Rogue-wave pair and dark-bright-rogue wave solutions of the coupled Hirota equations*

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Novel explicit rogue wave solutions of the coupled Hirota equations are obtained by using the Darboux transformation. In contrast to the fundamental Peregrine solitons and dark rogue waves, we present an interesting rogue-wave pair that involves four zero-amplitude holes for the coupled Hirota equations. It is significant that the corresponding expressions of the rogue-wave pair solutions contain polynomials of the fourth order rather than the second order. Moreover, dark-bright-rogue wave solutions of the coupled Hirota equations are given, and interactions between Peregrine solitons and dark-bright solitons are analyzed. The results further reveal the dynamical properties of rogue waves for the coupled Hirota equations.

Keywords: rogue-wave pair, dark-bright-rogue wave, coupled Hirota equations, Darboux transformation

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1. Introduction

Recently, the study of rogue waves has become a hot topic in the field of nonlinear science.[1–4] A rogue wave is localized in both space and time, and appears from nowhere and disappears without a trace.[3] So far, many important single-component systems, for example, the nonlinear Schrödinger (NLS) equation,[1–3] the derivative NLS equation,[5,6] the discrete NLS equation,[7] the Sasa–Satsuma equation,[8] the Hirota equation[9] and the variable coefficient NLS equation[10] have been proved to possess rogue wave solutions of diverse geometrical shapes. However, some complex systems, such as Bose–Einstein condensates, nonlinear optical fibers, and resonant interaction systems, always involve more than a single component.[11–13] So the latest research of rogue waves is gradually focused on the multi-component systems, and indeed the results of these coupled equations can reveal the nature and diversities of extreme waves more accurately than the scalar NLS model.[11]

It has been proved that rogue waves in coupled equations can present new structures apart from the fundamental Peregrine soliton, such as dark rogue waves in the coupled Gross–Pitaevskii equation,[4] dark-bright-rogue and breather-rogue waves in the Manakov system,[11,14] bright-dark-rogue waves, two similar and two different rogue waves in the coupled NLS equation,[15] and four-petaled flower rogue waves in the three-component NLS equations.[12] Moreover, interaction between rogue waves and the other nonlinear waves has attracted widespread attention [11–15]; it has been investigated from theoretical derivation to experimental analysis.[11]

In this paper, we consider the coupled Hirota (CH) equations[16–19]

$$\mathrm{i}u_t + \frac{1}{2}u_{xx} + (|u|^2 + |v|^2)u + \mathrm{i}\varepsilon[u_{xxx} + (6|u|^2 + 3|v|^2)u_x + 3uv^* v_x] = 0, \tag{1}$$

$$\mathrm{i}v_t + \frac{1}{2}v_{xx} + (|u|^2 + |v|^2)v + \mathrm{i}\varepsilon[v_{xxx} + (6|v|^2 + 3|u|^2)v_x + 3vu^* u_x] = 0. \tag{2}$$

Here $u(x,t)$ and $v(x,t)$ are the complex smooth envelops, and ε denotes the strength of high-order effects. The CH equations with high-order effects like the third dispersion, self-steepening, and inelastic Raman scattering were firstly proposed by Tasgal and Potasek to describe a non-relativistic boson field.[16] They are important in optics to illustrate the transmission process when high intensity ultra-short pulses pass through an optical glass fiber, while in this case the simple Manakov model is inadequate.[17] A series of important results have been reported on the CH equations, such as the Lax pair,[16,18] the Darboux transformation,[18] the Painlevé analysis,[17] bright and dark soliton solutions.[17,18] Especially, in Ref. [19], the fundamental and dark rogue waves in the CH equations were given by Chen and Song, which particularly indicates the striking structures of rogue waves for the CH equations. However, to the best of our knowledge, rogue-wave pairs and dark-bright-rogue waves in CH equations have never been investigated by any authors.

It is well known that several systematic methods have been developed to construct explicit solutions for nonlinear integrable equations, such as the bilinear method,[20–22] the symmetry approach,[23–33] the extended-tanh function method,[34–36] and the Darboux transformation. [37–44] Among

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them, the Darboux transformation has been extensively applied to construct Akhmediev breathers and high-order rogue wave solutions for many models.[2,3,9,14]

In this paper, based on the Darboux transformation, we present an interesting rogue-wave pair that involves four zero-amplitude holes for the CH equations. It is significant that the corresponding expressions of the rogue-wave pair solutions contain polynomials of the fourth order rather than the second order given in Ref. [19]. Moreover, dark-bright-rogue wave solutions of the CH equations are given, and the interactions between Peregrine solitons and dark-bright solitons are analyzed. The results further reveal the dynamical properties of rogue waves for the CH equations.

Our paper is organized as follows. In Section 2, we present the Lax pair and the Darboux transformation of the CH equations. In Section 3, a rogue-wave pair in the CH equations is obtained. In Section 4, the dark-bright-rogue wave in the CH equations is studied. The final section is a discussion.

2. Lax pair and Darboux transformation

In this section, we firstly present the Lax pair of the CH equations[16]

$$\phi_x = \boldsymbol{U}\phi, \quad \boldsymbol{U} = \lambda \boldsymbol{U}_0 + \boldsymbol{U}_1, \tag{3}$$

$$\phi_t = \boldsymbol{V}\phi, \quad \boldsymbol{V} = \lambda^3 \boldsymbol{V}_0 + \lambda^2 \boldsymbol{V}_1 + \lambda \boldsymbol{V}_2 + \boldsymbol{V}_3, \tag{4}$$

where

$$\boldsymbol{U}_0 = \frac{1}{12\varepsilon}\begin{pmatrix} -2\mathrm{i} & 0 & 0 \\ 0 & \mathrm{i} & 0 \\ 0 & 0 & \mathrm{i} \end{pmatrix}, \quad \boldsymbol{U}_1 = \begin{pmatrix} 0 & -u & -v \\ u^* & 0 & 0 \\ v^* & 0 & 0 \end{pmatrix},$$

$$\boldsymbol{V}_0 = \frac{1}{16\varepsilon}\boldsymbol{U}_0, \quad \boldsymbol{V}_1 = \frac{1}{8\varepsilon}\boldsymbol{U}_0 + \frac{1}{16\varepsilon}\boldsymbol{U}_1,$$

$$\boldsymbol{V}_2 = \frac{1}{4}\begin{pmatrix} \mathrm{i}e & -\frac{u}{2\varepsilon} - \mathrm{i}u_x & -\frac{v}{2\varepsilon} - \mathrm{i}v_x \\ \frac{u^*}{2\varepsilon} - \mathrm{i}u_x^* & -\mathrm{i}|u|^2 & -\mathrm{i}vu^* \\ \frac{v^*}{2\varepsilon} - \mathrm{i}v_x^* & -\mathrm{i}uv^* & -\mathrm{i}|v|^2 \end{pmatrix},$$

$$\boldsymbol{V}_3 = \begin{pmatrix} \varepsilon(e_1+e_2) + \frac{\mathrm{i}}{2}e & \varepsilon e_3 - \frac{\mathrm{i}}{2}u_x & \varepsilon e_4 - \frac{\mathrm{i}}{2}v_x \\ -\varepsilon e_3^* - \frac{\mathrm{i}}{2}u_x^* & -\varepsilon e_1 - \frac{\mathrm{i}}{2}|u|^2 & \varepsilon e_5 - \frac{\mathrm{i}}{2}vu^* \\ -\varepsilon e_4^* - \frac{\mathrm{i}}{2}v_x^* & -\varepsilon e_5^* - \frac{\mathrm{i}}{2}uv^* & -\varepsilon e_2 - \frac{\mathrm{i}}{2}|v|^2 \end{pmatrix},$$

with

$$e = |u|^2 + |v|^2, \quad e_1 = uu_x^* - u^*u_x,$$
$$e_2 = vv_x^* - v^*v_x, \quad e_3 = u_{xx} + 2eu,$$
$$e_4 = v_{xx} + 2ev, \quad e_5 = u^*v_x - vu_x^*.$$

Assume $(\phi_1, \phi_2, \phi_3)^{\mathrm{T}}$ is a basic solution of the Lax pair equations (3) and (4) with $\lambda = \lambda_1$ and $(u,v)^{\mathrm{T}} = (u[0], v[0])^{\mathrm{T}}$, then based on the Darboux transformation of the AKNS system,[37] the following explicit formulas

$$u[1] = u[0] + \mathrm{i}(\lambda_1 - \lambda_1^*)\frac{\phi_1\phi_2^*}{4\varepsilon(|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2)}, \tag{5}$$

$$v[1] = v[0] + \mathrm{i}(\lambda_1 - \lambda_1^*)\frac{\phi_1\phi_3^*}{4\varepsilon(|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2)} \tag{6}$$

give new special solutions of the CH equations. Next, the interesting rogue-wave pair and dark-bright-rogue wave solutions of the CH equations can be presented by means of formulas (5) and (6).

3. Rogue-wave pair of the CH equations

Now we start from the non-zero seed solutions of the CH equations

$$u[0] = c_1 \exp[\mathrm{i}\theta_1] = c_1 \exp[\mathrm{i}(a_1 x + b_1 t)],$$
$$v[0] = c_2 \exp[\mathrm{i}\theta_2] = c_2 \exp[\mathrm{i}(a_2 x + b_2 t)], \tag{7}$$

where

$$b_1 = (c_1^2 + c_2^2) - \frac{a_1^2}{2} + \varepsilon a_1^3 - 6\varepsilon a_1 c_1^2 - 3\varepsilon c_2^2(a_1 + a_2),$$
$$b_2 = (c_1^2 + c_2^2) - \frac{a_2^2}{2} + \varepsilon a_2^3 - 6\varepsilon a_2 c_2^2 - 3\varepsilon c_1^2(a_1 + a_2).$$

By comparing with the recent work of Chen and Song,[19] in order to solve the Lax pair equations (3) and (4) with variable coefficients, we introduce a gauge transformation[11,14] to convert them into a constant coefficient differential system, instead of directly substituting Eq. (7) into the Lax pair equations.

Let

$$\psi = \boldsymbol{M}\phi,$$
$$\boldsymbol{M} = \mathrm{diag}\left\{\exp\left[-\frac{\mathrm{i}}{3}(\theta_1+\theta_2)\right], \exp\left[\frac{\mathrm{i}}{3}(2\theta_1-\theta_2)\right], \exp\left[\frac{\mathrm{i}}{3}(2\theta_2-\theta_1)\right]\right\}, \tag{8}$$

then the Lax pair (3) and (4) with $\lambda = \lambda_1$ and the seed solution (7) can be transformed into

$$\psi_x = \widetilde{\boldsymbol{U}}\psi,$$
$$\widetilde{\boldsymbol{U}} = \lambda_1 \boldsymbol{U}_0 + \begin{pmatrix} -\frac{\mathrm{i}}{3}(a_1+a_2) & -c_1 & -c_2 \\ c_1 & \frac{\mathrm{i}}{3}(2a_1 - a_2) & 0 \\ c_2 & 0 & \frac{\mathrm{i}}{3}(2a_2 - a_1) \end{pmatrix}, \tag{9}$$

$$\psi_t = \widetilde{\boldsymbol{V}}\psi, \; \widetilde{\boldsymbol{V}} = \lambda_1^3\widetilde{\boldsymbol{V}}_0 + \lambda_1^2\widetilde{\boldsymbol{V}}_1 + \lambda_1\widetilde{\boldsymbol{V}}_2 + \widetilde{\boldsymbol{V}}_3, \tag{10}$$

where

$$\widetilde{\boldsymbol{V}}_0 = \frac{1}{16\varepsilon}\boldsymbol{U}_0, \quad \widetilde{\boldsymbol{V}}_1 = \frac{1}{8\varepsilon}\boldsymbol{U}_0 + \frac{1}{16\varepsilon}\begin{pmatrix} 0 & -c_1 & -c_2 \\ c_1 & 0 & 0 \\ c_2 & 0 & 0 \end{pmatrix},$$

$$\widetilde{\boldsymbol{V}}_2 = \frac{1}{4}\begin{pmatrix} \mathrm{i}(c_1^2+c_2^2) & -\frac{c_1}{2\varepsilon}(1-2\varepsilon a_1) & -\frac{c_2}{2\varepsilon}(1-2\varepsilon a_2) \\ \frac{c_1}{2\varepsilon}(1-2\varepsilon a_1) & -\mathrm{i}c_1^2 & -\mathrm{i}c_1c_2 \\ \frac{c_2}{2\varepsilon}(1-2\varepsilon a_2) & -\mathrm{i}c_1c_2 & -\mathrm{i}c_2^2 \end{pmatrix},$$

$$\widetilde{\boldsymbol{V}}_3 = \begin{pmatrix} -\frac{\mathrm{i}}{6}(f_1+f_2) & -\frac{c_1}{2}f_3 & -\frac{c_2}{2}f_4 \\ \frac{c_1}{2}f_3 & \frac{\mathrm{i}}{6}f_1 & \frac{\mathrm{i}}{2}c_1c_2f_5 \\ \frac{c_2}{2}f_4 & \frac{\mathrm{i}}{2}c_1c_2f_5 & \frac{\mathrm{i}}{6}f_2 \end{pmatrix},$$

with

$$f_1 = 12\varepsilon c_1^2 a_1 - 3c_1^2 + 4b_1 - 2b_2,$$

$$
\begin{aligned}
f_2 &= 12\varepsilon c_2^2 a_2 - 3c_2^2 + 4b_2 - 2b_1,\\
f_3 &= -4\varepsilon(c_1^2 + c_2^2) - a_1 + 2\varepsilon a_1^2,\\
f_4 &= -4\varepsilon(c_1^2 + c_2^2) - a_2 + 2\varepsilon a_2^2,\\
f_5 &= 2\varepsilon(a_1 + a_2) - 1.
\end{aligned}
$$

It is easy to prove that the matrices $\widetilde{U}$ and $\widetilde{V}$ cannot be reduced to a diagonal form. However, under the following conditions of the parameters

$$
c_1 = c_2, \quad \mathrm{Im}(\lambda_1) = 6\sqrt{3}c_2\varepsilon,
$$

$$
\mathrm{Re}(\lambda_1) = 2\varepsilon(c_2 - 2a_2), \quad a_1 = a_2 - c_2, \tag{11}
$$

they are both similar to a Jordan form.[11,14] In this case, the basic solution of the Lax pair equations (3) and (4) under the seed solution (7) and the requirements of the parameters (11) follows as

$$
\phi(x,t,\lambda_1) = M^{-1}\Omega(C_1,C_2,C_3)^{\mathrm{T}}, \tag{12}
$$

where C_1, C_2, C_3 are three arbitrary complex constants and

$$
\Omega = \begin{pmatrix}
1 & \gamma & \frac{\gamma^2}{2} + \frac{\sqrt{3}}{2}\mathrm{i}\kappa t\\
\frac{(\sqrt{3}-\mathrm{i})}{2} & \frac{(\sqrt{3}-\mathrm{i})}{2}\gamma + \frac{(\sqrt{3}\mathrm{i}-1)}{2c_2} & \frac{(\sqrt{3}-\mathrm{i})}{4}\gamma^2 + \frac{(\sqrt{3}\mathrm{i}-1)}{2c_2}\gamma + \frac{\sqrt{3}(\sqrt{3}\mathrm{i}+1)}{4}\kappa t - \frac{\mathrm{i}}{c_2^2}\\
\frac{(\sqrt{3}+\mathrm{i})}{2} & \frac{(\sqrt{3}+\mathrm{i})}{2}\gamma - \frac{(\sqrt{3}\mathrm{i}+1)}{2c_2} & \frac{(\sqrt{3}+\mathrm{i})}{4}\gamma^2 - \frac{(\sqrt{3}\mathrm{i}+1)}{2c_2}\gamma + \frac{\sqrt{3}(\sqrt{3}\mathrm{i}-1)}{4}\kappa t + \frac{\mathrm{i}}{c_2^2}
\end{pmatrix},
$$

with

$$
\begin{aligned}
\gamma &= x + (\alpha + \mathrm{i}\beta)t, \quad \kappa = \frac{2\beta}{3c_2} + 3\mathrm{i}\varepsilon c_2,\\
\alpha &= \frac{c_2}{2} - a_2 - \frac{15}{2}\varepsilon c_2^2 - 3\varepsilon c_2 a_2 + 3\varepsilon a_2^2,\\
\beta &= \frac{\sqrt{3}}{2}(1 - 6\varepsilon a_2 + 3\varepsilon c_2)c_2.
\end{aligned}
$$

Next, by making use of the Darboux transformation (5) and (6), the rational solutions of the CH equations can be directly given.

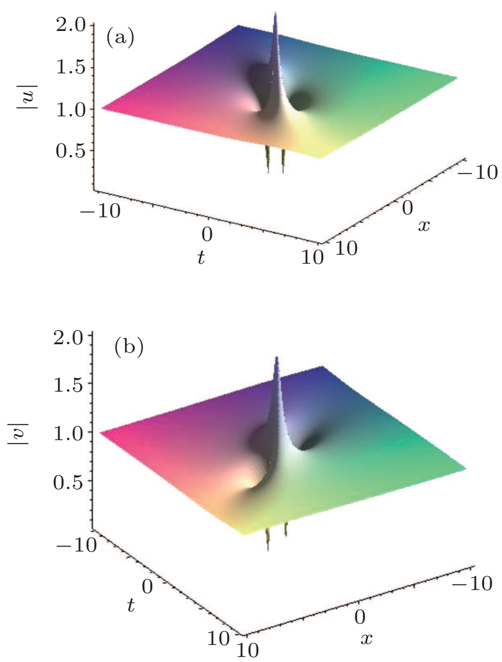

Fig. 1. (color online) Rogue wave solution (13) of the CH equations: (a) rogue waves in the u component, and (b) rogue waves in the v component. The parameters are $\varepsilon = 1/10$, $c_2 = 1$, and $a_2 = 1/2$.

If $C_1 = 0$, $C_2 = 1$, and $C_3 = 0$, we obtain the rogue wave solutions containing polynomials of the second order in γ, which are nothing but the fundamental rogue wave solutions of the CH equations, see Fig. 1. In this case, $u[1]$ or $v[1]$ has the same dynamics of the first-order rogue wave of the decoupled Hirota equation

$$
\begin{aligned}
u[1] &= c_2\left[1 + \frac{1}{2\varepsilon}\frac{(G_1 + \mathrm{i}H_1)}{D}\right]\exp[\mathrm{i}\theta_1],\\
v[1] &= c_2\left[1 + \frac{1}{2\varepsilon}\frac{(G_2 + \mathrm{i}H_2)}{D}\right]\exp[\mathrm{i}\theta_2],
\end{aligned} \tag{13}
$$

where

$$
\begin{aligned}
G_1 &= 9\varepsilon c_2^2\delta^2 - 3\sqrt{3}c_2\varepsilon\delta + 9\varepsilon c_2^2\beta^2 t^2 + 9\varepsilon c_2\beta t,\\
H_1 &= 3\sqrt{3}\varepsilon c_2^2\delta^2 - 9\varepsilon c_2\delta + 3\sqrt{3}\varepsilon c_2^2\beta^2 t^2 - 3\sqrt{3}\varepsilon c_2\beta t,\\
G_2 &= 9\varepsilon c_2^2\delta^2 - 3\sqrt{3}c_2\varepsilon\delta + 9\varepsilon c_2^2\beta^2 t^2 - 9\varepsilon c_2\beta t,\\
H_2 &= -3\sqrt{3}\varepsilon c_2^2\delta^2 + 9\varepsilon c_2\delta - 3\sqrt{3}\varepsilon c_2^2\beta^2 t^2 - 3\sqrt{3}\varepsilon c_2\beta t,\\
D &= -3c_2^2\delta^2 + 2\sqrt{3}c_2\delta - 3c_2^2\beta^2 t^2 - 2, \ \delta = x + \alpha t.
\end{aligned}
$$

If $C_1 = 0$, $C_2 = 0$, and $C_3 = 1$, we arrive at the rogue-wave pair solutions containing polynomials of the fourth order in γ rather than the second order for the CH equations, see Fig. 2. From Fig. 2, we see that the rogue-wave pair in the u component and the v component both have two humps along with four zero-amplitude points on the nonzero background, each hump in the rogue-wave pair is close to the other, and they have different shapes while keeping stable structures. Similar properties can also be found in the two-component NLS equations[15] and the Sass–Satsuma equation[8]

$$
\begin{aligned}
u[1] &= c_2\left[1 + \frac{1}{2\varepsilon}\frac{(\widetilde{G}_1 + \mathrm{i}\widetilde{H}_1)}{\widetilde{D}}\right]\exp(\mathrm{i}\theta_1),\\
v[1] &= c_2\left[1 + \frac{1}{2\varepsilon}\frac{(\widetilde{G}_2 + \mathrm{i}\widetilde{H}_2)}{\widetilde{D}}\right]\exp(\mathrm{i}\theta_2),
\end{aligned} \tag{14}
$$

where

$$
\begin{aligned}
\widetilde{D} = &-3c_2^4\delta^4 + 4\sqrt{3}c_2^3\delta^3 - (12c_2^2 + 6c_2^4\beta^2 t^2 - 18\sqrt{3}\varepsilon c_2^5 t)\delta^2\\
&+ (8\sqrt{3}c_2 - 36\varepsilon c_2^4 t - 4\sqrt{3}c_2^3\beta^2 t^2)\delta - 3c_2^4\beta^4 t^4
\end{aligned}
$$

$$
\begin{aligned}
& -18\sqrt{3}\varepsilon c_2^5\beta^2t^3-81\varepsilon^2c_2^6t^2+12\sqrt{3}\varepsilon c_2^3t-8,\\
\widetilde{G}_1 = {} & 9\varepsilon c_2^4\delta^4-6\sqrt{3}\varepsilon c_2^3\delta^3\\
& +18(\varepsilon c_2^4\beta^2t^2+\varepsilon c_2^3\beta t-3\sqrt{3}\varepsilon^2c_2^5t)\delta^2\\
& +6(9\varepsilon^2c_2^4t-2\sqrt{3}\varepsilon c_2^3\beta t+3\sqrt{3}\varepsilon c_2^3\beta^2t^2)\delta\\
& +9\varepsilon c_2^4\beta^4t^4+18(\varepsilon c_2^3\beta^3+3\sqrt{3}\varepsilon^2c_2^5\beta^2)t^3\\
& +27(2\sqrt{3}\varepsilon^2c_2^4\beta+9\varepsilon^3c_2^6)t^2-24c_2\varepsilon\beta t,\\
\widetilde{H}_1 = {} & 9\sqrt{3}\varepsilon c_2^4\delta^4-54\varepsilon c_2^3\delta^3+18(2\sqrt{3}\varepsilon c_2^2-9\varepsilon c_2^5t\\
& +\sqrt{3}\varepsilon c_2^4\beta^2t^2-\sqrt{3}\varepsilon c_2^3\beta t)\delta^2-18(2\varepsilon c_2^2\beta t+\varepsilon c_2^3\beta^2t^2\\
& +9\sqrt{3}\varepsilon^3c_2^4t)\delta+9\sqrt{3}\varepsilon c_2^4\beta^4t^4\\
& +18(9\varepsilon^2c_2^5\beta^2-\sqrt{3}\varepsilon c_2^3\beta^3)t^3\\
& -3(20\sqrt{3}\varepsilon c_2^2\beta^2+54\varepsilon^2c_2^4\beta-81\sqrt{3}\varepsilon^3c_2^6)t^2\\
& -324\varepsilon^2c_2^3t,\\
\widetilde{G}_2 = {} & 9\varepsilon c_2^4\delta^4-6\sqrt{3}\varepsilon c_2^3\delta^3\\
& -18(\varepsilon c_2^3\beta t-\varepsilon c_2^4\beta^2t^2+3\sqrt{3}\varepsilon^2c_2^5t)\delta^2\\
& +6(9\varepsilon^2c_2^4t+2\sqrt{3}\varepsilon c_2^3\beta t+3\sqrt{3}\varepsilon c_2^3\beta^2t^2)\delta\\
& +9\varepsilon c_2^4\beta^4t^4-18(\varepsilon c_2^3\beta^3-3\sqrt{3}\varepsilon^2c_2^5\beta^2)t^3\\
& +27(9\varepsilon^3c_2^6-2\sqrt{3}\varepsilon^2c_2^4\beta)t^2+24\varepsilon c_2\beta t,\\
\widetilde{H}_2 = {} & -9\sqrt{3}\varepsilon c_2^4\delta^4+54\varepsilon c_2^3\delta^3-18(2\sqrt{3}\varepsilon c_2^2-9\varepsilon^2c_2^5t\\
& +\sqrt{3}\varepsilon c_2^3\beta t+\sqrt{3}\varepsilon c_2^4\beta^2t^2)\delta^2-18(\varepsilon c_2^3\beta^2t^2+2\varepsilon c_2^2\beta t\\
& +9\sqrt{3}\varepsilon^2c_2^4t)\delta-9\sqrt{3}\varepsilon c_2^4\beta^4t^4\\
& -18(9\varepsilon^2c_2^5\beta^2+\sqrt{3}\varepsilon c_2^3\beta^3)t^3\\
& +3(20\varepsilon\sqrt{3}c_2^2\beta^2-54\varepsilon^2c_2^4\beta-81\sqrt{3}\varepsilon^3c_2^6)t^2\\
& +324\varepsilon^2c_2^3t.
\end{aligned}
$$

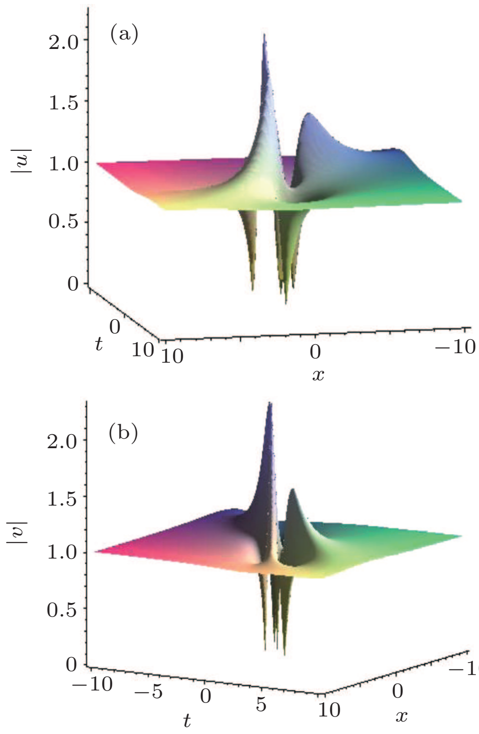

Fig. 2. (color online) Rogue-wave pair solution (14) of the CH equations: (a) rogue waves in the u component, (b) rogue waves in the v component. The parameters are $\varepsilon = 1/10$, $c_2 = 1$, and $a_2 = 1/2$.

4. Dark-bright-rogue waves of the CH equations

In this section, we choose

$$u[0] = c\exp[\mathrm{i}\theta] = c\exp[\mathrm{i}(ax+bt)],\ v[0]=0 \tag{15}$$

as the seed solution of the CH equations, here $b = -a^2/2 + c^2 + \varepsilon a^3 - 6\varepsilon c^2 a$. Similarly, denoting $\boldsymbol{M} = \mathrm{diag}\{1, \exp[\mathrm{i}\theta], 1\}$, hence under the gauge transformation $\psi = \boldsymbol{M}\phi$, the Lax pair (3) and (4) with $\lambda = \lambda_1$ and the seed solution (15) can be transformed into

$$\psi_x = \widetilde{\boldsymbol{U}}\psi,\quad \widetilde{\boldsymbol{U}} = \lambda_1\boldsymbol{U}_0 + \begin{pmatrix} 0 & -c & 0\\ c & \mathrm{i}a & 0\\ 0 & 0 & 0\end{pmatrix}, \tag{16}$$

$$\psi_t = \widetilde{\boldsymbol{V}}\psi,\quad \widetilde{\boldsymbol{V}} = \lambda_1^3\widetilde{\boldsymbol{V}}_0 + \lambda_1^2\widetilde{\boldsymbol{V}}_1 + \lambda_1\widetilde{\boldsymbol{V}}_2 + \widetilde{\boldsymbol{V}}_3, \tag{17}$$

where

$$\widetilde{\boldsymbol{V}}_0 = \frac{1}{16\varepsilon}\boldsymbol{U}_0,\quad \widetilde{\boldsymbol{V}}_1 = \frac{1}{8\varepsilon}\boldsymbol{U}_0 + \frac{1}{16\varepsilon}\begin{pmatrix} 0 & -c & 0\\ c & 0 & 0\\ 0 & 0 & 0\end{pmatrix},$$

$$\widetilde{\boldsymbol{V}}_2 = \frac{1}{4}\begin{pmatrix} \mathrm{i}c^2 & -\frac{c}{2\varepsilon}(1-2\varepsilon a) & 0\\ \frac{c}{2\varepsilon}(1-2\varepsilon a) & -\mathrm{i}c^2 & 0\\ 0 & 0 & 0\end{pmatrix},$$

$$\widetilde{\boldsymbol{V}}_3 = \begin{pmatrix} -\frac{\mathrm{i}}{2}c^2(4\varepsilon a-1) & -\frac{c}{2}(2\varepsilon a^2-4\varepsilon c^2-a) & 0\\ \frac{c}{2}(2\varepsilon a^2-4\varepsilon c^2-a) & \frac{\mathrm{i}}{2}(4\varepsilon c^2a-c^2+2b) & 0\\ 0 & 0 & 0\end{pmatrix}.$$

In the following, in order to obtain dark-bright-rogue wave solutions of the CH equations, we suppose $\lambda_1 = 4\varepsilon(2\mathrm{i}c-a)$, thus, the basic solution of the Lax pair equations (16) and (17) holds

$$\phi = \boldsymbol{M}^{-1}\boldsymbol{\Theta}(C_1, C_2, C_3)^{\mathrm{T}}, \tag{18}$$

where

$$\boldsymbol{\Theta} = \begin{pmatrix} \exp(\eta_1) & \exp(\eta_1)/\Gamma & 0\\ \exp(\eta_1) & \exp(\eta_1)/\Gamma & 0\\ 0 & 0 & \exp(\eta_2)\end{pmatrix},$$

with

$$
\begin{aligned}
& \eta_1 = m_1 + \mathrm{i}n_1,\quad \eta_2 = m_2 + \mathrm{i}n_2,\\
& \Gamma = k_1 + \mathrm{i}k_2,\quad \Gamma_1 = (k_1+\mathrm{i}k_2)t - x,\\
& \Gamma_2 = (k_1+\mathrm{i}k_2)t - x + 1/c,\\
& m_1 = \left(\varepsilon ca^2 - \frac{1}{3}ca - \frac{4}{3}\varepsilon c^3\right)t + \frac{1}{3}cx,\\
& n_1 = \left(\frac{5}{6}c^2 - \frac{1}{3}a^2 - 5\varepsilon c^2a + \frac{2}{3}\varepsilon a^3\right)t + \frac{2}{3}ax,\\
& k_1 = -3\varepsilon a^2 + 6\varepsilon c^2 + a,\\
& m_2 = \left(\frac{2}{3}ca + \frac{8}{3}\varepsilon c^3 - 2\varepsilon ca^2\right)t - \frac{2}{3}cx,\\
& n_2 = \left(4\varepsilon c^2a - \frac{2}{3}c^2 - \frac{1}{3}\varepsilon a^3 + \frac{1}{6}a^2\right)t - \frac{1}{3}ax,\\
& k_2 = 6\varepsilon ca - c.
\end{aligned}
$$

Next we set $C_1 = 1$, $C_2 = \mathrm{i}$, and $C_3 = 1$ to generate the dark-bright-rogue wave solutions of the CH equations, see Fig. 3

$$u[1] = c\left\{1 - 4\frac{F_1 \exp(2m_1)}{[D_1 \exp(2m_1) + D_2 \exp(2m_2)]}\right\}\exp[\mathrm{i}\theta],$$
$$v[1] = -4c\frac{F_2 \exp[m_1 + m_2 + \mathrm{i}(n_1 - n_2)]}{D_1 \exp(2m_1) + D_2 \exp(2m_2)}, \quad (19)$$

where

$$\begin{aligned} D_1 &= 2c^2(k_1^2 + k_2^2)(1 + t^2) + 2c(1 - 2cx) \\ &\quad \times (k_1 t + k_2) + 1 - 2cx + 2c^2x^2, \\ D_2 &= c^2(k_1^2 + k_2^2), \\ F_1 &= c^2(k_1^2 + k_2^2)(t^2 + 1) + c(k_1 + \mathrm{i}k_2)(t - \mathrm{i}) \\ &\quad - 2c^2x(k_1 t + k_2) - cx + c^2x^2, \\ F_2 &= c^2(k_1^2 + k_2^2)(\mathrm{i}t + 1) - c^2(\mathrm{i}k_1 + k_2)x. \end{aligned}$$

From Fig. 3, we see that a fundamental rogue wave collides with a dark soliton in the u component and a bright soliton in the v component. If $\mathrm{Re}(C_2) \neq 0$, the dark-bright soliton and the Peregrine soliton separate, here we omit the concrete expression of this solution and just show the interesting structures through Fig. 4. Similarly, if $\mathrm{Im}(C_1) \neq 0$ or $\mathrm{Im}(C_3) \neq 0$, the dark-bright-rogue wave solution can also be obtained, and the parameter C_2 still controls the mergence or separation of the Peregrine solitons and the dark-bright solitons like in Figs. 3 and 4. The results can help us better understand the attractive interactions between Peregrine solitons and dark-bright solitons in the CH equations.

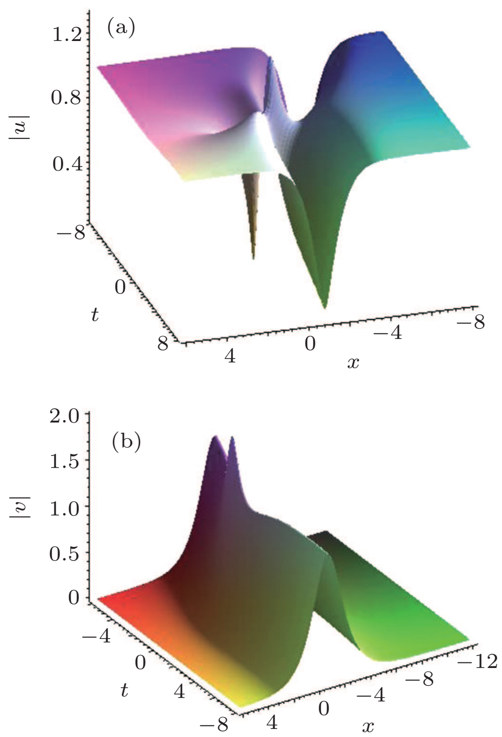

Fig. 3. (color online) Dark-bright-rogue wave solution (19) of the CH equations: (a) dark-rogue waves in the u component, (b) bright-rogue waves in the v component. The parameters are $\varepsilon = 1/20$, $c = 1$, and $a = 0$.

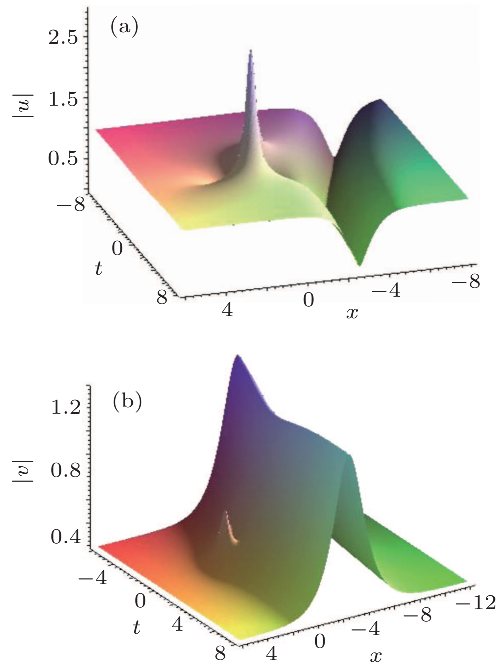

Fig. 4. (color online) Dark-bright-rogue waves of the CH equations: (a) dark-rogue waves in the u component, (b) bright-rogue waves in the v component. The parameters are $C_1 = 1$, $C_2 = 5 + \mathrm{i}$, $C_3 = 1$, $\varepsilon = 1/20$, $c = 1$, and $a = 0$.

5. Discussion and conclusion

In this paper, we investigate coupled Hirota equations. Based on the Darboux transformation, some novel explicit rogue wave solutions of the CH equations are given, including the rogue-wave pair solutions and the dark-bright-rogue wave solutions. It is significant that the rogue-wave pair solutions contain polynomials of the fourth order rather than the second order, and the dark-bright-rogue wave solutions are generated by satisfying some special requirements for the spectral parameters. Our results prove that rogue waves of the CH equations have some striking structures like the Manakov system, the two-component or three-component NLS equations, and the Sasa–Satsuma equation, which are distinct from rogue waves of the decoupled Hirota equation. We should mention that, in Section 4, if we choose $\lambda_1 \neq 4\varepsilon(2\mathrm{i}c - a)$, we can arrive at breather solutions of the coupled equations, and may further get the interactional solutions between breathers and Peregrine solitons like the Manakov model.[11] This is also interesting and we will investigate it in our future paper. The results in this paper further reveal the dynamical properties of rogue waves for the CH equations, and we hope the results will be verified by the physical experiments in the future.

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