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Adaptive Control and Function Projective Synchronization in 2D Discrete-Time Chaotic Systems*

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Abstract *This study addresses the adaptive control and function projective synchronization problems between 2D Rulkov discrete-time system and Network discrete-time system. Based on backstepping design with three controllers, a systematic, concrete and automatic scheme is developed to investigate the function projective synchronization of discrete-time chaotic systems. In addition, the adaptive control function is applied to achieve the state synchronization of two discrete-time systems. Numerical results demonstrate the effectiveness of the proposed control scheme.*

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Key words: adaptive function projective synchronization, backstepping design, adaptive control, discrete-time chaotic system

1 Introduction

Chaos has aroused considerable interests in many areas of science and technology due to its potential application to secure communications. Recently, chaos control and synchronization attract more and more attention from various fields. Over the last decades, many methods and techniques for chaos control and synchronization have been produced,[1−7] such as feedback approach,[1] adaptive method,[1] time-delay feedback approach,[2] backstepping design technique,[3] OGY method,[6] PC method,[7] etc.

Since the pioneering works of Fujisaka and Yamada,[8] Pecora and Carroll,[9] Ott and Pyragas,[10] Grebogi and Yorke,[11] up to now, there exist many types of chaos synchronization in dynamical systems such as complete synchronization, partial synchronization, phase synchronization, lag synchronization, anticipated synchronization, generalized slag, anticipated, and completed synchronization, synchronization, antiphase synchronization, etc.[12−18]

In particular, amongst all kinds of chaos synchronization, projective synchronization in partially linear systems reported by Mainieri and Rehacek[19] is one of the most noticeable ones that the drive and response vectors evolve in a proportional scale — the vectors become proportional. Recently, some researchers[20−25] extended the projective synchronization to non-partially-linear systems, and based on their work, we have proposed function projective synchronization in the continuous-time systems which the drive and response vectors evolve in a proportional scaling function matrix.[23] Many powerful methods have been reported to investigate some types of chaos (hyperchaotic) synchronization in continuous-time systems. In fact, many mathematical models of neural networks, biological process, physical process, and chemical process were defined using discrete-time dynamical systems.[24−27] Recently, more and more attentions were paid to the chaos (hyperchaos) control and synchronization in discrete-time dynamical systems.[28,35−38]

Backstepping design[28−32] has become a systematic and powerful method for the construction of both feedback controllers and associated Lyapunov functions. The design method has been applied to investigate control and synchronization of many continuous-time dynamical systems.[31−34] Up until now, some articles have been reported to extend the backstepping design to deduce some proper controllers to investigate chaos control and synchronization in some discrete-time dynamical systems.[28,35−38]

In this paper, based on the function synchronization method,[23,24] we would like to define a type of adaptive control and function projective synchronization (AFPS) in discrete-time dynamical systems. Based on the backstepping design method, we present a systematic and automatic algorithm to investigate simultaneously AFPS, via controllers between discrete-time drive system and response system, whether with strict-feedback form or not. With the aid of symbolic-numeric computation, the proposed scheme is used to illustrate AFPS between 2D discrete-time Rulkov system and Network system. Moreover numerical simulations are used to verify the effectiveness of the proposed scheme.

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The paper is organized as follows. In Sec. 2, the so-called AFPS in discrete-time systems, and consequently only one controller is obtained via backstepping design procedure. In Sec. 3, we simulate adaptive function projective synchronization in 2D Rulkov discrete-time system and Network discrete-time system. In Sec. 4, we simulate AFPS between the Rulkov discrete-time system and Network system by different function control. Finally, conclusions are drawn.

2 Adaptive Function Projective Synchronization of Discrete-Time Chaotic Systems

In the following, similar to the definitions of function projective synchronization[23] in continuous-time dynamical systems, we define a AFPS in discrete-time dynamical systems, and then give a Lyapunov stability theory for discrete-time dynamical systems.

Definition For two discrete-time dynamical systems (i) $x(k+1) = F(x(k))$ and (ii) $y(k+1) = G(y(k)) + u(x(k), y(k))$, where $(x(k), y(k)) \in R^{m+m}$, $k \in Z/Z^-$, and $u(x(k), y(k)) \in R^m$, let (iii)

$$\begin{aligned} E(k) &= (E_1(k), E_2(k), \ldots, E_m(k)) \\ &= (x_1(k) - f_1(x(k))y_1(k), x_2(k) - f_2(x(k))y_2(k), \\ &\quad \ldots, x_m(k) - f_m(x(k))y_m(k)) \\ \text{or } &(x_1(k) - y_1(k), x_2(k) - y_2(k), \ldots, x_m(k) \\ &- f_m(x(k))y_m(k)) \\ \text{or } &(x_1(k) - f_1(x(k))y_1(k), x_2(k) - y_2(k), \ldots, x_m(k) \\ &- y_m(k)) \end{aligned}$$

be boundary vector functions, if there exists proper controllers $u(x(k), y(k)) = (u_1(x(k), y(k)), u_2(x(k), y(k)), \ldots, u_m(x(k), y(k)))^{\mathrm{T}}$ such that $\lim_{k\to\infty}(E(k)) = 0$, we say that there exists **adaptive function projective synchronizaton (AFPS)** between the systems (i) and (ii).

Based on the Lyapunov stability theory, for the error discrete-time (iii) generated by drive system (i) and response system (ii), let

$$L(E_1(k), E_2(k), \ldots, E_m(k))|_{E_i(k)\equiv 0\ (i=1,2,\ldots,m)} = 0\,,$$

if $\triangle L(k) = L(k+1) - L(k) \le 0$, with the equality holding if and only if $E_i(k) \equiv 0$ $(i = 1, 2, \ldots, m)$, it is said that systems (i) and (ii) are adaptive function projective synchronized.

Then based on the backstepping design method, we would like to present a systematic, generalized and constructive scheme to seek the controllers such that 2D Rulkov discrete-time system and 2D Network discrete-time system with strict-feed form are adaptive function projective synchronized.

3 AFPS of 2D Rulkov Discrete-Time System and Network Discrete-Time System

Consider Rulkov[39] discrete-time system,

$$x_1(k+1) = \frac{4.3}{1 + x_1(k)^2} + x_2(k)\,,$$

$$x_2(k+1) = x_2(k) - 0.01(x_1(k) + 1)\,. \tag{1}$$

and Network[40] system with controllers $u(x, y)$

$$\begin{aligned} y_1(k+1) &= y_2(k) + u_1(x, y)\,, \\ y_2(k+1) &= -y_1(k) - k\sin(y_2(k)) + u_2(x, y)\,. \end{aligned} \tag{2}$$

as the drive system and response system, respectively.

Firstly we plot Figs. 1(a) and 1(b) to show the two systems with initial valuables $[x_1(0) = 0.1, x_2(0) = 0.2]$ and $[y_1(0) = -0.5, y_2(0) = -0.3]$, respectively.

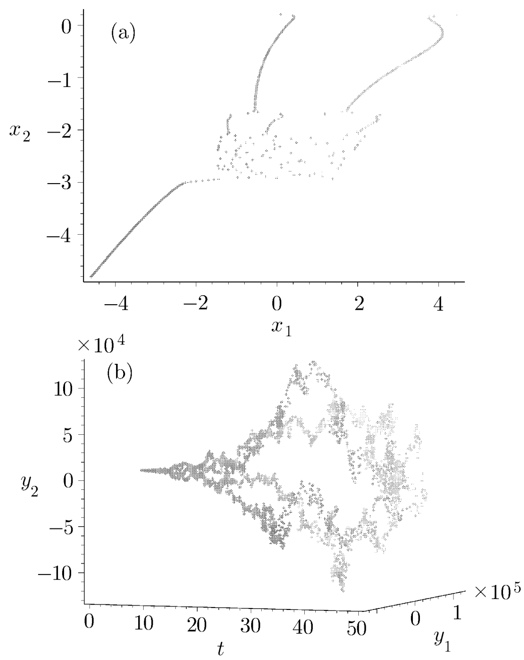

Fig. 1 (a) Rulkov discrete-time attractor; (b) Network discrete-time attractor.

In the following, we would like to realize the AFPS of Rulkov discrete-time system and Network discrete-time system by backstepping design method.

(i) Let the error states be $E_1(k) = x_1(k) - y_1(k)$, $E_2(k) = x_2(k) - \exp(x_2(k))y_2(k)$. Then from Eqs. (1) and (2), we have the discrete-time error dynamical system,

$$\begin{aligned} E_1(k+1) &= \frac{4.3}{1 + x_1(k)^2} + x_2(k) - y_2(k) - u_1(x, y)\,, \\ E_2(k+1) &= x_2(k) - 0.01x_1(k) - 0.01 \\ &\quad - \exp(x_2(k) - 0.01x_1(k) - 0.01) \\ &\quad \times (-y_1(k) + u_2(x, y) - k\sin(y_2(k)))\,. \end{aligned} \tag{3}$$

In the following based on the backstepping design and the improved ideas of Refs. [35] and [37], we give a systematic and constructive algorithm to derive the controllers $u(x, y)$ step by step such that systems (1) and (2) are synchronized together.

Step 1 Let the first partial Lyapunov function be $L_1(k) = |E_1(k)|$ and the second error variable be

$$E_2(k+1) = E_1(k+1) - c_{11}E_1(k)\,, \tag{4}$$

where $c_{11} \in R$. Then we have the derivative of $L_1(k)$

$$\Delta L_1(k) = |E_1(k+1)| - |E_1(k)|$$

$$\leq (|c_{11}|-1)|E_1(k)|+|E_2(k)|\,. \tag{5}$$

Step 2 Let

$$E_2(k+1)-c_{21}E_1(k)-c_{22}E_2(k)=0\,. \tag{6}$$

Then with the aid of symbolic computation, from the above equations (4) and (6) we obtain the controllers

$$\begin{aligned} u_1(x,y)=&-0.1(-43+10y_2(k)+10x_1(k)^2y_2(k)\\ &-2x_1(k)-2x_1(k)^3+2y_1(k)\\ &+2y_1(k)x_1(k)^2-10\exp(x_2(k))y_2(k)\\ &-\frac{10\exp(x_1(k))^2y_2(k))}{1+x_1(k)^2}\,,\\ u_2(x,y)=&\,0.01(60x_2(k)+24x_1(k)-1+100\exp(x_2(k)\\ &-0.01x_1(k)-0.01)+100\exp(x_2(k)\\ &-0.01x_1(k)-0.01)k\sin(y_2(k))-25y_1(k)\\ &+\frac{40\exp(x_2(k))y_2(k)}{\exp(x_2(k)-0.01x_1(k)-0.01)}\,. \end{aligned} \tag{7}$$

Let the second partial Lyapunov function be $L_2(k)=L_1(k)+d_1|E_2(k)|$, where $d_1>1$, then the derivative of $L(k)$ is

$$\begin{aligned} \Delta L(k)&=L_2(k+1)-L_2(k)\\ &=\Delta L_1(k)+d_1(|E_2(k+1)|-|E_2(k)|)\\ &\leq (|c_{11}|-1+d_1|c_{21}|)|E_1(k)|\\ &\quad+(1-d_1+d_1|c_{22}|)|E_2(k)|\,. \end{aligned} \tag{8}$$

It follows that the right-hand side of Eq. (8) is negative definite, if the following conditions hold,

$$|c_{11}|+d_1|c_{21}|<1\,,\quad d_1-d_1|c_{22}|>1\,. \tag{9}$$

Obviously, there exist many sets of solutions $[c_{11},c_{21},c_{22}]$ that satisfy Eq. (9). In the following we use numerical simulations to verify the effectiveness of the above-mentioned controllers. The parameters are chosen as $d_1=2$, $c_{11}=-0.2$, $c_{21}=-0.25$, $c_{22}=0.4$, and the initial values of system (1) and (2) with $u=0$ are taken as those in Fig. 1. The graphs of AFPS error states and the globally picture of the drive and response systems are displayed in Figs. 2 and 3.

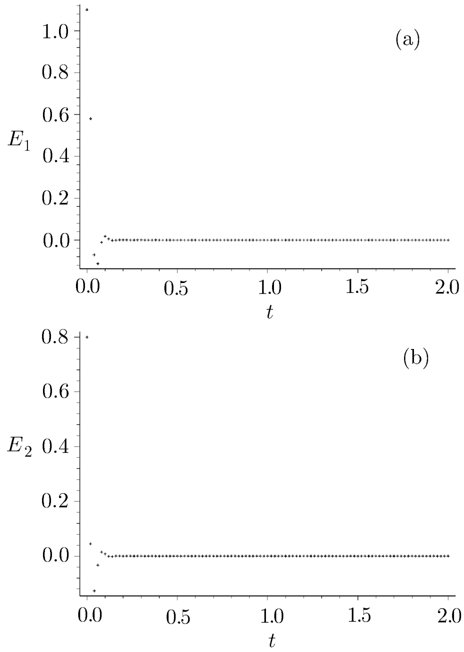

Fig. 2 The orbits of the error states.

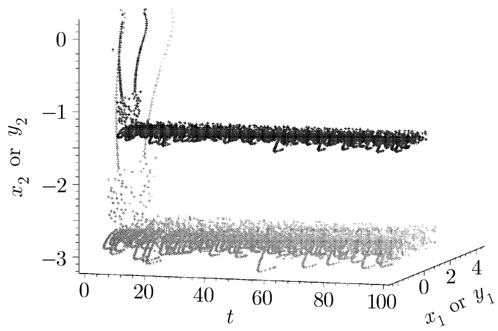

Fig. 3 The two attractors after being synchronized with $(f_1(x),f_2(x))=(4,4)$: the dark one is the response system with the controllers, and the other is the drive system.

(ii) Let the error states be $E_1(k)=x_1(k)-\exp(x_1(k))y_1(k)$, $E_2(k)=x_2(k)-y_2(k)$. Similarly, from Eqs. (1) and (2), we have the discrete-time error dynamical system

$$\begin{aligned} E_1(k+1)&=[4.3/(1+x_1(k)^2)]+x_2(k)-\exp[4.3/(1+x_1(k)^2)](y_2(k)+u_1(x,y))\,,\\ E_2(k+1)&=x_2(k)-0.01x_1(k)-0.01+y_1(k)-u_2(x,y)+k\sin(y_2(k))\,. \end{aligned} \tag{10}$$

Repeat the process in (i), we can get the attractors

$$\begin{aligned} u_1(x,y)=&\frac{-0.1}{(1+x_1(k)^2)\exp\big(\frac{0.1(4.3+10x_2(k)+10x_2(k)x_1(k)^2)}{1+x_1(k)^2}\big)}\Big[-43+10y_2(k)\exp\Big(\frac{0.1(4.3+10x_2(k)+10x_2(k)x_1(k)^2)}{1+x_1(k)^2}\Big)\\ &+10\exp\Big(\frac{0.1(4.3+10x_2(k)+10x_2(k)x_1(k)^2)}{1+x_1(k)^2}\Big)y_2(k)x_1(k)^2-2x_1(k)-2x_1(k)^3+2\exp(x_1(k))y_1(k)\\ &+2\exp(x_1(k))y_1(k)x_1(k)^2-10y_2(k)-10y_2(k)x_1(k)^2\Big]\,,\\ u_2(x,y)=&\,0.6x_2(k)+0.24x_1(k)-0.01+y_1(k)+k\sin(y_2(k))-0.25\exp(x_1(k))y_1(k)+0.4y_2(k)\,. \end{aligned} \tag{11}$$

Take the same values of $[c_{11},c_{21},c_{22},d_1]$ and the same initial values, we also use numerical simulations to verify the effectiveness of the above-mentioned controllers. The graphs of AFPS error states and the globally picture of the drive and response systems are displayed in Figs. 4 and 5.

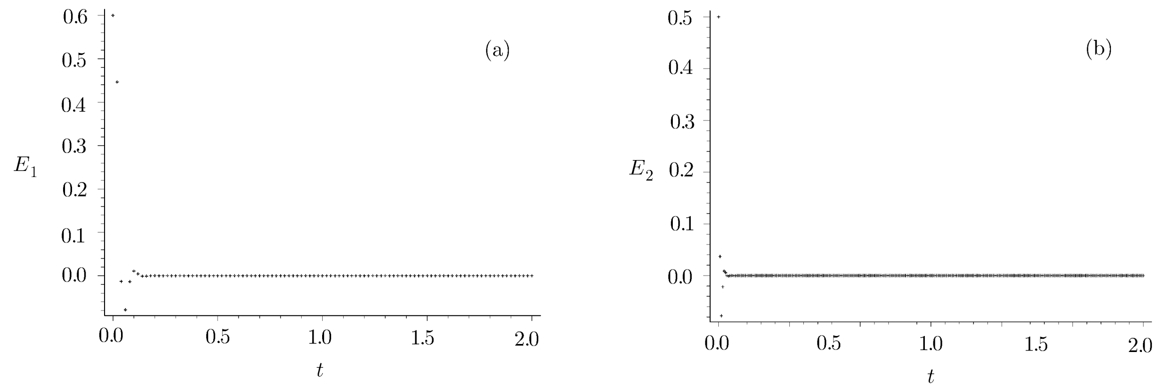

Fig. 4 The orbits of the error states.

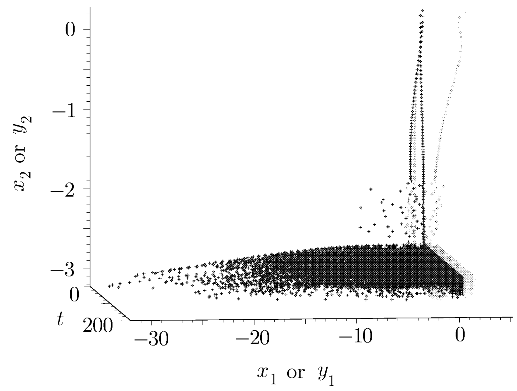

Fig. 5 The two attractors after being synchronized with $(f_1(x), f_2(x)) = (\exp(x_2(k)), 1)$: the dark one is the response system with the controllers, and the other is the drive system.

(iii) Let the error states be $E_1(k) = x_1(k) - \exp(x_1(k))y_1(k)$, $E_2(k) = x_2(k) - \exp(x_2(k))y_2(k)$. Similarly, from Eqs. (1) and (2), we have the discrete-time error dynamical system

$$\begin{aligned} E_1(k+1) &= \frac{4.3}{1+x_1(k)^2} + x_2(k) - \exp\Big(\frac{4.3}{1+x_1(k)^2} + x_2(k)\Big)(y_2(k) + u_1(x,y))\,, \\ E_2(k+1) &= x_2(k) - 0.01x_1(k) - 0.01 - \exp(x_2(k) - 0.01x_1(k) - 0.01)(-y_1(k) - k\sin(y_2(k)) + u_2(x,y)\,. \end{aligned} \tag{12}$$

Repeat the process in (i), we can get the attractors

$$\begin{aligned} u_1(x,y) =& -0.1\Big[-43 + 10\exp\Big(\frac{0.1(4.3 + 10x_2(k) + 10x_2(k)x_1(k)^2)}{1+x_1(k)^2}\Big)y_2(k) \\ &+ 10\exp\Big(\frac{0.1(4.3 + 10x_2(k) + 10x_2(k)x_1(k)^2)}{1+x_1(k)^2}\Big)y_2(k)x_1(k)^2 - 2x_1(k) \\ &- 2x_1(k)^3 + 2\exp(x_1(k))y_1(k) + 2\exp(x_1(k))y_1(k)x_1(k)^2 \\ &- 10\exp(x_2(k))y_2(k) - 10\exp(x_2(k))y_2(k)x_1(k)^2\Big]\,, \\ u_2(x,y) =& \frac{1}{100\exp(x_2(k) - 0.1x_1(k) - 0.01)}\big[60x_2(k) + 24x_1(k) - 1 + 100\exp(x_2(k) - 0.1x_1(k) - 0.01)y_1(k) \\ &+ 100\exp(x_2(k) - 0.1x_1(k) - 0.01)k\sin(y_2(k)) - 25\exp(x_1(k))y_1(k) + 40\exp(x_2(k))y_2(k))\big]\,. \end{aligned} \tag{13}$$

Take the same values of $[c_{11}, c_{21}, c_{22}, d_1]$ and the same initial values, we also use numerical simulations to verify the effectiveness of the above-mentioned controllers. The graphs of AFPS error states and the globally picture of the drive and response systems are displayed in Figs. 6 and 7.

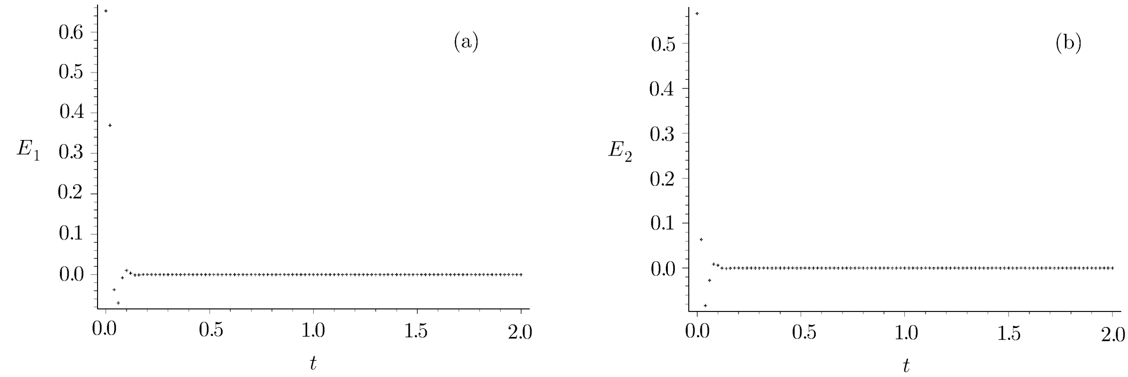

Fig. 6 The orbits of the error states.

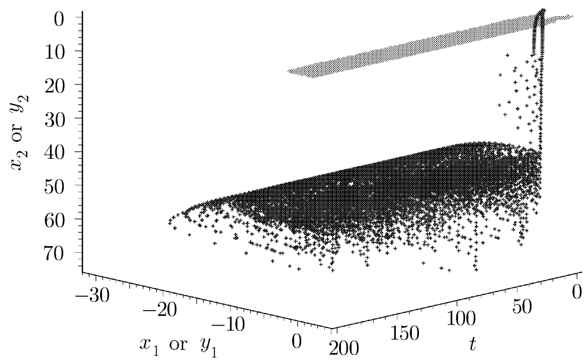

Fig. 7 The two attractors after being synchronized with $(f_1(x), f_2(x)) = (\exp(x_1(k)), \exp(x_2(k)))$: the dark one is the response system with the controllers, and the other is the drive system.

(iv) Let the error states be $E_1(k) = x_1(k) - 4y_1(k)$, $E_2(k) = x_2(k) - 4y_2(k)$. Similarly, from Eqs. (1) and (2), we have the discrete-time error dynamical system

$$E_1(k+1) = \frac{4.3}{1 + x_1(k)^2} + x_2(k) - 4y_2(k) - 4u_1(x, y)\,,$$
$$E_2(k+1) = x_2(k) - 0.01x_1(k) - 0.01 + 4y_1(k) - 4u_2(x, y) + 4k\sin(y_2(k))\,. \tag{14}$$

Repeat the process in (i), we can get the attractors,

$$u_1(x, y) = -0.025(-43 - 2x_1(k) - 2x_1(k)^3 + 8y_1(k) + 8y_1(k)x_1(k)^2/(1 + x_1(k)^2)\,,$$
$$u_2(x, y) = 0.15x_2(k) + 0.6x_1(k) - 0.0025 + 0.75y_1(k) + k\sin(y_2(k)) + 0.4y_2(k)\,. \tag{15}$$

Take the same values of $[c_{11}, c_{21}, c_{22}, d_1]$ and the same initial values, we also use numerical simulations to verify the effectiveness of the above-mentioned controllers. The graphs of AFPS error states and the globally picture of the drive and response systems are displayed in Figs. 8 and 9.

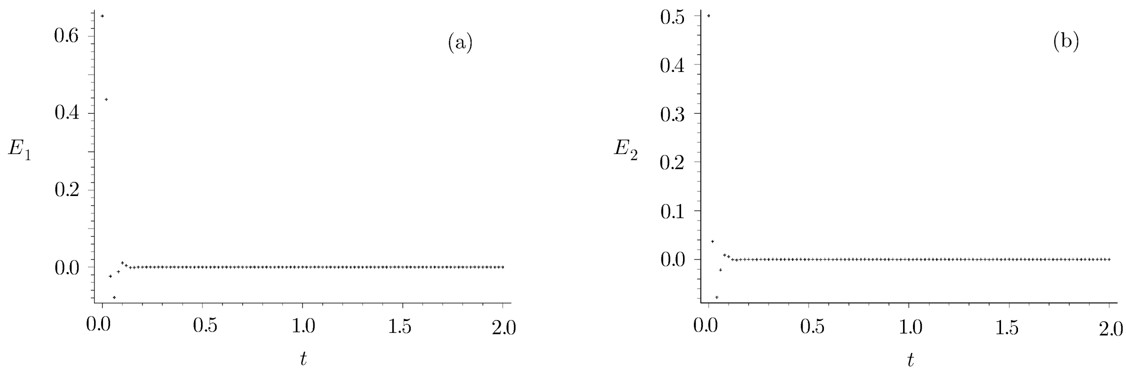

Fig. 8 The orbits of the error states.

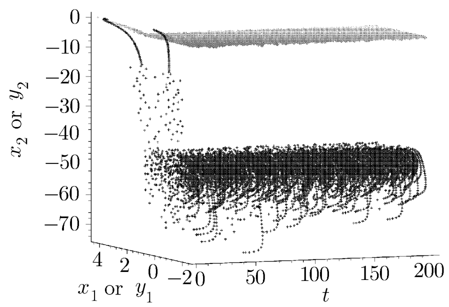

Fig. 9 The two attractors after being synchronized with $(f_1(x), f_2(x)) = (1, \exp(x_2(k)))$: the dark one is the response system with the controllers, and the other is the drive system.

4 Different Function Control of 2D Rulkov Discrete-Time System and Network Discrete-Time System

Consider Rulkov discrete-time system

$$x_1(k+1) = \frac{4.3}{1+x_1(k)^2} + x_2(k)\,,$$
$$x_2(k+1) = x_2(k) - 0.01(x_1(k)+1)\,. \tag{16}$$

and Network system with controllers $u(x,y)$

$$y_1(k+1) = y_2(k) + u_1(x,y),$$
$$y_2(k+1) = -y_1(k) - k\sin(y_2(k)) + u_2(x,y)\,. \tag{17}$$

as the drive system and response system, respectively.

In the following, we would like to realize the AFPS of Rulkov discrete-time system and Network discrete-time system by backstepping design method.

(i) Let the error states be $E_1(k) = x_1(k) - \cot(x_1(k))y_1(k)$, $E_2(k) = x_2(k) - y_2(k)$. Then from Eqs. (17) and (18), we have the discrete-time error dynamical system

$$\begin{aligned}E_1(k+1) &= \frac{4.3}{1+x_1(k)^2} + x_2(k) - \coth\Bigl(\frac{4.3}{1+x_1(k)^2} \\ &\quad + x_2(k)\Bigr)(y_2(k) + u_1(x,y))\,,\\ E_2(k+1) &= x_2(k) - 0.01x_1(k) - 0.01 + y_1(k) \\ &\quad + k\sin(y_2(k)) - u_2(x,y)\,.\end{aligned} \tag{18}$$

In the following based on the backstepping design and the improved ideas of Refs. [28] and [30], we give a systematic and constructive algorithm to derive the controllers $u(x,y)$ step by step such that systems (17) and (18) are synchronized together.

Step 1 Let the first partial Lyapunov function be $L_1(k) = |E_1(k)|$ and the second error variable be

$$E_2(k+1) = E_1(k+1) - c_{11}E_1(k)\,, \tag{19}$$

where $c_{11} \in R$. Then we have the derivative of $L_1(k)$

$$\begin{aligned}\Delta L_1(k) &= |E_1(k+1)| - |E_1(k)| \le (|c_{11}| - 1)|E_1(k)| \\ &\quad + |E_2(k)|\,.\end{aligned} \tag{20}$$

Step 2 Let

$$E_2(k+1) - c_{21}E_1(k) - c_{22}E_2(k) = 0\,. \tag{21}$$

Then with the aid of symbolic computation, from Eqs. (19) and (21) we obtain the controllers

$$\begin{aligned}u_1(x,y) &= \frac{-1}{10(1+x_1(k)^2))\coth\bigl(\frac{0.1(4.3+10x_2(k)+10x_2(k)x_1(k)^2)}{1+x_1(k)^2}\bigr)}\Bigl[-43+10\coth\Bigl(\frac{0.1(4.3+10x_2(k)+10x_2(k)x_1(k)^2)}{1+x_1(k)^2}\Bigr)y_2(k) \\ &\quad + 10\coth\Bigl(\frac{0.1(4.3+10x_2(k)+10x_2(k)x_1(k)^2)}{1+x_1(k)^2}\Bigr)x_1(k)^2y_2(k) - 2x_1(k) - 2x_1(k)^3 + 2\coth(x_1(k))y_1(k) \\ &\quad + 2y_1(k)x_1(k)^2\coth(x_1(k)) - 10y_2(k) - 10x_1(k)^2y_2(k)\Bigr]\,,\\ u_2(x,y) &= 0.6x_2(k) + 0.24x_1(k) - 0.01 + y_1(k) + k\sin(y_2(k)) - 0.25\coth(x_1(k))y_1(k) + 0.4y_2(k)\,.\end{aligned} \tag{22}$$

Let the second partial Lyapunov function be $L_2(k) = L_1(k) + d_1|E_2(k)|$, where $d_1 > 1$, then the derivative of $L(k)$ is

$$\begin{aligned}\Delta L(k) = L_2(k+1) - L_2(k) &= \Delta L_1(k) + d_1(|E_2(k+1)| - |E_2(k)|) \\ &\le (|c_{11}| - 1 + d_1|c_{21}|)|E_1(k)| + (1 - d_1 + d_1|c_{22}|)|E_2(k)|\,.\end{aligned} \tag{23}$$

It follows that the right-hand side of Eq. (23) is negative definite, if the following conditions hold:

$$|c_{11}| + d_1|c_{21}| < 1\,, \quad d_1 - d_1|c_{22}| > 1\,. \tag{24}$$

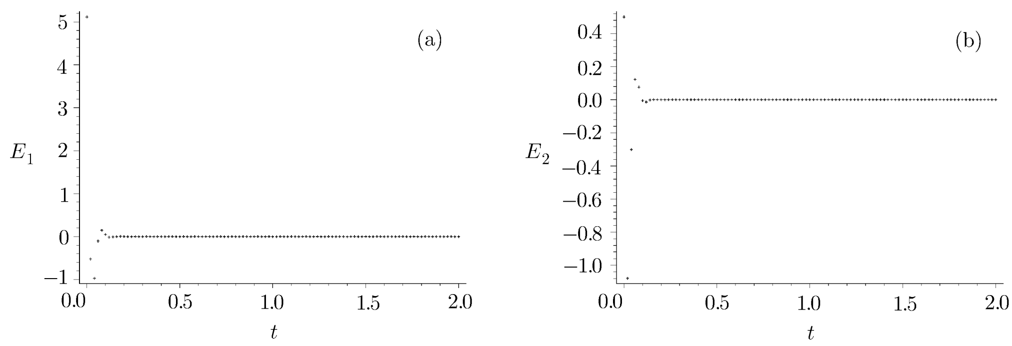

Fig. 10 The orbits of the error states.

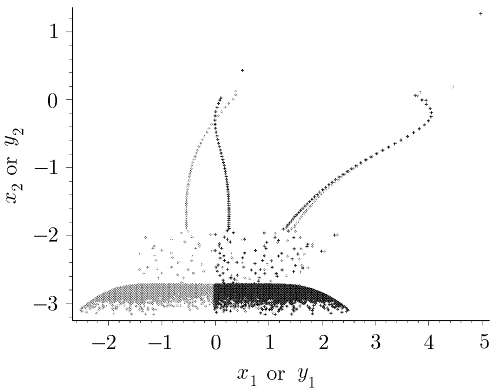

Fig. 11 The two attractors after being synchronized with $(f_1(x), f_2(x)) = (\cot(x_1(k)), 1)$: the dark one is the response system with the controllers, and the other is the drive system.

Obviously, there exist many sets of solutions $[c_{11}, c_{21}, c_{22}]$ that satisfy (24). In the following we use numerical simulations to verify the effectiveness of the above-mentioned controllers. The parameters are chosen as $d_1 = 2$, $c_{11} = -0.2$, $c_{21} = -0.25$, $c_{22} = 0.4$, and the initial values of system (16) and (17) with $u = 0$ are taken as those in Fig. 1. The graphs of AFPS error states and the globally picture of the drive and response systems are displayed in Figs. 10 and 11.

(ii) Let the error states be $E_1(k) = x_1(k) - y_1(k)$, $E_2(k) = x_2(k) - \coth(x_2(k))y_2(k)$. Similarly, from Eqs. (16) and (17), we have the discrete-time error dynamical system

$$\begin{aligned} E_1(k+1) &= \frac{4.3}{1+x_1(k)^2} + x_2(k) - y_2(k) - u_1(x,y)\,, \\ E_2(k+1) &= x_2(k) - 0.01x_1(k) - 0.01 + \coth(x_2(k) \\ &\quad - 0.01x_1(k) - 0.01)(-y_1(k) - k\sin(y_2(k)) \\ &\quad + u_2(x,y))\,. \end{aligned} \tag{25}$$

Repeat the process in (i), we can get the attractors

$$\begin{aligned} u_1(x,y) = &-0.1(-43 + 10y_2(k) + 10y_2(k)x_1(k)^2 - 2x_1(k) - 2x_1(k)^3 + 2y_1(k) + 2y_1(k)x_1(k)^2 \\ &- 10\coth(x_2(k))y_2(k) - 10\coth(x_2(k))y_2(k)x_1(k)^2)/(1+x_1(k)^2))\,, \\ u_2(x,y) = &\,0.01(60x_2(k) + 24x_1(k) - 1 + 100\coth(x_2(k) - 0.01x_1(k) - 0.01)y_1(k) + 100\coth(x_2(k) - 0.01x_1(k) - 0.01) \\ &\times k\sin(y_2(k)) - 25y_1(k) + 40\coth(x_2(k))y_2(k))/\coth(x_2(k) - 0.01x_1(k) - 0.01)\,. \end{aligned} \tag{26}$$

Take the same values of $[c_{11}, c_{21}, c_{22}, d_1]$ and the same initial values, we also use numerical simulations to verify the effectiveness of the above-mentioned controllers. The graphs of APS error states and the globally picture of the drive and response systems are displayed in Figs. 12 and 13.

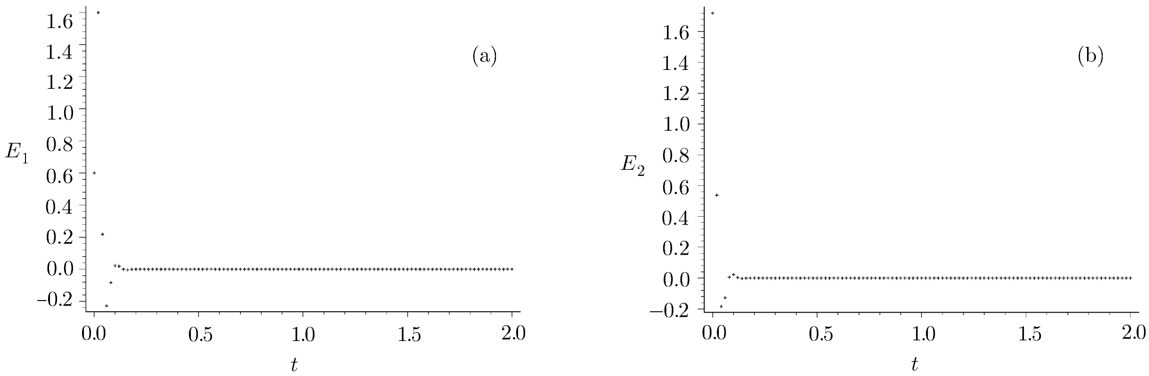

Fig. 12 The orbits of the error states.

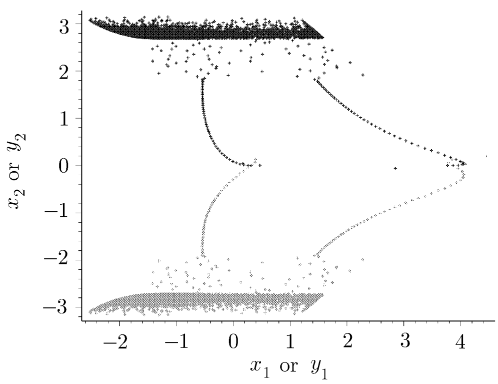

Fig. 13 The two attractors after being synchronized with $(f_1(x), f_2(x)) = (1, \coth(x_2(k)))$: the dark one is the response system with the controllers, and the other is the drive system.

(iii) Let the error states be $E_1(k) = x_1(k) - \coth(x_1(k))y_1(k)$, $E_2(k) = x_2(k) - \coth(x_2(k))y_2(k)$. Similarly, from Eqs. (16) and (17), we have the discrete-time error dynamical system

$$E_1(k+1) = \frac{4.3}{1+x_1(k)^2} + x_2(k) - \coth\Big(\frac{4.3}{1+x_1(k)^2} + x_2(k)\Big)(y_2(k) + u_1(x,y))\,,$$

$$E_2(k+1) = x_2(k) - 0.01x_1(k) - 0.01 - \coth(x_2(k) - 0.01x_1(k) - 0.01)(-y_1(k) - k\sin(y_2(k)) + u_2(x,y)\,. \qquad (27)$$

Repeat the process in (i), we can get the attractors

$$u_1(x,y) = \frac{-1}{10(1+x_1(k)^2))\coth\big(\frac{0.1(4.3+10x_2(k)+10x_2(k)x_1(k)^2)}{1+x_1(k)^2}\big)}\Big[-43+10\coth\Big(\frac{0.1(4.3+10x_2(k)+10x_2(k)x_1(k)^2)}{1+x_1(k)^2}\Big)y_2(k)$$
$$+ 10\coth\Big(\frac{0.1(4.3+10x_2(k)+10x_2(k)x_1(k)^2)}{1+x_1(k)^2}\Big)x_1(k)^2y_2(k) - 2x_1(k) - 2x_1(k)^3 + 2\coth(x_1(k))y_1(k)$$
$$+ 2y_1(k)x_1(k)^2\coth(x_1(k)) - 10\coth(x_2(k))y_2(k) - 10\coth(x_2(k))x_1(k)^2y_2(k))\Big]\,,$$
$$u_2(x,y) = \frac{1}{100(\coth(x_2(k) - 0.01x_1(k) - 0.01))}\big[60x_2(k) + 24x_1(k) - 1 + 100\coth(x_2(k) - 0.01x_1(k) - 0.01)y_1(k)$$
$$+ 100\coth(x_2(k) - 0.01x_1(k) - 0.01)k\sin(y_2(k)) - 25\coth(x_1(k))y_1(k) + 40\coth(x_2(k))y_2(k)\big]\,. \qquad (28)$$

Take the same values of $[c_{11}, c_{21}, c_{22}, d_1]$ and the same initial values, we also use numerical simulations to verify the effectiveness of the above-mentioned controllers. The graphs of AFPS error states and the globally picture of the drive and response systems are displayed in Figs. 14 and 15.

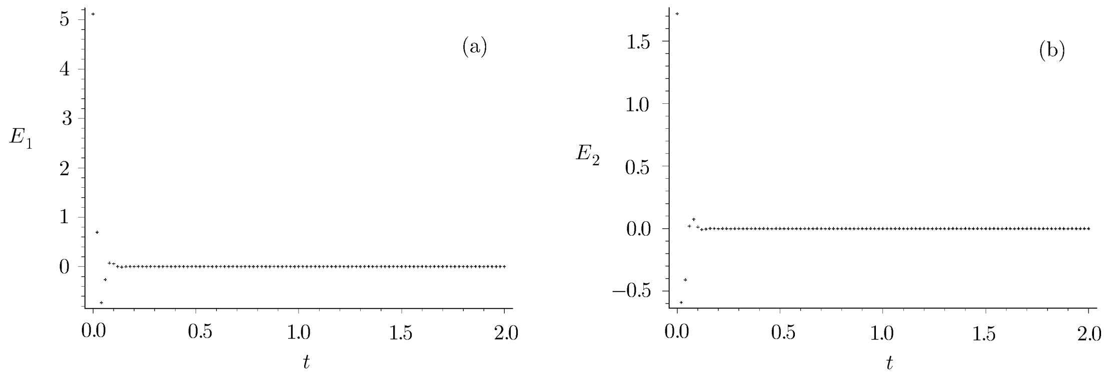

Fig. 14 The orbits of the error states.

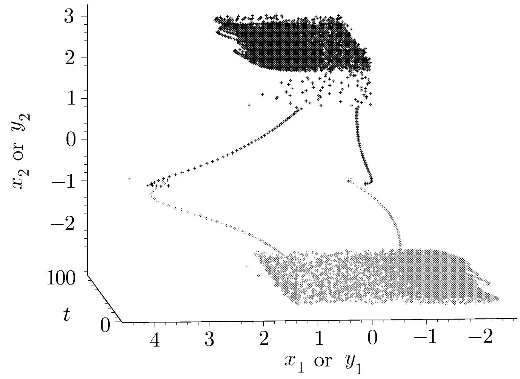

Fig. 15 Two attractors after being synchronized with $(f_1(x), f_2(x)) = (\coth(x_1(k)), \coth(x_2(k)))$: the dark one is the response system with the controllers, and the other is the drive system.

5 Summary and Conclusions

In this paper,we have defined adaptive function projective synchronization in discrete-time dynamical systems. And then backstepping control method is proposed for achieving adaptive function projective synchronization in a general class of the so-called strict-feedback chaotic systems, such as discrete-time Rulkov and Network systems. Only one controller is obtained via backstepping design technique that recursively interlaces the choice of a Lyapunov function with the design of feedback control. This control method allows us to arbitrarily amplify or reduce the scale of the dynamics of the slave system through a control. Numerical simulations are used to verify the effectiveness of the proposed scheme.

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