. \tag{28}$$

The exact solutions of the relativistic Toda lattice equation is[23]

$$\begin{aligned} u(n,t) &= -\frac{1}{\alpha} - c\coth(k) + c\tanh(kn + ct + c_0)\,, \\ v(n,t) &= \frac{c}{\alpha}\coth(k) - \frac{c}{\alpha}\tanh(kn + ct + c_0)\,, \end{aligned} \tag{29}$$

In order to verify whether the numerical solutions obtained by us achieve high accuracy, we give the figures. Here we set $\alpha = 1$, $c = 0.2$, $k = 0.5$, and $c_0 = 0$. Figure 1 shows that the series solutions of ADM give good approximation in small interval of convergence, and outside it high error is obtained.

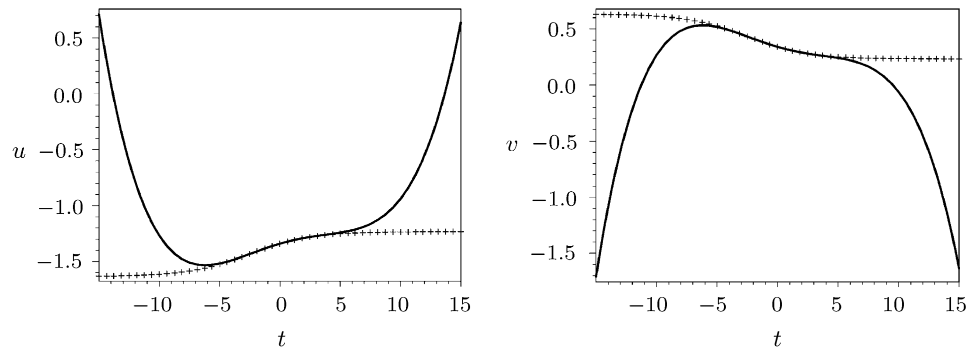

Fig. 1 The comparison between the ADM solutions and the exact solutions of the relativistic Toda lattice at $n = 1$. Line stands for the numerical solution figure and point for the exact one.

Using ADM-Padé approximants at $n = 1$, the rational approximants [2/2] is

$$\begin{aligned} u[2/2](1,t) &= \frac{-1.340367251 - 0.09242346073\,t - 0.01787156510\,t^2}{1.0 + 0.09242345331\,t + 0.01333333513\,t^2}\,, \\ v[2/2](1,t) &= \frac{0.3403672511 + 0.000000006146422623\,t + 0.004538229938\,t^2}{1.0 + 0.09242344891\,t + 0.01333333472\,t^2}\,. \end{aligned} \tag{30}$$

Figure 2 shows that ADM-Padé technique can enlarge the convergence domain of the series solution at $n = 1$. It is clear that the interval of convergence has increased by ADM–Padé technique. From Fig. 3 we can see that the absolute errors between the [2/2] ADM-Padé solutions and the exact solutions is very smaller than that between the ADM solutions and the exact one. Figure 4 shows that the ADM-Padé approximate solution of $u(n,t)$ and $v(n,t)$.

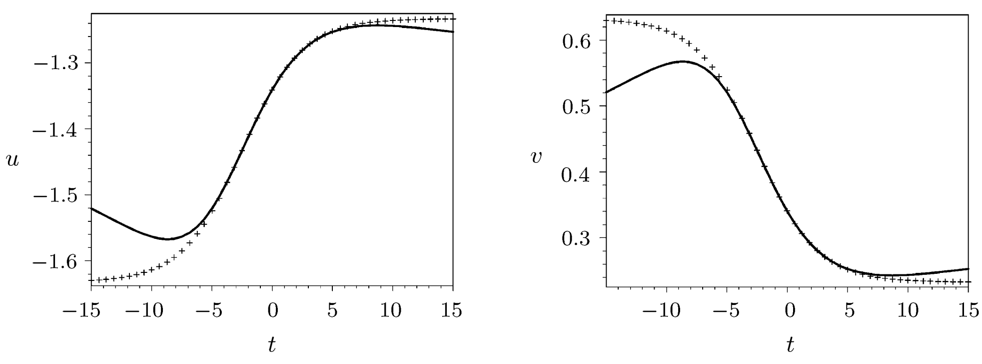

Fig. 2 The comparison between the [2/2] ADM-Padé solutions and the exact solutions of the relativistic Toda lattice at $n = 1$. Line stands for the [2/2] ADM–Padé solutions and point for the exact one.

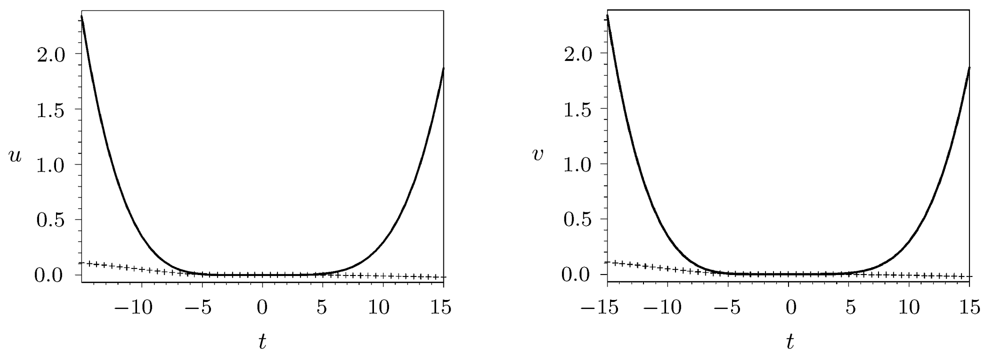

Fig. 3 The comparison of the error between the ADM solutions and [2/2] ADM–Padé solutions of the relativistic Toda lattice equation at $n=1$, Line stands for the error between ADM solution and exact solution; point for the error between the [2/2] ADM-Padé solution and the exact one.

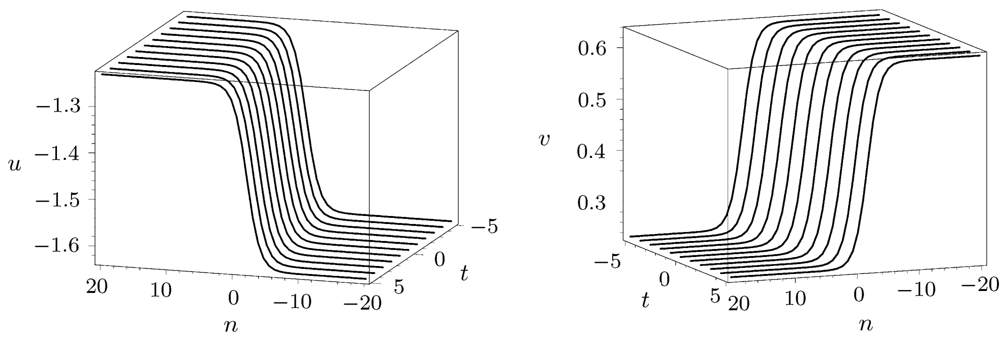

Fig. 4 The graph shows the ADM-Padé approximate solution of $u(n,t)$, $v(n,t)$ at $-5 \le t \le 5, -20 \le n \le 20$.

4 Soliton Solution of Modified Volterra Lattice Equation

Take the modified Volterra lattice equation (2) as:

$$u_t(n,t) = (u(n,t)^2 - 1)(u(n+1,t) - u(n-1,t)),$$

and the initial condition

$$u(n,0) = g(n), \tag{31}$$

where

$$g(n) = m\,\mathrm{sn}(k,m)\,\mathrm{sn}(kn+c,m), \tag{32}$$

The operator form of Eq. (2) is

$$L_t u(n,t) = u(n,t)^2 u(n+1,t) - u(n,t)^2 u(n-1,t) + u(n-1,t) - u(n+1,t), \tag{33}$$

where $L_t(\cdot) = \partial(\cdot)/\partial t$. For convenience, we set the nonlinear terms as:

$$M(u(n,t)^2, u(n+1,t)) = u(n,t)^2 u(n+1,t),$$
$$N(u(n,t)^2, u(n-1,t)) = u(n,t)^2 u(n-1,t). \tag{34}$$

Operating L_t^{-1} on both sides of Eq. (33) and using the initial condition, we obtain

$$u(n,t) = g(n) + L_t^{-1}(M(u(n,t)^2, u(n+1,t)) - N(u(n,t)^2, u(n-1,t)) + L_t^{-1}(u(n-1,t) - u(n+1,t)). \tag{35}$$

We assume the expressions of $u(n,t)$, in the decomposition forms:

$$u(n,t) = \sum_{m=0}^{\infty} u_m(n,t). \tag{36}$$

Samely, the nonlinear terms (34) can be expressed in terms of Adomian polynomial as follows:

$$M(u(n,t)^2, u(n+1,t)) = \sum_{m=0}^{\infty} A_m,$$
$$N(u(n,t)^2, u(n-1,t)) = \sum_{m=0}^{\infty} B_m. \tag{37}$$

Also, we can get the first components of Adomian polynomial as follows:

$$A_0 = u_0(n,t)^2 u_0(n+1,t),$$
$$B_0 = u_0(n,t)^2 u_0(n-1,t),$$
$$A_1 = 2u_0(n,t)u_0(n+1,t)u_1(n,t) + u_0(n,t)^2 u_1(n+1,t),$$
$$B_1 = 2u_0(n,t)u_0(n-1,t)u_1(n,t) + u_0(n,t)^2 u_1(n-1,t),$$
$$A_2 = u_1(n,t)^2 u_0(n+1,t) + 2u_0(n,t)u_1(n+1,t)u_1(n,t) + 2u_0(n,t)u_0(n+1,t)u_2(n,t) + u_0(n,t)^2 u_2(n+1,t),$$
$$B_2 = u_1(n,t)^2 u_0(n-1,t) + 2u_0(n,t)u_1(n-1,t)u_1(n,t) + 2u_0(n,t)u_0(n-1,t)u_2(n,t) + u_0(n,t)^2 u_2(n-1,t). \tag{38}$$

The components $u_m(n,t)$ can be determined by using recursive relations given by:

$$u_0(n,t) = g(n)\,,$$
$$u_m(n,t) = L_t^{-1}(A_{m-1} - B_{m-1}) + L_t^{-1}(u_{m-1}(n-1,t) - u_{m-1}(n+1,t))\,, \quad (39)$$

so the r-term approximate solution is evaluated as follows:

$$\phi_r = \sum_{m=0}^{r} u_m(n,t)\,. \quad (40)$$

Therefore, we can get sixth-order approximation

$$\phi_6 = \sum_{m=0}^{6} u_m(n,t)\,. \quad (41)$$

The exact solutions of this problem is[25]

$$u(n,t) = m\,\mathrm{sn}(k,m)\,\mathrm{sn}(kn - 2\,\mathrm{sn}(k,m)t + c, m)\,, \quad (42)$$

we set $k = 0.5$, $m = 0.25$, and $c = 0$. Using ADM–Padé approximation at $n = 1$, the rational approximations [3/3] is

$$u[3/3](1,t) = \frac{0.05720203865 - 0.09964049754\,t - 0.02325966601\,t^2 + 0.008599089448\,t^3}{1.0 + 0.001842353272\,t + 0.06971860967\,t^2 + 0.01298099644\,t^3}\,.$$

Figure 5(a) shows that the series solutions of ADM give good approximation in small interval of convergence, and outside it high error is obtained. Figure 5(b) shows that ADM–Padé technique can enlarge the convergence domian of the series solution at $n = 1$. It is clear that the interval of convergence has increased by ADM-Padé technique. Figure 6(a) shows that the absolute errors between the [3/3] ADM-Padé solutions and the exact solutions is very smaller than that between the ADM solutions and the exact one at $n = 1$. Figure 6(b) shows that the ADM approximate solution of $u(n,t)$.

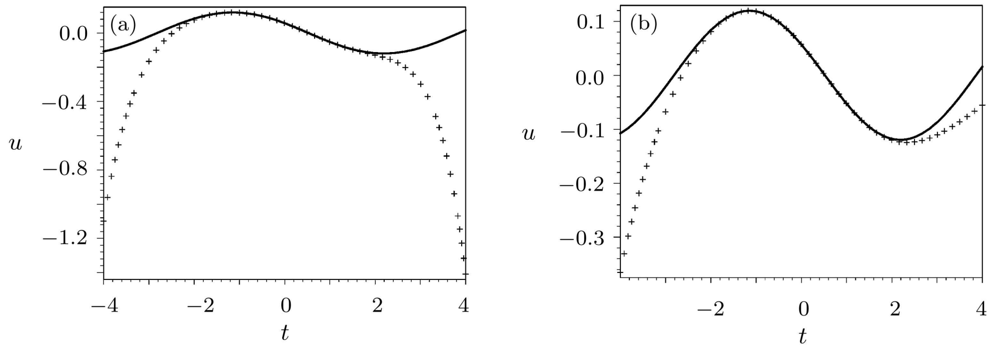

Fig. 5 The comparison between the ADM solutions and the exact solutions of the modified Volterra lattice equation at $n = 1$. Line stands for the exact solution and the point for ADM solution (a) and the [3/3] ADM–Padé solutions (b).

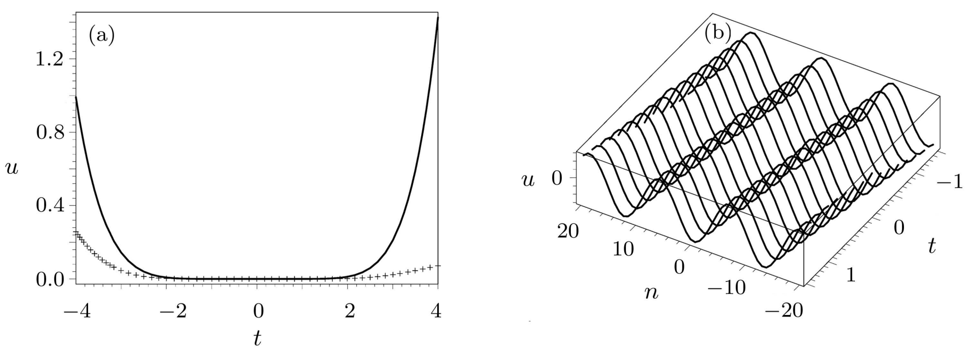

Fig. 6 The comparison of the error between the exact solution and the ADM solutions as well as the [3/3]ADM-Padé solutions of the modified Volterra lattice equation at $n = 1$, respectively. Line stands for the error between ADM solution and the exact solution and the point for the error between the [3/3] ADM-Padé solution and the exact one. The right graph shows the ADM approximate solution of $u(n,t)$, at $-1.5 \le t \le 1.5, -20 \le n \le 20$.

5 Conclusions

Combining Adomian decomposition method (ADM) with Padé approximants, we successfully solve the coupled Relativistic Toda lattice equations and the modified Volterra lattice equation. Numerical simulations show that ADM-Padé technique is an effective tool to solve nonlinear equations. It not only can improve the accuracy but also enlarge convergence domain of the truncated ADM solution. At the same time, the numerical solutions obtained can convergent the exact solutions with minimal calculation. To see approximate effective of the numerical solution by ADM-Padé technique directly, the discrete three-dimentional ADM-Padé approximate solution is drawn in three-dimentional space. However, in some cases, due to complex and tedious computation, we only drawn the figures of the ADM approximate solution, the figures of the ADM-Padé approximate solution cannot be easily given. Whether existing other methods to accelerate its convergent speed, the problem will be further studied.

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