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# Symbolic computation and construction of soliton-like solutions to the (2 + 1)-dimensional dispersive long-wave equations

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**Abstract**

By means of a new Riccati equation expansion method, we consider the (2 + 1)-dimensional dispersive long-wave equations $u_{yt} + \eta_{xx} + \frac{1}{2}(u^2)_{xy} = 0$, $\eta_t + (u\eta + u + u_{xy})_x = 0$. As a result, we not only can successfully recover the previously known formal solutions obtained by known tanh function methods but also construct new and more general formal solutions. The solutions obtained include the nontravelling wave and coefficient functions' soliton-like solutions, singular soliton-like solutions, triangular functions solutions.


*Keywords:* Generalized Riccati equation expansion method; Symbolic computation; (2 + 1)-dimensional dispersive long-wave equation: soliton-like solutions; Solitons

## 1. Introduction

In recent years, the study of solitons and the related issue of the construction of solutions to a wide variety of nonlinear evolution equations (NEEs) has become one of the most exciting and extremely active areas of research. There are a wealth of methods for finding special solutions of NEEs, such as, inverse scattering method, Bäcklund transformation, Darboux transformation,

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Cole-Hopf transformation, Hirota method, Painlevé method [1], tanh method [2–4], extended tanh-function method [5–7], modified extended tanh-function method [8], generalized hyperbolic-function method [9–11], variable separation approach [12]. But finding more powerful methods is still a significant subject in solving NEEs in soliton theory and its applications. With the rapid development of computerized symbolic computation, the application of symbolic computation to the physical and mathematical sciences appears to have a bright future.

One of most effectively straightforward methods to construct exact solutions of NEEs is tanh method [2–4]. Recently, Fan [5,6] has proposed an extended tanh-function method. More recently, Fan [7], Yan [13,14], Li and Chen [15–17] further developed this idea and made it much more lucid and straightforward for a class of NEEs. Most recently, Elwakil et al. [8] modified extended tanh-function method and obtain some new exact solutions. Gao and Tian [9–11] presented the generalized tanh method and generalized hyperbolic-function method by introducing coefficient functions. As is known, when applying direct method, the choice of an appropriate ansätz is of great importance. In this paper, based on the above work [2–11,13–17] and with the aid of symbolic computation software *Maple*, by introducing a new more general ansätz than the ansätz in the above methods, we present the generalized Riccati equation expansion method. To illustrate our algorithm, we take the $(2+1)$-dimensional dispersive long-wave equations (DLWS) [18–21] as a simple example, which reads

$$u_{yt} + \eta_{xx} + \frac{1}{2}(u^2)_{xy} = 0, \tag{1.1a}$$

$$\eta_t + (u\eta + u + u_{xy})_x = 0. \tag{1.1b}$$

The $(2+1)$-dimensional DLWS (1.1) was first derived by Boiti et al. [18] as a compatibility for a "weak" Lax pair. Recently considerable effort has been devoted to the study of this system. In [19], Paquin and Winternitz showed that the symmetry algebra of $(2+1)$-dimensional DLWS (1.1) is infinite-dimensional and possesses a Kac–Moody–Virasoro structure. Some special similarity solutions are also given in [19] by using symmetry algebra and the classical theoretical analysis. The more general symmetry algebra, $w_\infty$ symmetry algebra, is given in [20]. Lou [21] has given nine types of the two-dimensional partial differential equation reductions and 13 types of the ordinary differential equation reductions by means of the direct and nonclassical method. The system (1.1) have no Painlevé property though they are Lax or IST integrable [22]. More recently, Tang et al. [23], by means of the variable separation approach, the abundant localized coherent structures of the system (1.1) are derived. In [24], the possible chaotic and fractal localized structures are revealed for the system (1.1). Zhang [25], starting from the homogeneous balance method, found that the richness of the localized coherent structures of the model is caused by the appearance of two variable-separated arbitrary functions.

The plan of the paper is as follows. In Section 2, we describe briefly the generalized Riccati equation expansion method. In Section 3, we apply the method to Eq. (1.1) and bring out rich new families of the exact solutions of system (1.1), including the nontravelling wave and coefficient functions' soliton-like solutions, singular soliton-like solutions, triangular functions solutions. Conclusions will be presented finally.

## 2. Generalized Riccati equation expansion method

Let us simply describe the generalized Riccati equation expansion method, as follows: Consider a given system of NEEs in three independent variables $x$, $y$, $t$

$$E_1(u, v, u_t, v_t, u_x, v_x, u_y, v_y, u_{xx}, v_{xx}, u_{xt}, v_{xt}, u_{xy}, v_{xy}, u_{yt}, v_{yt}, \dots,) = 0, \tag{2.1a}$$

$$E_2(u, v, u_t, v_t, u_x, v_x, u_y, v_y, u_{xx}, v_{xx}, u_{xt}, v_{xt}, u_{xy}, v_{xy}, u_{yt}, v_{yt}, \dots,) = 0. \tag{2.1b}$$

We seek the following formal solutions of the given system by the new more general ansätz

$$u(x,y,t) = a_0 + \sum_{i=1}^{m}\left[a_i\phi^i(\xi) + b_i\phi^{i-1}(\xi)\sqrt{R+\phi^2(\xi)} + g_i\phi^{-i}(\xi)\right], \tag{2.2a}$$

$$v(x,y,t) = A_0 + \sum_{j=1}^{n}\left[A_j\phi^i(\xi) + B_j\phi^{j-1}(\xi)\sqrt{R+\phi^2(\xi)} + G_j\phi^{-j}(\xi)\right], \tag{2.2b}$$

where $m$, $n$ are integers to be determined by balancing the highest order derivative terms with the nonlinear terms in (2.1), $R$ is a real constant, while $a_0 = a_0(x,y,t)$, $A_0 = A_0(x,y,t)$, $a_i = a_i(x,y,t)$, $b_i = b_i(x,y,t)$, $g_i = g_i(x,y,t)$, $A_j = A_i(x,y,t)$, $B_j = B_j(x,y,t)$, $G_j = G_j(x,y,t)$ $(i = 1,\dots,m; j = 1,\dots,n)$, $\xi = \xi(x,y,t)$ are all differentiable functions and $\phi(\xi)$ satisfies

$$\frac{d\phi(\xi)}{d\xi} = R + \phi^2(\xi). \tag{2.3}$$

It is easy to see that the ansätz (2.2) is more general than the ansätz in the generalized hyperbolic-function method [9–11], tanh method [2–4], extended tanh-function method [5–7,13–17], modified extended tanh-function method [8]. Firstly, compared with the tanh method, extended tanh-function, as well as the modified extended tanh-function method, the restriction on $\xi(x,y,t)$ as merely a linear function $x$, $y$, $t$ and the restriction on the coefficients $a_i$, $b_i$, $g_i$, $A_j$, $B_j$, $G_j$ $(i = 0,\dots,m; j = 0,\dots,n)$ and $\xi$ as constants are removed. Secondly, compared with the generalized hyperbolic-function method [9–11], we cannot only recover the exact solutions for a given NEE which are the superposition of different powers of the $\operatorname{sech}\xi$ function, $\tanh\xi$ function or their combinations, but also we can, with no extra effort, find other new and more general types of solutions, such as singular soliton-like solutions, coth-type solutions and triangular periodic-like solutions, tan-type solutions, and these formal functions' combination, even rational solutions etc. More importantly, we add terms $g_i\phi^{-i}(\xi)$ in the new ansätz (2.2), so more types of solutions would be expected for some equations.

There exists the following steps to be considered further:

*Step 1.* Determined the values of $m$ and $n$ of system (2.2) by respectively balancing the highest-order partial derivative terms and the nonlinear terms in system (2.1).

*Step 2.* Substituting (2.2) along with (2.3) into (2.1), multiplying by the most simple common denominator in the obtained system, setting the coefficients of $\phi^p(\xi)\left(\sqrt{R+\phi^2(\xi)}\right)^q$

$(p = 0, 1, \ldots; q = 0, 1)$ $\Big($*Note*: where $\phi^p(\xi)$ denotes $p$ power of $\phi(\xi)$ and $\left(\sqrt{R+\phi^2(\xi)}\right)^q$ denotes $q$ power of $\sqrt{R+\phi^2(\xi)}\Big)$ to zero, we obtain a set of over-determined partial differential equations with regard to differentiable functions $a_i$, $b_i$, $g_i$, $A_j$, $B_j$, $G_j$ $(i = 0, \ldots, m; j = 0, \ldots, n)$ and $\xi$.

*Step 3.* Solving the over-determined partial differential equations by use of the PDEtools package of *Maple*, we would end up with the explicit expressions for $a_i$, $b_i$, $g_i$, $A_j$, $B_j$, $G_j$ $(i = 0, \ldots, m; j = 0, \ldots, n)$ and $\xi$ or the constraints among them.

*Step 4.* It is well-known that the general solutions of Riccati equation (2.3) are

$$\phi(\xi) = \begin{cases} -\sqrt{-R}\tanh\left(\sqrt{-R}\xi\right), & R < 0, \\ -\sqrt{-R}\coth\left(\sqrt{-R}\xi\right), & R < 0, \\ \sqrt{R}\tan\left(\sqrt{R}\xi\right), & R > 0, \\ -\sqrt{R}\cot\left(\sqrt{R}\xi\right), & R > 0, \\ -\frac{1}{\xi}, & R = 0. \end{cases} \tag{2.4}$$

Thus according to (2.2), (2.4) and the conclusions in Step 3, the soliton-like solutions of (2.1) can be obtained.

For the generalization of the ansätz, naturally more complicated computation is expected than ever before. Even if the availability of computer symbolic systems like *Maple* or *Mathematica* allows us to perform the complicated and tedious algebraic calculation and differential calculation on a computer, in general, it is very difficult, sometime impossible, to solve the set of over-determined partial differential equations in Step 2. As the calculation goes on, in order to drastically simplify the work or make the work feasible, we often choose special function forms for $a_i$, $b_i$, $g_i$, $A_j$, $B_j$, $G_j$ $(i = 0, \ldots, m; j = 0, \ldots, n)$ and $\xi$, on a trial-and-error basis.

## 3. Application

In this section, by use of the generalized Riccati equation expansion method, we investigate a (2+1)-dimensional dispersive long-wave system [18–25], i.e., Eq. (1.1). By balancing the highest-order contributions from both the linear and nonlinear terms in Eq. (1.1), we obtain $m = 1$, $n = 2$ in (2.2). Therefore we assume the solutions of Eq. (1.1) in the form

$$u(x,y,t) = a_0 + a_1\phi(\xi) + b_1\sqrt{R+\phi^2(\xi)} + g_1\phi^{-1}(\xi), \tag{3.1a}$$

$$\eta(x,y,t) = A_0 + A_1\phi(\xi) + A_2\phi^2(\xi) + B_1\sqrt{R+\phi^2(\xi)} + B_2\phi(\xi)\sqrt{R+\phi^2(\xi)} + G_1\phi^{-1}(\xi) + G_2\phi^{-2}(\xi) \tag{3.1b}$$

where $a_0 = a_0(y,t)$, $a_1 = a_1(y,t)$, $b_1 = b_1(y,t)$, $g_1 = g_1(y,t)$, $A_0 = A_0(y,t)$, $A_1 = A_1(y,t)$, $A_2 = A_2(y,t)$, $B_1 = B_1(y,t)$, $B_2 = B_2(y,t)$, $G_1 = G_1(y,t)$, $G_2 = G_2(y,t)$ and $\xi = xp(y,t) + q(y,t)$ all differentiable functions and $\phi(\xi)$ satisfies (2.3). The aim of choosing these functions to be special forms, i.e., the $x$ independence of $a_0$, $a_1$ etc., is to make calculation feasible.

Substituting (3.2) along with (2.3) into (1.1), multiplying by $\phi(\xi)^4$ and $\phi^4(\xi)\sqrt{R+\phi^2(\xi)}$ in the first equation and the second equation respectively, collecting coefficients of monomials of $\phi(\xi)$, $\sqrt{R+\phi(w)^2}$ and $x$ (Notice that $a_0$, $a_1$, $b_1$, $g_1$, $A_0$, $A_1$, $A_2$, $B_1$, $B_2$, $G_1$, $G_2$, $p$ and $q$ are independent of $x$.) with the aid of *Maple*, then setting each coefficients to zero, we can deduce a set of over-determined partial differential equations with respect to the unknown functions $a_0$, $a_1$, $b_1$, $g_1$, $A_0$, $A_1$, $A_2$, $B_1$, $B_2$, $G_1$, $G_2$, $p$ and $q$. Because the set includes 69 equations, for simplification, we do not list them in the paper.

Using the powerful PDEtools package of *Maple*, solving the set of partial differential equations, we can obtain the following nontrivial results. (*Note*: in the rest of this paper, $c_1$, $C_i$ $(i=1,\dots,5)$ are arbitrary constants, $F_3(y)$ is an arbitrary function with respect to $y$, and so on.)

*Case 1.*

$$\begin{aligned}
&a_0=-\frac{1}{C_1}\left[\int\frac{-F_3(y)}{2C_1}\,\mathrm{d}y+F_6(t)\right],\quad q=\int\left[-\frac{F_3(y)t+F_4(y)}{2C_1}\right]\mathrm{d}y+F_6(t),\\
&A_2=F_3(y)t+F_4(y),\quad A_0=\frac{1}{2C_1^2}[-F_3(y)+2C_1^2RF_3(y)t+2C_1^2RF_4(y)-2C_1^2],\\
&a_1=2C_1,\quad p=C_1,\quad g_1=A_1=B_1=G_1=B_2=b_1=G_2=0.
\end{aligned}\tag{3.2}$$

*Case 2.*

$$\begin{aligned}
&g_1=A_1=B_1=G_1=B_2=b_1=G_2=0,\quad q=\int-\frac{F_1(y)t+F_2(y)}{2C_1}\,\mathrm{d}y+F_5(t),\\
&a_0=-\frac{1}{C_1}\left[\int-\frac{1}{2C_1}F_1(y)\,\mathrm{d}y+F_5(t)\right],\quad A_2=F_1(y)t+F_2(y),\quad a_1=-2C_1,\\
&p=C_1,\quad A_0=\frac{F_1(y)+2C_1^2RF_1(y)t+2C_1^2RF_2(y)-2C_1^2}{2C_1^2}=0.
\end{aligned}\tag{3.3}$$

*Case 3.*

$$\begin{aligned}
&g_1=2RC_1,\quad G_2=F_3(y)t+F_4(y),\quad q=\int-\frac{F_3(y)t+F_4(y)}{2R^2C_1}\,\mathrm{d}y+F_6(t),\\
&A_2=a_1=A_1=B_1=G_1=B_2=b_1=0,\quad a_0=-\frac{1}{C_1}\left(\int-\frac{F_3(y)}{2R^2C_1}\,\mathrm{d}y+F_6(t)\right),\\
&A_0=\frac{F_3(y)-2R^2C_1^2+2C_1^2RF_3(y)t+2C_1^2RF_4(y)}{2R^2C_1^2},\quad p=C_1.
\end{aligned}\tag{3.4}$$

*Case 4.*

$$\begin{aligned}
&A_2=a_1=A_1=B_1=G_1=B_2=b_1=0,\quad a_0=-\frac{1}{C_1}\left[\int-\frac{F_1(y)}{2R^2C_1}\,\mathrm{d}y+F_5(t)\right],\\
&q=\int-\frac{F_1(y)t+F_2(y)}{2R^2C_1}\,\mathrm{d}y+F_5(t),\quad G_2=F_1(y)t+F_2(y),\quad g_1=-2RC_1,\\
&A_0=\frac{-F_1(y)-2R^2C_1^2+2C_1^2RF_1(y)t+2C_1^2RF_2(y)}{2R^2C_1^2},\quad p=C_1.
\end{aligned}\tag{3.5}$$

*Case 5.*

$$a_1 = 2C_1, \quad g_1 = 2RC_1, \quad p = C_1, \quad A_1 = B_1 = G_1 = B_2 = b_1 = 0,$$
$$a_0 = -\frac{F_4(t)}{C_1}, \quad q = \int -\frac{F_2(y)}{2C_1}\,\mathrm{d}y + F_4(t), \quad G_2 = F_2(y)R^2, \quad A_0 = -1, \quad A_2 = F_2(y). \tag{3.6}$$

*Case 6.*

$$g_1 = -2RC_1, \quad p = C_1, \quad A_1 = B_1 = G_1 = B_2 = b_1 = 0, \quad a_1 = -2C_1,$$
$$a_0 = -\frac{F_3(t)}{C_1}, \quad A_2 = F_1(y), \quad G_2 = F_1(y)R^2, \quad q = \int -\frac{F_1(y)}{2C_1}\,\mathrm{d}y + F_3(t), \quad A_0 = -1. \tag{3.7}$$

*Case 7.*

$$a_0 = -\frac{1}{C_1}\left[\int -\frac{F_3(y)}{2C_1}\,\mathrm{d}y + F_6(t)\right], \quad q = \int -\frac{F_3(y)t + F_4(y)}{2C_1}\,\mathrm{d}y + F_6(t),$$
$$A_2 = F_3(y)t + F_4(y), \quad g_1 = 2RC_1, \quad p = C_1, \quad A_1 = B_1 = G_1 = B_2 = b_1 = 0, \quad a_1 = -2C_1,$$
$$G_2 = R^2F_3(y)t + R^2F_4(y), \quad A_0 = \frac{4C_1^2RF_3(y)t + 4C_1^2RF_4(y) - 2C_1^2 + F_3(y)}{2C_1^2}. \tag{3.8}$$

*Case 8.*

$$a_1 = 2C_1, \quad g_1 = -2RC_1, \quad p = C_1, \quad A_1 = B_1 = G_1 = B_2 = b_1 = 0,$$
$$a_0 = -\frac{1}{C_1}\left[\int -\frac{F_1(y)}{2C_1}\,\mathrm{d}y + F_5(t)\right], \quad A_2 = F_1(y)t + F_2(y), \quad q = \int -\frac{(F_1(y)t + F_2(y))}{2C_1}\,\mathrm{d}y + F_5(t),$$
$$G_2 = R^2F_1(y)t + F_2(y)R^2, \quad A_0 = \frac{4C_1^2RF_1(y)t + 4C_1^2RF_2(y) - 2C_1^2 - F_1(y)}{2C_1^2}. \tag{3.9}$$

*Case 9.*

$$a_1 = g_1 = A_1 = B_1 = G_1 = B_2 = G_2 = 0, \quad p = C_1, \quad a_0 = -\frac{F_4(t)}{C_1},$$
$$q = \int -\frac{F_2(y)}{2C_1}\,\mathrm{d}y + F_4(t), \quad A_2 = F_2(y), \quad b_1 = 2C_1, \quad A_0 = -1 + \frac{1}{2}F_2(y)R. \tag{3.10}$$

*Case 10.*

$$a_1 = g_1 = A_1 = B_1 = G_1 = B_2 = G_2 = 0, \quad p = C_1, \quad a_0 = -\frac{F_3(t)}{C_1},$$
$$A_2 = F_1(y), \quad q = \int -\frac{F_1(y)}{2C_1}\,\mathrm{d}y + F_3(t), \quad A_0 = -1 + \frac{1}{2}F_1(y)R, \quad b_1 = -2C_1. \tag{3.11}$$

*Case 11.*

$$\begin{aligned}&g_1 = A_1 = B_1 = G_1 = G_2 = 0, \quad p = C_1, \quad q = \int -\frac{F_{11}(y)t + F_{12}(y)}{C_1}\,\mathrm{d}y + F_{20}(t),\\&b_1 = C_1, \quad a_1 = C_1, \quad A_2 = F_{11}(y)t + F_{12}(y), \quad a_0 = -\frac{1}{C_1}\left[\int -\frac{F_{11}(y)}{C_1}\,\mathrm{d}y + F_{20}(t)\right],\\&A_0 = \frac{C_1^2RF_{11}(y)t + C_1^2RF_{12}(y) - F_{11}(y) - C_1^2}{C_1^2}, \quad B_2 = F_{11}(y)t + F_{12}(y).\end{aligned} \tag{3.12}$$

*Case 12.*

$$\begin{aligned}&g_1 = A_1 = B_1 = G_1 = G_2 = 0, \quad p = C_1, \quad b_1 = C_1, \quad a_0 = -\frac{1}{C_1}\left[\int \frac{F_5(y)}{C_1}\,\mathrm{d}y + F_{19}(t)\right],\\&a_1 = -C_1, \quad A_0 = -\frac{C_1^2RF_5(y)t + C_1^2RF_6(y) + F_5(y) + C_1^2}{C_1^2}, \quad B_2 = F_5(y)t + F_6(y),\\&A_2 = -F_5(y)t - F_6(y), \quad q = \int \frac{F_5(y)t + F_6(y)}{C_1}\,\mathrm{d}y + F_{19}(t).\end{aligned} \tag{3.13}$$

*Case 13.*

$$\begin{aligned}&g_1 = A_1 = B_1 = G_1 = G_2 = 0, \quad p = C_1, \quad a_1 = C_1, \quad a_0 = -\frac{1}{C_1}\left[\int \frac{F_3(y)}{C_1}\,\mathrm{d}y + F_{18}(t)\right],\\&A_0 = -\frac{C_1^2RF_3(y)t + C_1^2RF_4(y) - F_3(y) + C_1^2}{C_1^2}, \quad b_1 = -C_1, \quad A_2 = -F_3(y)t - F_4(y),\\&q = \int \frac{F_3(y)t + F_4(y)}{C_1}\,\mathrm{d}y + F_{18}(t), \quad B_2 = F_3(y)t + F_4(y).\end{aligned} \tag{3.14}$$

*Case 14.*

$$\begin{aligned}&g_1 = A_1 = B_1 = G_1 = G_2 = 0, \quad p = C_1, \quad A_2 = F_1(y)t + F_2(y), \quad b_1 = -C_1, \quad a_1 = -C_1,\\&a_0 = -\frac{1}{C_1}\left[\int -\frac{F_1(y)}{C_1}\,\mathrm{d}y + F_{17}(t)\right], \quad q = \int \frac{-(F_1(y)t + F_2(y))}{C_1}\,\mathrm{d}y + F_{17}(t),\\&A_0 = \frac{C_1^2RF_1(y)t + C_1^2RF_2(y) + F_1(y) - C_1^2}{C_1^2}, \quad B_2 = F_1(y)t + F_2(y).\end{aligned} \tag{3.15}$$

From (3.1), (2.4) and (3.2)–(3.15), we can obtain the following solutions for the $(2+1)$-dimensional dispersive long-wave equation.

*Type 1*. From Case 1–2, we can obtain the following solutions

$$\begin{cases} u_{11} = a_0 - a_1\sqrt{-R}\tanh\left[\sqrt{-R}(xp+q)\right], \\ \eta_{11} = A_0 + A_2R\tanh^2\left[\sqrt{-R}(xp+q)\right], \quad R<0 \end{cases} \tag{3.16}$$

$$\begin{cases} u_{12} = a_0 - a_1\sqrt{-R}\coth\left[\sqrt{-R}(xp+q)\right], \\ \eta_{12} = A_0 + A_2R\coth^2\left[\sqrt{-R}(xp+q)\right], \quad R<0 \end{cases} \tag{3.17}$$

$$\begin{cases} u_{13} = a_0 + a_1\sqrt{R}\tan\left[\sqrt{R}(xp+q)\right], \\ \eta_{13} = A_0 + A_2R\tan^2\left[\sqrt{R}(xp+q)\right], \quad R>0 \end{cases} \tag{3.18}$$

$$\begin{cases} u_{14} = a_0 - a_1\sqrt{R}\cot\left[\sqrt{R}(xp+q)\right], \\ \eta_{14} = A_0 + A_2R\cot^2\left[\sqrt{R}(xp+q)\right], \quad R>0 \end{cases} \tag{3.19}$$

where $a_0$, $a_1$, $p$, $q$, $A_0$ and $A_2$ are determined by (3.2) and (3.3), respectively. At the same time, due to the arbitrariness of functions $F_i(y)$ $(i=1,\ldots,6)$, $F_5(t)$, $F_6(t)$, the solutions obtained by Case 3–4 are just the same as the solutions (3.16)–(3.19) .

*Type 2*. From Case 5–8, we can obtain the following solutions

$$\begin{cases} u_{21} = a_0 \pm a_1\sqrt{-R}\left[\tanh\left[\sqrt{-R}(xp+q)\right] \mp \coth\left[\sqrt{-R}(xp+q)\right]\right], \\ \eta_{21} = A_0 + A_2R\left[\tanh^2\left[\sqrt{-R}(xp+q)\right] + \coth^2[-R(xp+q)]\right], \quad R<0 \end{cases} \tag{3.20}$$

$$\begin{cases} u_{22} = a_0 + a_1\sqrt{R}\left[\tan\left[\sqrt{R}(xp+q)\right] \mp \cot\left[\sqrt{R}(xp+q)\right]\right], \\ \eta_{22} = A_1 + A_2)R\left[\tan^2\left[\sqrt{R}(xp+q)\right] + \cot^2[R(xp+q)]\right], \quad R>0 \end{cases} \tag{3.21}$$

where $a_0$, $A_2$, $p$, $q$ are determined by (3.6)–(3.9), respectively.

*Type 3*. From Cases 9–10, we can obtain the following solutions

$$\begin{cases} u_{31} = a_0 + b_1\sqrt{R}\,\mathrm{sech}\left[\sqrt{-R}(xp+q)\right], \\ \eta_{31} = A_0 + A_2R\tanh^2\left[\sqrt{-R}(xp+q)\right], \quad R<0 \end{cases} \tag{3.22}$$

$$\begin{cases} u_{32} = a_0 + b_1\sqrt{-R}\,\mathrm{csch}\left[\sqrt{-R}(xp+q)\right], \\ \eta_{32} = A_0 + A_2R\coth^2\left[\sqrt{-R}(xp+q)\right], \quad R<0 \end{cases} \tag{3.23}$$

$$\begin{cases} u_{33} = a_0 + b_1\sqrt{R}\sec\left[\sqrt{R}(xp+q)\right], \\ \eta_{33} = A_0 + A_2R\tan^2\left[\sqrt{R}(xp+q)\right], \quad R>0 \end{cases} \tag{3.24}$$

$$\begin{cases} u_{34} = a_0 - b_1\sqrt{R}\csc\left[\sqrt{R}(xp+q)\right], \\ \eta_{34} = A_0 + A_2R\cot^2\left[\sqrt{R}(xp+q)\right], \quad R<0 \end{cases} \tag{3.25}$$

where $a_0$, $b_1$, $A_0$, $A_2$, $p$ and $q$ are determined by (3.10), (3.11), respectively.

*Type 4.* From Cases 11–14, we can obtain the following solutions

$$\begin{cases} u_{41} = a_0 - a_1\sqrt{-R}\tanh\left(\sqrt{-R}\xi\right) + b_1\sqrt{R}\,\mathrm{sech}\left(\sqrt{-R}\xi\right), \\ \eta_{41} = A_0 + A_2R\tanh^2\left(\sqrt{-R}\xi\right) + \mathrm{i}B_2R\tanh\left(\sqrt{-R}\xi\right)\mathrm{sech}\left(\sqrt{-R}\xi\right), \quad R < 0 \end{cases} \tag{3.26}$$

$$\begin{cases} u_{42} = a_0 - a_1\sqrt{-R}\coth\left(\sqrt{-R}\xi\right) + b_1\sqrt{-R}\,\mathrm{csch}\left(\sqrt{-R}\xi\right), \\ \eta_{42} = A_0 + A_2R\coth^2\left(\sqrt{-R}\xi\right) + B_2R\coth\left(\sqrt{-R}\xi\right)\mathrm{csch}\left(\sqrt{-R}\xi\right), \quad R < 0 \end{cases} \tag{3.27}$$

$$\begin{cases} u_{43} = a_0 + a_1\sqrt{R}\tan\left(\sqrt{R}\xi\right) + b_1\sqrt{R}\sec\left(\sqrt{R}\xi\right), \\ \eta_{43} = A_0 + A_2R\tan^2\left(\sqrt{R}\xi\right) + B_2R\tan\left(\sqrt{R}\xi\right)\sec\left(\sqrt{R}\xi\right), \quad R > 0 \end{cases} \tag{3.28}$$

$$\begin{cases} u_{43} = a_0 - a_1\sqrt{R}\cot\left(\sqrt{R}\xi\right) + b_1\sqrt{R}\csc\left(\sqrt{R}\xi\right), \\ \eta_{43} = A_0 + A_2R\cot^2\left(\sqrt{R}\xi\right) + B_2R\cot\left(\sqrt{R}\xi\right)\csc\left(\sqrt{R}\xi\right), \quad R > 0 \end{cases} \tag{3.29}$$

where $\xi = xp + q$ and $a_0$, $a_1$, $b_1$, $A_0$, $A_2$, $B_2$, $p$, $q$ are determined by (3.12)–(3.15), respectively.

## 4. Conclusions

In summary, based on the computerized symbolic computation and a Riccati equation, by introducing a new more general ansätz than the ansätz in the extended tanh-function method, modified extended tanh-function method, and generalized hyperbolic-function method, we have proposed the generalized Riccati equation expansion method for searching for exact solutions of NEEs and implemented in computer symbolic systems. Making use of our method and with the aid of *Maple*, we study the $(2+1)$-dimensional dispersive long-wave equation and obtain some new families of the exact solutions. In our obtained exact solutions the restriction on $\xi(x, y, t)$ as merely a linear function $x$, $y$, $t$ and the restriction on the coefficients, such as $a_0$, $a_i$, $b_i$, $g_i$ $(i = 1, \ldots, m)$, etc., as constants are removed and, with no extra effort, the singular solitonic solution and triangular function solutions, even rational formal solutions could be obtained. To make the work feasible, how to choose the forms for $a_0$, $a_i$, $b_i$, $g_i$ $(i = 1, \ldots, m)$, $\xi$, etc., in the ansätz would be the key step in the computation of our method. It is shown that the method, proposed in this paper for a system of NEEs, may be extended to find exact solutions of other mathematical and physical equation(s).

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