Stabilizing of Two-Dimensional Discrete Lorenz Chaotic System and Three-Dimensional Discrete Rössler Hyperchaotic System *

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A method is used to stabilize the unstable discrete system: two-dimensional discrete Lorenz system and three-dimensional discrete Rössler system.

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A large number of control methods have been developed and are being applied to real systems.[1−10] The method given by Ott, Grebogi, and Yorke (OGY)[1] is to stabilize an unstable orbit in the neighborhood of a hyperbolic fixed point by forcing the orbit onto the stable manifold. The method proposed by Romeiras, Grebogi, Ott, and Dayawansa (RGOD)[2] is not yet suitable for controlling hyperchaos since the method changes the stability property of the fixed point completely. However, the method proposed by Yang *et al.*[11] gives a new idea to stabilize unstable orbits even if there is no preexisting stable manifold nearby. For a finite-dimensional dynamical system, whose governing equations may or may not be analytically available, Yang *et al.*[11] show how to stabilize an unstable orbit in a neighborhood of a "fully" unstable fixed point. The advantage of this method is such that only one of the unstable directions is to be stabilized via time-dependent adjustments of control parameters. The parameter adjustments can be optimized. Recently, Bu *et al.*[12] developed a method which does not require any adjustable control parameters of the system. In this Letter, we use the method to stabilize two-dimensional discrete Lorenz systems[13] and three-dimensional discrete Rössler systems[14] to fixed points respectively.

Here we employ the method to study an n-dimensional dynamical system defined by

$$x_{k+1} = F(x_k), \tag{1}$$

where $x \in R^n$ is an n-dimensional vector, F is a nonlinear vector valued function. Let x_f be the fixed point of the map (1). To stabilize a chaotic orbit to this fixed point, take a variable feedback control described by

$$x_{k+1} = F(x_k) + M(F(x_k) - x_k), \tag{2}$$

Define an infinitesimal deviation of x_k from x_f as $\delta x_k = x_k - x_f$. Then from Eq. (2), one has

$$\delta x_{k+1} \approx J\delta x_k + M(J - I)\delta x_k, \tag{3}$$

where $J = (\partial F/\partial x_k)\,|_{x_k = x_f}$ is the Jacobian matrix of the original system F evaluated at the fixed point x_f and I is the $n \times n$ identity matrix. The goal of controlling here is to make $\lim_{k\to\infty} |\delta x_k| \to 0$ (which implies that $x_k \to x_f$, as $k \to \infty$). For this aim, one requires

$$\delta x_{k+1} = Q\delta x_k, \tag{4}$$

where Q is an $n \times n$ matrix and takes the form

$$Q = \begin{pmatrix} q_1 & 0 \\ 0 & q_2 \end{pmatrix}, \tag{5}$$

where $q_1, q_2 \in (-1, 1)$ are constants.Substituting Eqs. (4) and (5) into Eq. (3) and eliminating δx_k, choosing one special form of the matrix $Q = qI, q \in (-1, 1)$, one have

$$M = (qI - J)(J - I)^{-1}. \tag{6}$$

Using this method, we stabilize the two-dimensional discrete Lorenz system represented as

$$\begin{aligned} x_{k+1} &= (1 + \alpha\beta)x_k - \beta x_k y_k, \\ y_{k+1} &= (1 - \beta)y_k + \beta x_k^2, \end{aligned} \tag{7}$$

where α and β are the parameters, and we choose $\alpha = 1.25$, and $\beta = 0.75$.

In the following based on the method mentioned above, we will make the Lorenz system stabilize at a fixed point. There are three different fixed points (x_f, y_f) of map (7): (0,0), (1.11803,1.25) and $(-1.11803, 1.25)$. We choose (1.11803,1.25) as our research object. The Jacobian matrix corresponding the fixed point (x_f, y_f) is

$$J = \begin{pmatrix} 1 + \alpha\beta - \beta y_f & -\beta x_f \\ 2\beta x_f & 1 - \beta \end{pmatrix}. \tag{8}$$

From Eq. (6) one can have

$$M = \begin{pmatrix} \frac{q-1-\alpha\beta+\beta y_f+2x_f^2\beta}{\beta(\alpha-y_f-2x_f^2)} & -\frac{x_f(q-1)}{\beta(\alpha-y_f-2x_f^2)} \\ \frac{2x_f(q-1)}{\beta(\alpha-y_f-2x_f^2)} & \mathcal{X} \end{pmatrix},$$

$$\mathcal{X} = \frac{2x_f^2\beta+q\alpha-qy_f-\alpha+y_f+\alpha\beta-\beta y_f}{\beta(\alpha-y_f-2x_f^2)}, \tag{9}$$

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where (x_f, y_f)=(1.11803,1.25), $\alpha = 1.25$, and $\beta = 0.75$. Choosing the parameter $q = 0.5$, and $q = 0.3$ respectively, one obtains

$$M_1 = \begin{pmatrix} -0.73333 & -0.29814 \\ 0.59628 & -1.0 \end{pmatrix},$$
$$M_2 = \begin{pmatrix} -0.62667 & -0.41740 \\ 0.83480 & -1.0 \end{pmatrix}. \quad (10)$$

From Eq. (2), substituting Eq. (10) into Eq. (7), we can obtain

$$x_{k+1} = 0.26667[(1+\alpha\beta)x_k - \beta x_k y_k] + 0.73333x_k - 0.29841[(1-\beta)y_k + \beta x_k^2 - y_k],$$
$$y_{k+1} = 0.59628[(1+\alpha\beta)x_k - \beta x_k y_k - x_k] + y_k, \quad (11)$$

$$x_{k+1} = 0.37333[(1+\alpha\beta)x_k - \beta x_k y_k] + 0.62667x_k - 0.41740[(1-\beta)y_k + \beta x_k^2 - y_k],$$
$$y_{k+1} = 0.83480[(1+\alpha\beta)x_k - \beta x_k y_k - x_k] + y_k. \quad (12)$$

In the following, we give the orbit of two-dimensional discrete Lorenz system before being stabilized in Fig. 1(a). In Fig. 1(b), three orbits starting from different initial points are stabilized to the fixed point (1.11803, 1.25). It is shown that the unstable orbit is stabilized to the desired fixed point monotonically. Then the orbits stabilized of x_k and y_k versus t_k are depicted contrasting with the ones before stabilized in Figs. 2 and 3, respectively.

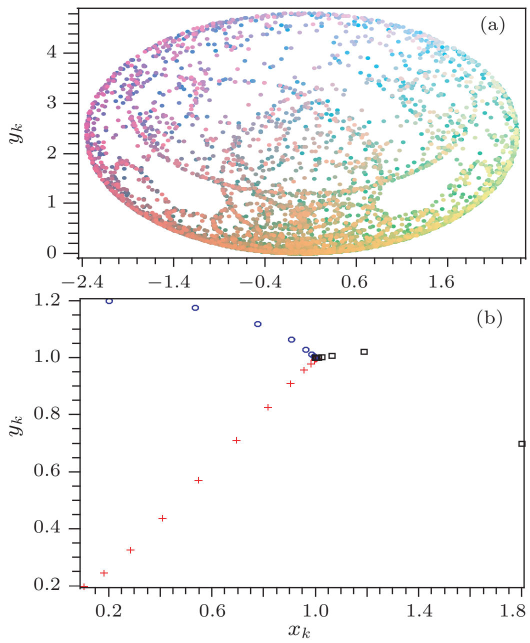

Fig. 1. (a) Two-dimensional discrete Lorenz system, (b) three orbits starting from different initial points are stabilized to the fixed point (1.11803, 1.25), for $q = 0.5$.

Next, we consider a three-dimensional discrete Rössler hyperchaotic system

$$x_{k+1} = \alpha x_k(1-x_k) - \beta(z_k+\gamma)(1-2y_k),$$
$$y_{k+1} = \delta y_k(1-y_k) + \xi z_k,$$
$$z_{k+1} = \eta((z_k+\gamma)(1-2y_k)-1)(1-\theta x_k), \quad (13)$$

where $\alpha = 3.8$, $\beta = 0.05$, $\gamma = 0.35$, $\delta = 3.78$, $\xi = 0.2$,$\eta = 0.1$ and $\theta = 1.9$. There are nine fixed points including (0.00495, 0.05201, −0.07179). Here we just consider the condition at the fixed point $(x_f, y_f, z_f) = (0.00495, 0.05201, -0.07179)$. Following the above procedure, the Jacobian matrix of map (13) is

$$J = \begin{pmatrix} 3.8-7.6x_f & 0.10z_f+0.035 & -0.05+0.1y_f \\ 0 & 3.78-7.56y_f & 0.2 \\ \mathcal{C}_{31} & \mathcal{C}_{32} & \mathcal{C}_{33} \end{pmatrix},$$
$$\mathcal{C}_{31} = 0.19[(z_f+0.35)(2y_f-1)+1],$$
$$\mathcal{C}_{32} = (0.19x_f - 0.1)(2z_f+0.7),$$
$$\mathcal{C}_{33} = (0.1-0.19x_f)(1-2y_f). \quad (14)$$

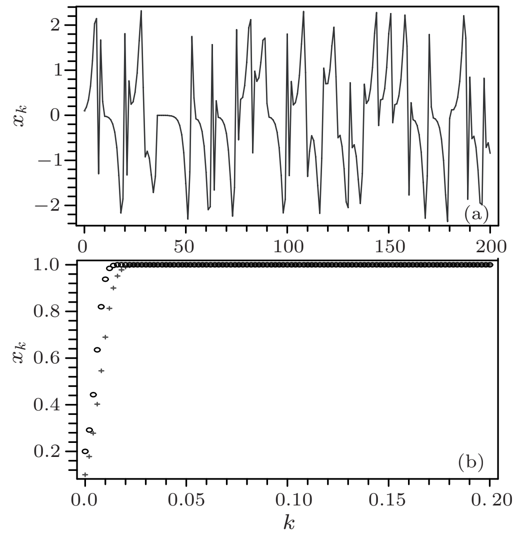

Fig. 2. (a) Characteristics of x_k versus k before stabilized, (b) x_k versus k after stabilized for $q = 0.5$ and $q = 0.3$.

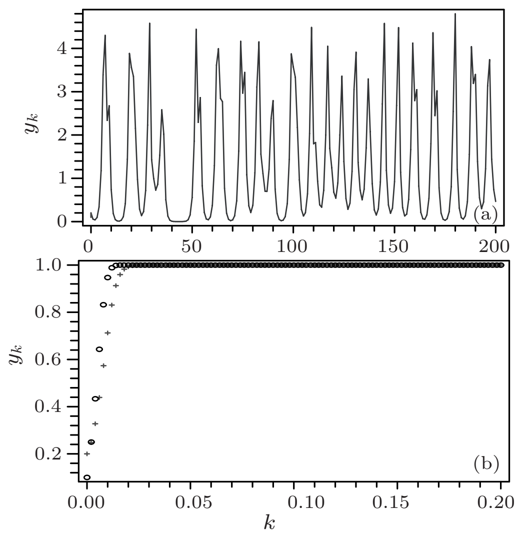

Fig. 3. (a) Characteristics of y_k versus k before stabilized, (b) y_k versus k after stabilized for $q = 0.5$ and $q = 0.3$.

From Eq. (6), choosing $q = 0.5$ and $q = -0.5$ respectively, the matrix M at the fixed point

$(0.00495, 0.05201, -0.07179)$ is correspondingly obtained as follows:

$$M = \begin{pmatrix} 1.18149 & 0.00233 & 0.00943 \\ 0.00239 & -1.21058 & -0.04634 \\ -0.02855 & 0.01310 & -0.44702 \end{pmatrix}, \quad (15)$$

$$M = \begin{pmatrix} -1.54447 & 0.007 & 0.0283 \\ 0.00718 & -1.63175 & -139 \\ -0.08566 & 0.03931 & 0.65894 \end{pmatrix}. \quad (16)$$

From Eq. (2), substituting Eq. (15) and (16) into Eq. (13), one can obtain

$$\begin{aligned} x_{k+1} &= 2.18149[\alpha x_k(1-x_k) - \beta(z_k+\gamma)(1-2y_k)] \\ &\quad - 1.18149x_k + 0.00233(\delta y_k(1-y_k) \\ &\quad + \xi z_k - y_k) + 0.00943[\eta((z_k+\gamma) \\ &\quad \cdot (1-2y_k) - 1)(1-\theta x_k) - z_k], \\ y_{k+1} &= 0.00239[\alpha x_k(1-x_k) - \beta(z_k+\gamma)(1-2y_k) \\ &\quad - x_k] - 1.21058[\delta y_k(1-y_k) + \xi z_k - y_k] \\ &\quad + \delta y_k(1-y_k) + \xi z_k - 0.04634[\eta((z_k+\gamma) \\ &\quad \cdot (1-2y_k) - 1)(1-\theta x_k) - z_k], \\ z_{k+1} &= \eta[(z_k+\gamma)(1-2y_k) - 1](1-\theta x_k) \\ &\quad - 0.002855[\alpha x_k(1-x_k) - \beta(z_k+\gamma)(1-2y_k) \\ &\quad - x_k] + 0.01310[\delta y_k(1-y_k) + \xi z_k - y_k] \\ &\quad - 0.44702[\eta((z_k+\gamma)(1-2y_k) - 1) \\ &\quad \cdot (1-\theta x_k) - z_k], \end{aligned} \quad (17)$$

$$\begin{aligned} x_{k+1} &= \alpha x_k(1-x_k) - \beta(z_k+\gamma)(1-2y_k) \\ &\quad - 1.54447(\alpha x_k(1-x_k) - \beta(z_k+\gamma) \\ &\quad \cdot (1-2y_k) - x_k) + 0.007[\delta y_k(1-y_k) \\ &\quad + \xi z_k - y_k] + 0.0283[\eta((z_k+\gamma) \\ &\quad \cdot (1-2y_k) - 1)(1-\theta x_k) - z_k], \\ y_{k+1} &= \delta y_k(1-y_k) + \xi z_k + 0.00718(\alpha x_k(1-x_k) \\ &\quad - \beta(z_k+\gamma)(1-2y_k) - x_k) \\ &\quad - 1.63175(\delta y_k(1-y_k) + \xi z_k - y_k) \\ &\quad - 0.139[\eta((z_k+\gamma)(1-2y_k) - 1) \\ &\quad \cdot (1-\theta x_k) - z_k], \\ z_{k+1} &= \eta[(z_k+\gamma)(1-2y_k) - 1](1-\theta x_k) \\ &\quad - 0.08566[\alpha x_k(1-x_k) - \beta(z_k+\gamma) \\ &\quad \cdot (1-2y_k) - x_k] + 0.03931[\delta y_k(1-y_k) \\ &\quad + \xi z_k - y_k] + 0.65894[\eta((z_k+\gamma) \\ &\quad \cdot (1-2y_k) - 1)(1-\theta x_k) - z_k]. \end{aligned} \quad (18)$$

The numerical results are presented in the following. The orbit of three-dimensional discrete time Rössler system is given by Fig. 4(a). In Fig. 4(b), three orbits starting from different initial points are stabilized to the fixed point $(0.00495, 0.05201, -0.07179)$. We can also obtain the result that the three-dimensional discrete time Rössler hyperchaotic system is stabilized. In Figs. 5–7, the orbits stabilized of x_k, y_k and z_k versus t_k are plotted in comparison with the ones before stabilized, respectively.

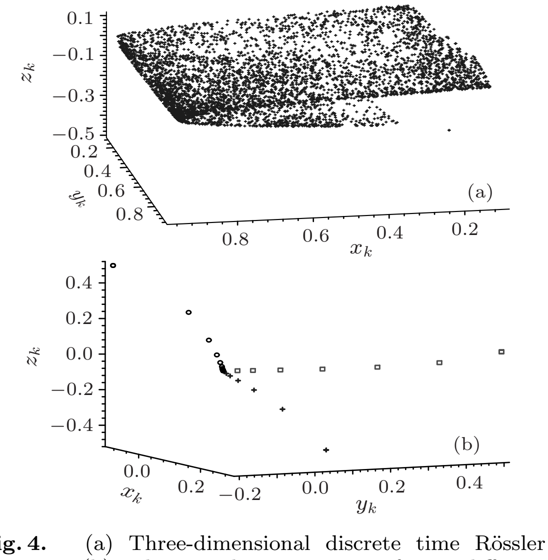

Fig. 4. (a) Three-dimensional discrete time Rössler system, (b) three orbits starting from different initial points are stabilized to the fixed point $(0.00495, 0.05201, -0.07179)$, for $q = 0.5$.

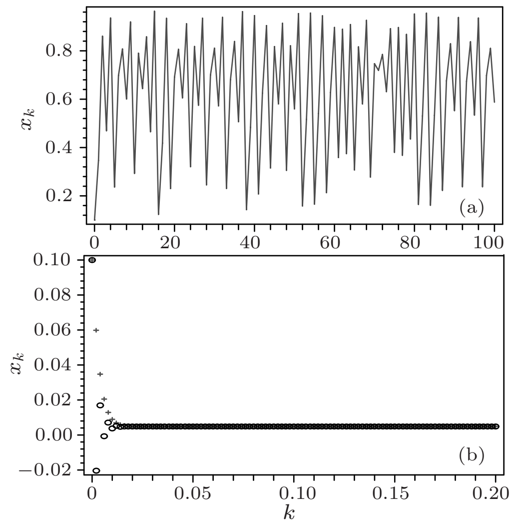

Fig. 5. (a) Characteristics of x_k versus k before stabilized, (b) x_k versus k after stabilized for $q = 0.5$ and $q = -0.5$.

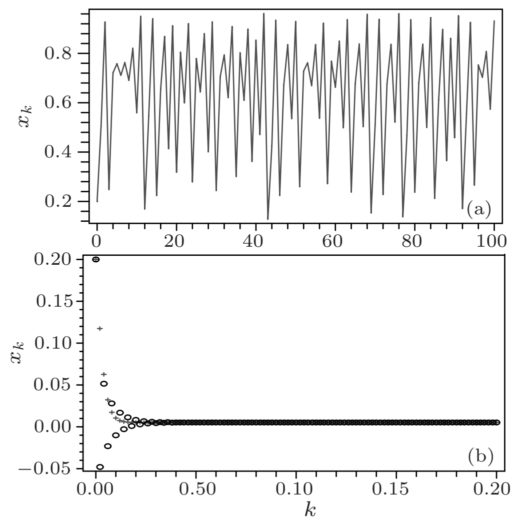

Fig. 6. (a) Characteristics of y_k versus k before stabilized, (b) y_k versus k after stabilized for $q = 0.5$ and $q = -0.5$.

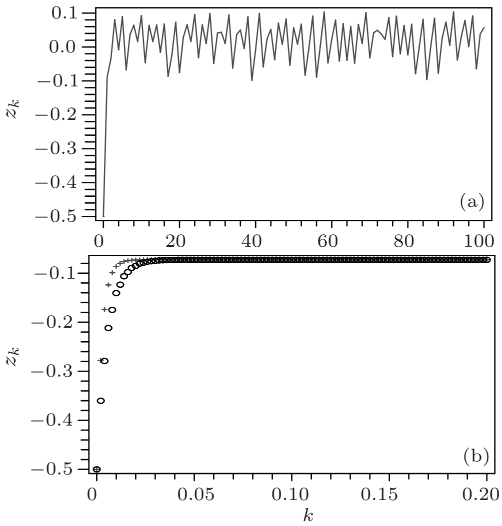

Fig. 7. (a) Characteristics of z_k versus k before stabilized, (b) z_k versus k after stabilized for $q = 0.5$ and $q = -0.5$.

In summary, a two-dimensional discrete Lorenz system and a three-dimensional discrete Rössler system are stabilized to fixed points, respectively. From the process carried out, it is shown that stabilizing the unstable discrete systems neither requires a prior analytical knowledge of the underlying system nor need any adjustable control parameters in advance.

References

[1] Ott E, Grebogi C and York J A 1990 *Phys. Rev. Lett.* **64** 1196
[2] Zhou C S and Chen T L 1996 *Chin. Phys. Lett.* **13** 572
[3] Tan W and Wang Y N 2005 *Chin. Phys.* **14** 72
[4] Chen G R and Lü J H 2003 (Beijing: Science Press) (in Chinese)
[5] Peng B, Petro V and Showalter K 1991 *J. Phys. Chem.* **95** 4957
[6] Wang J G and Zhao Y 2005 *Chin. Phys. Lett.* **22** 2508
[7] Pyragas K 1992 *Phys. Lett.* A **170** 421
[8] Abed F H, Wang H O and Chen R C 1994 *Physica* D **70** 154
[9] Mirus K A and Sprott J C 1999 *Phys. Rev.* E **59** 5313
[10] Guo L X, Xu Z Y and Hu M F 2008 *Chin. Phys.* B **17** 4067
[11] Yang L, Liu Z and Mao J 2000 *Phys. Rev. Lett.* **84** 67
[12] Bu S, Wang S and Ye H 2001 *Phys. Rev.* E **64** 046209
[13] Lorenz E N 1963 *J. Atmos. Sci.* **20** 130
[14] Rössler O E 1979 *Phys. Lett.* A **71** 397