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FUNCTION PROJECTIVE SYNCHRONIZATION BETWEEN TWO IDENTICAL CHAOTIC SYSTEMS

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First, a function projective synchronization of two identical systems is defined. Second, based on the active control method and symbolic computation *Maple*, the scheme of function projective synchronization is developed to synchronize two identical chaotic systems (two identical classic Lorenz systems) up to a scaling function matrix with different initial values. Numerical simulations are used to verify the effectiveness of the scheme.

Keywords: Function projective synchronization; active control method; identical classic Lorenz systems.

1. Introduction

The synchronization problem in chaotic systems has been intensively and extensively studied in recent decades, since Pecora and Carroll[1] discovered the synchronization of the chaotic system by means of nonlinear driving. On the one hand, it is because of both the surprising nature of the phenomenon and its prospective practical applications, particularly in communication technology, biological networks, artificial neural networks, etc, and on the other hand, the availability of computer systems allow us to perform some complicated and tedious calculation on a computer. Up to now, there exist many types of chaos synchronization in dynamical systems such as complete synchronization, partial synchronization, phase synchronization, lag synchronization, anticipated synchronization, generalized slag, anticipated, and completed synchronization, synchronization, antiphase synchronization, etc.[2–5] In particular, among all kinds of chaos synchronization, projective synchronization reported by Mainieri and Rehacek[6] is one of the most noticable ones that the drive and response vectors evolve in a proportional scale — the vectors become proportional.

The early projective synchronization is usually observable only in a class of systems with partial-linearity,[7] but recently some researchers[8,9] have achieved control of the projective synchronization in a general class of chaotic systems including non-partially-linear systems, and termed this projective synchronization as "generalized projective synchronization" (GPS). Li[10] shows GPS between Lorenz system and Chen's system. Recently, the modify of projective synchronization is proposed in Ref. 11 to synchronize two identical systems up to a scaling constant matrix.

In this paper, based on the work,[6–10] and by introducing a new more general synchronization form than the one in the above scheme, we propose a new and more definition: function projective synchronization (FPS), where the response of the synchronized dynamical states synchronize up to a scaling function matrix f. Numerical simulation results are given for illustration and verification.

2. Function Projective Synchronization Between Two Identical Systems

Based on the previous projective synchronization and modified projective synchronization proposed by Li,[11] function projective synchronization is characterized by a scaling function matrix. At the first, we give the instruction of FPS as follows:

Let $\dot{x} = F(x,t)$ be the drive chaotic system, and $\dot{y} = F(y,t) + U$ is the response system, where $x = (x_1(t), x_2(t), \ldots, x_m(t))^T$, $y = (y_1(t), y_2(t), \ldots, y_m(t))^T$, $U = (u_1(x,y), u_2(x,y), \ldots, u_m(x,y))$ is a controller to be determined later. Denote $e_i = x_i - f_i(x)y_i$, $(i = 1, 2, \ldots, m)$, $f_i(x)(i = 1, 2, \ldots, m)$ are functions of x. If $\lim_{t\to\infty} \|e\| = 0$, $e = (e_1, e_2, \ldots, e_m)$, we call these two identical chaotic systems *FPS*, and we call f a "scaling function matrix".

Consider the drive system in the form of

$$\dot{x} = Ax + h(x,t) \tag{1}$$

where $x \in R^n$, A is an $m \times m$ constant matrix, and $h : R^n \to R^n$ is a nonlinear function. Assume that the response system coupled with Eq. (1) is as follows:

$$\dot{y} = Ay + h(y,t) + U \tag{2}$$

where $y \in R^n$, U is a controller to be determined later.

Theorem 1. *For an invertible diagonal function matrix f, function projective synchronization between the two systems* (1) *and* (2) *will occur, if the following conditions are satisfied*:

(i) $U = f^{-1}h(x,t) + (f^{-1}Af - A)y + f^{-1}B(x - fy) - h(y,t) - f^{-1}gy$, *where* $g = \mathrm{diag}(\dot{f}_1, \dot{f}_2, \ldots, \dot{f}_m)$, *and* $B \in R^{m\times m}$;

(ii) *The real parts of all the eigenvalues of* $(A - B)$ *are negative.*

Proof. From $e = x - fy$ in definition of FPS, one can get $\dot{e} = \dot{x} - f\dot{y} - gy = Ax + h(x,t) - f(Ay + h(y,t) + U) - gy = Ax + h(x,t) - fAy - fh(y,t) - gy - h(x,t) - Afy + fAy - B(x - fy) + fh(y,t) + gy = (A - B)e$.

For a feasible control, the feedback B must be selected such that all the eigenvalues of $(A-B)$, if any, have negative real parts. Thus, if the matrix $(A-B)$ is in full rank, the system $\dot{e}$ is asymptotically stable at the origin, which implies Eqs. (1) and (2) are in the state of function projective synchronization.

In this case, the active control method[13] is usually adopted to obtain the gain matrix B for any specified eigenvalues of $(A-B)$. □

Remark 1. When $f_1 = f_2 = \cdots = f_m = 1$, $f_1 = f_2 = \cdots = f_m = \alpha$ and $f_1 = \alpha_1$, $f_2 = \alpha_2, \ldots, f_m = \alpha_m$, CS, projective synchronization and modified projective synchronization will appear, respectively. And the scaling function matrix f has no effect on the eigenvalues of $(A-B)$, that is to say, one can adjust the scaling function matrix arbitrarily during control without worrying about the control robustness.

3. FPS of Two Identical Chaotic Systems

First, we introduce a new unified chaotic system (called unified Lorenz-Chen-Lü system[12]) in the following form

$$\begin{cases} \dot{x_1} = (25\alpha + 10)(x_2 - x_1)\,, \\ \dot{x_2} = (28 - 35\alpha)x_1 - x_1x_3 + (29\alpha - 1)x_2\,, \\ \dot{x_3} = x_1x_2 - \dfrac{8+\alpha}{3}x_3\,. \end{cases}$$

When $\alpha = 0$, this system is called *classic Lorenz system*, that is

$$\begin{cases} \dot{x_1} = 10(x_2 - x_1)\,, \\ \dot{x_2} = 28x_1 - x_1x_3 - x_2\,, \\ \dot{x_3} = x_1x_2 - \dfrac{8}{3}x_3 \end{cases}$$

In this section, we use active control method to make two identical classic Lorenz systems globally synchronized.

The drive and response systems are defined as follows:

$$\begin{cases} \dot{x_1} = 10(x_2 - x_1)\,, \\ \dot{x_2} = 28x_1 - x_1x_3 - x_2\,, \\ \dot{x_3} = x_1x_2 - \dfrac{8}{3}x_3 \end{cases} \tag{3}$$

and

$$\begin{cases} \dot{y_1} = 10(y_2 - y_1) + u_1\,, \\ \dot{y_2} = 28y_1 - y_1y_3 - y_2 + u_2\,, \\ \dot{y_3} = y_1y_2 - \dfrac{8}{3}y_3 + u_3\,. \end{cases} \tag{4}$$

Here is written as $\dot{x} = Ax + h(x,t)$, where $A = \begin{pmatrix} -10 & 10 & 0 \\ 28 & -1 & 0 \\ 0 & 0 & -\frac{8}{3} \end{pmatrix}$, and $h(x,t) = (0, -x_1x_3, x_1x_2)^T$.

Referring to the original methods of active control, we choose $f(x) = \mathrm{diag}(\tanh x_3, \tanh x_3, m)$, that is

$$\begin{cases} e_1 = x_1 - y_1 \tanh x_3\,, \\ e_2 = x_2 - y_2 \tanh x_3\,, \\ e_3 = x_3 - m y_3\,. \end{cases} \tag{5}$$

By feedback stepping method, we can choose

$$B = \begin{pmatrix} 0 & 10 & 0 \\ 28 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

It is easy to see that, the matrix

$$A - B = \begin{pmatrix} 10 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -\dfrac{8}{3} \end{pmatrix}$$

has all eigenvalues with negative real parts. And correspondingly the three control functions $u_i (i = 1, 2, 3)$ are in the form as follows:

$$U = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} \dfrac{(\tanh^2 x_3 - 1)(x_1 x_2 - 8/3 x_3) y_1 + 10(x_2 - y_2 \tanh x_3)}{\tanh x_3} \\ \dfrac{28(x_1 - y_1 \tanh x_3) - x_1 x_3}{\tanh x_3} + y_1 y_3 - \dfrac{(1 - \tanh^2 x_3)(x_1 x_2 - 8/3 x_3) y_2}{\tanh x_3} \\ \dfrac{x_1 x_2}{m} - y_1 y_2 \end{pmatrix} \tag{6}$$

where we let $m = 1$. Based on symbolic computation system *Maple* and Lyapunov stability theory, we give some numerical simulations to illustrate our results. Figure 1 shows the numerical simulation of the error e of the two identical Lorenz systems, and Fig. 2 reveals the numerical global synchronization between them with initial values (0.1, 0.2, 0.3) and (0.2, 0.3, 0.5) respectively.

Remark 2. The active method does not to calculate the Lyapunov exponents and the eigenvalues of the Jacobian matrix, which makes it simple and convenient. Recently, the active methods has been extended to study the chaos synchronization of real Van der Pol oscillators and complex Van der Pol oscillators.[14] Based on the active method, Park proposed the adaptive synchronization method for Rössler system with uncertain parameters.[15] Wang used the adaptive synchronization method to study the adaptive synchronization and parameters identification problem for a class of high-dimensional autonomous uncertain chaotic systems.[16] Our scheme will be naturally extended to investigate the problem of chaos synchronization to

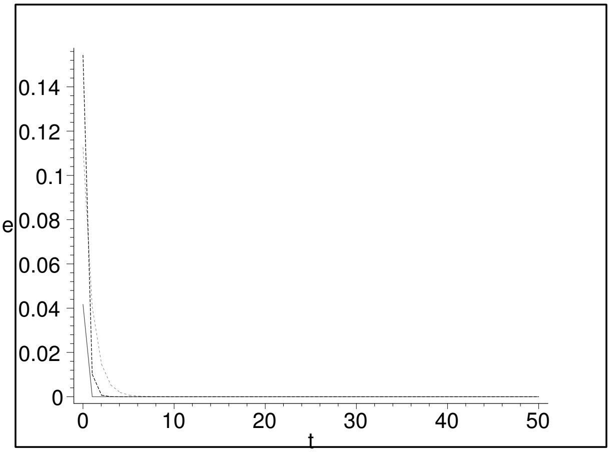

Fig. 1. "–" denotes e_1, "– · –" denotes e_2 and "– –" denotes e_3.

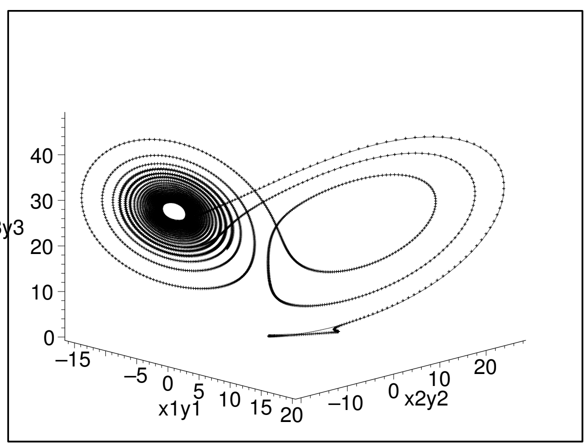

Fig. 2. "–" denotes the drive system, "· · ·" denotes the response system.

chaotic system with three uncertain parameters: the adaptive synchronization in further work.

4. Summary and Conclusions

In summary, we have defined a FPS of two identical systems, which is more general than the definition known. It extends the control capability to achieve a full range synchronization of all state variables in a functional proportional way. Based on the active control method and symbolic computation *Maple*, we show how to synchronize two identical chaotic systems with the active control method, and make them globally synchronize in one coordinate with a scaling function matrix f. Numerical simulations are carried out to verify the effectiveness of the proposed controller.

With the aid of symbolic-numeric computation, the scheme can be used for other chaotic systems, hyperchaotic systems and uncertain chaotic systems.

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