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Symmetry Reduction, Exact Solutions, and Conservation Laws of the (2+1)-Dimensional Dispersive Long Wave Equations

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Z. Naturforsch. **64a,** 1 – 7 (2009); received November 8, 2008 / revised December 17, 2008

By means of the generalized direct method, we investigate the (2+1)-dimensional dispersive long wave equations. A relationship is constructed between the new solutions and the old ones and we obtain the full symmetry group of the (2+1)-dimensional dispersive long wave equations, which includes the Lie point symmetry group $\mathcal{S}$ and the discrete groups $\mathcal{D}$. Some new forms of solutions are obtained by selecting the form of the arbitrary functions, based on their relationship. We also find an infinite number of conservation laws of the (2+1)-dimensional dispersive long wave equations.

Key words: Dispersive Long Wave Equations; Generalized Direct Method; Explicit Solution; Conservation Laws.
PACS numbers: 02.20.-a, 02.30.Jr, 04.20.Jb, 11.30.-j

1. Introduction

Symmetry structure and the conservation law structure are two basic aspects in mathematical physics. Symmetry group techniques provide one method to obtain solutions of partial differential equations [1 – 4]. Since Sophus Lie set up the theory of Lie point symmetry group, a standard method had been widely used to find Lie point symmetry algebras and groups for almost all the known integrable systems [1]. Recently, a simple direct method presented by Clarkson and Kruskal [2, 3] was used to find all the possible similarity reductions of a nonlinear system without using any group theory. Lou and Ma [4] modified their direct method to find the generalized Lie and non-Lie symmetry groups for the well-known nonlinear equation. The expressions of the exact finite transformations of the Lie groups are much simpler than those obtained via the standard approaches. Symmetries are infinitesimal transformations of the fields under which all solutions are mapped into solutions. If the symmetry of the differential equation is obtained, we will know the transformation structure of the equation. In some cases, conservation laws express conservation of physical quantities. Conservation laws can be used in many ways, such as to prove existence and uniqueness theorems, to derive shock conditions, and to check that numerical methods are not producing spurious results (at least qualitatively) [5]. Because of the importance of the relationship between symmetries and conservation laws [6], this paper studies the conservation laws using the direct method [7]. Kara and Mahomed [6] have recently established a close connection between conservation laws of a differential equation and the Lie point symmetries of the differential equation via the Lie-Bäcklund operators. Using them, we can connect the obtained conservation laws with the given symmetries.

In this paper, using the generalized direct method, we investigate the (2+1)-dimensional dispersive long wave (DLW) equations

$$\begin{aligned} &u_{yt}+v_{xx}+\frac{1}{2}(u^2)_{xy}=0,\\ &v_t+(uv+u+u_{xy})_x=0, \end{aligned} \tag{1}$$

which were first obtained by Boiti et al. [8]. Equations (1) have been studied by many authors (see e. g. [9 – 15]). Lou [9] gave a set of generalized symmetries by a simple constructible formula. In [10 – 13], Tang et al. obtained a lot of localized coherent or excitation structures such as the solitofs, dromions, lumps, breathers, instantons, ring solitons, peakons, compactons, localized chaotic, fractal patterns, and so on by means of the variable separation approach and defined a general type of localized excitations, folded solitary waves, and foldons. Zhang [14] obtained multisoliton-like solutions by the Bäcklund transformation. Authors of [15] obtained some exact solutions expressed by the Jacobi elliptic functions by using the F-expansion method and modified F-expansion method. While [16] considered the symmetry groups of the simpler formula of (1) by a method to directly find finite symmetry transformation groups and then symmetries of Lax integrable nonlinear physical systems.

This paper is arranged as follows: In section 2 we get a relationship between the new solutions and the old ones of the (2+1)-dimensional DLW equations by using the generalized direct method. Taking a special case, the symmetry of the DLW equations is obtained. In section 3, some new solutions are given by using the obtained relationship. Section 4 gives the infinite number of conservation laws of the (2+1)-dimensional DLW equations. Finally, some conclusions and discussions are given in section 5.

2. Symmetry Group of the Dispersive Long Wave Equations

Suppose that the solution of (1) has the following form by using the generalized direct method [4]:

$$\begin{aligned} u(x,y,t) &= \alpha + \beta U(\xi,\eta,\tau), \\ v(x,y,t) &= \phi + \theta V(\xi,\eta,\tau), \end{aligned} \tag{2}$$

where $\alpha = \alpha(x,y,t)$, $\beta = \beta(x,y,t)$, $\phi = \phi(x,y,t)$, $\theta = \theta(x,y,t)$, $\xi = \xi(x,y,t)$, $\eta = \eta(x,y,t)$, and $\tau = \tau(x,y,t)$ are functions of x, y, and t to be determined by requiring $U(\xi,\eta,\tau)$ and $V(\xi,\eta,\tau)$ to satisfy the dispersive long wave equations as $u(x,y,t)$ and $v(x,y,t)$ under the transformation $\{u,x,y,t\} \to \{U,\xi,\eta,\tau\}$ and $\{v,x,y,t\} \to \{V,\xi,\eta,\tau\}$.

Restrict U and V to satisfy the same equations as (1), i. e.

$$\begin{aligned} &U_{\eta\tau} + V_{\xi\xi} + \frac{1}{2}(U^2)_{\xi\eta} = 0, \\ &V_\tau + (UV + U + U_{\xi\eta})_\xi = 0. \end{aligned} \tag{3}$$

Substituting (2) with (3) into (1) and let the coefficients of U, V and their derivatives be zero, we obtain some equations to be solved. By solving these equations, we have

$$\begin{aligned} &\xi = \delta_1\sqrt{\delta_2\tau_t}x + \xi_0, \quad \alpha = -\frac{1}{2}\frac{\tau_{tt}}{\tau_t}x - \frac{\delta_1\xi_{0t}}{\sqrt{\delta_2\tau_t}}, \\ &\beta = \delta_1\delta_2\sqrt{\delta_2\tau_t}, \quad \phi = -1 + \delta_1\sqrt{\delta_2\tau_t}\eta_y, \\ &\theta = \delta_1\sqrt{\delta_2\tau_t}\eta_y, \end{aligned} \tag{4}$$

where $\eta \equiv \eta(y)$, $\tau \equiv \tau(t)$, and $\xi_0 \equiv \xi_0(t)$ are arbitrary functions, while δ_1 and δ_2 are determined by

$$\delta_1 = \pm 1, \quad \delta_2 = \pm 1.$$

From the symmetry theorem we know that the Lie point symmetry group $\mathcal{S}$ of the DLW equations corresponds to $\delta_1 = \delta_2 = 1$. For the dispersive long wave equations, the full symmetry group $\mathcal{G}$ is the product of the usual Lie point symmetry group $\mathcal{S}$ and the discrete group $\mathcal{D}$

$$\mathcal{G} = \mathcal{D} \otimes \mathcal{S}, \quad \mathcal{D} \equiv \{I, R_1, R_2, R_1^2\},$$

where I is the identity transformation and

$$\begin{aligned} R_1 : &u(x,y,t) \to -\mathrm{i}u(\mathrm{i}x,y,t), \\ &v(x,y,t) \to \mathrm{i}v(\mathrm{i}x,y,t), \\ R_2 : &u(x,y,t) \to \mathrm{i}u(-\mathrm{i}x,y,t), \\ &v(x,y,t) \to -\mathrm{i}v(-\mathrm{i}x,y,t). \end{aligned}$$

If we set

$$\eta(y) = y + \varepsilon f(y), \quad \tau = t + \varepsilon g(t), \quad \xi_0(t) = \varepsilon h(t), \quad \delta_1 = 1, \quad \delta_2 = 1,$$

with an infinitesimal parameter ε, where $f(y)$, $g(t)$, and $h(t)$ are arbitrary functions, then (2) can be written as

$$\begin{aligned} u &= U + \varepsilon\sigma(U), \\ v &= V + \varepsilon\sigma(V). \end{aligned}$$

Further we obtain the symmetry

$$\begin{aligned} \sigma(U) &= \left(\frac{1}{2}g_t x + h(t)\right)U_x + f(y)U_y + g(t)U_t \\ &\quad + \frac{1}{2}g_t U - \frac{1}{2}g_{tt}x - h_t, \\ \sigma(V) &= \left(\frac{1}{2}g_t x + h(t)\right)V_x + f(y)V_y \\ &\quad + g(t)V_t + \left(f_y + \frac{1}{2}g_t\right)V + f_y + \frac{1}{2}g_t. \end{aligned} \tag{5}$$

The equivalent vector expression of the above symmetry is

$$\begin{aligned} \mathcal{V} &= \left(\frac{1}{2}g_t x + h(t)\right)\frac{\partial}{\partial x} + f(y)\frac{\partial}{\partial y} + g(t)\frac{\partial}{\partial t} \\ &\quad - \left(\frac{1}{2}g_t U - \frac{1}{2}g_{tt}x - h_t\right)\frac{\partial}{\partial U} \\ &\quad - \left[\left(f_y + \frac{1}{2}g_t\right)V + f_y + \frac{1}{2}g_t\right]\frac{\partial}{\partial V}, \end{aligned} \tag{6}$$

which was also obtained in [9].

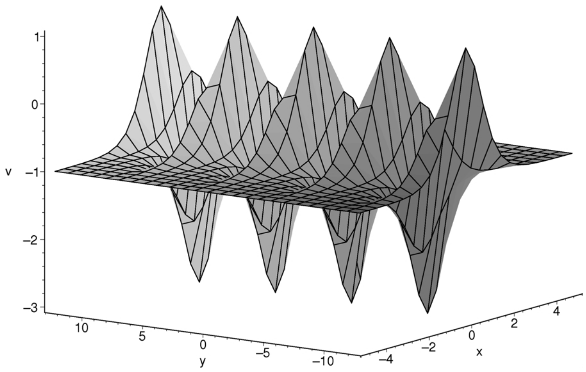

Fig. 1. Plot of $v(x,y,t)$ with $\eta(y) = \sin(y)$, $\tau(t) = t$, $\delta_1 = 1$, $\delta_2 = 1$, $k = 1$, $l = 1$, $p = 1$ at the time $t = 0$: Oscillated dromions solution.

3. Solutions of the Dispersive Long Wave Equations

In this section, we will use (2) to get the new solutions of the DLW equations from the given one. In [15] the authors gave the solution of the DLW equations as follows:

$$\begin{aligned} u(x,y,t) &= -\frac{p}{k} \pm 2k\tanh(kx+ly+pt), \\ v(x,y,t) &= 2kl\operatorname{sech}^2(kx+ly+pt) - 1. \end{aligned} \tag{7}$$

Then, from (2) we can get the solution of the DLW equations:

$$\begin{aligned} u(x,y,t) &= -\frac{1}{2}\frac{\tau_{tt}}{\tau_t}x - \frac{\delta_1\xi_{0t}}{\sqrt{\delta_2\tau_t}} + \delta_1\delta_2\sqrt{\delta_2\tau_t} \\ &\cdot\left[-\frac{p}{k} \pm 2k\tanh\left(k(\delta_1\sqrt{\delta_2\tau_t}x+\xi_0)+l\eta+p\tau\right)\right], \\ v(x,y,t) &= -1 + \delta_1\sqrt{\delta_2\tau_t}\eta_y + \delta_1\sqrt{\delta_2\tau_t}\eta_y \\ &\cdot\left[2kl\operatorname{sech}^2\left(k(\delta_1\sqrt{\delta_2\tau_t}x+\xi_0)+l\eta+p\tau\right)-1\right]. \end{aligned} \tag{8}$$

From our following figures analysis, we can see that some new types of localized excitations, like oscillated dromions, multi-dromions, breathers solutions, multi-string soliton solutions or amplitude soliton solutions are found by selecting appropriate functions as $\eta(y)$ and $\tau(t)$. This is because the solution (8) includes all the group invariant solutions corresponding to the solution (7).

Figures 1 – 6 exhibits plots of $v(x,y,t)$ for different examples. The solutions discussed in this paper can not be obtained by [redacted] (MLVSA) and their shapes do not change with time. However, you can see some interaction behaviours between the localized excitations in other papers, e. g. [10, 11, 13].

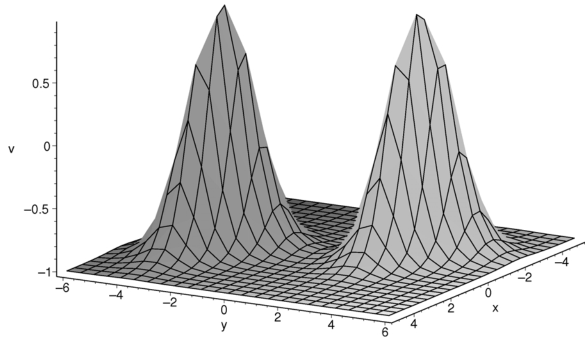

Fig. 2 Plot of $v(x,y,t)$ with $\eta(y) = \tanh(y-3)+\tanh(y+3)$, $\tau(t) = t$, $\delta_1 = 1$, $\delta_2 = 1$, $k = 1$, $l = 1$, $p = 1$ at the time $t = 0$: Two-dromions solution.

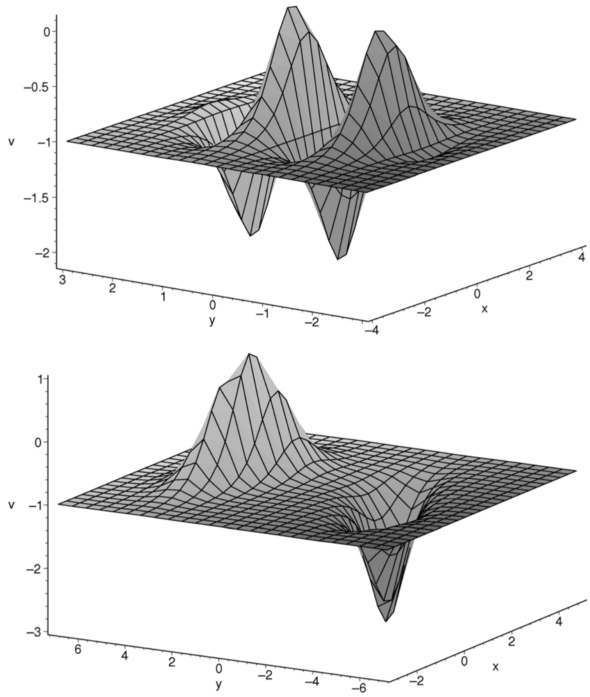

Fig. 3. Plot of $v(x,y,t)$ with $\eta(y) = \exp(-y^2)\sin(y^2)$, $\tau(t) = t$, $\delta_1 = 1$, $\delta_2 = 1$, $k = 1$, $l = 1$, $p = 1$ at the time $t = 0$ and $\eta(y) = \tanh(y-3)+\tanh(-y-3)$, $\tau(t) = t$, $\delta_1 = 1$, $\delta_2 = 1$, $k = 1$, $l = 1$, $p = 1$ at the time $t = 0$: Breathers solutions.

4. Conservation Laws of the Dispersive Long Wave Equations

Now we construct the following conservation laws of (1):

$$D_tT + D_xX + D_yY = 0, \tag{9}$$

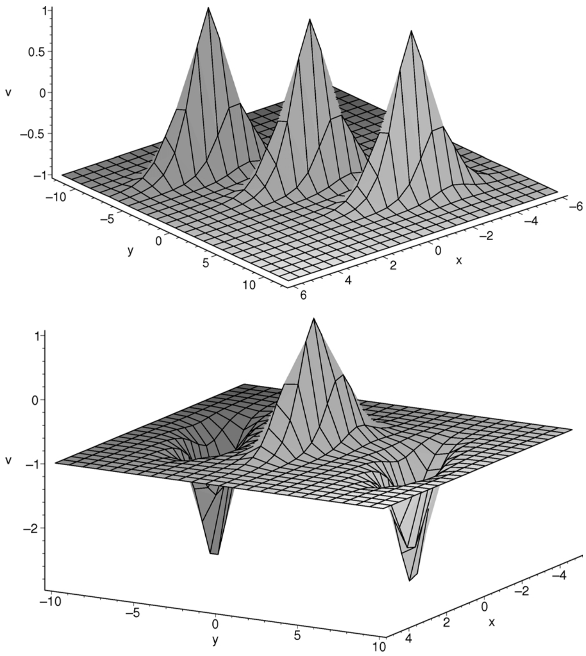

Fig. 4. Plot of $v(x,y,t)$ with $\eta(y) = \tanh(y-6)+\tanh(y)+\tanh(y+6)$, $\tau(t)=t$, $\delta_1=1$, $\delta_2=1$, $k=1$, $l=1$, $p=1$ at the time $t=0$ and $\eta(y)=\tanh(-y+6)+\tanh(y)+\tanh(-y-6)$, $\tau(t)=t$, $\delta_1=1$, $\delta_2=1$, $k=1$, $l=1$, $p=1$ at the time $t=0$: Three-dromions solution.

where $X = X(x,y,t,u,v,u_x,u_y,u_t,v_x,v_y,v_t,u_{xx},u_{xy},u_{xt},u_{yy},u_{yt},u_{tt})$, $Y = Y(x,y,t,u,v,u_x,u_y,u_t,v_x,v_y,v_t,u_{xx},u_{xy},u_{xt},u_{yy},u_{yt},u_{tt})$, and $T = T(x,y,t,u,v,u_x,u_y,u_t,v_x,v_y,v_t,u_{xx},u_{xy},u_{xt},u_{yy},u_{yt},u_{tt})$. By using the method presented in [7] (the process is listed in Appendix), we can have

$$\begin{aligned}
X ={}& [f_1(u_x,u_y)+f_2(u_y)u_t+f_3(u_y)x](u_{xy}u_{yt}-u_{xt}u_{yy})\\
&+f_4(y)u_{xy}+[c_1u+f_{11}(v)]u_yv_tu_{yt}-[f_2(u_y)u_x+f_3(u_y)t\\
&-f_5(u_y,u_t)](u_{yy}u_{tt}-u_{yt}^2)+\Big[c_1v-\int f_8(t)\mathrm{d}t\Big]u_t^2u_{yy}\\
&+c_5u_tv_tu_{yt}-(\dot f_8(t)y^2+2\dot f_9(t)y+f_{10}(t))u_y^2u_{yy}\\
&+(c_1u+f_{11}(v))u_tv_tu_{yy}-2(f_8(t)y+f_9(t))u_yu_tu_{yy}\\
&+\Big[\frac{f_{12}(u_y)v}{u_y}-f_{23}(y,t)-f_{24}(u)-f_{25}(u,u_y)\\
&+c_3x-c_4\Big]u_tu_{yy}-(c_1u+f_{11}(v))u_tv_yu_{yt}+\Big[\frac{f_{12}(u_y)u}{u_y}\\
&+f_{13}(v)+f_{14}(v,u_y)\Big]v_tu_{yy}-\Big[\int f_{23t}(y,t)+f_{27t}(y,t)\mathrm{d}y\\
&-f_6(y)x+f_{29}(t)\Big]u_yu_{yy}+\Big[f_7(y)x-\int f_{28t}(y,t)\mathrm{d}y
\end{aligned}$$

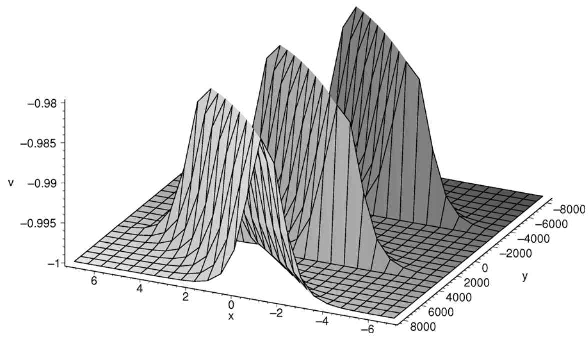

Fig. 5. Plot of $v(x,y,t)$ with $\eta(y) = \mathrm{am}(\frac{y}{100},0.0001)$, $\tau(t)=t$, $\delta_1=1$, $\delta_2=1$, $k=1$, $l=1$, $p=1$ at the time $t=0$: Multi-string soliton solutions.

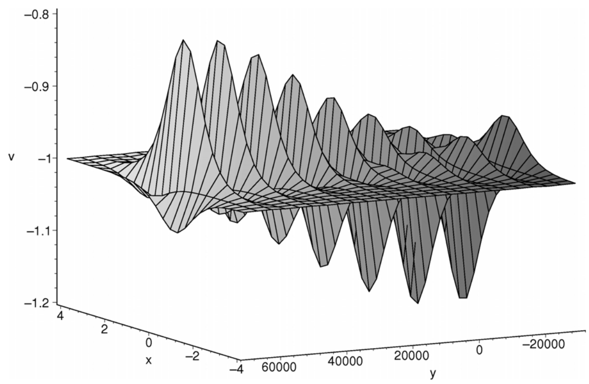

Fig. 6. Plot of $v(x,y,t)$ with $\eta(y) = \mathrm{sn}(\frac{y}{10},0.9)$, $\tau(t)=t$, $\delta_1=1$, $\delta_2=1$, $k=1$, $l=1$, $p=1$ at the time $t=0$: Amplitude soliton solutions.

$$\begin{aligned}
&-f_{26}(t)\Big]u_{yy}-\Big[\frac{f_{12}(u_y)u}{u_y}+f_{13}(v)+f_{14}(v,u_y)\Big]v_yu_{yt}\\
&-\Big[c_2u_y+\int f_{30y}(x,y)\mathrm{d}x-\int f_{23y}(y,t)\mathrm{d}t-f_{31}(y)\\
&+f_{32}(x,y,t)\Big]u_tu_{yt}+(f_8(t)y+f_9(t))u_y^2u_{yt}+\Big[f_{24}(u)\\
&+f_{25}(u,u_y)-\int f_{33y}(x,y)\mathrm{d}x-\dot f_6(y)xt+f_{27}(y,t)+c_3x\\
&+c_4\Big]u_yu_{yt}-f_{12}(u_y)u_{yt}v+\Big[\int f_{34}(x)-f_{54y}(x,y)\mathrm{d}x\\
&-\dot f_7(y)xt+f_{28}(y,t)-f_{39}(x,y,t)-f_{40}(y,t)\Big]u_{yt}-\Big[c_1v\\
&-\int f_8(t)\mathrm{d}t+\frac{1}{2}c_2\Big]u_y^2u_{tt}-(c_1u+f_{11}(v))u_yv_yu_{tt}\\
&-c_5u_tv_yu_{tt}-f_{37}(x,y)u_t^2u_{tt}+\Big[\int f_{23y}(y,t)\mathrm{d}t\\
&-\int f_{30y}(x,y)\mathrm{d}x+f_{31}(y)\Big]u_yu_{tt}+\Big[\int f_{32y}(x,y,t)\mathrm{d}t\\
&-f_{38}(x,y)\Big]u_tu_{tt}+\Big[\int f_{39y}(x,y,t)+f_{40y}(y,t)\mathrm{d}t
\end{aligned}$$

$$-f_{41}(x,y)\Big]u_{tt}+[\dot f_{42}(t)x+f_{43}(y)]uu_y+[\dot f_{42}(t)x+f_{43}(y)]v_x-[\dot f_{42}(t)x+f_{43}(y)]v_y+[f_{44}(x,t)u+f_{45}(x,t)]u_y-[f_{46}(x,y)+f_{47}(y)]vv_t+[f_{42}(t)+\dot f_{43}(y)t-f_{48}(x,y)+f_{49}(x)-f_{50}(y)]v_t+f_4(y)uv+f_4(y)u+f_{51}(y,t)-\int f_{52t}(x,y,t)+f_{53y}(x,y,t)\mathrm{d}x,$$

$$Y=-[f_1(u_x,u_y)+f_2(u_y)u_t+f_3(u_y)x](u_{xx}u_{yt}-u_{xt}u_{xy})+2(f_8(t)y+f_9(t))u_yu_tu_{xy}+[f_2(u_y)u_x+f_3(u_y)t-f_5(u_y,u_t)](u_{xy}u_{tt}-u_{xt}u_{yt})-\Big[c_1v-\int f_8(t)\mathrm{d}t\Big]u_t^2u_{xy}+(\dot f_8(t)y^2+2\dot f_9(t)y+f_{10}(t))u_y^2u_{xy}+\Big[f_{63}(x)u_x-c_1v+c_2+2\int f_8(t)\mathrm{d}t\Big]u_yu_tu_{xt}-(c_1u+f_{11}(v))u_tv_tu_{xy}+\Big[f_{23}(y,t)-\frac{f_{12}(u_y)v}{u_y}-c_3x+c_4+f_{24}(u)+f_{25}(u,u_y)\Big]u_tu_{xy}+\Big[-f_6(y)x+\int f_{23t}(y,t)+f_{27t}(y,t)\mathrm{d}y+f_{29}(t)\Big]u_yu_{xy}+[f_8(t)y+f_9(t)]u_y^2u_{xt}-\Big[\frac{f_{12}(u_y)u}{u_y}+f_{13}(v)+f_{14}(v,u_y)\Big]v_tu_{xy}+\Big[f_7(y)x+\int f_{28t}(y,t)\mathrm{d}y+f_{26}(t)\Big]u_{xy}+[f_{32}(x,y,t)+f_{58}(x)]u_tu_{xt}-c_5u_tv_tu_{xt}-[c_1u+f_{11}(v)]u_yv_tu_{xt}+f_{23}(y,t)u_yu_{xt}+\frac{1}{2}f_{56}(x)u_t^2u_{xt}+[f_{61}(x)+f_{62}(x,u_t)]u_xu_{xt}+[f_{39}(x,y,t)+f_{40}(y,t)]u_{xt}+[c_1v+c_2]u_xu_tu_{yt}+[c_1u+f_{11}(v)]u_tv_xu_{yt}+\Big[\frac{f_{12}(u_y)u}{u_y}+f_{13}(v)+f_{14}(v,u_y)\Big]v_xu_{yt}+\frac{1}{2}f_{63}(x)u_x^2u_tu_{yt}+[f_{30}(x,y)+c_3t]u_tu_{yt}+\Big[\frac{f_{12}(u_y)v}{u_y}+c_3x-c_4-f_{24}(u)-f_{25}(u,u_y)\Big]u_xu_{yt}+[f_6(y)t+f_{33}(x,y)]u_yu_{yt}+[f_{54}(x,y)+f_7(y)t]u_{yt}+\Big[\frac{1}{2}f_{63}(x)u_y+\frac{1}{2}f_{62u_t}(x,u_t)\Big]u_x^2u_{tt}+[c_1v+c_2]u_xu_yu_{tt}+f_{56}(x)u_xu_tu_{tt}+f_{58}(x)u_xu_{tt}+[c_1u+f_{11}(v)]u_yv_xu_{tt}+c_5u_tv_xu_{tt}+[f_{30}(x,y)+c_3t]u_yu_{tt}+\Big[\int f_{38x}(x,y)\mathrm{d}y-\int f_{32x}(x,y,t)\mathrm{d}t+f_{65}(x)\Big]u_tu_{tt}+\Big[\int f_{37x}(x,y)\mathrm{d}y+f_{64}(x)\Big]u_t^2u_{tt}+\Big[f_{34}(x)t-\int f_{39x}(x,y,t)\mathrm{d}t+\int f_{41x}(x,y)\mathrm{d}y+f_{66}(x)\Big]u_{tt}-[f_{44}(x,t)u+f_{45}(x,t)]u_x+\Big[\int f_{46x}(x,y)\mathrm{d}y-f_{69}(x)\Big]vv_t+(\dot f_{42}(t)x+f_{43}(y))v_x+\Big[f_{49}(x)+f_{42}(t)+\int f_{44x}(x,t)\mathrm{d}t\Big]uu_t+\Big[f_{67}(x)v+\int f_{45x}(x,t)\mathrm{d}t+\dot f_{42}(t)x+f_{68}(x)\Big]u_t+\Big[f_{67}(x)u+\int f_{48x}(x,y)-\dot f_{49}(x)\mathrm{d}y-f_{49}(x)-f_{42}(t)-f_{70}(x)\Big]v_t+f_{53}(x,y,t),$$

$$T=[f_1(u_x,u_y)+f_2(u_y)u_t+f_3(u_y)x](u_{xx}u_{yy}-u_{xy}^2)+[c_1u+f_{11}(v)]v_yu_tu_{xy}-[f_2(u_y)u_x+f_3(u_y)t-f_5(u_y,u_t)](u_{xy}u_{yt}-u_{yy}u_{xt})+\Big[c_1v-2\int f_8(t)\mathrm{d}t-f_{63}(x)u_x\Big]u_yu_tu_{xy}-\Big[\int f_{23y}(y,t)\mathrm{d}t-\int f_{30y}(x,y)\mathrm{d}x+f_{31}(y)+f_{58}(x)\Big]u_tu_{xy}+2(f_8(t)y+f_9(t))u_y^2u_{xy}+\Big[\frac{f_{12}(u_y)u}{u_y}+f_{13}(v)+f_{14}(v,u_y)\Big]v_yu_{xy}+[f_{60}(x,y,t,u)-f_{61}(x)-f_{62}(x,u_t)]u_xu_{xy}+\Big[\dot f_6(y)xt-f_{23}(y,t)-f_{24}(u)-f_{25}(u,u_y)-f_{27}(y,t)+\int f_{33y}(x,y)\mathrm{d}x-c_3x-c_4\Big]u_yu_{xy}-f_{60}(x,y,t,u)u_xu_{xy}+\Big[\dot f_7(y)xt-f_{28}(y,t)-\int f_{34}(x)-f_{54y}(x,y)\mathrm{d}x+f_{12}(u_y)v\Big]u_{xy}-\frac{1}{2}f_{56}(x)u_t^2u_{xy}+\Big[c_1v-\int f_8(t)\mathrm{d}t+\frac{1}{2}c_2\Big]u_y^2u_{xt}+[c_1u+f_{11}(v)]u_yv_yu_{xt}+f_{37}(x,y)u_t^2u_{xt}+\Big[\int f_{30y}(x,y)\mathrm{d}x-\int f_{23y}(y,t)\mathrm{d}t-f_{31}(y)\Big]u_yu_{xt}-\Big[\int f_{32y}(x,y,t)\mathrm{d}t-f_{38}(x,y)\Big]u_tu_{xt}+\Big[f_{41}(x,y)-\int f_{39y}(x,y,t)+f_{40y}(y,t)\mathrm{d}t\Big]u_{xt}-\Big[\frac{f_{12}(u_y)u}{u_y}+f_{13}(v)+f_{14}(v,u_y)\Big]v_xu_{yy}+c_5v_yu_tu_{xt}-[c_1u+f_{11}(v)]u_tv_xu_{yy}-\frac{1}{2}f_{63}(x)u_x^2u_tu_{yy}-[c_1v+f_{16}(x,y,t,u)]u_xu_tu_{yy}+[f_{16}(x,y,t,u)-c_2]u_xu_tu_{yy}-\Big[\frac{f_{12}(u_y)v}{u_y}-f_{24}(u)-f_{25}(u,u_y)+c_3x-c_4\Big]u_xu_{yy}-[f_6(y)t+f_{33}(x,y)]u_yu_{yy}-[c_3t+f_{30}(x,y)]u_tu_{yy}-[f_{54}(x,y)+f_7(y)t]u_{yy}-\frac{1}{2}f_{63}(x)u_x^2u_yu_{yt}-\frac{1}{2}f_{62u_t}(x,u_t)u_x^2u_{yt}-(c_1v+c_2)u_xu_yu_{yt}-f_{56}(x)u_xu_tu_{yt}-f_{58}(x)u_xu_{yt}-[c_1u+f_{11}(v)]u_yv_xu_{yt}-c_5u_tv_xu_{yt}-\Big[\int f_{37x}(x,y)\mathrm{d}y$$

$$+f_{64}(x)\Big]u_t^2u_{yt}-f_{67}(x)vu_y-[f_{30}(x,y)+c_3t]u_yu_{yt}$$
$$-\Big[\int f_{38x}(x,y)\mathrm{d}y-\int f_{32x}(x,y,t)\mathrm{d}t+f_{65}(x)\Big]u_tu_{yt}$$
$$-\Big[f_{34}(x)t-\int f_{39x}(x,y,t)\mathrm{d}t+\int f_{41x}(x,y)\mathrm{d}y$$
$$+f_{66}(x)\Big]u_{yt}+[f_{46}(x,y)+f_{47}(y)]vv_x-\Big[f_{49}(x)$$
$$+f_{42}(t)+\int f_{44x}(x,t)\mathrm{d}t\Big]uu_y+\Big[f_{43}(y)-\int f_{45x}(x,t)\mathrm{d}t$$
$$-f_{68}(x)\Big]u_y+[f_{48}(x,y)-f_{49}(x)-f_{42}(t)-\dot{f}_{43}(y)t$$
$$+f_{50}(y)]v_x-\Big[\int f_{46x}(x,y)\mathrm{d}y-f_{69}(x)\Big]vv_y$$
$$+\Big[-f_{67}(x)u-\int f_{48x}(x,y)-\dot{f}_{49}(x)\mathrm{d}y+f_{49}(x)$$
$$+f_{42}(t)+f_{70}(x)\Big]v_y+f_4(y)v+f_{52}(x,y,t), \qquad (10)$$

where f_i $(i=1,2,\ldots,70)$ are arbitrary functions and c_i $(i=1,2,\ldots,5)$ are arbitrary constants, and we have

$$D_tT+D_xX+D_yY=$$
$$(f_{42t}(t)x+f_{43}(y))(u_{yt}+v_{xx}+uu_{xy}+u_xu_y)$$
$$+f_4(y)(v_t+u_x+u_xv+uv_x+u_{xxy})=0.$$

It shows that the vector (T,X,Y) is a conservation vector of the DLW equations. By (1.8) in [7], $V_h=h(t)\frac{\partial}{\partial x}+h_t\frac{\partial}{\partial U}$ is associated with (10) if $c_1=c_2=c_3=c_5=0$, $f_2(u_y)=f_3(u_y)=f_5(u_y,u_t)=f_6(y)=f_7(y)=f_8(t)=f_9(t)=f_{11}(v)=f_{12}(u_y)=f_{23}(y,t)=f_{30}(x,y)=f_{31}(y)=f_{32}(x,y,t)=f_{33}(x,y)=f_{34}(x)=f_{37}(x,y)=f_{38}(x,y)=f_{41}(x,y)=f_{44}(x,t)=f_{54}(x,y)=f_{56}(x)=f_{58}(x)=f_{63}(x)=f_{64}(x)=f_{65}(x)=f_{66}(x)=f_{67}(x)=f_{68}(x)=f_{69}(x)=f_{70}(x)=0$ and $f_{25}(u,u_y)=-f_{24}(u)-c_4, f_{39}(x,y,t)=-f_{40}(y,t), f_{42}(t)=-a_1, f_{45}(x,t)=f_{45}(t), f_{46}(x,y)=f_{46}(y), f_{48}(x,y)=f_{48}(y), f_{49}(x)=a_1, f_{52}(x,y,t)=f_4(y), f_{62}(x,u_t)=-f_{61}(x)$, $f_{53}(x,y,t)=f_{53}(t)$, where a_1 is an arbitrary constant. Giving the associated vector of the symmetry of (1), we can connect it with (10) by (1.8) in [7].

5. Conclusion

In summary, the relationship is set up between the new solutions and the old ones of the DLW equations, and the symmetry of the DLW equations is obtained by means of the generalized direct method. Both the Lie point symmetry groups and the non-Lie symmetry groups are obtained without using any group theory. The Lie symmetry groups obtained via traditional Lie approaches are only special cases. Based on a given solution, one can construct another new one with the help of the obtained relationship by selecting the form of the arbitrary functions. As results, rich solutions of the DLW equations are constructed which contain oscillated dromions, multi-dromions, breathers solutions, multi-string soliton solutions, and amplitude soliton solutions. To illustrate the properties of obtained solutions, some figures are given. At last, conservation laws of the (2+1)-dimensional DLW equations are given by using the direct method.

Acknowledgements

The work is supported by the National Natural Science Foundation of China (Grant No. 10735030), the National Natural Science Foundation of China (Grant No. 90718041), Shanghai Leading Academic Discipline Project (No. B412), Program for Changjiang Scholars and Innovative Research Team in University (IRT0734) and K. C. Wong Magna Fund in Ningbo University.

Appendix: Process for finding T, X and Y

If u and v is a solution of (1), the conserved form (9) along u and v with the help of (1) separates by the highest derivative terms in u and v as

$$u_{ttt}: T_{u_{tt}}=0, \quad u_{ytt}: T_{u_{yt}}+Y_{u_{tt}}=0,$$
$$u_{yyt}: T_{u_{yy}}+Y_{u_{yt}}=0, \quad u_{yyy}: Y_{u_{yy}}=0,$$
$$u_{xtt}: T_{u_{xt}}+X_{u_{tt}}=0, \quad u_{xyt}: T_{u_{xy}}+X_{u_{yt}}+Y_{u_{xt}}=0,$$
$$u_{xyy}: Y_{u_{xy}}+X_{u_{yy}}=0, \quad u_{xxt}: T_{u_{xx}}+X_{u_{xt}}=0,$$
$$u_{xxx}: X_{u_{xx}}=0, \quad v_{tt}: T_{v_t}=0,$$
$$v_{yt}: T_{v_y}+Y_{v_t}=0, \quad v_{yy}: Y_{v_y}=0,$$
$$v_{xt}: T_{v_x}+X_{v_t}=0, \quad v_{xy}: Y_{v_x}+X_{v_y}=0,$$
$$v_{xx}: -T_{u_y}-Y_{u_t}+X_{v_x}=0,$$

where we substitute $u_{yt}=-v_{xx}-uu_{xy}-u_xu_y$ into (9). Solving the above equations and substituting the solutions with $u_{xxy}=-v_t-u_x-u_xv-uv_x$ into (9), we can get another equation. Separating the equation by the lower derivative terms, one get other equations. Solving all the equations, we can get the form of T, X, and Y.

[1] P. J. Olver, Application of Lie Group to Differential Equation Springer, New York 1986; G. W. Bluman and S. C. Anco, Symmetry and integration methods for differential equations Springer, New York 2002.
[2] P. A. Clarkson and M Kruskal, J. Math. Phys. **30**, 2201 (1989).
[3] P. A. Clarkson, Chaos, Solitons and Fractals **5 (12)**, 2261 (1995).
[4] S. Y. Lou and H. C. Ma, J. Phys. A: Math. Gen. **38**, L129 (2005); H. C. Ma, Chin. Phys. Lett. **22**, 554 (2005); H. C. Ma, Commun. Theor. Phys. (Beijing, China) **43**, 1047 (2005); H. C. Ma and S. Y. Lou, Commun. Theor. Phys. (Beijing, China) **46**, 1005 (2006); H. C. Ma, S. Y. Lou, and A. P. Deng, Commun. Theor. Phys. (Beijing, China) **50**, 685 (2008).
[5] P. E. Hydon, J. Phys. A: Math. Gen. **34**, 10347 (2001).
[6] A. H. Kara and F. M. Mahomed, Int. J. Theor. Phys. **39**, 23 (2000).
[7] A. H. Kara and C. M. Khalique, Int. J. Non-Linear Mech. **36**, 1041 (2001).
[8] M. Boiti, J. J. P. Leon, and F Pempinelli, Inverse Problem **3**, 371 (1987).
[9] S. Y. Lou, J. Phys. A: Math. Gen. **27**, 3235 (1994).
[10] X. Y. Tang and S. Y. Lou, Chaos, Solitons and Fractals **14**, 1451 (2002).
[11] X. Y. Tang, S. Y. Lou, and Y Zhang, Phys. Rev. E **66**, 046601 (2002); X. Y. Tang, S. Y. Lou, and Y Zhang, Commun. Theor. Phys. (Beijing, China) **39**, 129 (2003).
[12] X. Y. Tang, C. L. Chen, and S. Y. Lou, J. Phys. A: Math. Gen. **35**, L293 (2002).
[13] X. Y. Tang and S. Y. Lou, Commun. Theor. Phys. (Beijing, China) **40**, 62 (2003).
[14] J. F. Zhang, Commun. Theor. Phys. (Beijing, China) **33**, 557 (2000).
[15] Y. H. Tian, H. L. Chen, and X. Q. Liu, Commun. Theor. Phys. (Beijing, China) **44**, 8 (2005); Y. H. Tian, H. L. Chen, and X. Q. Liu, Commun. Theor. Phys. (Beijing, China) **45**, 207 (2006).
[16] S. Y. Lou and H. C. Ma, Chaos, Solitons and Fractals **30**, 804 (2006).