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New exact solutions for modified nonlinear dispersive equations $mK(m, n)$ in higher dimensions spaces

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Abstract

With the use of some proper transformations and symbolic computation, we present a general and unified method for investigating the general modified nonlinear dispersive equations $mK(m, n)$ in higher dimensions spaces. The work formally shows how to construct the general solutions and some special exact-solutions for $mK(m, n)$ equations in higher dimensional spatial domains. The general solutions not only contain the solutions by Wazwaz [Math. Comput. Simulation 59 (2002) 519] but also contain many new compact and noncompact solutions.

Keywords: $mK(m, n)$ equations; Travelling wave solution; Symbolic computation

1. Introduction

Since the soliton phenomena was first observed by Scott Russell in 1834 and KdV equation was solved by the inverse scattering method by Garder et al. in 1972 [1,2], the study of solutions and the related issue of the construction of solution to a wide class of nonlinear equations have become one of the most exciting and extremely active areas of research and investigation. Directly searching for exact solutions of nonlinear evolution equations (NEEs) has become more and more attractive, partly due to the availability of computer systems like *Maple* or *Mathematica* which allow us to perform some complicated and tedious algebraic calculation on a computer, as well as help us to find new exact solutions of NEEs. Rosenau and Hyman [3] investigated the role of nonlinear dispersion in the formation of patterns in liquid drops by introducing and studying a family of nonlinear KdV like equation of the form

$$u_t + a(u^m)_x + (u^n)_{xxx} = 0, \quad m > 1, \ \ 1 \le n \le 3, \tag{1.1}$$

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they found that nonlinear dispersion can compactify solitary waves and generate compactons, and introduced a class of solitary waves with compact support, which they called compactons, that collide elastically and vanish identically outside a finite core region [3–7]. They discovered that solitary waves may compactify under the influence of nonlinear dispersion which is capable of causing deep qualitative changes in the native of genuinely nonlinear phenomena.

Equations of this type with values of m and n are denoted by $K(m, n)$. In [3], four cases $m, n = 2, 3$ were studied thoroughly. The studies continued in this direction and the general case where $m = n$ was examined and a general formula that satisfies (1) was derived. Several other papers by Rosenau [5–7] and by Rosenau and coworkers [3,4] investigated the new discovery thoroughly. Olver and Rosenau [4] investigated the tri-Hamiltonian duality between solitons and compactons. Ismail and Taha [8] implemented a finite difference method and a finite element method to study the two type $K(2, 2)$ and $K(3, 3)$ equations. Ludu and Draayer [9] introduced a useful work on patterns on liquid surfaces where cnoidal waves compactons and sealing wave discussed. In [10], Dinda and Remoissenet demonstrated the existence of a breacher with a compact support, i.e., a breather compacton, in a nonlinear Klein–Gorden lattic with a soft on site substrate potential.

For more details about the role of nonlinear dispersion in pattern formation and for more insight through the compacton behavior, the reader is advised to see the remarkable achievements in [3–10].

Wazwaz [11–16] has devoted considerable effort to study the $K(n, n)$ equation and make new developments in this regard. In [15], two sets of entirely new formulas that produce compactons and anticompacton for any integer $n, n \geq 1$ are established. Wazwaz presented a general and unified approach for analyzing the genuinely nonlinear dispersive $mK(n, n)$ equations in one-, two- and three-dimensional spatial domain given by

$$u^{n-1}u_t + a(u^m)_x + (u^n)_{xxx} = 0, \tag{1.2}$$

$$u^{n-1}u_t + a(u^m)_x + b(u^n)_{xxx} = 0, \tag{1.3}$$

$$u^{n-1}u_t + a(u^m)_x + b(u^n)_{xxx} + k(u^n)_{yyy} = 0, \tag{1.4}$$

$$u^{n-1}u_t + a(u^m)_x + b(u^n)_{xxx} + k(u^n)_{yyy} + r(u^n)_{zzz} = 0, \tag{1.5}$$

where a, b, k, r are constants. Eqs. (1.2)–(1.5) differ from those studied in [3–14] only in the addition of term u^{n-1} that multiplies u_t. Wazwaz [15] formally shows how to construct compact and noncompact solutions in one-, two- and three-dimensional spatial domains. Two distinct general formulaes for each model, that are of substantial interest, are developed for all positive integers, $n, n > 1$.

It is worth noting that three main methods, namely, the pseudo-spectral method, the tri-Hamiltonian operators and Adomain decomposition method, have emerged as appropriate algorithms to handle the $K(n, n)$ equation effectively.

The present work is motivated by the desire to extend the work made in [15] and with the use of some proper transformations and the improved tanh-method, we consider the general modified nonlinear dispersive equations $mK(m, n)$ in higher dimensional space. Our approach stems mainly from the extended tanh-method presented by Fan and Zhang [17,18]. Recently, Fan and coworkers [19–21], and Yan [22] further developed this idea and made it much more lucid and straightforward for a class of NEEs. By introducing more general proper transformation we improve the extended tanh-method. As a result, the general solutions for $mK(n, n)$ equations and special exact solutions for $mK(m, n)$ equations in higher dimensional spatial domains, even in N-dimensional space.

2. The one-dimensional equation

We consider the travelling solutions of Eq. (1.1) in the form $u(x,t) = u(\xi)$, $\xi = \mu(x-ct)$, where μ is a constant. Then (1.1) reduces to

$$-cu^{n-1}u' + a(u^m)' + \mu^2(u^n)''' = 0. \tag{2.1}$$

We make the transformation

$$u(\xi) = v^{1/n}(\xi), \tag{2.2}$$

then substituting (2.2) into (2.1) yields

$$-cv' + amv^{-1+(m/n)}v' + n\mu^2 v''' = 0. \tag{2.3}$$

We discuss two cases, respectively

Case 1. $n = m$.

When $m = n$, Eq. (2.3) changes into

$$(na-c)v' + n\mu^2 v''' = 0. \tag{2.4}$$

The general solution of linear ordinary differential equation (2.4) is

$$v(\xi) = -\frac{\mu\sqrt{n}c_1}{\sqrt{c-an}}\exp\left[-\frac{\sqrt{c-an}}{\mu\sqrt{n}}\xi\right] + \frac{\mu\sqrt{n}c_2}{\sqrt{c-an}}\exp\left[\frac{\sqrt{c-an}}{\mu\sqrt{n}}\xi\right] + c_3, \tag{2.5}$$

where μ, c_1, c_2, c_3 are arbitrary constants.

Therefore from (2.2) and (2.5), we obtain a family of solutions of (1.1)

$$u(x,t) = \left\{-\frac{\mu\sqrt{n}c_1}{\sqrt{c-an}}\exp\left[-\frac{\sqrt{c-an}}{\sqrt{n}}(x-ct)\right] + \frac{\mu\sqrt{n}c_2}{\sqrt{c-an}}\exp\left[\frac{\sqrt{c-an}}{\sqrt{n}}(x-ct)\right] + c_3\right\}^{1/n}. \tag{2.6}$$

Setting $c_1 = c_2$ and $c_3 = 0$, from (2.5) we obtain the following set of general compactons solutions

$$u(x,t) = \begin{cases} \left\{\dfrac{2\mu\sqrt{n}c_1}{\sqrt{an-c}}\sin\left[\dfrac{\sqrt{na-c}}{\sqrt{n}}(x-ct)\right]\right\}^{1/n}, & |x-ct| \le \dfrac{\sqrt{n}}{\sqrt{na-c}}\pi, \\ 0, & \text{otherwise}. \end{cases} \tag{2.7}$$

Setting $c_1 = -c_2$ and $c_3 = 0$, we obtain the following set of general compactons solutions

$$u(x,t) = \begin{cases} \left\{-\dfrac{2\mu\sqrt{n}c_1}{\sqrt{c-an}}\cos\left[\dfrac{\sqrt{na-c}}{\sqrt{n}}(x-ct)\right]\right\}^{1/n}, & |x-ct| \le \dfrac{\sqrt{n}}{2\sqrt{na-c}}\pi, \\ 0, & \text{otherwise}. \end{cases} \tag{2.8}$$

In view of the arbitrariness of the constants μ, c_1, c, the solutions of (2.7), (2.8) cover the solutions in [15].

Case 2. $m \not\equiv n$.

We firstly define the degree of $v(\xi)$ as $D[v(\xi)] = k$ which gives rise to the degree of the other expressions as

$$D\left[\frac{\mathrm{d}^q v}{\mathrm{d}\xi^q}\right] = k + q, \qquad D\left[v^p \left(\frac{\mathrm{d}^q v}{\mathrm{d}\xi^q}\right)^s\right] = kp + s(k+q). \tag{2.9}$$

Therefore by balancing $v^{-1+(m/n)}v'$ with v''' in (2.3), we obtain the balance constant $k_1 = 2n/(m-n)$. Therefore we make transformation

$$v(\xi) = w^{2n/(m-n)}(\xi), \tag{2.10}$$

then substituting (2.10) into (2.3) yields

$$am(m-n)^2 w^4 w' + 2n(m^2 - 5mn + 6n^2)\mu^2 (w')^3 - 3n(m^2 - 4mn + 3n^2)\mu^2 w w' w'' + (m-n)^2 w^2(-cw' + n\mu^2 w''') = 0. \tag{2.11}$$

Balancing $w^2 w'''$ with $w^4 w'$ (or $(w')^3$, or $ww'w''$) in (2.11), we obtain the balance constant $k_2 = 1$. Therefore we assume the solutions of (2.11) are of the form

$$w(\xi) = A + B\phi + C\sqrt{R + \phi^2} \tag{2.12}$$

and $\phi = \phi(\xi)$ satisfies

$$\phi'(\xi) = R + \phi(\xi)^2. \tag{2.13}$$

With the aid of *Mathematica*, substituting (2.12)–(2.13) into (2.11) and collecting all terms with the same power in $\phi^k(\sqrt{R+\phi^2})^j$ $(j = 0, 1; k = 0, \ldots, 6)$ yields a system of equations with respect to $\phi^k(\sqrt{R+\phi^2})^j$. Setting the coefficients of $\phi^k(\sqrt{R+\phi^2})^j$ $(j = 0, 1; k = 0, \ldots, 6)$ in the obtained system of equations to zero, we deduce the following set of over-determined algebraic polynomials with respect to the unknowns A, B, C, R, μ, c:

$$BR(aA^4 m(m-n)^2 + A^2(m-n)^2(-c + 6aC^2 mR + 2nR\mu^2) + R(-cC^2(m-n)^2 + R(aC^4 m(m-n)^2 + n(2B^2(m^2 - 5mn + 6n^2) - C^2(m^2 - 8mn + 7n^2))\mu^2))) = 0, \tag{2.14}$$

$$ABC(m-n)R(-2c(m-n) + 4am(m-n)(A^2 + C^2 R) + n(m+5n)R\mu^2) = 0, \tag{2.15}$$

$$A(m-n)R(C^2(-2c(m-n) + 4am(m-n)(A^2 + C^2 R) + (7m-n)nR\mu^2) + 2B^2(c(-m+n) + 2am(m-n)(A^2 + 3C^2 R) + n(-m+7n)R\mu^2)) = 0, \tag{2.16}$$

$$C(aA^4 m(m-n)^2 + A^2(m-n)^2(-c + R(6a(2B^2 + C^2)m + 5n\mu^2)) + R(C^2(m-n)(c(-m+n) + R(aC^2 m(m-n) + 2n(m+2n)\mu^2)) + B^2(-2c(m-n)^2 + R(4aC^2 m(m-n)^2 + n(m^2 - 2mn + 13n^2)\mu^2)))) = 0, \tag{2.17}$$

$$B(aA^4 m(m-n)^2 + A^2(m-n)^2(-c + 2R(3a(B^2 + 4C^2)m + 4n\mu^2)) + R(C^2(-4c(m-n)^2 + R(7aC^2 m(m-n)^2 + n(5m^2 + 14mn - 7n^2)\mu^2)) + B^2(-c(m-n)^2 + 2R(3aC^2 m(m-n)^2 + n(m^2 - 5mn + 10n^2)\mu^2)))) = 0, \tag{2.18}$$

$$ABC(m-n)(-4c(m-n) + 4am(m-n)(2A^2 + 3B^2 R + 5C^2 R) + n(11m + 19n)R\mu^2) = 0, \tag{2.19}$$

$$A(m-n)(4aB^4m(m-n)R + C^2(-2c(m-n) + am(m-n)(A^2+2C^2R) + n(13m+5n)R\mu^2) + B^2(-2c(m-n) + 4(am(m-n)(A^2+9C^2R) + n(m+5n)R\mu^2))) = 0, \tag{2.20}$$

$$C(-cC^2(m-n)^2 + 4aB^4m(m-n)^2R + 2(aC^2m(m-n)^2(3A^2+C^2R) + n(3A^2(m-n)^2 + 2C^2(m^2+mn-n^2)R)\mu^2) + B^2(-3c(m-n)^2 + 2(am(m-n)^2(9A^2+7C^2R) + 3n(m^2+mn+2n^2)R\mu^2))) = 0, \tag{2.21}$$

$$B(18aA^2C^2m^3 - 3cC^2(m-n)^2 - 36aA^2C^2m^2n + 18aA^2C^2mn^2 + 11aC^4m^3R + aB^4m(m-n)^2R - 22aC^4m^2nR + 11aC^4mn^2R + 6A^2m^2n\mu^2 - 12A^2mn^2\mu^2 + 6A^2n^3\mu^2 + 12C^2m^2nR\mu^2 + 12C^2mn^2R\mu^2 + B^2(-c(m-n)^2 + 2(am(m-n)^2(3A^2+8C^2R) + n(m^2+mn+4n^2)R\mu^2))) = 0, \tag{2.22}$$

$$4ABC(m-n)(4a(B^2+C^2)m(m-n) + 3n(m+n)\mu^2) = 0, \tag{2.23}$$

$$2A(m-n)(2a(B^4+6B^2C^2+C^4)m(m-n) + 3(B^2+C^2)n(m+n)\mu^2) = 0, \tag{2.24}$$

$$Cm(a(5B^4+10B^2C^2+C^4)(m-n)^2 + 2(3B^2+C^2)n(m+n)\mu^2) = 0, \tag{2.25}$$

$$Bm(a(B^4+10B^2C^2+5C^4)(m-n)^2 + 2(B^2+3C^2)n(m+n)\mu^2) = 0. \tag{2.26}$$

From (2.14)–(2.26), with the aid of *Mathematica*, we have

Case 1.

$$A = B = 0, \qquad C^2 = -\frac{2n(m+n)\mu^2}{a(m-n)^2}, \qquad R = -\frac{c(m-n)^2}{4n^3\mu^2}. \tag{2.27}$$

Case 2.

$$A = C = 0, \qquad B^2 = -\frac{6\mu^2}{a}, \qquad R = \frac{c}{8n\mu^2}, \qquad m = 2n. \tag{2.28}$$

Case 3.

$$A = 0, \qquad B = \pm C = \pm\sqrt{-\frac{3\mu^2}{2a}}, \qquad m = 2n. \tag{2.29}$$

Case 4.

$$A = C = 0, \qquad B^2 = -\frac{2\mu^2}{a}, \qquad R = \frac{c}{2n\mu^2}, \qquad m = 3n. \tag{2.30}$$

Case 5.

$$A = 0, \qquad B = \pm C = \pm\sqrt{-\frac{\mu^2}{2a}}, \qquad R = \frac{2c}{n\mu^2}, \qquad m = 3n. \tag{2.31}$$

Case 6.

$$A^2 = \frac{c}{4an}, \qquad B^2 = \frac{\mu^2}{4a}, \qquad C = 0, \qquad R = -\frac{c}{n\mu^2}, \qquad m = -3n. \tag{2.32}$$

Case 7.

$$A^2 = \frac{c}{4an}, \qquad B = \pm C = \pm\sqrt{\frac{\mu^2}{16a}}, \qquad R = -\frac{4c}{n\mu^2}, \qquad m = -3n. \tag{2.33}$$

It is well known that the general solutions of (2.13) are

1. When $R < 0$:

$$\phi(\xi) = -\sqrt{-R}\tanh(\sqrt{-R}\xi), \qquad \phi(\xi) = -\sqrt{-R}\coth(\sqrt{-R}\xi). \tag{2.34}$$

2. When $R = 0$:

$$\phi(\xi) = -\frac{1}{\xi}. \tag{2.35}$$

3. When $R > 0$:

$$\phi(\xi) = \sqrt{R}\tan(\sqrt{R}\xi), \qquad \phi(\xi) = -\sqrt{R}\cot(\sqrt{R}\xi). \tag{2.36}$$

Therefore, combining (2.2), (2.10), (2.12), (2.34), (2.36) along with Cases 1–7, we obtain the travelling wave solutions of $mK(m, n)$ equations (1.1).

Case 1. When $R = -c(m-n)^2/4n^3\mu^2 < 0$, i.e., $nc > 0$, from (2.27) we get

$$u_{11} = \left\{-\frac{m+n}{2an^2}\text{sech}^2\sqrt{\frac{c(m-n)^2}{4n^3}}(x-ct)\right\}^{1/(m-n)}, \tag{2.37}$$

$$u_{12} = \left\{\frac{m+n}{2an^2}\text{csch}^2\sqrt{\frac{c(m-n)^2}{4n^3}}(x-ct)\right\}^{1/(m-n)}. \tag{2.38}$$

When $R = -c(m-n)^2/4n^3\mu^2 > 0$, i.e., $nc < 0$

$$u_{13} = \left\{\frac{m+n}{2an^2}\sec^2\sqrt{-\frac{c(m-n)^2}{4n^3}}(x-ct)\right\}^{1/(m-n)}, \tag{2.39}$$

$$u_{14} = \left\{\frac{m+n}{2an^2}\csc^2\sqrt{-\frac{c(m-n)^2}{4n^3}}(x-ct)\right\}^{1/(m-n)}. \tag{2.40}$$

Note. To simplify, we omit the periodic solutions in the rest of this section.

Cases 2–3. From (2.28)–(2.29), the $mK(2n, n)$ equation, $u^{n-1}u_t+a(u^{2n})_x+(u^n)_{xxx} = 0$, has the following solutions

$$u_{21} = \left\{\frac{3c}{4an}\tanh^2\sqrt{-\frac{c}{8n}}(x-ct)\right\}^{1/n}, \tag{2.41}$$

$$u_{22} = \left\{\frac{3c}{4an}\coth^2\sqrt{-\frac{c}{8n}}(x-ct)\right\}^{1/n}, \tag{2.42}$$

$$u_{31} = \left\{\pm\sqrt{\frac{3c}{4an}}\left\{\tanh\left[\sqrt{-\frac{c}{2n}}(x-ct)\right]\pm i\,\mathrm{sech}\left[\sqrt{-\frac{c}{2n}}(x-ct)\right]\right\}\right\}^{2/n}, \tag{2.43}$$

$$u_{32} = \left\{\pm\sqrt{\frac{3c}{4an}}\left\{\coth\left[\sqrt{-\frac{c}{2n}}(x-ct)\right]\pm \mathrm{csch}\left[\sqrt{-\frac{c}{2n}}(x-ct)\right]\right\}\right\}^{2/n}. \tag{2.44}$$

Cases 4–5. From ((2.30)–(2.31), the $mK(3n, n)$ equation, $u^{n-1}u_t + a(u^{3n})_x + (u^n)_{xxx} = 0$, has the following solutions

$$u_{41} = \left\{\frac{c}{an}\tanh^2\left[\sqrt{-\frac{c}{n}}(x-ct)\right]\right\}^{1/2n}, \tag{2.45}$$

$$u_{41} = \left\{\frac{c}{an}\coth^2\left[\sqrt{-\frac{c}{n}}(x-ct)\right]\right\}^{1/2n}, \tag{2.46}$$

$$u_{51} = \left\{\pm\sqrt{\frac{c}{an}}\left\{\tanh\left[\sqrt{-\frac{2c}{n}}(x-ct)\right]\pm i\,\mathrm{sech}\left[\sqrt{-\frac{2c}{n}}(x-ct)\right]\right\}\right\}^{1/n}, \tag{2.47}$$

$$u_{52} = \left\{\pm\sqrt{\frac{c}{an}}\left\{\coth\left[\sqrt{-\frac{2c}{n}}(x-ct)\right]\pm \mathrm{csch}\left[\sqrt{-\frac{2c}{n}}(x-ct)\right]\right\}\right\}^{1/n}. \tag{2.48}$$

Cases 6–7. From (2.32)–(2.33), the $mK(-3n, n)$ equation, $u^{n-1}u_t + a(u^{-3n})_x + (u^n)_{xxx} = 0$, has the following solutions

$$u_{61} = \left\{\frac{c}{4an}\tanh^2\left[\sqrt{\frac{c}{n}}(x-ct)\right]\right\}^{-1/(4n)}, \tag{2.49}$$

$$u_{61} = \left\{\frac{c}{4an}\coth^2\left[\sqrt{\frac{c}{n}}(x-ct)\right]\right\}^{-1/(4n)}, \tag{2.50}$$

$$u_{71}=\left\{\pm\sqrt{\frac{c}{4an}}\left\{1\pm\tanh\left[\sqrt{\frac{4c}{n}}(x-ct)\right]\pm i\,\mathrm{sech}\left[\sqrt{\frac{4c}{n}}(x-ct)\right]\right\}\right\}^{-1/(2n)}, \tag{2.51}$$

$$u_{72}=\left\{\pm\sqrt{\frac{c}{4an}}\left\{1\pm\coth\left[\sqrt{\frac{4c}{n}}(x-ct)\right]\pm \mathrm{csch}\left[\sqrt{\frac{4c}{n}}(x-ct)\right]\right\}\right\}^{-1/(2n)}. \tag{2.52}$$

3. The N-dimensional equation

We set the nonlinear dispersive equation in an N-dimensional space

$$u^{n-1}u_t+a(u^m)_x+\sum_{i=1}^{N}k_i(u^n)_{x_ix_ix_i}=0, \tag{3.1}$$

where $x_1=x$, $u\equiv u(x_1,x_2,\ldots,x_N,t)$, and k_i are constants.

We now consider the travelling wave solutions of Eq. (3.1) in the form

$$u(x_1,x_2,\ldots,x_m,t)=u(\xi),\qquad \xi=k(x_1+x_2+\ldots+x_N-ct). \tag{3.2}$$

Case 1. $m=n$.

Proceeding as before, we obtain the following general solutions of Eq. (3.1)

$$u(x,t)=\left\{-\frac{k\sqrt{n\sum_{i=1}^{N}k_i}c_1}{\sqrt{c-an}}\exp\left[-\frac{\sqrt{c-an}}{\sqrt{n\sum_{i=1}^{N}k_i}}\xi\right]+\frac{k\sqrt{n\sum_{i=1}^{N}k_i}c_2}{\sqrt{c-an}}\exp\left[\frac{\sqrt{c-an}}{\sqrt{n\sum_{i=1}^{N}k_i}}\xi\right]+c_3\right\}^{1/n}. \tag{3.3}$$

It is not difficult to verify that from the solution (3.3), when setting parameters to be equal to proper values, we can obtain all of the general compacton solutions of the nonlinear $mK(n,n)$ equation in [15].

Case 2. $m\not\equiv n$.

Proceeding as before, we find seven families of special solution of Eq. (3.1).

1. When $R=-c(m-n)^2/4n^3k^2\sum_{i=1}^{N}k_i<0$, we get

$$u_{11}=\left\{-\frac{m+n}{2an^2}\mathrm{sech}^2\sqrt{\frac{c(m-n)^2}{4n^3\sum_{i=1}^{N}k_i}}\xi\right\}^{1/(m-n)}, \tag{3.4}$$

$$u_{12}=\left\{\frac{m+n}{2an^2}\mathrm{csch}^2\sqrt{\frac{c(m-n)^2}{4n^3\sum_{i=1}^{N}k_i}}\xi\right\}^{1/(m-n)}. \tag{3.5}$$

When $R=-c(m-n)^2/4n^3k^2\sum_{i=1}^{N}k_i>0$, we get

$$u_{13}=\left\{\frac{m+n}{2an^2}\sec^2\sqrt{-\frac{c(m-n)^2}{4n^3\sum_{i=1}^{N}k_i}}\xi\right\}^{1/(m-n)},\tag{3.6}$$

$$u_{14}=\left\{\frac{m+n}{2an^2}\csc^2\sqrt{-\frac{c(m-n)^2}{4n^3\sum_{i=1}^{N}k_i}}\xi\right\}^{1/(m-n)}.\tag{3.7}$$

Note. To simplify, we omit the periodic solutions in the rest of this paper.

2–3. The equation $u^{n-1}u_t+a(u^{2n})_{x_1}+\sum_{i=1}^{N}k_i(u^n)_{x_ix_ix_i}=0$ has the following solutions

$$u_{21}=\left\{\frac{3c}{4an}\tanh^2\sqrt{-\frac{c}{8n\sum_{i=1}^{N}k_i}}\xi\right\}^{1/n},\tag{3.8}$$

$$u_{22}=\left\{\frac{3c}{4an}\coth^2\sqrt{-\frac{c}{8n\sum_{i=1}^{N}k_i}}\xi\right\}^{1/n},\tag{3.9}$$

$$u_{31}=\left\{\pm\sqrt{\frac{3c}{4an}}\left\{\tanh\left[\sqrt{-\frac{c}{2n\sum_{i=1}^{N}k_i}}\xi\right]\pm i\,\mathrm{sech}\left[\sqrt{-\frac{c}{2n\sum_{i=1}^{N}k_i}}\xi\right]\right\}\right\}^{2/n},\tag{3.10}$$

$$u_{32}=\left\{\pm\sqrt{\frac{3c}{4an}}\left\{\coth\left[\sqrt{-\frac{c}{2n\sum_{i=1}^{N}k_i}}\xi\right]\pm \mathrm{csch}\left[\sqrt{-\frac{c}{2n\sum_{i=1}^{N}k_i}}\xi\right]\right\}\right\}^{2n}.\tag{3.11}$$

4–5. The equation $u^{n-1}u_t+a(u^{3n})_{x_1}+\sum_{i=1}^{N}k_i(u^n)_{x_ix_ix_i}=0$ has the following solutions

$$u_{41}=\left\{\frac{c}{an}\tanh^2\left[\sqrt{-\frac{c}{n\sum_{i=1}^{N}k_i}}\xi\right]\right\}^{1/(2n)},\tag{3.12}$$

$$u_{41}=\left\{\frac{c}{an}\coth^2\left[\sqrt{-\frac{c}{n\sum_{i=1}^{N}k_i}}\xi\right]\right\}^{1/(2n)},\tag{3.13}$$

$$u_{51}=\left\{\pm\sqrt{\frac{c}{an}}\left\{\tanh\left[\sqrt{-\frac{2c}{n\sum_{i=1}^{N}k_i}}\xi\right]\pm i\,\mathrm{sech}\left[\sqrt{-\frac{2c}{n\sum_{i=1}^{N}k_i}}\xi\right]\right\}\right\}^{1/n},\tag{3.14}$$

$$u_{52}=\left\{\pm\sqrt{\frac{c}{an}}\left\{\coth\left[\sqrt{-\frac{2c}{n\sum_{i=1}^{N}k_i}}\xi\right]\pm \mathrm{csch}\left[\sqrt{-\frac{2c}{n\sum_{i=1}^{N}k_i}}\xi\right]\right\}\right\}^{1/n}.\tag{3.15}$$

6–7. The equation $u^{n-1}u_t + a(u^{-3n})_{x_1} + \sum_{i=1}^{N}(u^n)_{x_i x_i x_i} = 0$ has the following solutions

$$u_{61} = \left\{\frac{c}{4an}\tanh^2\left[\sqrt{\frac{c}{n\sum_{i=1}^{N}k_i}}\xi\right]\right\}^{-1/(4n)}, \tag{3.16}$$

$$u_{61} = \left\{\frac{c}{4an}\coth^2\left[\sqrt{\frac{c}{n\sum_{i=1}^{N}k_i}}\xi\right]\right\}^{-1/(4n)}, \tag{3.17}$$

$$u_{71} = \left\{\pm\sqrt{\frac{c}{4an}}\left\{1 \pm \tanh\left[\sqrt{\frac{4c}{n\sum_{i=1}^{N}k_i}}\xi\right] \pm i\,\mathrm{sech}\left[\sqrt{\frac{4c}{n\sum_{i=1}^{N}k_i}}\xi\right]\right\}\right\}^{-1/(2n)}, \tag{3.18}$$

$$u_{72} = \left\{\pm\sqrt{\frac{c}{4an}}\left\{1 \pm \coth\left[\sqrt{\frac{4c}{n\sum_{i=1}^{N}k_i}}\xi\right] \pm \mathrm{csch}\left[\sqrt{\frac{4c}{n\sum_{i=1}^{N}k_i}}\xi\right]\right\}\right\}^{-1/(2n)}. \tag{3.19}$$

4. Conclusions

The phenomena of compactons shows a rich variety of concepts and properties that should be addressed and, therefore, more work should be invested in studying these newly developed structures [3–16]. Many scientific processes [9] other than fluid, such as super deformed nuclei, preformation of cluster in hydrodynamic models and the fission of liquid drops may be explained on the basis of the compacton concept. In this paper, we use the improved extended-tanh method, by introducing a more general ansatz and property transformation, to extend the previous work [15] on study for $mk(n, n)$ equations and $mK(m, n)$ equations. We obtained the general solutions for $mk(n, n)$ equations by a proper transformation and $mK(m, n)$ equations by applying the improved tanh-method. The solutions by us included the solutions obtained by Wazwaz [15]. To our knowledge, some new solution have not been found before. The method can be used to many other nonlinear equations or coupled ones. In addition, this method is also computerizable, which allows us to perform complicated and tedious algebraic calculation on a computer.

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