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Complexiton solutions of the (2+1)-dimensional dispersive long wave equation*

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In this pager a pure algebraic method implemented in a computer algebraic system, named multiple Riccati equations rational expansion method, is presented to construct a novel class of complexiton solutions to integrable equations and nonintegrable equations. By solving the (2+1)-dimensional dispersive long wave equation, it obtains many new types of complexiton solutions such as various combination of trigonometric periodic and hyperbolic function solutions, various combination of trigonometric periodic and rational function solutions, various combination of hyperbolic and rational function solutions, etc.

Keywords: multiple Riccati equations rational expansion method, complexiton solution

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1. Introduction

Recently, Ma[1] presented a novel class of explicit exact solutions, named complexiton solutions, to the Korteweg–de Vires (KdV) equation under the help of its bilinear form for the extent and diversity of the nonlinear waves described by the KdV equation. These complexiton solutions possess singularities of combination of trigonometric function and hyperbolic function. More recently, Lou *et al*[2] used the mapping relation among the $(n+1)$-dimensional sine–Gordon field equation and the cubic nonlinear Klein–Gordon equation to solve the $(n+1)$-dimensional sine–Gordon field equation, and found many new types of complexiton solutions. However, the above two methods were only applied to integrable equations. An interesting problem in solution theory is to set up a direct and uniform algebraic method for constructing more general form complexiton solutions for integrable equations and nonintegrable equations.

The present work is motivated by the desire to set up a new arithmetic, named multiple Riccati equations rational expansion (MRERE) method, to construct new types of complexiton solutions of integrable equations and nonintegrable equations, such as many new types of exact explicit solutions, some complexiton solutions such as various combination of trigonometric periodic and hyperbolic function solutions, various combination of trigonometric periodic and rational function solutions, various combination of hyperbolic and rational function solutions.

As is well known, the tanh method provides a straightforward and effective algorithm to obtain such particular solutions for a large number of integrable equations and nonintegrable equations. There are few works in the traditional tanh methods[3] and various extended methods[4−10] to construct the complexiton solutions of combination of trigonometric function and hyperbolic function solutions, rational function and trigonometric function solutions or rational function and hyperbolic function solutions. In our method, we use two or more variables which satisfy two different Riccati equations, in which different parameters is chosen independently. In this way, we can construct many families of novel complexiton solutions of some nonlinear Partial differential equations (PDEs), in which hyperbolic (solitary) function and triangular periodic functions or rational function wave and hyperbolic function waves can appear in a solution at same time. Although our method can not recover all complexiton solutions obtained by Ma's method

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and Lou *et al*'s method, other new types of complexiton solutions cannot be found by Ma's method and Lou *et al*'s method. On the other hand, it can recover the solutions by various improved and extended tanh methods[3–10] and find other new types of complexiton solutions which cannot be found by existing tanh function methods. In particular, our method is a straightforward and pure algebraic algorithm implemented in a computer algebraic system.

By use of our method and with the aid of symbolic computation, we solve the (2+1)-dimensional dispersive long wave equation, and obtain many new types of explicit exact solutions, some complexiton solutions such as various combination of trigonometric periodic and hyperbolic function solutions, various combination of trigonometric periodic and rational function solutions, various combination of hyperbolic and rational function solutions, etc.

This paper is organized as follows. In Section 2, the detailed derivation of the MRERE method will be given. In Section 3 the application of the MRERE method to the (2+1)-dimensional dispersive long wave equation is illustrated. The conclusion is given in Section 4.

2. Summary of the multiple Riccati equations rational expansion method

In the following we would like to outline the main steps of our method.

Step 1. For a given system of polynomial PDE with constant coefficients, with some physical fields $u_i\left(x,y,t\right)$ in three variables x, y, t,

$$\Delta\left(u_i,u_{it},u_{ix},u_{iy},u_{itt},u_{ixt},u_{iyt},u_{ixx},u_{iyy},u_{ixy},\ldots\right)=0, \tag{1}$$

we use the wave transformation

$$u_i\left(x,y,t\right)=U_i\left(\xi\right),\quad \xi=k(x+ly+\lambda t), \tag{2}$$

where k, l and λ are constants to be determined later. Then the nonlinear partial differential system (1) is reduced to a nonlinear ordinary differential system:

$$\Theta\left(\,U_i,\;U_i',\;U_i'',\ldots\right)=0. \tag{3}$$

Step 2. We introduce a new ansätz in terms of finite rational formal expansion in the following forms:

$$U_i(\xi)=a_{i0}+\sum_{j=1}^{m_i}\frac{\sum\limits_{r_{j1}+r_{j2}=j}a^j_{r_{j1}r_{j2}}\phi^{r_{j1}}\left(\xi\right)\psi^{r_{j2}}\left(\xi\right)}{\left(\mu_1\phi\left(\xi\right)+\mu_2\psi\left(\xi\right)+1\right)^j}, \tag{4}$$

where $a^j_{r_{j1}r_{j2}}$, μ_1 and μ_2 ($r_{jn}=1,2,\ldots,j$; $j=0,1,\ldots,m_i$; $n=1,2$; $i=1,2,\ldots$) are constants to be determined later and the new variables $\phi=\phi\left(\xi\right)$ and $\psi=\psi\left(\xi\right)$ satisfy the Riccati equation, i.e.,

$$\frac{\mathrm{d}\phi}{\mathrm{d}\xi}=h_1+h_2\phi^2,\quad \frac{\mathrm{d}\psi}{\mathrm{d}\xi}=h_3+h_4\psi^2, \tag{5}$$

where h_1, h_2, h_3 and h_4 are constants.

Step 3. Determine the m_i of the rational formal polynomial solutions Eq.(4) by respectively balancing the highest nonlinear terms and the highest-order partial derivative terms in the given system equations (see Refs.[3–10] for details), and then give the formal solutions.

Step 4. Substitute Eq.(4) into Eq.(3) along with Eq.(5) and then set all coefficients of $\phi^p\left(\xi\right)\psi^q\left(\xi\right)$ ($p=0,1,2,\ldots$; $q=0,1,2,\ldots$) of the resulting system's numerator to be zero to get an over-determined system of nonlinear algebraic equations with respect to k, μ_1, μ_2 and $a^j_{r_{j1}r_{j2}}$ ($r_{jn}=1,2,\ldots,j$; $j=1,2,\ldots,m_i$; $n=1,2$; $i=1,2,\ldots$).

Step 5. Solving the over-determined system of nonlinear algebraic equations by use of symbolic computation system Maple, we end up with the explicit expressions for k, μ_1, μ_2, and $a^j_{r_{j1}r_{j2}}$ ($r_{jn}=1,2,\ldots,j$; $j=1,2,\ldots,m_i$; $n=1,2$; $i=1,2,\ldots$).

Step 6. The general solutions of the Riccati equation (5)

$$\frac{\mathrm{d}F}{\mathrm{d}\xi}=R_1+R_2F^2$$

are

(1) when $R_1 = \dfrac{1}{2}$ and $R_2 = -\dfrac{1}{2}$,

$$F(\xi) = \tanh(\xi) \pm \mathrm{isech}(\xi), \quad F(\xi) = \coth(\xi) \pm \mathrm{csch}(\xi), \tag{6}$$

(2) when $R_1 = R_2 = \pm\dfrac{1}{2}$,

$$F(\xi) = \sec(\xi) \pm \tan(\xi), \quad F(\xi) = \csc(\xi) \pm \cot(\xi), \tag{7}$$

(3) when $R_1 = 1$ and $R_2 = -1$,

$$F(\xi) = \tanh(\xi), \quad F(\xi) = \coth(\xi), \tag{8}$$

(4) when $R_1 = R_2 = 1$,

$$F(\xi) = \tan(\xi), \tag{9}$$

(5) when $R_1 = R_2 = -1$,

$$F(\xi) = \cot(\xi), \tag{10}$$

(6) when $R_1 = 0$ and $R_2 \neq 0$,

$$F(\xi) = -\frac{1}{R_2\xi + c_0}. \tag{11}$$

According to Eqs.(2) and (4), the conclusions in Step 5 and the general solutions Eqs.(6)–(11), we can obtain many types of complexiton solution of Eq.(1) as follows.

(I) When $h_1 = 1$, $h_2 = -1$ and $h_3 = h_4 = \pm 1$, we can get combination of tanh (coth) and tan function solutions and combination of tanh (coth) and sec function solutions:

$$u_i = a_{i0} + \sum_{j=1}^{m_i} \frac{\sum\limits_{r_{j1}+r_{j2}=j} a^j_{r_{j1}r_{j2}} \tanh^{r_{j1}}(\xi)\tan^{r_{j2}}(\xi)}{(\mu_1\tanh(\xi) + \mu_2\tan(\xi) + 1)^j}, \tag{12a}$$

$$u_i = a_{i0} + \sum_{j=1}^{m_i} \frac{\sum\limits_{r_{j1}+r_{j2}=j} a^j_{r_{j1}r_{j2}} \tanh^{r_{j1}}(\xi)\sec^{r_{j2}}(\xi)}{(\mu_1\tanh(\xi) + \mu_2\sec(\xi) + 1)^j}, \tag{12b}$$

$$u_i = a_{i0} + \sum_{j=1}^{m_i} \frac{\sum\limits_{r_{j1}+r_{j2}=j} a^j_{r_{j1}r_{j2}} \coth^{r_{j1}}(\xi)\tan^{r_{j2}}(\xi)}{(\mu_1\coth(\xi) + \mu_2\tan(\xi) + 1)^j}, \tag{12c}$$

$$u_i = a_{i0} + \sum_{j=1}^{m_i} \frac{\sum\limits_{r_{j1}+r_{j2}=j} a^j_{r_{j1}r_{j2}} \coth^{r_{j1}}(\xi)\sec^{r_{j2}}(\xi)}{(\mu_1\coth(\xi) + \mu_2\sec(\xi) + 1)^j}. \tag{12d}$$

(II) When $h_1 = 1$, $h_2 = -1$ and $h_3 = h_4 = \pm\dfrac{1}{2}$, we can get combination of tanh (coth), sec and tan function solutions and combination of tanh (coth), csc, sec, cot function solutions:

$$u_i = a_{i0} + \sum_{j=1}^{m_i} \frac{\sum\limits_{r_{j1}+r_{j2}=j} a^j_{r_{j1}r_{j2}} \tanh^{r_{j1}}(\xi)\left(\sec(\xi) \pm \tan(\xi)\right)^{r_{j2}}}{(\mu_1\tanh(\xi) + \mu_2(\sec(\xi) \pm \tan(\xi)) + 1)^j}, \tag{13a}$$

$$u_i = a_{i0} + \sum_{j=1}^{m_i} \frac{\sum\limits_{r_{j1}+r_{j2}=j} a^j_{r_{j1}r_{j2}} \tanh^{r_{j1}}(\xi)\left(\csc(\xi) \pm \cot(\xi)\right)^{r_{j2}}}{(\mu_1\tanh(\xi) + \mu_2(\sec(\xi) \pm \tan(\xi)) + 1)^j}, \tag{13b}$$

$$u_i = a_{i0} + \sum_{j=1}^{m_i} \frac{\sum\limits_{r_{j1}+r_{j2}=j} a^j_{r_{j1}r_{j2}} \coth^{r_{j1}}(\xi)\left(\sec(\xi) \pm \tan(\xi)\right)^{r_{j2}}}{(\mu_1\coth(\xi) + \mu_2(\sec(\xi) \pm \tan(\xi)) + 1)^j}, \tag{13c}$$

$$u_i = a_{i0} + \sum_{j=1}^{m_i} \frac{\sum\limits_{r_{j1}+r_{j2}=j} a^j_{r_{j1}r_{j2}} \coth^{r_{j1}}(\xi)\left(\csc(\xi) \pm \cot(\xi)\right)^{r_{j2}}}{(\mu_1\coth(\xi) + \mu_2(\sec(\xi) \pm \tan(\xi)) + 1)^j}. \tag{13d}$$

(III) When $h_1 = 1$, $h_2 = -1$, $h_3 = 0$, and $h_4 \neq 0$, we can get combination of tanh (coth) and rational function solutions:

$$u_i = a_{i0} + \sum_{j=1}^{m_i} \frac{\sum\limits_{r_{j1}+r_{j2}=j} a^j_{r_{j1}r_{j2}} \tanh^{r_{j1}}(\xi) \left(-\frac{1}{h_4\xi+c_0}\right)^{r_{j2}}}{(\mu_1 \tanh(\xi) - \frac{\mu_2}{h_4\xi+c_0} + 1)^j}, \tag{14a}$$

$$u_i = a_{i0} + \sum_{j=1}^{m_i} \frac{\sum\limits_{r_{j1}+r_{j2}=j} a^j_{r_{j1}r_{j2}} \coth^{r_{j1}}(\xi) \left(-\frac{1}{h_4\xi+c_0}\right)^{r_{j2}}}{(\mu_1 \coth(\xi) - \frac{\mu_2}{h_4\xi+c_0} + 1)^j}. \tag{14b}$$

(IV) When $h_1 = -\dfrac{1}{2}$, $h_2 = \dfrac{1}{2}$ and $h_3 = h_4 = \pm 1$, we can get combination of tanh, sech and tan (cot) function solutions and combination of coth, csch and tan (cot) function solutions:

$$u_i = a_{i0} + \sum_{j=1}^{m_i} \frac{\sum\limits_{r_{j1}+r_{j2}=j} a^j_{r_{j1}r_{j2}} (\tanh(\xi) \pm i\operatorname{sech}(\xi))^{r_{j1}} \tan^{r_{j2}}(\xi)}{(\mu_1 (\tanh(\xi) \pm i\operatorname{sech}(\xi)) + \mu_2 \tan(\xi) + 1)^j}, \tag{15a}$$

$$u_i = a_{i0} + \sum_{j=1}^{m_i} \frac{\sum\limits_{r_{j1}+r_{j2}=j} a^j_{r_{j1}r_{j2}} (\coth(\xi) \pm \operatorname{csch}(\xi))^{r_{j1}} \tan^{r_{j2}}(\xi)}{(\mu_1 (\coth(\xi) \pm \operatorname{csch}(\xi)) + \mu_2 \tan(\xi) + 1)^j}, \tag{15b}$$

$$u_i = a_{i0} + \sum_{j=1}^{m_i} \frac{\sum\limits_{r_{j1}+r_{j2}=j} a^j_{r_{j1}r_{j2}} (\tanh(\xi) \pm i\operatorname{sech}(\xi))^{r_{j1}} \cot^{r_{j2}}(\xi)}{(\mu_1 (\tanh(\xi) \pm i\operatorname{sech}(\xi)) + \mu_2 \cot(\xi) + 1)^j}, \tag{15c}$$

$$u_i = a_{i0} + \sum_{j=1}^{m_i} \frac{\sum\limits_{r_{j1}+r_{j2}=j} a^j_{r_{j1}r_{j2}} (\coth(\xi) \pm \operatorname{csch}(\xi))^{r_{j1}} \cot^{r_{j2}}(\xi)}{(\mu_1 (\coth(\xi) \pm \operatorname{csch}(\xi)) + \mu_2 \cot(\xi) + 1)^j}. \tag{15d}$$

(V) When $h_1 = -\frac{1}{2}$, $h_2 = \frac{1}{2}$ and $h_3 = h_4 = \pm\frac{1}{2}$, we can get combination of tanh, sech (coth, csch), sec and tan function solutions and combination of tanh, sech (coth, csch), csc and cot function solutions:

$$u_i = a_{i0} + \sum_{j=1}^{m_i} \frac{\sum\limits_{r_{j1}+r_{j2}=j} a^j_{r_{j1}r_{j2}} (\tanh(\xi) \pm i\operatorname{sech}(\xi))^{r_{j1}} (\sec(\xi) \pm \tan(\xi))^{r_{j2}}}{(\mu_1 (\tanh(\xi) \pm i\operatorname{sech}(\xi)) + \mu_2 (\sec(\xi) \pm \tan(\xi)) + 1)^j}, \tag{16a}$$

$$u_i = a_{i0} + \sum_{j=1}^{m_i} \frac{\sum\limits_{r_{j1}+r_{j2}=j} a^j_{r_{j1}r_{j2}} (\coth(\xi) \pm \operatorname{csch}(\xi))^{r_{j1}} (\sec(\xi) \pm \tan(\xi))^{r_{j2}}}{(\mu_1 (\coth(\xi) \pm \operatorname{csch}(\xi)) + \mu_2 (\sec(\xi) \pm \tan(\xi)) + 1)^j}, \tag{16b}$$

$$u_i = a_{i0} + \sum_{j=1}^{m_i} \frac{\sum\limits_{r_{j1}+r_{j2}=j} a^j_{r_{j1}r_{j2}} (\tanh(\xi) \pm i\operatorname{sech}(\xi))^{r_{j1}} (\csc(\xi) \pm \cot(\xi))^{r_{j2}}}{(\mu_1 (\tanh(\xi) \pm i\operatorname{sech}(\xi)) + \mu_2 (\csc(\xi) \pm \cot(\xi)) + 1)^j}, \tag{16c}$$

$$u_i = a_{i0} + \sum_{j=1}^{m_i} \frac{\sum\limits_{r_{j1}+r_{j2}=j} a^j_{r_{j1}r_{j2}} (\coth(\xi) \pm \operatorname{csch}(\xi))^{r_{j1}} (\csc(\xi) \pm \cot(\xi))^{r_{j2}}}{(\mu_1 (\coth(\xi) \pm \operatorname{csch}(\xi)) + \mu_2 (\csc(\xi) \pm \cot(\xi)) + 1)^j}. \tag{16d}$$

(VI) When $h_1 = 1$, $h_2 = -1$, $h_3 = 0$, and $h_4 \neq 0$, we can get combination of tanh, sech (coth, csch), rational function solutions:

$$u_i = a_{i0} + \sum_{j=1}^{m_i} \frac{\sum\limits_{r_{j1}+r_{j2}=j} a^j_{r_{j1}r_{j2}} \left(\tanh(\xi) \pm \mathrm{isech}(\xi)\right)^{r_{j1}} \left(-\frac{1}{h_4\xi+c_0}\right)^{r_{j2}}}{\left(\mu_1 \left(\tanh(\xi) \pm \mathrm{isech}(\xi)\right) - \frac{\mu_2}{\mathrm{h}_4\xi+\mathrm{c}_0} + 1\right)^j}, \tag{17a}$$

$$u_i = a_{i0} + \sum_{j=1}^{m_i} \frac{\sum\limits_{r_{j1}+r_{j2}=j} a^j_{r_{j1}r_{j2}} \left(\coth(\xi) \pm \mathrm{csch}(\xi)\right)^{r_{j1}} \left(-\frac{1}{h_4\xi+c_0}\right)^{r_{j2}}}{\left(\mu_1 \left(\coth(\xi) \pm \mathrm{csch}(\xi)\right) - \frac{\mu_2}{\mathrm{h}_4\xi+\mathrm{c}_0} + 1\right)^j}. \tag{17b}$$

(VII) When $h_1 = h_2 = \pm 1$, $h_3 = 0$, and $h_4 \neq 0$, we can get combination of tan (cot), rational function solutions:

$$u_i = a_{i0} + \sum_{j=1}^{m_i} \frac{\sum\limits_{r_{j1}+r_{j2}=j} a^j_{r_{j1}r_{j2}} \tan^{r_{j1}}(\xi) \left(-\frac{1}{h_4\xi+c_0}\right)^{r_{j2}}}{\left(\mu_1 \tan(\xi) - \frac{\mu_2}{h_4\xi+c_0} + 1\right)^j}, \tag{18a}$$

$$u_i = a_{i0} + \sum_{j=1}^{m_i} \frac{\sum\limits_{r_{j1}+r_{j2}=j} a^j_{r_{j1}r_{j2}} \cot^{r_{j1}}(\xi) \left(-\frac{1}{h_4\xi+c_0}\right)^{r_{j2}}}{\left(\mu_1 \cot(\xi) - \frac{\mu_2}{h_4\xi+c_0} + 1\right)^j}. \tag{18b}$$

(VIII) When $h_1 = h_2 = \pm\frac{1}{2}$, $h_3 = 0$, and $h_4 \neq 0$, we can get combination of tan (cot), sec (csc), rational function solutions:

$$u_i = a_{i0} + \sum_{j=1}^{m_i} \frac{\sum\limits_{r_{j1}+r_{j2}=j} a^j_{r_{j1}r_{j2}} \left(\sec(\xi) \pm \tan(\xi)\right)^{r_{j1}} \left(-\frac{1}{h_4\xi+c_0}\right)^{r_{j2}}}{\left(\mu_1 \left(\sec(\xi) \pm \tan(\xi)\right) - \frac{\mu_2}{h_4\xi+c_0} + 1\right)^j}, \tag{19a}$$

$$u_i = a_{i0} + \sum_{j=1}^{m_i} \frac{\sum\limits_{r_{j1}+r_{j2}=j} a^j_{r_{j1}r_{j2}} \left(\csc(\xi) \pm \cot(\xi)\right)^{r_{j1}} \left(-\frac{1}{h_4\xi+c_0}\right)^{r_{j2}}}{\left(\mu_1 \left(\csc(\xi) \pm \cot(\xi)\right) - \frac{\mu_2}{h_4\xi+c_0} + 1\right)^j}. \tag{19b}$$

Remark 1:

The method proposed here is more general than various existing methods[3−8] and the projective Riccati equation expansion method[9−10] for finding exact solutions of nonlinear PDEs. The appeal and success of the method lie in the fact that writing the exact solutions of a nonlinear equation as polynomials of ϕ and ψ whose derivations are in closed form, the equation can be changed into a nonlinear system of algebraic equations. The system can be solved with the help of symbolic computation.

Remark 2:

We can easily see that when $h_1 \neq h_3$ or $h_2 \neq h_4$, ϕ and ψ satisfy two different Riccati equations, the hyperbolic functions and triangular functions can appear in one solution at the same time. These solutions have not been obtained by any other Riccati equation expansion methods or projective Riccati equation expansion methods.

3. Complexiton solution of the (2+1)-dimensional dispersive long wave equation

Let us consider the (2+1)-dimensional dispersive long wave equation(DLWE),

$$\begin{aligned} u_{yt} + v_{xx} + (uu_x)_y &= 0, \\ v_t + u_x + (uv)_x + u_{xxy} &= 0. \end{aligned} \tag{20}$$

The (2+1)-dimensional DLWE (20) was first derived by Boiti *et al*[11] as a compatibility for a "weak" Lax pair. Recently considerable effort has been devoted to the study of this system.[12−16]

In order to get some families of rational form wave solutions to the (2+1)-dimensional DLWE, by considering the wave transformations $u(x,y,t)=U(\xi)$, $v(x,y,t)=V(\xi)$ and $\xi=k(x+ly+\lambda t)$, we change Eq.(20) to the form

$$\begin{aligned}&\lambda l\,U''+V''+lU'^2+lUU''=0,\\&\lambda V'+U'+(UV)'+k^2lU'''=0.\end{aligned}\tag{21}$$

For the (2+1)-dimensional DLWE, by balancing the highest nonlinear terms and the highest order partial derivative terms in Eq.(21), we suppose Eq.(21) have the following formal travelling wave solution:

$$U(\xi)=a_0+\frac{a_1\phi+b_1\psi}{\mu_1\phi+\mu_2\psi+1},$$

$$\begin{aligned}V(\xi)=\;&A_0+\frac{A_1\phi+B_1\psi}{\mu_1\phi+\mu_2\psi+1}\\&+\frac{A_2\phi^2+B_2\phi\psi+C_1\psi^2}{(\mu_1\phi+\mu_2\psi+1)^2},\end{aligned}\tag{22}$$

where μ_1, μ_2, a_0, a_i, b_i, A_0, A_i, B_i and $C_1(i=1,2)$ are constants to be determined later and the new variables ϕ and ψ satisfy Eq.(5).

With the aid of Maple, substituting Eq.(22) along with Eq.(5) into Eq.(21) and setting the coefficients of these terms $\phi^i\psi^j$ to be zero, we get a set of over-determined algebraic equations with respect to a_0, a_1, b_1, A_0, A_1, B_1 A_2, B_2, C_1, μ_1, μ_2, k, l and λ.

By use of the Maple soft package 'Charsets' by Wang Dongming, which is based on the Wu-elimination method,[17] solving the over-determined algebraic equations, we get the following results:

$$\begin{aligned}&A_1=-\left(\lambda+a_0\right)la_1,\quad B_1=-b_1a_0l-b_1\lambda\,l,\quad \mu_1=\pm1,\\&\mu_2=\pm1,\quad A_2=-\frac{1}{2}\,{a_1}^2l,\quad B_2=-b_1a_1l,\quad C_1=-\frac{1}{2}\,{b_1}^2l,\end{aligned}\tag{23}$$

where a_0, A_0, a_1, b_1, k, l and λ are arbitrary constants.

According to Eqs.(21)–(23) and the general solutions of Eq.(5) listed in the above section, we will obtain the following exact solutions for the (2+1)-dimensional DLWE.

Note that there are so many solutions obtained here, we just list some new solutions for the (2+1)-dimensional DLWE to illustrate the efficiency of our method.

Family 1. When $h_1=1$, $h_2=-1$ and $h_2=h_4=\pm1$, then we can get combination of tanh and tan function solution:

$$\begin{aligned}U_1=\;&a_0+\frac{a_1\tanh(\xi)}{\pm\tanh(\xi)\pm\tan(\xi)+1}+\frac{b_1\tan(\xi)}{\pm\tanh(\xi)\pm\tan(\xi)+1},\\V_1=\;&A_0-\frac{(\lambda+a_0)\,la_1\tanh(\xi)}{\pm\tanh(\xi)\pm\tan(\xi)+1}-\frac{(b_1a_0l+b_1\lambda\,l)\tan(\xi)}{\pm\tanh(\xi)\pm\tan(\xi)+1}\\&-\frac{{a_1}^2l\tanh^2(\xi)}{2\left(\pm\tanh(\xi)\pm\tan(\xi)+1\right)^2}-\frac{b_1a_1l\tanh(\xi)\tan(\xi)}{\left(\tanh(\xi)+\tan(\xi)+1\right)^2}\\&-\frac{{b_1}^2l\tan^2(\xi)}{2\left(\pm\tanh(\xi)\pm\tan(\xi)+1\right)^2},\end{aligned}\tag{24}$$

where $\xi=k\,(x+ly+\lambda t)$, a_0, A_0, a_1, b_1, k, l and λ are arbitrary constants.

Family 2. When $h_1=1$, $h_2=-1$ and $h_3=h_4=\pm\frac{1}{2}$, then we can get combination of tanh, sec and tan function solution:

$$\begin{aligned}U_2=\;&a_0+\frac{a_1\tanh(\xi)}{\pm\tanh(\xi)\pm\sec(\xi)\pm\tan(\xi)+1}+\frac{b_1\left(\sec(\xi)\pm\tan(\xi)\right)}{\pm\tanh(\xi)\pm\sec(\xi)\pm\tan(\xi)+1},\\V_2=\;&A_0-\frac{(\lambda+a_0)\,la_1\tanh(\xi)}{\pm\tanh(\xi)\pm\sec(\xi)\pm\tan(\xi)+1}+\frac{(-b_1a_0l-b_1\lambda\,l)\left(\sec(\xi)\pm\tan(\xi)\right)}{\pm\tanh(\xi)\pm\sec(\xi)\pm\tan(\xi)+1}\\&-\frac{{a_1}^2l\tanh^2(\xi)}{2\left(\pm\tanh(\xi)\pm\sec(\xi)\pm\tan(\xi)+1\right)^2}-\frac{b_1a_1l\tanh(\xi)\left(\sec(\xi)\pm\tan(\xi)\right)}{\left(\pm\tanh(\xi)\pm\sec(\xi)\pm\tan(\xi)+1\right)^2}\\&-\frac{{b_1}^2l\left(\sec(\xi)\pm\tan(\xi)\right)^2}{2\left(\pm\tanh(\xi)\pm\sec(\xi)\pm\tan(\xi)+1\right)^2},\end{aligned}\tag{25}$$

where $\xi = k(x+ly+\lambda t)$, a_0, A_0, a_1, b_1, k, l and λ are arbitrary constants.

Family 3. When $h_1 = 1$, $h_2 = -1$, $h_3 = 0$, and $h_4 \neq 0$, then we can get combination of tanh and rational function solution:

$$\begin{aligned} U_3 = &\ a_0 + \frac{a_1 (h_4\xi + c_0)\tanh(\xi)}{\pm(h_4\xi + c_0)\tanh(\xi) \mp 1 + h_4\xi + c_0} - \frac{b_1}{\pm(h_4\xi + c_0)\tanh(\xi) \mp 1 + h_4\xi + c_0}, \\ V_3 = &\ A_0 - \frac{(\lambda + a_0)\, l a_1 (h_4\xi + c_0)\tanh(\xi)}{\pm(h_4\xi + c_0)\tanh(\xi) \mp 1 + h_4\xi + c_0} + \frac{(b_1 a_0 l + b_1 \lambda l)}{\pm(h_4\xi + c_0)\tanh(\xi) \mp 1 + h_4\xi + c_0} \\ & - \frac{{a_1}^2 l \left((h_4\xi + c_0)\tanh(\xi)\right)^2}{2\left(\pm(h_4\xi + c_0)\tanh(\xi) \mp 1 + h_4\xi + c_0\right)^2} + \frac{b_1 a_1 l \tanh(\xi)(h_4\xi + c_0)}{\left(\pm(h_4\xi + c_0)\tanh(\xi) \mp 1 + h_4\xi + c_0\right)^2} \\ & - \frac{{b_1}^2 l}{2\left(\pm(h_4\xi + c_0)\tanh(\xi) \mp 1 + h_4\xi + c_0\right)^2}, \end{aligned} \tag{26}$$

where $\xi = k(x+ly+\lambda t)$, $h_4 \neq 0$, c_0, a_0, A_0, a_1, b_1, k, l and λ are arbitrary constants.

Family 4. When $h_1 = -\frac{1}{2}$, $h_2 = \frac{1}{2}$ and $h_3 = h_4 = \pm 1$, then we can get combination of tanh, sech and tan function solution:

$$\begin{aligned} U_4 = &\ a_0 + \frac{a_1 (\tanh(\xi) \pm \mathrm{isech}(\xi))}{\pm\tanh(\xi) \pm \mathrm{isech}(\xi) \pm \tan(\xi) + 1} + \frac{b_1 \tan(\xi)}{\pm\tanh(\xi) \pm \mathrm{isech}(\xi) \pm \tan(\xi) + 1}, \\ V_4 = &\ A_0 - \frac{(\lambda + a_0)\, l a_1 (\tanh(\xi) \pm \mathrm{isech}(\xi))}{\pm\tanh(\xi) \pm \mathrm{isech}(\xi) \pm \tan(\xi) + 1} - \frac{(b_1 a_0 l + b_1 \lambda l)\tan(\xi)}{\pm\tanh(\xi) \pm \mathrm{isech}(\xi) \pm \tan(\xi) + 1} \\ & - \frac{{a_1}^2 l (\tanh(\xi) \pm \mathrm{isech}(\xi))^2}{2\left(\pm\tanh(\xi) \pm \mathrm{isech}(\xi) \pm \tan(\xi) + 1\right)^2} - \frac{b_1 a_1 l (\tanh(\xi) \pm \mathrm{isech}(\xi))\tan(\xi)}{\left(\pm\tanh(\xi) \pm \mathrm{isech}(\xi) \pm \tan(\xi) + 1\right)^2} \\ & - \frac{{b_1}^2 l \tan^2(\xi)}{\left(\pm\tanh(\xi) \pm \mathrm{isech}(\xi) \pm \tan(\xi) + 1\right)^2}, \end{aligned} \tag{27}$$

where $\xi = k(x+ly+\lambda t)$, a_0, A_0, a_1, b_1, k, l and λ are arbitrary constants.

Family 5. When $h_1 = -\frac{1}{2}$, $h_2 = \frac{1}{2}$ and $h_3 = h_4 = \pm\frac{1}{2}$, then we combination of tanh, sech, sec and tan function solution:

$$\begin{aligned} U_5 = &\ a_0 + \frac{a_1 (\tanh(\xi) \pm \mathrm{isech}(\xi) + b_1 (\sec(\xi) \pm \tan(\xi)))}{\pm(\tanh(\xi) \pm \mathrm{isech}(\xi)) \pm (\sec(\xi) \pm \tan(\xi)) + 1}, \\ V_5 = &\ A_0 - \frac{(\lambda + a_0)\, l \left(a_1 (\tanh(\xi) \pm \mathrm{isech}(\xi)) + b_1 (\sec(\xi) \pm \tan(\xi))\right)}{\pm(\tanh(\xi) \pm \mathrm{isech}(\xi)) \pm (\sec(\xi) \pm \tan(\xi)) + 1} \\ & - \frac{{a_1}^2 l (\tanh(\xi) \pm \mathrm{isech}(\xi))^2}{2\left(\pm(\tanh(\xi) \pm \mathrm{isech}(\xi)) \pm (\sec(\xi) \pm \tan(\xi)) + 1\right)^2} \\ & - \frac{b_1 a_1 l (\tanh(\xi) \pm \mathrm{isech}(\xi))(\sec(\xi) \pm \tan(\xi))}{\left(\pm(\tanh(\xi) \pm \mathrm{isech}(\xi)) \pm (\sec(\xi) \pm \tan(\xi)) + 1\right)^2} \\ & + \frac{{b_1}^2 l (\sec(\xi) \pm \tan(\xi))^2}{2\left(\pm(\tanh(\xi) \pm \mathrm{isech}(\xi)) \pm (\sec(\xi) \pm \tan(\xi)) + 1\right)^2}, \end{aligned} \tag{28}$$

where $\xi = k(x+ly+\lambda t)$, a_0, A_0, a_1, b_1, k, l and λ are arbitrary constants.

Family 6. When $h_1 = 1$, $h_2 = -1$, $h_3 = 0$, and $h_4 \neq 0$, then we can get combination of tanh, sech, rational function solution:

$$\begin{aligned} U_6 = \ & a_0 + \frac{a_1\left(h_4\xi + c_0\right)\left(\tanh\left(\xi\right) \pm \operatorname{isech}\left(\xi\right)\right)}{\left(h_4\xi + c_0\right)\left(\tanh\left(\xi\right) \pm \operatorname{isech}\left(\xi\right)\right) \mp 1 + h_4\xi + c_0} \\ & - \frac{b_1}{\left(h_4\xi + c_0\right)\left(\tanh\left(\xi\right) \pm \operatorname{isech}\left(\xi\right)\right) \mp 1 + h_4\xi + c_0}, \\ V_6 = \ & A_0 - \frac{\left(\lambda + a_0\right) l a_1\left(h_4\xi + c_0\right)\left(\tanh\left(\xi\right) \pm \operatorname{isech}\left(\xi\right)\right)}{\left(h_4\xi + c_0\right)\left(\tanh\left(\xi\right) \pm \operatorname{isech}\left(\xi\right)\right) \mp 1 + h_4\xi + c_0} \\ & - \frac{\left(-b_1 a_0 l - b_1 \lambda l\right)}{\left(h_4\xi + c_0\right)\left(\tanh\left(\xi\right) \pm \operatorname{isech}\left(\xi\right)\right) \mp 1 + h_4\xi + c_0} \\ & - \frac{{a_1}^2 l\left(\tanh\left(\xi\right) \pm \operatorname{isech}\left(\xi\right)\right)^2}{2\left(\left(h_4\xi + c_0\right)\left(\tanh\left(\xi\right) \pm \operatorname{isech}\left(\xi\right)\right) \mp 1 + h_4\xi + c_0\right)^2} \\ & + \frac{b_1 a_1 l\left(\tanh\left(\xi\right) \pm \operatorname{isech}\left(\xi\right)\right)\left(h_4\xi + c_0\right)}{\left(\left(h_4\xi + c_0\right)\left(\tanh\left(\xi\right) \pm \operatorname{isech}\left(\xi\right)\right) \mp 1 + h_4\xi + c_0\right)^2} \\ & - \frac{{b_1}^2 l}{2\left(\left(h_4\xi + c_0\right)\left(\tanh\left(\xi\right) \pm \operatorname{isech}\left(\xi\right)\right) \mp 1 + h_4\xi + c_0\right)^2}, \end{aligned} \tag{29}$$

where $\xi = k\left(x + ly + \lambda t\right)$, $h_4 \neq 0$, c_0, a_0, A_0, a_1, b_1, k, l and λ are arbitrary constants.

Family 7. When $h_1 = h_2 = \pm 1$, $h_3 = 0$, and $h_4 \neq 0$, then we can get combination of tan and rational function solution:

$$\begin{aligned} U_7 = \ & a_0 + \frac{a_1\left(h_4\xi + c_0\right)\tan\left(\xi\right)}{\pm\left(h_4\xi + c_0\right)\tan\left(\xi\right) \mp 1 + h_4\xi + c_0} - \frac{b_1}{\pm\left(h_4\xi + c_0\right)\tan\left(\xi\right) \mp 1 + h_4\xi + c_0}, \\ V_7 = \ & A_0 - \frac{\left(\lambda + a_0\right) l a_1\left(h_4\xi + c_0\right)\tan\left(\xi\right)}{\pm\left(h_4\xi + c_0\right)\tan\left(\xi\right) \mp 1 + h_4\xi + c_0} + \frac{\left(b_1 a_0 l + b_1 \lambda l\right)}{\pm\left(h_4\xi + c_0\right)\tan\left(\xi\right) \mp 1 + h_4\xi + c_0} \\ & - \frac{{a_1}^2 l\left(\left(h_4\xi + c_0\right)\tan\left(\xi\right)\right)^2}{2\left(\pm\left(h_4\xi + c_0\right)\tanh\left(\xi\right) \mp 1 + h_4\xi + c_0\right)^2} + \frac{b_1 a_1 l \tan\left(\xi\right)\left(h_4\xi + c_0\right)}{\left(\pm\left(h_4\xi + c_0\right)\tan\left(\xi\right) \mp 1 + h_4\xi + c_0\right)^2} \\ & - \frac{{b_1}^2 l}{2\left(\pm\left(h_4\xi + c_0\right)\tan\left(\xi\right) \mp 1 + h_4\xi + c_0\right)^2}, \end{aligned} \tag{30}$$

where $\xi = k\left(x + ly + \lambda t\right)$, $h_4 \neq 0$, c_0, a_0, A_0, a_1, b_1, k, l and λ are arbitrary constants.

Family 8. When $h_1 = 1$, $h_2 = -1$, $h_3 = 0$, and $h_4 \neq 0$, then we can get combination of tan, sec, rational function solution:

$$\begin{aligned} U_8 = \ & a_0 + \frac{a_1\left(h_4\xi + c_0\right)\left(\sec\left(\xi\right) \pm \tan\left(\xi\right)\right)}{\left(h_4\xi + c_0\right)\left(\sec\left(\xi\right) \pm \tan\left(\xi\right)\right) \mp 1 + h_4\xi + c_0} \\ & - \frac{b_1}{\left(h_4\xi + c_0\right)\left(\sec\left(\xi\right) \pm \tan\left(\xi\right)\right) \mp 1 + h_4\xi + c_0}, \\ V_8 = \ & A_0 - \frac{\left(\lambda + a_0\right) l a_1\left(h_4\xi + c_0\right)\left(\sec\left(\xi\right) \pm \tan\left(\xi\right)\right)}{\left(h_4\xi + c_0\right)\left(\sec\left(\xi\right) \pm \tan\left(\xi\right)\right) \mp 1 + h_4\xi + c_0} \\ & - \frac{\left(-b_1 a_0 l - b_1 \lambda l\right)}{\left(h_4\xi + c_0\right)\left(\sec\left(\xi\right) \pm \tan\left(\xi\right)\right) \mp 1 + h_4\xi + c_0} \\ & - \frac{{a_1}^2 l\left(\sec\left(\xi\right) \pm \tan\left(\xi\right)\right)^2}{2\left(\left(h_4\xi + c_0\right)\left(\sec\left(\xi\right) \pm \tan\left(\xi\right)\right) \mp 1 + h_4\xi + c_0\right)^2} \\ & + \frac{b_1 a_1 l\left(\sec\left(\xi\right) \pm \tan\left(\xi\right)\right)\left(h_4\xi + c_0\right)}{\left(\left(h_4\xi + c_0\right)\left(\sec\left(\xi\right) \pm \tan\left(\xi\right)\right) \mp 1 + h_4\xi + c_0\right)^2} \\ & - \frac{{b_1}^2 l}{2\left(\left(h_4\xi + c_0\right)\left(\sec\left(\xi\right) \pm \tan\left(\xi\right)\right) \mp 1 + h_4\xi + c_0\right)^2}, \end{aligned} \tag{31}$$

where $\xi = k\left(x + ly + \lambda t\right)$, $h_4 \neq 0$, c_0, a_0, A_0, a_1, b_1, k, l and λ are arbitrary constants.

At the same time, we can also get some new solutions which are not the complexiton solutions and cannot be obtained by other tanh method, such as:

Family 9. When $h_1 = h_3 = \dfrac{1}{2}$ and $h_2 = h_4 = -\dfrac{1}{2}$, then we obtain following solutions.

$$\begin{aligned} U_9 = \ & a_0 + \frac{a_1 (\tanh(\xi) \pm \mathrm{isech}(\xi) + b_1 (\coth(\xi) \pm \mathrm{csch}(\xi)))}{\pm(\tanh(\xi) \pm \mathrm{isech}(\xi)) \pm (\coth(\xi) \pm \mathrm{csch}(\xi)) + 1}, \\ V_9 = \ & A_0 - \frac{(\lambda + a_0)\, l\, (a_1 (\tanh(\xi) \pm \mathrm{isech}(\xi)) + b_1 (\coth(\xi) \pm \mathrm{csch}(\xi)))}{\pm(\tanh(\xi) \pm \mathrm{isech}(\xi)) \pm (\coth(\xi) \pm \mathrm{csch}(\xi)) + 1} \\ & - \frac{{a_1}^2 l\, (\tanh(\xi) \pm \mathrm{isech}(\xi))^2}{2\,(\pm(\tanh(\xi) \pm \mathrm{isech}(\xi)) \pm (\coth(\xi) \pm \mathrm{csch}(\xi)) + 1)^2} \\ & - \frac{b_1 a_1 l\, (\tanh(\xi) \pm \mathrm{isech}(\xi))\,(\coth(\xi) \pm \mathrm{csch}(\xi))}{(\pm(\tanh(\xi) \pm \mathrm{isech}(\xi)) \pm (\coth(\xi) \pm \mathrm{csch}(\xi)) + 1)^2} \\ & + \frac{{b_1}^2 l\, (\coth(\xi) \pm \mathrm{csch}(\xi))^2}{2\,(\pm(\tanh(\xi) \pm \mathrm{isech}(\xi)) \pm (\coth(\xi) \pm \mathrm{csch}(\xi)) + 1)^2}, \end{aligned} \tag{32}$$

where $\xi = k(x + ly + \lambda t)$, a_0, A_0, a_1, b_1, k, l and λ are arbitrary constants.

Family 10. When $h_1 = h_2 = \pm\dfrac{1}{2}$ and $h_3 = h_4 = \pm\dfrac{1}{2}$, then we obtain following solutions.

$$\begin{aligned} U_{10} = \ & a_0 + \frac{a_1 (\sec(\xi) \pm \tan(\xi) + b_1 (\csc(\xi) \pm \cot(\xi)))}{\pm(\sec(\xi) \pm \tan(\xi)) \pm (\csc(\xi) \pm \cot(\xi)) + 1}, \\ V_{10} = \ & A_0 - \frac{(\lambda + a_0)\, l\, (a_1 (\sec(\xi) \pm \tan(\xi)) + b_1 (\csc(\xi) \pm \cot(\xi)))}{\pm(\sec(\xi) \pm \tan(\xi)) \pm (\csc(\xi) \pm \cot(\xi)) + 1} \\ & - \frac{{a_1}^2 l\, (\sec(\xi) \pm \tan(\xi))^2}{2\,(\pm(\sec(\xi) \pm \tan(\xi)) \pm (\csc(\xi) \pm \cot(\xi)) + 1)^2} \\ & - \frac{b_1 a_1 l\, (\sec(\xi) \pm \tan(\xi))\,(\csc(\xi) \pm \cot(\xi))}{(\pm(\sec(\xi) \pm \tan(\xi)) \pm (\csc(\xi) \pm \cot(\xi)) + 1)^2} \\ & + \frac{{b_1}^2 l\, (\csc(\xi) \pm \cot(\xi))^2}{2\,(\pm(\sec(\xi) \pm \tan(\xi)) \pm (\csc(\xi) \pm \cot(\xi)) + 1)^2}, \end{aligned} \tag{33}$$

where $\xi = k(x + ly + \lambda t)$, a_0, A_0, a_1, b_1, k, l and λ are arbitrary constants.

4. Conclusion

To construct exact solutions of nonlinear PDEs, a MRERE method is presented and a series of novel classes of complexiton solutions of the (2+1)-dimensional dispersive long wave equation are found. The novel solutions obtained by the MRERE method include various combination of trigonometric periodic and hyperbolic function solutions, various combination of trigonometric periodic and rational function solutions, various combination of hyperbolic and rational function solutions, etc. Further works about the extensions of the MRERE method have the following three options.

1) Mixed equation method: choose two different sub-equations such as the Riccati equation[4] and the generalized Elliptic equation[18] in the MRERE method.

2) Construct complexiton solutions which have different travelling waves.

3) For many (1+1) dimensional physically interesting systems, it is found that complexiton solution is one of newly found important nonlinear excitation, especially, non-singular complexiton solutions have also been discussed.[19]

The result of this paper indicates that the complexiton solutions are also existed for physically interested (2+1)-dimensional models. Whether the non-singular complexiton solutions exist in (2+1)-dimensional models, this problem will been studied in near future.

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