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Exact Analytical Solutions in Bose–Einstein Condensates with Time-Dependent Atomic Scattering Length*

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Abstract *In the paper, the generalized Riccati equation rational expansion method is presented. Making use of the method and symbolic computation, we present three families of exact analytical solutions of Bose–Einstein condensates with the time-dependent interatomic interaction in an expulsive parabolic potential. Then the dynamics of two anlytical solutions are demonstrated by computer simulations under some selectable parameters including the Feshbach-managed nonlinear coefficient and the hyperbolic secant function coefficient.*

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1 Introduction

With the experimental observation and theoretical studies of Bose–Einstein condensates (BECs),[1] there has been intense interest in the nonlinear excitations of the atomic matter waves, such as dark,[2] bright solitons,[3−6] vortices,[7] and the four-wave mixing.[8] Recent experiments have demonstrated that the variation of the effective scattering length, even including its sign, can be achieved by utilizing the so-called Feshbach resonance.[9,10] In Ref. [11], it has been demonstrated that the variation of nonlinearity of the Gross–Pitaevskii (GP) equation via Feshbach resonance provides a powerful tool for controlling the generation of bright and dark soliton trains starting from periodic waves.

At the mean-field level, the GP equation governs the evolution of the macroscopic wave function of BECs. In the physically important case of the cigar-shaped BECs, it is reasonable to reduce the GP equation into a one-dimensional nonlinear Schrödinger equation,[5,12−15]

$$\mathrm{i}\frac{\partial\psi(x,t)}{\partial t}+\frac{\partial^2\psi(x,t)}{\partial x^2}+2a(t)|\psi(x,t)|^2\psi(x,t)+\frac{1}{4}\lambda^2x^2\psi(x,t)=0\,,\tag{1}$$

In Eq. (1), time t and coordinate x are measured in units $2/\omega_\perp$ and $a_\perp$, where $a_\perp=(\hbar/m\omega_\perp)^{1/2}$ and $a_0=(\hbar/m\omega_0)^{1/2}$ are linear oscillator lengths in the transverse and cigar-axis directions, respectively. $\omega_\perp$ and ω_0 are corresponding harmonic oscillator frequencies, m is the atomic mass, and $\lambda=2|\omega_0|/\omega_\perp\ll 1$. The Feshbach-managed nonlinear coefficient reads $a(t)=|a_s(t)|/a_B=g_0\exp(\lambda t)$ (a_B is the Bohr radius).[14] The normalized macroscopic wave function $\psi(x,t)$ is connected to the original order parameter $\Psi(\boldsymbol{r},t)$ as follows:

$$\Psi(\boldsymbol{r},t)=\frac{1}{\sqrt{2\pi a_B}a_\perp}\psi\Big(\frac{x}{a_\perp},\frac{\omega_\perp t}{2}\Big)\times\exp\Big(-\mathrm{i}\omega_\perp t-\frac{y^2+z^2}{2a_\perp}\Big)\,.\tag{2}$$

In Ref. [16], Liang *et al.* presented a thorough analysis of the dynamics of a bright soliton of BECs with time-varying atomic scattering length in an expulsive parabolic potential by using the so-called Darboux transformation.[17,18] Their results show that, under a safe range of parameters, the bright soliton can be compressed into very high local matter densities by increasing the absolute value of the atomic scattering length. In Ref. [19], Zhang *et al.* obtained two families of solitons of Bose–Einstein condensates with a time-dependent atomic scattering length in an expulsive parabolic potential, and as an example, they selected the experimental parameter, i.e. the Feshbach-managed nonlinear coefficient reading $a(t)=g_0\exp(\lambda t)$ and obtained the results which can be recovered in Ref. [16].

The motivation for the present study lies in the physical importance of the BECs Eq. (1) and the need to have some exact solutions, especially solitons. To have some explicit analytical solutions of Eq. (1) may enable one to better understand the physical phenomena which it describes. The exact solutions, which are accurate and explicit, may help physicists and engineers to discuss and examine the sensitivity of the model to several physical parameters. In

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this work, we will present a method to construct some exact solutions for Eq. (1). With the help of symbolic computation, three families of solutions of Eq. (1) are derived.

2 Generalized Riccati Equation Rational Expansion Method

Now we establish the generalized Riccati equation rational expansion method as follows:

Given a nonlinear partial differential equations (PDEs) with two variables $\{x, t\}$

$$p(u_t, u_x, u_{xt}, u_{tt}, u_{xx}, \ldots) = 0\,. \tag{3}$$

Step 1 We assume that the solutions of Eq. (3) are as follows:

$$u(z,t) = A_0 + \sum_{i=1}^{m} \frac{A_i\phi^i(\xi) + B_i\phi^{i-1}(\xi)\sqrt{R+\phi^2(\xi)}}{(1+\mu_1\phi(\xi)+\mu_2\sqrt{R+\phi^2(\xi)})^i}\,, \tag{4}$$

where $A_0 = A_0(x,t)$, $A_i = A_i(x,t)$, $B_i = B_i(x,t)$, $(i = 1, \ldots, m)$, $\xi = \xi(x,t)$ are all differentiable functions of $\{x, t\}$, and the new variable $\phi = \phi(\xi)$ satisfies

$$\phi' - (R+\phi^2) = \frac{\mathrm{d}\phi(\xi)}{\mathrm{d}\xi} - (R+\phi^2(\xi)) = 0\,. \tag{5}$$

The parameter m can be determined by balancing the highest order derivative term and the nonlinear terms in Eq. (3). m is usually a positive integer, if not, some proper transformation $u(x,t) \to u^m(x,t)$ may be in order to satisfy this requirement.

Step 2 Substituting Eq. (4) along with Eq. (5) into Eq. (3), extracting the numerator of the obtained system, setting the coefficients of $\phi^j(\xi)(\sqrt{R+\phi^2(\xi)})^i$ $(j = 0, 1, \ldots;\ i = 0, 1)$ to zero, we obtain a set of over-determined PDEs with regard to differential functions A_0, A_i, B_i, $(i = 1, \ldots, m)$ and ξ.

Step 3 Solving the over-determined PDEs by use of a symbolic computation system — *Maple*, we would end up the explicit expressions for A_0, A_i, B_i $(i = 1, \ldots, m)$ and ξ or the constraints among them.

Step 4 It is well known that the general solutions of Riccati equation (5) are

$$\phi(\xi) = \begin{cases} -\sqrt{-R}\tanh(\sqrt{-R}\xi)\,, & R<0\,, \\ -\sqrt{-R}\coth(\sqrt{-R}\xi)\,, & R<0\,, \\ -1/\xi\,, & R=0\,, \\ \sqrt{R}\tan(\sqrt{R}\xi)\,, & R>0\,, \\ -\sqrt{R}\cot(\sqrt{R}\xi)\,, & R>0\,. \end{cases} \tag{6}$$

Thus according to Eqs. (4) and (6) and the conclusions in Step 3, some exact analytical solutions of Eq. (3) can be obtained.

Remark 1 The method proposed is more general than the tanh function method,[20,21] extended tanh function method,[22,23] improved extended tanh function method,[24] projective Riccati equation method,[25] general projective Riccati equation method,[26,27] and the Riccati equation rational expansion method.[28] Compared with the above-mentioned methods, the restriction on $\xi(x,t)$ as merely a linear function $\{x, t\}$ and the restriction on the coefficients A_0, A_i, B_i $(i = 1, 2, \ldots, m)$ as constants are removed.

Remark 2 The generalized hyperbolic-function method,[29] generalized Riccati equation method,[30,31] and generalized projective Riccati equation methods[32] can be recovered by selecting different parameters: A_0, A_i, B_i $(i = 1, 2, \ldots, m)$, $\xi(x,t)$, μ_1, and μ_2.

Remark 3 For the generalization of the ansatz, naturally more complicated computation is expected than ever before. Even if the availability of computer symbolic systems like *Maple* allows us to perform the complicated and tedious algebraic calculation and differential calculation on a computer. In general, it is very difficult, sometime impossible, to solve the set of over-determined PDEs in Step 2. As the calculation goes on, in order to drastically simplify the work or make the work feasible, we often choose special function forms for A_0, A_i, B_i $(i = 1, \ldots, m)$ and $\xi(x,t)$, on a trial-and-error basis.

3 Exact Analytical Solutions of BECs Equation (1)

We now investigate the BECs equation (1) with the generalized Riccati equation rational expansion method proposed here. In order to obtain some exact solutions of Eq. (1), we select the solutions of Eq. (1) as the following forms,

$$\begin{aligned} \psi(x,t) &= \Big(A_0(t) + \frac{A_1(t)\phi(\xi) + B_1(t)\sqrt{R+\phi^2(\xi)}}{1+\mu_1\phi(\xi)+\mu_2\sqrt{R+\phi^2(\xi)}}\Big)\exp(\mathrm{i}\Theta(x,t))\,, \\ \Theta(x,t) &= k_0(t) + k_1(t)x + k_2(t)x^2\,, \quad \xi = p(t)x + q(t)\,, \end{aligned} \tag{7}$$

where $A_0(t)$, $A_1(t)$, $B_1(t)$, $p(t)$, $q(t)$, $k_0(t)$, $k_1(t)$, $k_2(t)$ are functions of t to be determined, μ_1 and μ_2 are constants, and $\phi(\xi)$ satisfies Eq. (5).

Substituting Eqs. (5) and (7) into Eq. (1), removing the exponential term, collecting coefficients of monomials of

$\phi(\xi)$, $\sqrt{R+\phi^2(\xi)}$, and x of the resulting system, then separating each coefficient to the real part and imaginary part and setting each part to zero, we obtain an ordinary differential equation (ODE) system with respect to differentiable functions $a(t)$, $A_0(t)$, $A_1(t)$, $B_1(t)$, $p(t)$, $q(t)$, $k_0(t)$, $k_1(t)$, and $k_2(t)$. Because the ODE system includes 35 ODEs, for simplification, we omit them in the paper.

Solving the ODE system with symbolic computation system *Maple*, we can obtain the following results.

The following three cases all satisfy the following conditions,

$$p(t) = c_1 \exp\Bigl(\int -4k_2(t)\,\mathrm{d}t\Bigr)\,, \quad q(t) = -2c_1c_2 \int \exp\Bigl(\int -8k_2(t)\,\mathrm{d}t\Bigr)\mathrm{d}t + c_3\,,$$
$$k_1(t) = c_2 \exp\Bigl(\int -4k_2(t)\,\mathrm{d}t\Bigr), k_2(t) = \frac{1}{4}\frac{(\mathrm{e}^{2\lambda(t+c_0)}-1)\lambda}{1+\mathrm{e}^{2\lambda(t+c_0)}}\,, \quad \text{or } k_2(t) = \pm\frac{\lambda}{4}\,. \tag{8}$$

Case 1

$$\mu_2 = B_1(t) = 0\,, \quad a(t) = -\frac{\exp(-4\int k_2(t)\,\mathrm{d}t)c_1^2(1+R\mu_1^2)}{c_4^2}\,, \quad A_0(t) = -\frac{R\mu_1}{1+\mu_1^2R}A_1(t)\,,$$
$$A_1(t) = c_4 \exp\Bigl(\int -2k_2(t)\,\mathrm{d}t\Bigr)\,, \quad k_0(t) = (2Rc_1^2 - c_2^2)\int \exp\Bigl(-8\int k_2(t)\,\mathrm{d}t\Bigr)\mathrm{d}t + c_5\,. \tag{9}$$

Case 2

$$\mu_2 = A_0(t) = A_1(t) = 0\,, \quad k_0(t) = \int (-c_2^2 - Rc_1^2)\exp\Bigl(-8\int k_2(t)\,\mathrm{d}t\Bigr)\mathrm{d}t + c_5\,,$$
$$B_1(t) = c_4\mathrm{exp}\Bigl(\int -2k_2(t)\,\mathrm{d}t\Bigr)\,, \quad a(t) = -\frac{\exp(-4\int k_2(t)\,\mathrm{d}t)c_1^2(1+R\mu_1^2)}{c_4^2}\,. \tag{10}$$

Case 3

$$\mu_1 = \pm\mu_2\,, \quad A_1(t) = \pm B_1(t) = c_4\mathrm{exp}\Bigl(\int -2k_2(t)\,\mathrm{d}t\Bigr)\,, \quad A_0(t) = \mp\frac{R\mu_1}{R\mu_1^2+1}A_1(t)\,,$$
$$a(t) = -\frac{1}{4}\frac{(R\mu_1^2+1)^2\mathrm{exp}\bigl[-4\int k_2(t)\,\mathrm{d}t\bigr]c_1^2}{c_4^2}\,, \quad k_0(t) = \int\Bigl(\frac{1}{2}Rc_1^2 - c_2^2\Bigr)\mathrm{exp}\Bigl(-8\int k_2(t)\,\mathrm{d}t\Bigr)\mathrm{d}t + c_5\,, \tag{11}$$

Thus from Eqs. (6) and (7) and Cases 1 $\sim$ 3, three families of exact analytical solutions are obtained as follows. For simplification, we only list the tanh-type's solutions under $R<0$.

Family 1

$$\psi_1(x,t) = c_4 \exp\Bigl(\int -2k_2(t)\,\mathrm{d}t\Bigr)\Bigl[-\frac{\mu_1 R}{\mu_1^2R+1} - \frac{\sqrt{-R}\tanh(\sqrt{-R}\xi)}{1-\mu_1\sqrt{-R}\tanh(\sqrt{-R}\xi)}\Bigr]\exp\bigl[\mathrm{i}(k_0(t)+k_1(t)x+k_2(t)x^2)\bigr]\,, \tag{12}$$

where ξ, $k_1(t)$, and $k_2(t)$ are determined by Eq. (18) and

$$a(t) = -\frac{c_1^2(R\mu_1^2+1)^2}{c_4^2}\exp\Bigl(\int -4k_2(t)\,\mathrm{d}t\Bigr)\,, \quad k_0(t) = (2Rc_1^2 - c_2^2)\int \exp\Bigl(-8\int k_2(t)\,\mathrm{d}t\Bigr)\mathrm{d}t + c_5\,. \tag{13}$$

Family 2

$$\psi_2(x,t) = c_4\mathrm{exp}\Bigl(\int -2k_2(t)\,\mathrm{d}t\Bigr)\frac{\sqrt{R}\,\mathrm{sech}(\sqrt{-R}\xi)}{1-\mu_1\sqrt{-R}\,\tanh(\sqrt{-R}\xi)}\exp\bigl[\mathrm{i}(k_0(t)+k_1(t)x+k_2(t)x^2)\bigr]\,, \tag{14}$$

where ξ, $k_1(t)$, and $k_2(t)$ are determined by Eq. (18) and

$$a(t) = -\frac{\exp(-4\int k_2(t)\,\mathrm{d}t){c_1}^2(1+R\mu_1^2)}{c_4^2}\,, \quad k_0(t) = \int (c_2^2 - Rc_1^2)\exp\Bigl(-8\int k_2(t)\,\mathrm{d}t\Bigr)\mathrm{d}t + c_5\,. \tag{15}$$

Family 3

$$\psi_3(x,t) = \Bigl[\mp\frac{R\mu_1}{\mu_1^2R+1} + \frac{-\sqrt{-R}\tanh(\sqrt{-R}\xi) \pm \sqrt{R}\,\mathrm{sech}(\sqrt{-R}\xi)}{1-\mu_1\sqrt{-R}\tanh(\sqrt{-R}\xi) \pm \mu_1\sqrt{R}\,\mathrm{sech}(\sqrt{-R}\xi)}\Bigr]$$
$$\times\, c_4\mathrm{exp}\Bigl(\int -2k_2(t)\,\mathrm{d}t\Bigr)\exp\bigl[\mathrm{i}(k_0(t)+k_1(t)x+k_2(t)x^2)\bigr]\,, \tag{16}$$

where ξ, $k_1(t)$, and $k_2(t)$ are determined by Eq. (18) and

$$a(t) = -\frac{1}{4}\frac{(R\mu_1^2+1)^2\mathrm{exp}\bigl[-4\int k_2(t)\,\mathrm{d}t\bigr]{c_1}^2}{c_4^2}\,, \quad k_0(t) = \Bigl(\frac{1}{2}c_1^2R - c_2^2\Bigr)\int \exp\Bigl(-8\int k_2(t)\,\mathrm{d}t\Bigr)\mathrm{d}t + c_5\,. \tag{17}$$

In the above three families, besides satisfying the corresponding conditions, each family also satisfies the following conditions at the same time.

$$\xi = c_1 \exp\Big(\int -4k_2(t)\,\mathrm{d}t\Big)x - 2c_1c_2 \int \exp\Big(\int -8k_2(t)\,\mathrm{d}t\Big)\,\mathrm{d}t + c_3\,,$$
$$k_1(t) = c_2 \exp\Big(\int -4k_2(t)\,\mathrm{d}t\Big)\,,\quad k_2(t) = \frac{1}{4}\frac{(\mathrm{e}^{2\lambda(t+c_0)}-1)\lambda}{1+\mathrm{e}^{2\lambda(t+c_0)}} \quad \Big(\text{or } k_2(t) = \pm\frac{\lambda}{4}\Big)\,. \tag{18}$$

Remark 4 If setting $\mu_1 = 0$ in Family 1 and Family 2, the dark soliton (11) and bright soliton (12) obtained in Ref. [19] can be recovered. But to our knowledge, the other solutions have not been reported earlier.

If we select $k_2(t) = -\lambda/4$ in Family 1 and Family 2 and set $\mu_1 = 0$, then the scattering length

$$a(t) = -\frac{c_1^2}{c_4^2}c_5 \exp(\lambda t) = g_0 \exp(\lambda t)\,,$$

i.e., the Feshbach-managed nonlinear coefficient[14] (because of $\lambda \ll 1$, the nonlinear coefficient $a(t) = g_0 \exp(\lambda t)$ can be expressed as $a(t) = g_0[1+\lambda t + o^2(\lambda t)]$). Under this condition, we can obtain well-known dark and bright solitons.[16,19] If we select

$$k_2(t) = \frac{1}{4}\frac{(\mathrm{e}^{2\lambda(t+c_0)}-1)\lambda}{1+\mathrm{e}^{2\lambda(t+c_0)}}$$

and $\mu_1 = 0$, then the scattering length is

$$a(t) = -\frac{c_1^2}{2c_4^2}c_5\,\mathrm{sech}(\lambda t) = g_1\,\mathrm{sech}(\lambda t)$$

under $c_0 = 0$, i.e., the hyperbolic secant function coefficient.

In order to understand the significance of these solutions expressed by Eqs. (12) $\sim$ (15) and (18), the main soliton features of them were investigated by using direct computer simulations with the accuracy as high as 10^{-9}.

Two figures (Figs. 1(a) and 1(b)) are plotted to show the dynamics of the Feshbach-resonance-managed soliton in the expulsive parabolic potential given by Eq. (12) under some special parameters (**Note:** in the figures of this paper: $M = |\psi(x,t)|^2$). As is shown in Figs. 1(a) and 1(b), the height of the dark soliton increases exponentially and its width gets compressed during its propagation. From Fig. 2 we can see that, with the propagation of the dark soliton $\psi_1(x,t)$, the dark solitons have a decrease in the height and an expansion in the width. At the same time, four figures are plotted to show the dynamics of the bright soliton $\psi_2(x,t)$ under some selectable parameters. As is shown in Figs. 3 and 4, the properties of bright soliton $\psi_2(x,t)$ are similar to the dark soliton $\psi_1(x,t)$.

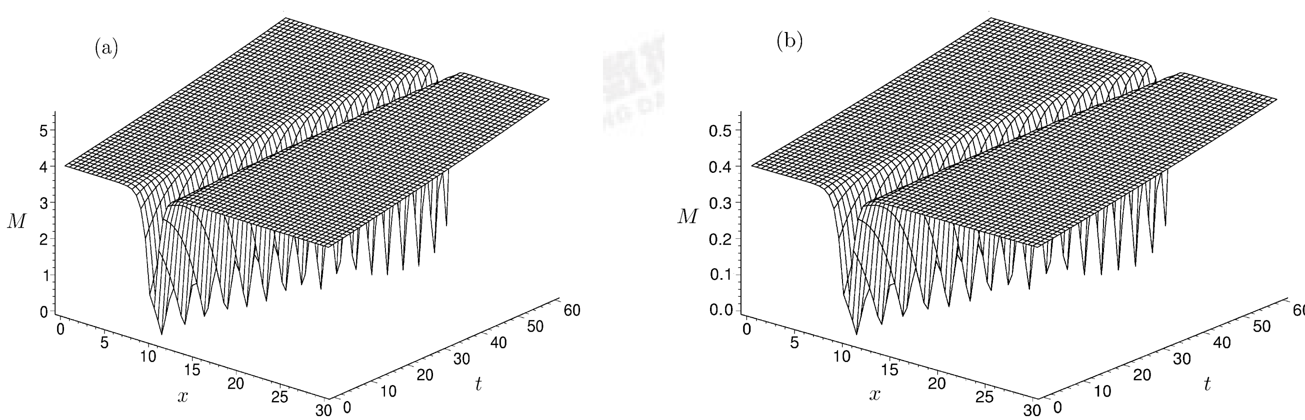

Fig. 1 The dynamics of the Feshbach-resonance-managed soliton in the expulsive parabolic potential given by Eq. (12). The parameters are given as follows: $R = -1$, $k_2(t) = -\lambda/4$, $\lambda = 0.01$, $c_1 = 1$, $c_2 = 0.1$, $c_3 = 0.04$, $c_4 = 40$, and $\mu_1 = 0$ in (a) ($\mu_1 = 0.0001$ in (b)).

It is necessary to point out that the properties shown in Fig. 1(a) and Fig. 3(a) are in agreement with Refs. [19] and [16]. Only in the conditon $0 \neq \mu_1 \ll 1$, can the solutions $\psi_1(x,t)$ and $\psi_2(x,t)$ possess the properties of solitons.

With regard to the solution $\psi_3(x,t)$, the property of soliton is not found by us through computer simulation.

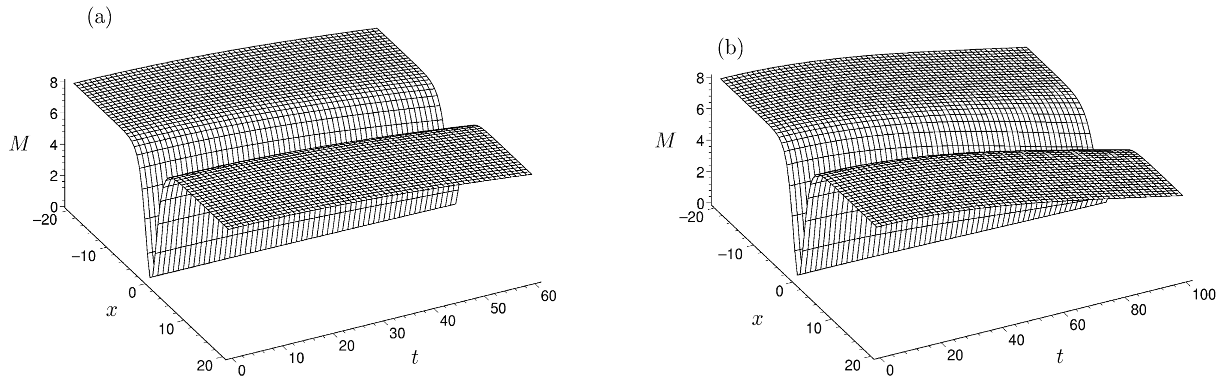

Fig. 2 The dynamics of the solitons given by Eq. (12) under $k_2(t) = (\mathrm{e}^{2\lambda(t+c_0)} - 1)\lambda/(4(1 + \mathrm{e}^{2\lambda(t+c_0)}))$. The concrete parameters are as follows: $R = -1$, $\lambda = 0.01$, $c_0 = 0$, $c_1 = 1$, $c_2 = 0$, $c_3 = 0.04$, $c_4 = 0.4$, and $\mu_1 = 0$ in (a) ($\mu_1 = 0.0001$ in (b)).

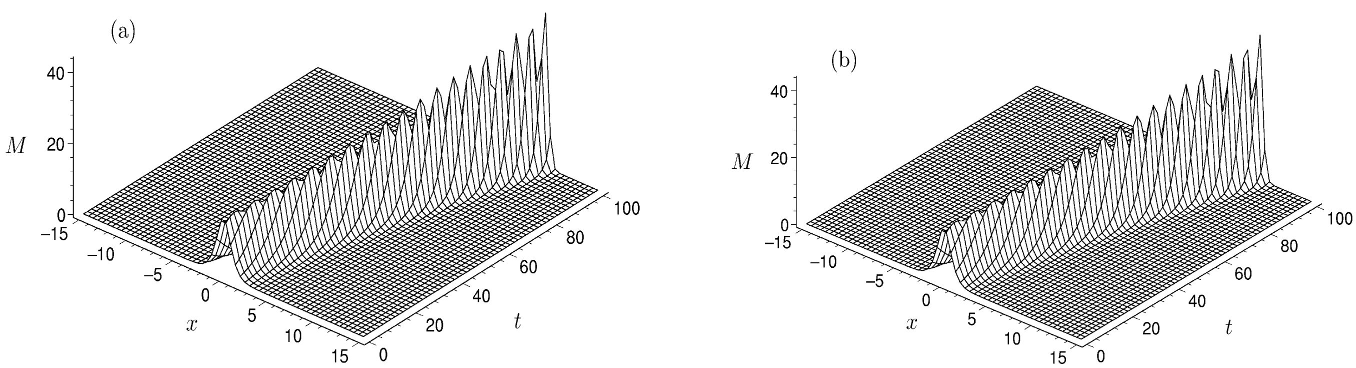

Fig. 3 The dynamics of the Feshbach-resonance-managed soliton in the expulsive parabolic potential given by Eq. (13). The parameters are given as follows: $R = -1$, $k_2(t) = -\lambda/4$, $\lambda = 0.01$, $c_1 = 1$, $c_2 = 0.04$, $c_3 = 4$, $c_4 = 4$, and $\mu_1 = 0$ in (a) ($\mu_1 = 0.0001$ in (b)).

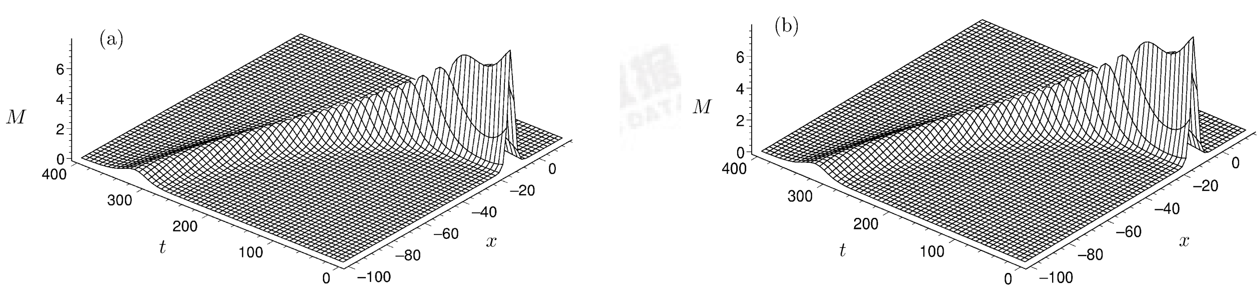

Fig. 4 The dynamics of the solitons given by Eq. (13) under $k_2(t) = (\mathrm{e}^{2\lambda(t+c_0)} - 1)\lambda/(4(1 + \mathrm{e}^{2\lambda(t+c_0)}))$. The concrete parameters are as follows: $R = -1$, $\lambda = 0.01$, $c_0 = 0$, $c_1 = 1$, $c_2 = 0.04$, $c_3 = 4$, $c_4 = 4$, and $\mu_1 = 0$ in (a) ($\mu_1 = 0.0001$ in (b)).

4 Summary and Discussion

In the paper, the generalized Riccati equation rational expansion method is presented. Making use the method and symbolic computation, we present three families of exact solutions of Bose–Einstein condensates with the time-dependent interatomic interaction in an expulsive parabolic potential. Then the dynamics of two solutions are demonstrated by computer simulations under some selectable parameters including the Feshbach-managed nonlinear coefficient $a(t) = g_0 \exp(\lambda t)$ and the hyperbolic secant function coefficient $a(t) = g_1 \operatorname{sech}(\lambda t)$. The method proposed here can be applied to other PDEs and coupled ones.

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