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Symbolic computation and solitons of the nonlinear Schrödinger equation in inhomogeneous optical fiber media

Biao Li [a,b,c,*], Yong Chen [b,c]

[a] *Department of Physics, Shanghai Jiao Tong University, Shanghai 200030, China*
[b] *Nonlinear Science Center, Ningbo University, Ningbo 315211, China*
[c] *MM Key Lab, Chinese Academy of Sciences, Beijing 100080, China*

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Abstract

In this paper, the inhomogeneous nonlinear Schrödinger equation with the loss/gain and the frequency chirping is investigated. With the help of symbolic computation, three families of exact analytical solutions are presented by employing the extended projective Riccati equation method. From our results, many previous known results of nonlinear Schrödinger equation obtained by some authors can be recovered by means of some suitable selections of the arbitrary functions and arbitrary constants. Of optical and physical interests, soliton propagation and soliton interaction are discussed and simulated by computer, which include snake-soliton propagation and snake-solitons interaction, boomerang-like soliton propagation and boomerang-like solitons interaction, dispersion managed (DM) bright (dark) soliton propagation and DM solitons interaction.

1. Introduction

As is known, the nonlinear Schrödinger equation (NLS) model is one of the most important and "universal" nonlinear models of modern science. In particular, NLSE optical solitons are regarded as the natural data bits and as an important alternative for the next generation of ultrahigh speed optical telecommunication systems. The milestone works are: the possibility of solitons in optical fibers proved by Hasegawa and Tappert [1]; the inverse scattering transform scheme for NLSE reported by Zakharov and Shabat [2]; the experimental evidence of solitons in optical fibers shown by Mollenauer et al. [3]. Since then, the dynamics of the soliton propagation in optical fibers has become a major area of research given its potential applicability in the optical communication systems, and have been extensively studied theoretically by various methods [4–28]. However, in a real fiber, in general, the core medium is not homogeneous [6,7]. Considering the inhomogeneities in the fiber, the dynamics of the optical pulse propagation is governed by the following inhomogeneous nonlinear Schrödinger [6,7] equation (INLS)

* Corresponding author. Address: Department of Physics, Shanghai Jiao Tong University, Shanghai 200030, China.
E-mail address: libiao@dlut.edu.cn (B. Li).

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$$\mathrm{i}\frac{\partial u}{\partial z}+\frac{\alpha(z)}{2}\frac{\partial^2 u}{\partial t^2}+\beta(z)|u|^2u+M(z)t^2u+\mathrm{i}F(z)u=0, \tag{1}$$

where $u(z,t)$ is the complex envelope of the electrical field in a comoving frame, z is the normalized distance and t is the retarded time, $\alpha(z)$ is the group velocity dispersion parameter and $\beta(z)$ is related to the Kerr nonlinearity, and $M(z)$ and $F(z)$ are inhomogeneous parameters related to phase modulation and loss/gain, which are the functions of the propagation distance z, respectively. The study of nonlinear wave propagation for Eq. (1) is of great interest and have wide range of applications. It is not only restricted for optical pulse propagating in inhomogeneous optical fiber media, which has found application in pulse compression, but also for the core of a dispersion-managed soliton [8]. Recently, the application of Eq. (1) with various forms of inhomogeneities has been studied in various papers [9–13]. It should be pointed out that without the residual loss/gain term Eq. (1) has been studied in different context in [9,10]. With the loss/gain term, Eq. (1) has been reported in [10–14] from the integrability point of view, where by choosing a special parameter, one soliton solution has been obtained by Bäcklund transformation. Without the phase modulation term $M(z)$, Eq. (1) has been studied by many authors, such as, in [10,17,18], Serkin et al. developed an effective mathematical algorithm to discover and investigate infinite number of novel soliton solutions and discussed soliton management and femtosecond soliton amplification; Ruan et al. [22] reported some exact solutions by the symmetry approach; Hong et al. [23] proposed an intrinsic conservation law; Li et al. [26] develop a system method and construct six families of exact analytical solutions for it.

In [6], by the transformation $u=\mu\sqrt{\alpha(z)/\beta(z)}q(z,t)$, $z=\frac{1}{2}\int\alpha(\tau)\,\mathrm{d}\tau$, Eq. (1) become the following nonintegrable form:

$$\mathrm{i}\frac{\partial q}{\partial z}+\frac{\partial^2 q}{\partial t^2}+2\mu^2|q|^2q+M_1(z)t^2q+\mathrm{i}F_1(z)q=0, \tag{2}$$

here $M_1(z)=2M(z)/\alpha(z)$ and $F_1(z)=2F(z)/\alpha(z)+(\beta(z)\alpha(z)_z-\ \beta(z)_z\alpha)/\alpha(z)^2\beta(z)$ are related to the phase modulation and the loss/gain, respectively, and the subscript "z" represents the derivative with respect to z, and the parameter μ is the real constant. Under the conditions $F_1(z)=\beta=\mathrm{const}$ and $M_1(z)=\beta^2$, Li et al. [7] obtained N-solitary solution of Eq. (2) by Darboux transformation and investigated two exact analytical solutions that describe the modulation instability and the soliton propagation on a continuous wave background with the loss/gain and the frequency chirping. In [27], three families of analytical solutions for Eq. (2) are derived by us, and the soliton propagation and solitons interaction scenario are discussed and simulated by computer.

The motivation in this paper lies in the optical and physical importance of the INLSE (1) and the need to have some exact analytical solutions. To have some explicit analytical solutions of INLSE (1) may enable one to better understand the optical and physical phenomena which it describes. The exact solutions, which are accurate and explicit, may help physicists and engineers to discuss and examine the sensitively of the model to several physical parameters. In this work, we will work on INLSE (1) to construct a series exact analytical solutions by symbolic computation and the extended projective Riccati equation method proposed by us [25–28]. As a result, three families of analytical solutions for INLSE (1) are derived. Then based on these analytical solutions, soliton propagation and soliton interaction are discussed and simulated by computer.

2. Solitons of INLSE in inhomogeneous optical fiber media

We now investigate NLSE (1) with the extended projective Riccati equation method proposed by us [25–28]. In order to obtain some exact solutions of INLSE (1), firstly we make the transformation

$$u(z,t)=[a_0(z)+a_1(z)\sigma(\xi)+b_1(z)\tau(\xi)]\exp\left\{\mathrm{i}[t^2\lambda_2(z)+t\lambda_1(z)+\lambda_0(z)]\right\}, \tag{3}$$

where

$$\xi=t\Omega(z)+\delta(z), \tag{4}$$

and $a_0(z)$, $a_1(z)$, $b_1(z)$, $\Omega(z)$, $\delta(z)$, $\lambda_2(z)$, $\lambda_1(z)$ and $\lambda_0(z)$ are functions of z to be determined, $\tau(\xi)$ and $\sigma(\xi)$ satisfy the following projective Riccati equation:

$$\frac{\mathrm{d}\sigma(\xi)}{\mathrm{d}\xi}=-\sigma(\xi)\tau(\xi),\quad \frac{\mathrm{d}\tau(\xi)}{\mathrm{d}\xi}=1-\mu\sigma(\xi)-\tau^2(\xi), \tag{5}$$

where μ is constant, and $\sigma(\xi)$ and $\tau(\xi)$ satisfy the following equation:

$$\tau^2(\xi)=1-2\mu\sigma(\xi)+(\mu^2-1)\sigma^2(\xi). \tag{6}$$

Substituting (3)–(6) into INSLE (1), removing the exponential term, collecting coefficients of monomials of $\tau(\xi)$, $\sigma(\xi)$ and t of the resulting system, then separating each coefficients to the real part and imaginary part and setting each part to zero, we obtain an ordinary differential equation (ODE) system with respect to differentiable functions α, $\beta(z)$, $M(z)$, $F(z)$, $a_0(z)$, $a_1(z)$, $b_1(z)$, $\lambda_0(z)$, $\lambda_1(z)$ and $\lambda_2(z)$. Because the ODE system include 21 ODEs, for simplification, we omit them in the paper.

Solving the ODE system with symbolic computation system-*Maple*, we can obtain the following results:

Case 1.

$$\begin{aligned}
&\mu=0,\quad \lambda_0(z)=-\frac{1}{2}\int\alpha(z)\mathrm{e}^{-4\int\lambda_2(z)\alpha(z)\,\mathrm{d}z}\mathrm{d}zC_1^2-\int\alpha(z)\mathrm{e}^{-4\int\lambda_2(z)\alpha(z)\,\mathrm{d}z}\mathrm{d}zC_2^2+C_3,\\
&a_0(z)=a_1(z)=0,\quad \lambda_1(z)=C_1\mathrm{e}^{-2\int\lambda_2(z)\alpha(z)\,\mathrm{d}z},\quad F(z)=-\frac{\frac{\mathrm{d}}{\mathrm{d}z}b_1(z)+\alpha(z)\lambda_2(z)b_1(z)}{b_1(z)},\\
&M(z)=2\alpha(z)(\lambda_2(z))^2+\frac{\mathrm{d}}{\mathrm{d}z}\lambda_2(z),\quad \delta(z)=-C_2C_1\int\alpha(z)\mathrm{e}^{-4\int\lambda_2(z)\alpha(z)\mathrm{d}z}\mathrm{d}z+C_4,\\
&\Omega(z)=C_2\mathrm{e}^{-2\int\lambda_2(z)\alpha(z)\,\mathrm{d}z},\quad \beta(z)=-\frac{\alpha(z)C_2^2\mathrm{e}^{-4\int\lambda_2(z)\alpha(z)\,\mathrm{d}z}}{(b_1(z))^2},
\end{aligned}\tag{7}$$

where C_1, C_2, C_3, C_4 are arbitrary constants, $\alpha(z)$, $\lambda_2(z)$ and $b_1(z)$ are arbitrary functions of z.

Case 2.

$$\begin{aligned}
&\mu=a_0(z)=b_1(z)=0,\quad \lambda_1(z)=C_1\mathrm{e}^{-2\int\lambda_2(z)\alpha(z)\,\mathrm{d}z},\quad M(z)=2\alpha(z)(\lambda_2(z))^2+\frac{\mathrm{d}}{\mathrm{d}z}\lambda_2(z),\\
&\delta(z)=-C_2C_1\int\alpha(z)\mathrm{e}^{-4\int\lambda_2(z)\alpha(z)\,\mathrm{d}z}\mathrm{d}z+C_4,\quad \Omega(z)=C_2\mathrm{e}^{-2\int\lambda_2(z)\alpha(z)\,\mathrm{d}z},\\
&\lambda_0(z)=\frac{1}{2}\int\alpha(z)\mathrm{e}^{-4\int\lambda_2(z)\alpha(z)\,\mathrm{d}z}\mathrm{d}zC_2^2-\frac{1}{2}\int\alpha(z)\mathrm{e}^{-4\int\lambda_2(z)\alpha(z)\,\mathrm{d}z}\mathrm{d}zC_1^2+C_3,\\
&\beta(z)=\frac{\alpha(z)C_2^2\mathrm{e}^{-4\int\lambda_2(z)\alpha(z)\,\mathrm{d}z}}{(a_1(z))^2},\quad F(z)=-\frac{\frac{\mathrm{d}}{\mathrm{d}z}a_1(z)+\alpha(z)\lambda_2(z)a_1(z)}{a_1(z)},
\end{aligned}\tag{8}$$

where C_1, C_2, C_3, C_4 are arbitrary constants, $\alpha(z)$, $\lambda_2(z)$ and $a_1(z)$ are all arbitrary function of z, $a_1'=\frac{\mathrm{d}}{\mathrm{d}z}a_1(z)$.

Case 3.

$$\begin{aligned}
&a_0(z)=0,\quad a_1(z)=-b_1(z)\sqrt{\mu^2-1},\quad F(z)=-\frac{\alpha(z)\lambda_2(z)b_1(z)+\frac{\mathrm{d}}{\mathrm{d}z}b_1(z)}{b_1(z)},\\
&M(z)=2(\lambda_2(z))^2\alpha(z)+\frac{\mathrm{d}}{\mathrm{d}z}\lambda_2(z),\quad \lambda_1(z)=C_1\mathrm{e}^{-2\int\alpha(z)\lambda_2(z)\,\mathrm{d}z},\\
&\Omega(z)=C_3\mathrm{e}^{-2\int\alpha(z)\lambda_2(z)\,\mathrm{d}z},\quad \beta(z)=-\frac{1}{4}\frac{\alpha(z)C_3^2\mathrm{e}^{-4\int\alpha(z)\lambda_2(z)\,\mathrm{d}z}}{(b_1(z))^2},\\
&\lambda_0(z)=-\frac{1}{2}\int\alpha(z)\mathrm{e}^{-4\int\alpha(z)\lambda_2(z)\,\mathrm{d}z}\mathrm{d}zC_1^2-\frac{1}{4}\int\alpha(z)\mathrm{e}^{-4\int\alpha(z)\lambda_2(z)\,\mathrm{d}z}\mathrm{d}zC_3^2+C_6,\\
&\delta(z)=-C_3C_1\int\alpha(z)\mathrm{e}^{-4\int\alpha(z)\lambda_2(z)\,\mathrm{d}z}\mathrm{d}z+C_5,
\end{aligned}\tag{9}$$

where C_1, C_2, C_3, C_4 are arbitrary constants, $\alpha(z)$, $\lambda_2(z)$ and $b_1(z)$ are arbitrary functions of z.

We know that Eqs. (5) and (6) have the following solutions:

$$\sigma(\xi)=\frac{1}{\mu+\cosh\xi},\quad \tau(\xi)=\frac{\sinh\xi}{\mu+\cosh\xi}.\tag{10}$$

Thus according to (3), (4), (7)–(10), we can obtain three families of exact analytical solutions for INLSE (1) as follows:

Family 1–2. From Cases 1, 2, two families of solutions for INLSE (1) are as follows:

$$u_1(z,t) = C_2 p(z)\sqrt{-\frac{\alpha(z)}{\beta(z)}}\tanh\left\{C_2\left[p(z)t + C_1\int \alpha(z)p(z)^2\,\mathrm{d}z\right] + C_4\right\} \times \exp\left\{\mathrm{i}\left[\lambda_2(z)t^2 + C_1 p(z)t - \frac{C_1^2+2C_2^2}{2}\int \alpha(z)p(z)^2\,\mathrm{d}z + C_3\right]\right\}, \tag{11}$$

$$u_2(z,t) = C_2 p(z)\sqrt{\frac{\alpha(z)}{\beta(z)}}\operatorname{sech}\left\{C_2\left[p(z)t + C_1\int \alpha(z)p(z)^2\,\mathrm{d}z\right] + C_4\right\} \times \exp\left\{\mathrm{i}\left[\lambda_2(z)t^2 + C_1 p(z)t + \frac{C_2^2-C_1^2}{2}\int \alpha(z)p(z)^2\,\mathrm{d}z + C_3\right]\right\}, \tag{12}$$

where

$$\begin{aligned} &p(z) = \mathrm{e}^{-2\int \lambda_2(z)\alpha(z)\,\mathrm{d}z}, \quad M(z) = 2\alpha(z)(\lambda_2(z))^2 + \frac{\mathrm{d}}{\mathrm{d}z}\lambda_2(z), \\ &\beta(z)C_0^2 = \alpha(z)C_2^2 p(z)\mathrm{e}^{2\int F(z)\,\mathrm{d}z}, \end{aligned} \tag{13}$$

and C_0, C_1, C_2, C_3, C_4 are arbitrary constants.

Family 3.

$$u_3(z,t) = \frac{C_3 p(z)}{2}\sqrt{-\frac{\alpha(z)}{\beta(z)}}\left[\frac{\sqrt{\mu^2-1}}{\mu+\cosh(\xi)} \pm \frac{\sinh(\xi)}{\mu+\cosh(\xi)}\right] \times \exp\left\{\mathrm{i}\left[\lambda_2(z)t^2 + C_1 p(z)t - \frac{2C_1^2+C_3^2}{4}\int \alpha(z)p(z)^2\,\mathrm{d}z + C_3\right]\right\}, \tag{14}$$

where

$$\begin{aligned} &p(z) = \mathrm{e}^{-2\int \lambda_2(z)\alpha(z)\,\mathrm{d}z}, \quad M(z) = 2\alpha(z)(\lambda_2(z))^2 + \frac{\mathrm{d}}{\mathrm{d}z}\lambda_2(z), \\ &4\beta(z)C_0^2 = -\alpha(z)C_3^2 p(z)\mathrm{e}^{2\int F(z)\,\mathrm{d}z}, \quad \xi = C_3 p(z)t - C_3 C_1\int \alpha(z)p(z)^2\,\mathrm{d}z + C_5, \end{aligned} \tag{15}$$

and μ, C_0, C_1, C_3, C_5 are arbitrary constants.

Remark: (1) When setting $C_1 = 0$, $M(z) = 0$, the *Theorems 1–2* [17] and *Theorems 1–2* [18] can be reproduced by our solutions (11) and (12); (2) When setting $M(z) = 0$, the solutions obtained in [23] can also be recovered by selecting arbitrary functions and arbitrary constants suitably; (3) The solutions obtained in [27] can be recovered by setting $\alpha(z) = \text{const}$, $\beta(z) = \text{const}$; (4) When setting $M(z) = 0$, the solutions in [26] can be recovered. But to our knowledge, the other solutions have not been reported earlier.

In order to understand the significance of these solutions expressed by (11)–(15), the main soliton features of them were investigated by using direct computer simulations with the accuracy as high as 10^{-9}. We have investigated the interaction dynamics of particle-like solutions obtained and the influence of high-order effects on the dynamics of dispersion and amplification management. As follows from numerical investigations elastic character of chirped solitons interacting does not depend on a number of interacting solitons and their phases. For simplicity, we only consider some examples for each solution under some special parameters. Here, $U = |u(z,t)|^2$ denotes the intensity of solution.

Figs. 1a,b and 2a,b shows the snake-shaped dark solitons and boomerange-like solitons scenario given by $u_1(z,t)$. Dispersion managed (DM) soliton $u_1(z,t)$ with periodic dispersion coefficient are shown in Figs. 3a,b and 4a,b. Fig. 5a and b depict one snake-shaped bright soliton propagation and two snake-shaped bright solitons interaction given by $u_1(z,t)$, respectively. Fig. 6a and b represents one DM bright soliton evolution and two DM bright solitons interaction given by $u_2(z,t)$. Fig. 7a and b plot one periodic DM dark soliton propagation and two periodic DM solitons interaction.

In Fig. 8a and b, we consider some periodical chirped soliton solutions of $u_3(z,t)$. Suppose that the intensity of solitons varies periodically as

$$\alpha(z) = 1 - 0.8\sin(z)^2, \quad \beta(z) = 1, \quad p(z) = \frac{1}{2z + \widetilde{C_0}}. \tag{16}$$

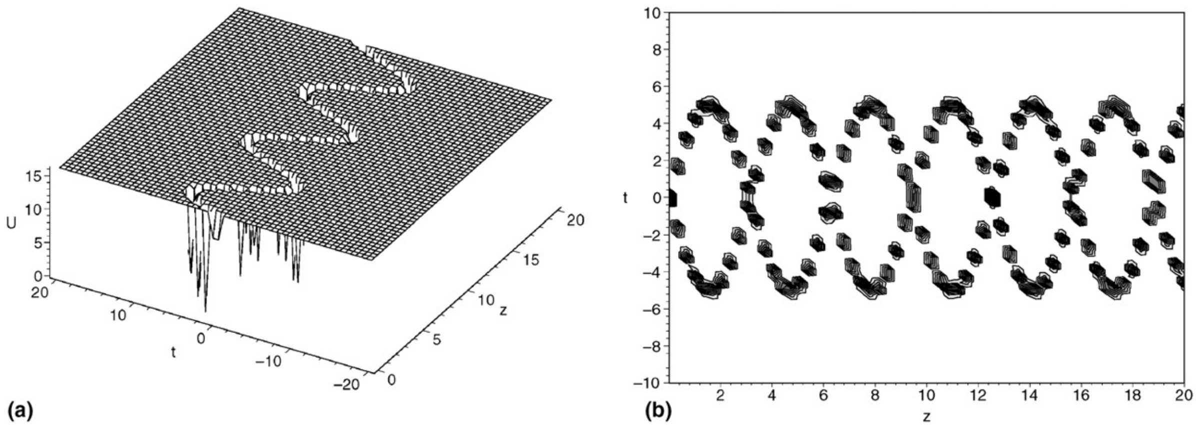

Fig. 1. Snake-soliton propagation and contour plot of two snake-solitons interaction scenario given by dark solitons u_1. Input conditions: $\lambda_2(z)=0$, $\alpha(z)=-\cos(z)$, $\beta(z)=\cos(z)$, $C_1=5$, $C_2=4$, $C_4=0$ in (a).

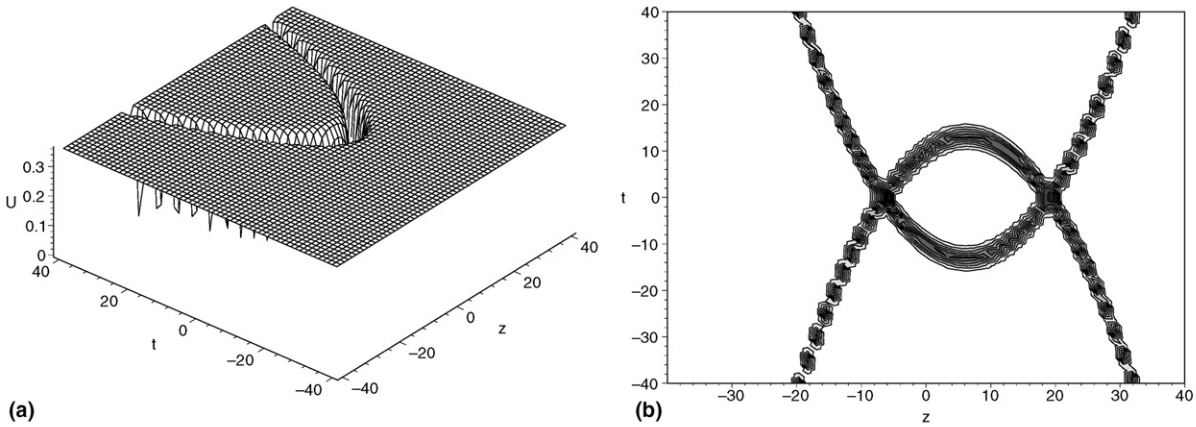

Fig. 2. Boomerang-like soliton propagation and interaction scenario given by dark solitons u_1. Input conditions: $\lambda_2(z)=0$, $\alpha=\beta=1-0.14z$, $C_1=1$, $C_2=0.6$, $C_4=0$.

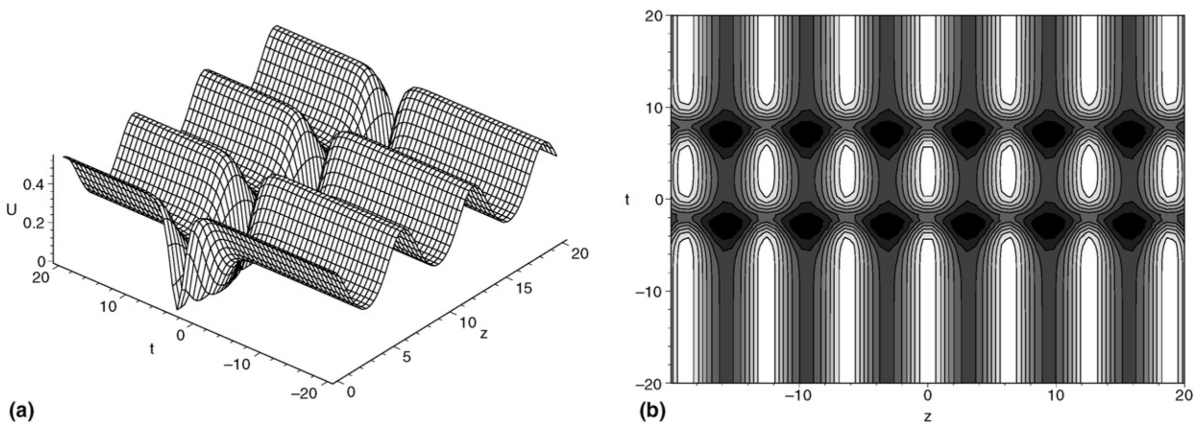

Fig. 3. Evolution of the periodic dark solitary wave and contour plot of two periodic dark solitary wave given by u_1 with $\lambda_2(z)=0.1$, $\alpha(z)=\sin(z)$, $\beta(z)=-\sin(z)$, $C_1=1$, $C_2=0.6$, $C_4=0$.

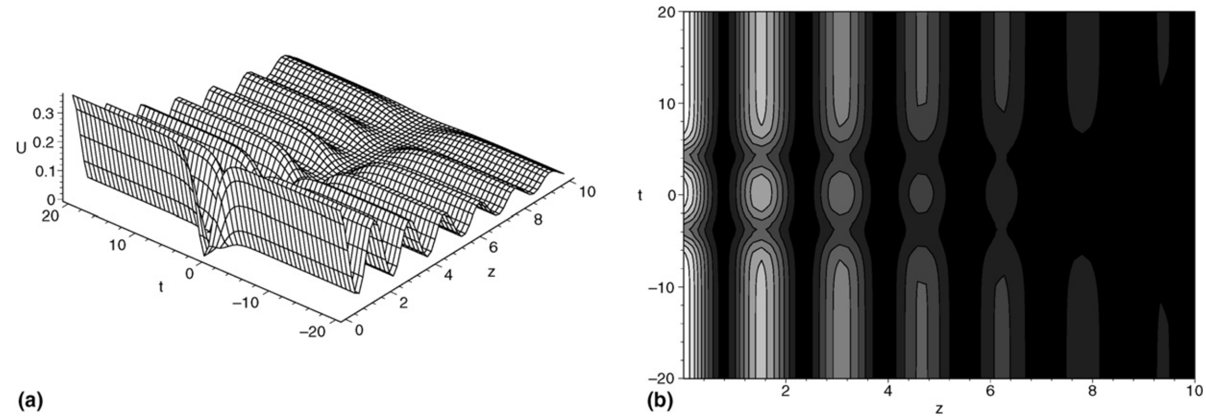

Fig. 4. (a) depicts one DM dark soliton propagation and (b) simulates two DM dark solitons interaction given by u_1 with $\lambda_2(z) = 0.1$, $\alpha(z) = 1 - 0.9\sin(2z)^2$, $\beta(z) = 1$, $C_1 = 0.1$, $C_2 = 0.6$, $C_4 = 0$.

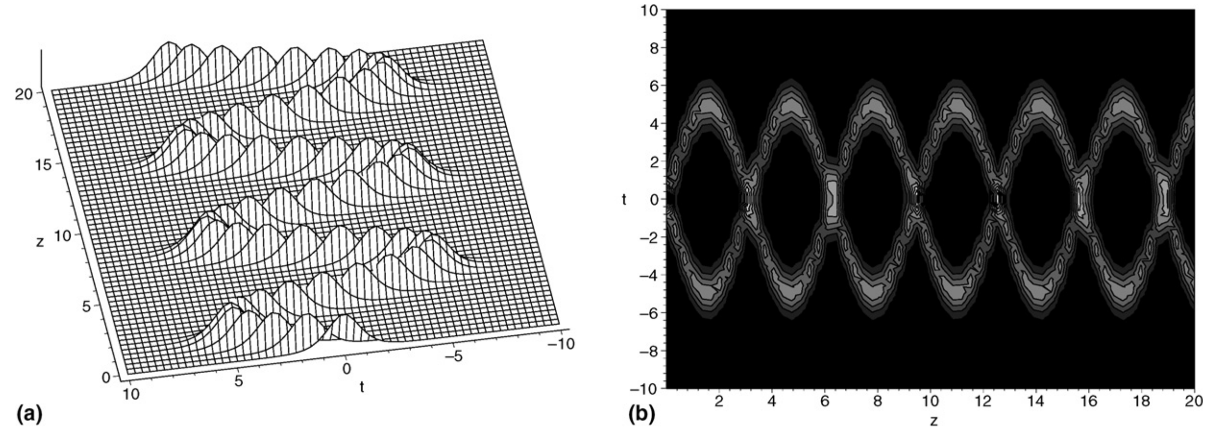

Fig. 5. (a) and (b) depict snake-soliton propagation and interaction given by u_2 with $\lambda_2(z) = 0$, $\alpha(z) = -\beta(z) = -\cos(z)$, $C_1 = 5$, $C_2 = 0.4$, $C_4 = 0$.

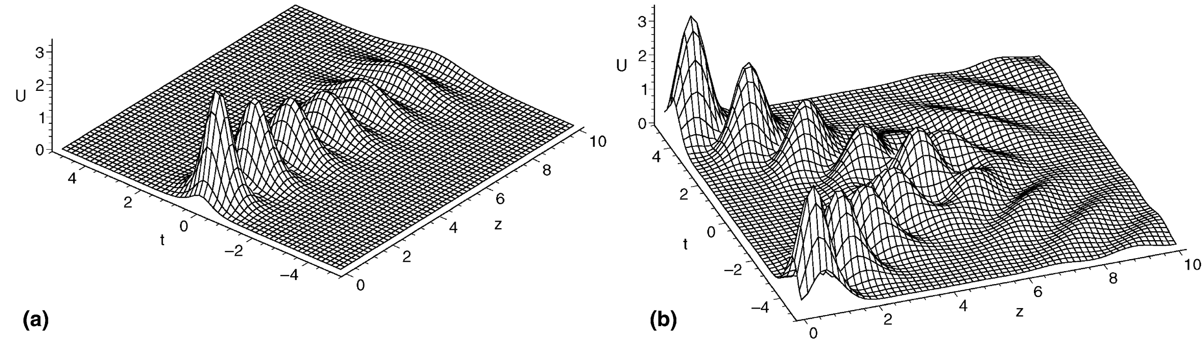

Fig. 6. (a) and (b) depict one DM bright soliton propagation and interaction scenario given by u_2, respectively. Input conditions: $\lambda_2(z) = 0.1$, $\alpha(z) = 1 - 0.9\cos(2z)^2$, $\beta(z) = 1$, $C_1 = 0.01$, $C_2 = 2$, $C_4 = 0$.

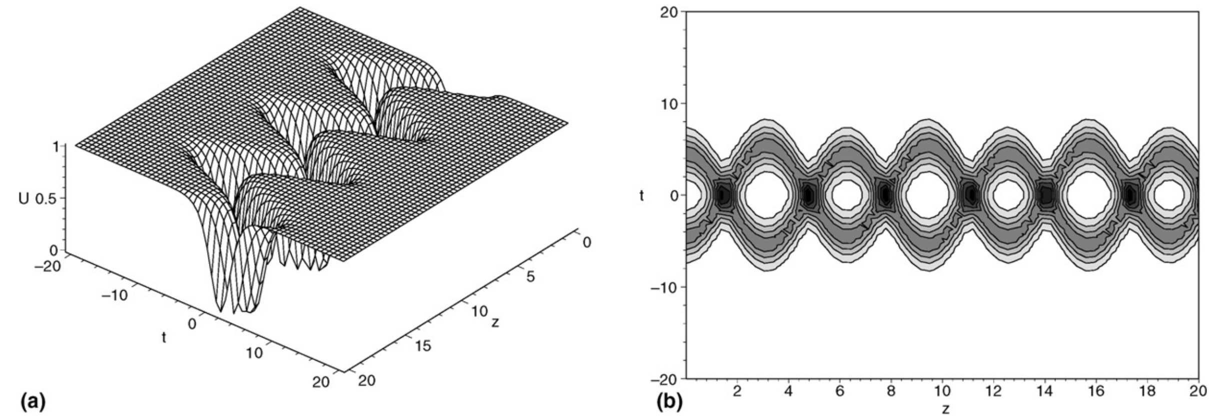

Fig. 7. (a) and (b) depict one dark soliton propagation and two dark solitons interaction scenario given by u_3, respectively. Input conditions: $\lambda_2(z)=0$, $\alpha(z)=-\beta(z)=-\sin(z)$, $C_1=5$, $C_3=2$, $C_4=0$, $\mu=1.1$.

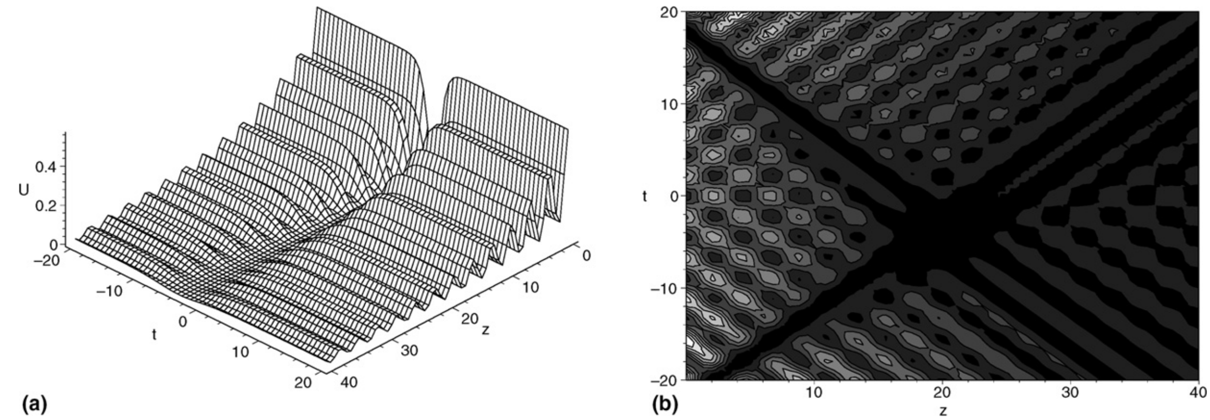

Fig. 8. (a) and (b) depict one periodic DM dark soliton propagation and two periodic DM dark solitons interaction scenario given by u_3, respectively. Input conditions: $\alpha(z)=1-0.8\sin(z)^2$, $\beta(z)=1$, $\widetilde{C_0}=20$, $C_1=1$, $C_3=-30$, $C_5=0$, $\mu=2$.

Then we can deduce the parameters $\lambda_2(z)$, $M(z)$ and $F(z)$, for example

$$\lambda_2(z)=\frac{-5.0}{(-10.0z-5.0\widetilde{C_0}+8.0(\sin(z))^2z+4.0(\sin(z))^2\widetilde{C_0})}. \tag{17}$$

It is necessary to point out that we plot some figures for each solutions under some special parameters. In fact, under different parameters, the feature of solutions by $u_1(z,t)$, $u_2(z,t)$ and $u_3(z,t)$ are rich and colorful. In the future work, we will further study the various NLS equations and make great efforts to reveal some significative phenomena in physics and optics

3. Summary and discussion

In this paper, the inhomogeneous nonlinear Schrödinger equation with the loss/gain and the frequency chirping is investigated. With the help of symbolic computation, three families of exact analytical solutions are presented by employing the extended projective Riccati equation method. From our results, many previous known results of nonlinear Schrödinger equation obtained by some authors can be recovered by means of some suitable selections of the arbitrary functions and arbitrary constants. Of optical and physical interests, soliton propagation and soliton interaction are discussed and simulated by computer.

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