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Bright-Dark Mixed *N*-Soliton Solution of Two-Dimensional Multicomponent Maccari System

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Abstract: Based on the KP hierarchy reduction method, we construct the general bright-dark mixed N-soliton solution of the two-dimensional (2D) (M+1)-component Maccari system comprised of M-component short waves (SWs) and one-component long wave (LW) with all possible combinations of nonlinearities. We firstly consider two types of mixed N-soliton solutions (two-bright-one-dark and one-bright-two-dark solitons in SW components) to the (3+1)-component Maccari system in detail. Then by extending our analysis to the (M+1)-component Maccari system, its general m-bright-(M−m)-dark mixed N-soliton solution is obtained. The formula obtained also contains the general all-bright and all-dark N-soliton solutions as special cases. For the two-bright-one-dark mixed soliton solution of the (3+1)-component Maccari system, it can be shown that solioff excitation and solioff interaction take place in the two SW components supporting bright solitons, whereas the SW component supporting dark solitons and the LW component possess V-type solitary and interaction.

Keywords: Bright-Dark N-Soliton; KP Hierarchy Reduction Method; Multi-Component Maccari System; τ Function.

1 Introduction

It is of great interest to study the multicomponent nonlinear systems as the nonlinear interaction of multiple waves may result in some new physical phenomenons [1–5]. One of the most celebrated examples is the multicomponent generalisation of the nonlinear Schrödinger (NLS) equation [6–12], which is considered as the basic model in describing the evolution of slowly varying wave packets in nonlinear wave systems. On the other hand, in recent years, the investigation of nonlinear systems describing the interaction of multiple short wave (SW) packets with a long wave (LW) in nonlinear dispersive media has draw more and more attentions [13–18]. For some multicomponent systems [14, 15, 17], it has been found that the solitons exhibit certain nontrivial inelastic (shape-changing) collision behaviours, which have not been found in the single-component counterparts and may be used for realizing multistate logic and soliton collision-based computing.

As a matter of fact, resonance often takes place in nonlinear systems if special criteria among frequencies and wavenumbers are met. The Maccari system is a representative example, where the group velocity of the SW (high-frequency wave) and the phase velocity of the LW (low-frequency wave) match with each other. The two-dimensional (2D) Maccari system takes the following form [19]

$$\mathrm{i}A_t + A_{xx} + LA = 0, \tag{1}$$

$$\mathrm{i}B_t + B_{xx} + LB = 0, \tag{2}$$

$$L_y = (AA^* + BB^*)_x, \tag{3}$$

where $A(x, y, t)$ and $B(x, y, t)$ are complex while $L(x, y, t)$ is real, the asterisk means complex conjugate and the subscripts denote partial differentiations hereafter. This system is usually used to describe the motion of isolated waves, localised in a small part of space, in some fields such as plasma physics, nonlinear optics, and hydrodynamics. It is not difficult to find the relationships between the 2D Maccari system (1)–(3) and some other models. For instance, it reduces to the NLS equation [20] if we take $y=x$. The reduction $y=t$ leads to the coupled long-wave resonance system [21]. When $A=B^*$, it becomes to the so-called simplest (2+1)-dimensional extension of the NLS equation proposed by Fokas [22]. Many studies have been done to study the Maccari system. For example, the two-dromion solutions are obtained in [23] by virtue of the technique of coalescence of wavenumbers. By using the variable separation approach [24, 25], some coherent soliton structures such as dromions, breathers, foldons, etc., are obtained in [26, 27]. In a very recent work [28],

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with the aid of Hirota's bilinear method, the lump and rogue-wave solutions are investigated.

It is desirable to extend the studies to multicomponent cases since a variety of complex systems such as Bose-Einstein condensates, nonlinear optical fibres, etc., usually involve more than one component. For these physical systems, according to the physical situations, the nonlinearities are positive or negative. For instance, in Bose-Einstein condensates [29], the nonlinear coefficients take positive or negative when the interaction between the atoms is repulsive or attractive. In the current paper, we consider a general multicomponent Maccari system

$$\mathrm{i}A_t^{(k)}+A_{xx}^{(k)}+LA^{(k)}=0,\quad k=1,\,2,\,\cdots,\,M, \tag{4}$$

$$L_y=\left(\sum_{k=1}^{M}\sigma_k A^{(k)}A^{(k)*}\right)_x, \tag{5}$$

where $\sigma_k=\pm 1$, $A^{(k)}(x,\,y,\,t)$ are complex functions. This system is referred as the 2D $(M+1)$-component Maccari system hereafter. The goal of this study is to construct the general bright-dark mixed *N*-soliton solution to the 2D multicomponent Maccari system (4) and (5) with all possible combinations of nonlinearity coefficients including all-positive, all-negative and mixed types by virtue of the KP hierarchy reduction technique. Compared with the standard 2D Maccari system (1)–(3), the mixed solitons in the multicomponent counterpart exhibit some new dynamical behaviours, such as solioff excitation and solioff interaction, which have not been found in the original system. For the dynamics of the mixed soliton solution to the (3+1)-component Maccari system, it can be shown that in the two-bright-one-dark mixed soliton solution, solioff excitation and solioff interaction occur in the two SW components supporting bright solitons, while V-type solitary and interaction take place in the SW component supporting dark solitons and the LW component. This novel phenomenon is consistent with the fact that the arbitrariness of nonlinearity coefficients σ_k provides an additional freedom resulting in rich soliton dynamics. In addition, for the one-bright-two-dark mixed soliton solution, the two-soliton bound state is also discussed. The formula of the mixed *N*-soliton solution to the $(M+1)$-component Maccari system also contains the general all-bright and all-dark *N*-soliton solutions as special cases.

It is worth noting that the KP hierarchy reduction technique for deriving soliton solutions of soliton equations is an effective and elegant method, which is firstly introduced by the Kyoto school [30] in the 1970s. This method has been used to derive soliton solutions of the NLS equation, the modified KdV equation and the Davey-Stewartson (DS) equation. Indeed, the pseudo-reduction of the 2D Toda lattice hierarchy to constrained KP systems with dark soliton solutions is proposed in [31], while the reduction to constrained KP systems with bright soliton solutions from multicomponent KP hierarchy is developed in [32]. By virtue of this method, the general *N*-dark-dark soliton solution of the two-component focusing-defocusing NLS equations (Manakov system) is obtained by Ohta et al. [33]. Also using this method, the general bright-dark mixed *N*-soliton solution of the vector NLS equations with all possible combinations of nonlinearities is studied by Feng [8]. In some recent works, this technique is applied to construct the *N*-dark soliton [16] and bright-dark mixed *N*-soliton [17, 18] solutions to the multicomponent Yajima-Oikawa (YO) system. More recently, the KP hierarchy reduction technique has been used to construct various rational (lump and rogue wave) solutions of soliton equations [34–36], see also the literatures [37–39].

The rest of this paper is organised as follows. In Section 2, the general two-bright-one-dark and one-bright-two-dark mixed *N*-soliton solutions of the 2D (3+1)-component Maccari system are constructed by virtue of the KP hierarchy reduction method in detail. Besides, the dynamics of the single and two solitons are also discussed. Section 3 devotes to extend the similar analysis to obtain the general *m*-bright-$(M-m)$-dark mixed *N*-soliton solution of the 2D $(M+1)$-component Maccari system. The last section is allotted for conclusion.

2 Mixed *N*-Soliton Solution of the 2D (3+1)-Component Maccari System

We first restrict our treatment to the 2D (3+1)-component Maccari system

$$\mathrm{i}A_t^{(k)}+A_{xx}^{(k)}+LA^{(k)}=0,\quad k=1,\,2,\,3, \tag{6}$$

$$L_y=(\sigma_1 A^{(1)}A^{(1)*}+\sigma_2 A^{(2)}A^{(2)*}+\sigma_3 A^{(3)}A^{(3)*})_x, \tag{7}$$

where $\sigma_k=\pm 1$ for $k=1,\,2,\,3$. For the system (6) and (7), the mixed-type vector solitons in the three SW components consist of two types: two-bright-one-dark soliton and one-bright-two-dark soliton. Therefore, in the subsequent two subsections, we will construct these two types of soliton solutions, respectively.

2.1 Two-Bright-One-Dark Soliton in the SW Components

For this case, assuming $A^{(1)}$ and $A^{(2)}$ are of bright type while $A^{(3)}$ is of dark type, we introduce the dependent variable transformations

$$A^{(1)}=\frac{g^{(1)}}{f},\quad A^{(2)}=\frac{g^{(2)}}{f},\quad A^{(3)}=\rho_1 e^{i(\alpha_1 x-\alpha_1^2 t)}\frac{h^{(1)}}{f},$$
$$L=2(\log f)_{xx}, \tag{8}$$

then the (3+1)-component Maccari system (6) and (7) is converted into the bilinear form

$$(D_x^2+\mathrm{i}D_t)g^{(k)}\cdot f=0,\quad k=1,\,2, \tag{9}$$

$$(D_x^2+2i\alpha_1 D_x+\mathrm{i}D_t)h^{(1)}\cdot f=0, \tag{10}$$

$$D_xD_yf\cdot f=\sum_{k=1}^{2}\sigma_k g^{(k)}g^{(k)*}-\sigma_3\rho_1^2(f^2-h^{(1)}h^{(1)*}), \tag{11}$$

where $g^{(1)}(x,y,t)$, $g^{(2)}(x,y,t)$, and $h^{(1)}(x,y,t)$ are complex, $f(x,y,t)$ is real, α_1 and ρ_1 are real constants. The Hirota's bilinear differential operator is defined as

$$D_x^lD_y^mD_t^n f(x,y,t)\cdot g(x,y,t)=\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\right)^l\left(\frac{\partial}{\partial y}-\frac{\partial}{\partial y'}\right)^m\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial t'}\right)^n f(x,y,t)\cdot g(x',y',t')\Big|_{x=x',y=y',t=t'}. \tag{12}$$

In what follows, the mixed N-soliton solution of the (3+1)-component Maccari system (6) and (7) is constructed in detail. For this purpose, we consider the τ function solution to the three-component KP hierarchy

$$\tau_{0,0}(k_1)=\begin{vmatrix}\mathbf{A} & I\\ -I & \mathbf{B}\end{vmatrix}, \tag{13}$$

$$\tau_{1,0}(k_1)=\begin{vmatrix}\mathbf{A} & I & \Omega^T\\ -I & \mathbf{B} & \mathbf{0}^T\\ \mathbf{0} & -\bar{\Psi} & 0\end{vmatrix},\quad \tau_{-1,0}(k_1)=\begin{vmatrix}\mathbf{A} & I & \mathbf{0}^T\\ -I & \mathbf{B} & \Psi^T\\ -\bar{\Omega} & \mathbf{0} & 0\end{vmatrix}, \tag{14}$$

$$\tau_{0,1}(k_1)=\begin{vmatrix}\mathbf{A} & I & \Omega^T\\ -I & \mathbf{B} & \mathbf{0}^T\\ \mathbf{0} & -\bar{\Upsilon} & 0\end{vmatrix},\quad \tau_{0,-1}(k_1)=\begin{vmatrix}\mathbf{A} & I & \mathbf{0}^T\\ -I & \mathbf{B} & \Upsilon^T\\ -\bar{\Omega} & \mathbf{0} & 0\end{vmatrix}, \tag{15}$$

where $\mathbf{0}$ is an N-component zero-row vector, I is an $N\times N$ identity matrix, $\mathbf{A}$ and $\mathbf{B}$ are $N\times N$ matrices, $\Omega,\Psi,\Upsilon,\bar{\Omega},\bar{\Psi}$ and $\bar{\Upsilon}$ are N-component row vectors defined respectively as

$$a_{ij}(k_1)=\frac{1}{p_i+\bar{p}_j}\left(-\frac{p_i-c_1}{\bar{p}_j+c_1}\right)^{k_1}e^{\xi_i+\bar{\xi}_j},$$
$$b_{ij}=\frac{\mu_1}{q_i+\bar{q}_j}e^{\eta_i+\bar{\eta}_j}+\frac{\mu_2}{r_i+\bar{r}_j}e^{\chi_i+\bar{\chi}_j},$$
$$\Omega=(e^{\xi_1},e^{\xi_2},\cdots,e^{\xi_N}),\quad \Psi=(e^{\eta_1},e^{\eta_2},\cdots,e^{\eta_N}),$$
$$\Upsilon=(e^{\chi_1},e^{\chi_2},\cdots,e^{\chi_N}),$$
$$\bar{\Omega}=(e^{\bar{\xi}_1},e^{\bar{\xi}_2},\cdots,e^{\bar{\xi}_N}),\quad \bar{\Psi}=(e^{\bar{\eta}_1},e^{\bar{\eta}_2},\cdots,e^{\bar{\eta}_N}),$$
$$\bar{\Upsilon}=(e^{\bar{\chi}_1},e^{\bar{\chi}_2},\cdots,e^{\bar{\chi}_N}),$$

with

$$\xi_i=\frac{1}{p_i-c_1}x_{-1}^{(1)}+p_ix_1+p_i^2x_2+\xi_{i0},$$
$$\bar{\xi}_j=\frac{1}{\bar{p}_j+c_1}x_{-1}^{(1)}+\bar{p}_jx_1-\bar{p}_j^2x_2+\bar{\xi}_{j0},$$
$$\eta_i=q_iy_1^{(1)}+\eta_{i0},\quad \bar{\eta}_j=\bar{q}_jy_1^{(1)}+\bar{\eta}_{j0},$$
$$\chi_i=r_iy_1^{(2)}+\chi_{i0},\quad \bar{\chi}_j=\bar{r}_jy_1^{(2)}+\bar{\chi}_{j0},$$

where $p_i,\bar{p}_j,q_i,\bar{q}_j,r_i,\bar{r}_j,\xi_{i0},\bar{\xi}_{j0},\eta_{i0},\bar{\eta}_{j0},\chi_{i0},\bar{\chi}_{j0}$ and c_1 are complex constants. Based on the Sato theory [30], the τ functions (13)–(15) satisfy the bilinear equations

$$(D_{x_1}^2-D_{x_2})\tau_{1,0}(k_1)\cdot\tau_{0,0}(k_1)=0, \tag{16}$$

$$(D_{x_1}^2-D_{x_2})\tau_{0,1}(k_1)\cdot\tau_{0,0}(k_1)=0, \tag{17}$$

$$(D_{x_1}^2-D_{x_2}+2c_1D_{x_1})\tau_{0,0}(k_1+1)\cdot\tau_{0,0}(k_1)=0, \tag{18}$$

$$D_{x_1}D_{y_1^{(1)}}\tau_{0,0}(k_1)\cdot\tau_{0,0}(k_1)=-2\mu_1\tau_{1,0}(k_1)\tau_{-1,0}(k_1), \tag{19}$$

$$D_{x_1}D_{y_1^{(2)}}\tau_{0,0}(k_1)\cdot\tau_{0,0}(k_1)=-2\mu_2\tau_{0,1}(k_1)\tau_{0,-1}(k_1), \tag{20}$$

$$(D_{x_1}D_{x_{-1}^{(1)}}-2)\tau_{0,0}(k_1)\cdot\tau_{0,0}(k_1)=-2\tau_{0,0}(k_1+1)\tau_{0,0}(k_1-1). \tag{21}$$

These bilinear equations can be proved by using the Grammian technique [40, 41], which are omitted here. By assuming x_1, $x_{-1}^{(1)}$, $y_1^{(1)}$, $y_1^{(2)}$, μ_1, μ_2 are real; x_2 and c_1 are pure imaginary and taking $p_j^*=\bar{p}_j$, $q_j^*=\bar{q}_j=r_j^*=\bar{r}_j$, $\xi_{j0}^*=\bar{\xi}_{j0}$, $\eta_{j0}^*=\bar{\eta}_{j0}$ and $\chi_{j0}^*=\bar{\chi}_{j0}$, it is easy to check that

$$a_{ij}(k_1)=a_{ji}^*(k_1),\quad b_{ij}=b_{ji}^*.$$

Moreover, we let

$$f=\tau_{0,0}(0),\quad g^{(1)}=\tau_{1,0}(0),\quad g^{(2)}=\tau_{0,1}(0),\quad h^{(1)}=\tau_{0,0}(1),$$

therefore, f is real and

$$g^{(1)*}=-\tau_{-1,0}(0),\quad g^{(2)*}=-\tau_{0,-1}(0),\quad h^{(1)*}=\tau_{0,0}(-1),$$

thus the equations (16)–(21) become to

$$(D_{x_1}^2-D_{x_2})g^{(k)}\cdot f=0,\quad k=1,\,2, \tag{22}$$

$$(D_{x_1}^2-D_{x_2}+2c_1D_{x_1})h^{(1)}\cdot f=0, \tag{23}$$

$$D_{x_1}D_{y_1^{(k)}}f\cdot f=2\mu_k g^{(k)}g^{(k)*},\quad k=1,\,2, \tag{24}$$

$$(D_{x_1}D_{x_{-1}^{(1)}}-2)f\cdot f=-2h^{(1)}h^{(1)*}. \tag{25}$$

Furthermore, by using the independent variable transformations

$$x_1=x,\quad x_2=\mathrm{i}t,\quad x_{-1}^{(1)}=-\frac{1}{2}\sigma_3\rho_1^2y,$$
$$y_1^{(k)}=\frac{1}{2}\nu_k y,\quad k=1,\,2, \tag{26}$$

the bilinear equations (22)–(25) are recast to the bilinear equations (9)–(11) with $c_1=i\alpha_1$ and $\sigma_k=\mu_k\nu_k$ for $k=1, 2$.

Finally, we have obtained the two-bright-one-dark mixed *N*-soliton solution to the 2D (3+1)-component Maccari system

$$f=\begin{vmatrix}\mathbf{A} & I\\ -I & \mathbf{B}\end{vmatrix},\quad g^{(k)}=\begin{vmatrix}\mathbf{A} & I & \Omega^T\\ -I & \mathbf{B} & \mathbf{0}^T\\ \mathbf{0} & \Psi^{(k)} & 0\end{vmatrix},\quad h^{(1)}=\begin{vmatrix}\mathbf{A}^{(1)} & I\\ -I & \mathbf{B}\end{vmatrix}, \tag{27}$$

where the entries in the matrices $\mathbf{A}$, $\mathbf{A}^{(1)}$, and $\mathbf{B}$ are given by

$$a_{ij}=\frac{1}{p_i+p_j^*}e^{\xi_i+\xi_j^*},\quad a_{ij}^{(1)}=\frac{1}{p_i+p_j^*}\left(-\frac{p_i-i\alpha_1}{p_j^*+i\alpha_1}\right)e^{\xi_i+\xi_j^*},$$
$$b_{ij}=\frac{1}{q_i+q_j^*}\sum_{k=1}^{2}\mu_k e^{\eta_i^{(k)}+\eta_j^{(k)*}}, \tag{28}$$

meanwhile, Ω and $\Psi^{(k)}$ are *N*-component row vectors

$$\Omega=(e^{\xi_1},\,e^{\xi_2},\,\cdots,\,e^{\xi_N}),\quad \Psi^{(k)}=-(e^{\eta_1^{(k)*}},\,e^{\eta_2^{(k)*}},\,\cdots,\,e^{\eta_N^{(k)*}}), \tag{29}$$

with $\xi_i=p_ix+ip_i^2t-\dfrac{\sigma_3\rho_1^2}{2(p_i-i\alpha_1)}y+\xi_{i0},\quad \eta_i^{(k)}=\dfrac{1}{2}\nu_kq_iy+\eta_{i0}^{(k)}$, where p_i, q_i, ξ_{i0}, and $\eta_{i0}^{(k)}$, $(k=1,\,2;\ i=1,\,2,\cdots,\,N)$, are complex constants.

2.1.1 One-Soliton Solution

Taking $N=1$ in the formula (27), we can get one-soliton solution. In this case, the τ functions read

$$f=\begin{vmatrix}a_{11} & 1\\ -1 & b_{11}\end{vmatrix},\quad g^{(k)}=\begin{vmatrix}a_{11} & 1 & e^{\xi_1}\\ -1 & b_{11} & 0\\ 0 & -e^{\eta_1^{(k)*}} & 0\end{vmatrix},\quad h^{(1)}=\begin{vmatrix}a_{11}^{(1)} & 1\\ -1 & b_{11}\end{vmatrix}, \tag{30}$$

where a_{11}, $a_{11}^{(1)}$, and b_{11} can be obtained from (28), and $\xi_1=p_1x+\mathrm{i}p_1^2t-\dfrac{\sigma_3\rho_1^2}{2(p_1-\mathrm{i}\alpha_1)}y+\xi_{10},\quad \eta_1^{(k)}=\dfrac{1}{2}\nu_kq_1y+\eta_{10}^{(k)}$ for $k=1, 2$. The τ function f in (30) can be written as

$$f=1+\frac{e^{2\xi_{1R}}}{4p_{1R}q_{1R}}\sum_{k=1}^{2}\mu_ke^{2\eta_{1R}^{(k)}}, \tag{31}$$

in which $\xi_1=\xi_{1R}+\mathrm{i}\xi_{1I}$ and $\eta_1^{(k)}=\eta_{1R}^{(k)}+\mathrm{i}\eta_{1I}^{(k)}$, the suffixes R and I denote the real and imaginary parts, respectively.

To get nonsingular solution, we take $\mu_1=\mu_2=1$ and $p_{1R}q_{1R}>0$. According to the values of σ_1 and σ_2, the one-soliton solution can be classified into three cases: (i) positive nonlinearities $(\sigma_1, \sigma_2)=(1, 1)$, (ii) negative nonlinearities $(\sigma_1, \sigma_2)=(-1, -1)$, and (iii) mixed-type. For the third case, without loss of generality, we choose $(\sigma_1, \sigma_2)=(1, -1)$ and take $\sigma_3=1$. Thus, we have $\nu_1=-\nu_2=1$ and the τ functions (30) can be rewritten as

$$f=1+\sum_{k=1}^{2}e^{\Phi_k+\Phi_k^*+\theta_k}, \tag{32}$$

$$g^{(k)}=e^{\Phi_k+\mathrm{i}\Lambda_k+\xi_{10}+\eta_{10R}^{(k)}},\quad k=1,\,2, \tag{33}$$

$$h^{(1)}=1+\sum_{k=1}^{2}e^{\Phi_k+\Phi_k^*+\theta_k+2\mathrm{i}\varphi_1}, \tag{34}$$

with $\Phi_k=\Phi_{kR}+\mathrm{i}\Phi_{kI}$, $\Phi_{kR}=p_{1R}x+\Pi_{kR}y-2p_{1R}p_{1I}t$, $\Pi_{kR}=\dfrac{1}{2}\left(\dfrac{-\rho_1^2p_{1R}}{(p_{1I}-\alpha_1)^2+p_{1R}^2}+(-1)^{k+1}q_{1R}\right)$, $\Phi_{kI}=p_{1I}x+\Pi_{kI}y+(p_{1R}^2-p_{1I}^2)t$,

$\Pi_{kI}=\dfrac{1}{2}\left(\dfrac{\rho_1^2(p_{1I}-\alpha_1)}{(p_{1I}-\alpha_1)^2+p_{1R}^2}+(-1)^{k+1}q_{1I}\right)$, $\quad\Lambda_k=(-1)^kq_{1I}y-\eta_{10I}^{(k)}$,

$e^{\theta_k}=\dfrac{1}{4p_{1R}q_{1R}}e^{2\xi_{10R}+2\eta_{10R}^{(k)}}$ and $e^{2\mathrm{i}\varphi_1}=-\dfrac{p_1-\mathrm{i}\alpha_1}{p_1^*+\mathrm{i}\alpha_1}$ for $k=1, 2$.

Next, the asymptotic analysis of the one-soliton solution is carried out. From the above τ functions, it is obvious that the one-soliton solution obtained actually represents two-soliton resonance solution. The asymptotic forms of solitons (s_1 and s_2) before and after collision are of the following form.

(i) Before collision $(x, y\to-\infty)$

Soliton s_1 $(\Phi_{1R}\simeq 0,\ \Phi_{2R}\to-\infty)$

$$A^{(1)} \simeq -\frac{\sqrt{p_{1R}q_{1R}}}{e^{\xi_{10R}}} e^{i(\Phi_{1I}+\Lambda_1)}\operatorname{sech}\left(\Phi_{1R}+\frac{\theta_1}{2}\right),$$
$$A^{(2)} \simeq 0,$$
$$A^{(3)} \simeq \frac{1}{2}\rho_1 e^{i(\alpha_1 x-\alpha_1^2 t)}\left[1+e^{2i\phi_1}+(e^{2i\phi_1}-1)\tanh\left(\Phi_{1R}+\frac{\theta_1}{2}\right)\right],$$
$$L \simeq 2p_{1R}^2\operatorname{sech}^2\left(\Phi_{1R}+\frac{\theta_1}{2}\right).$$

Soliton s_2 ($\Phi_{2R}\simeq 0$, $\Phi_{1R}\to -\infty$)

$$A^{(1)} \simeq 0,$$
$$A^{(2)} \simeq -\frac{\sqrt{p_{1R}q_{1R}}}{e^{\xi_{10R}}} e^{i(\Phi_{2I}+\Lambda_2)}\operatorname{sech}\left(\Phi_{2R}+\frac{\theta_2}{2}\right),$$
$$A^{(3)} \simeq \frac{1}{2}\rho_1 e^{i(\alpha_1 x-\alpha_1^2 t)}\left[1+e^{2i\phi_1}+(e^{2i\phi_1}-1)\tanh\left(\Phi_{2R}+\frac{\theta_2}{2}\right)\right],$$
$$L \simeq 2p_{1R}^2\operatorname{sech}^2\left(\Phi_{2R}+\frac{\theta_2}{2}\right).$$

(ii) After collision ($x, y\to +\infty$))

Soliton s_1 ($\Phi_{1R}\simeq 0$, $\Phi_{2R}\to +\infty$)

$$A^{(1)} \simeq 0,$$
$$A^{(2)} \simeq 0,$$
$$A^{(3)} \simeq \rho_1 e^{i(\alpha_1 x-\alpha_1^2 t+2\phi_1)},$$
$$L \simeq 0.$$

Soliton s_2 ($\Phi_{2R}\simeq 0$, $\Phi_{1R}\to +\infty$)

$$A^{(1)} \simeq 0,$$
$$A^{(2)} \simeq 0,$$
$$A^{(3)} \simeq \rho_1 e^{i(\alpha_1 x-\alpha_1^2 t+2\phi_1)},$$
$$L \simeq 0.$$

This kind of one-soliton solution is displayed in Figure 1 with the parametric choice $p_1=\frac{1}{2}+\frac{1}{2}i$, $q_1=3+2i$, $\mu_1=\mu_2=\nu_1=-\nu_2=\sigma_3=\rho_1=\alpha_1=1$ and $\xi_{10}=\eta_{10}^{(1)}=\eta_{10}^{(2)}=0$. From the figures, one can see that the bright solitons in $A^{(1)}$ and $A^{(2)}$ are solitoffs, while the dark soliton in $A^{(3)}$ and the bright soliton in L are V-type solitary wave. As pointed in [14, 15], the formation of this special soliton solution is ascribe to the mixed-type nonlinearity coefficients in multi-component systems.

2.1.2 Two-Soliton Solution

To get two-soliton solution, we take $N=2$ in the formula (27), then the τ functions read

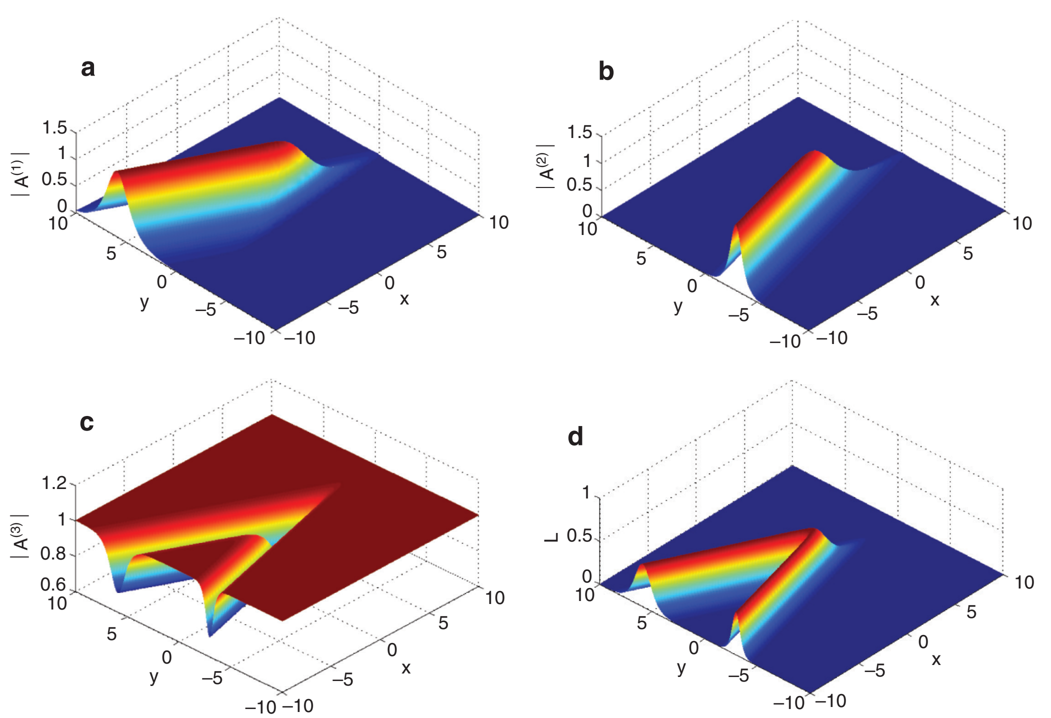

Figure 1: Two-bright-one-dark mixed one-soliton of the (3+1)-component Maccari system at time $t=0$. (a) The $A^{(1)}$ component, (b) the $A^{(2)}$ component, (c) the $A^{(3)}$ component, and (d) the L component.

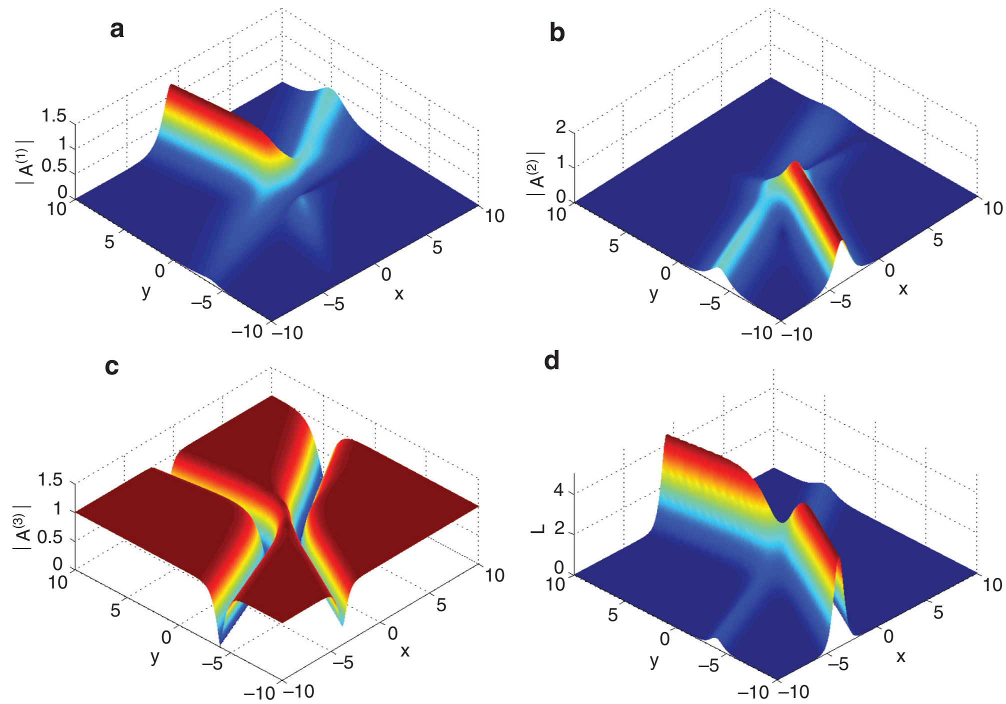

Figure 2: Two-bright-one-dark mixed two-soliton of the (3+1)-component Maccari system at time $t=0$. (a) The $A^{(1)}$ component, (b) the $A^{(2)}$ component, (c) the $A^{(3)}$ component, and (d) the L component.

$$f=\begin{vmatrix} a_{11} & a_{12} & 1 & 0 \\ a_{21} & a_{22} & 0 & 1 \\ -1 & 0 & b_{11} & b_{12} \\ 0 & -1 & b_{21} & b_{22} \end{vmatrix},\quad g^{(k)}=\begin{vmatrix} a_{11} & a_{12} & 1 & 0 & e^{\xi_1} \\ a_{21} & a_{22} & 0 & 1 & e^{\xi_2} \\ -1 & 0 & b_{11} & b_{12} & 0 \\ 0 & -1 & b_{21} & b_{22} & 0 \\ 0 & 0 & -e^{\eta_1^{(k)*}} & -e^{\eta_2^{(k)*}} & 0 \end{vmatrix},$$

$$h^{(1)}=\begin{vmatrix} a_{11}^{(1)} & a_{12}^{(1)} & 1 & 0 \\ a_{21}^{(1)} & a_{22}^{(1)} & 0 & 1 \\ -1 & 0 & b_{11} & b_{12} \\ 0 & -1 & b_{21} & b_{22} \end{vmatrix}, \tag{35}$$

where a_{ij}, $a_{ij}^{(1)}$, and b_{ij} can be obtained from (28), and $\xi_i = p_i x + \mathrm{i}p_i^2 t - \frac{\sigma_3\rho_1^2}{2(p_i-\mathrm{i}\alpha_1)}y+\xi_{i0}$, $\eta_i^{(k)}=\frac{1}{2}\nu_k q_i y+\eta_{i0}^{(k)}$ for $i=1, 2; k=1, 2$.

This kind of two-soliton solution is illustrated in Figure 2 with the parameters chosen as $p_1=\frac{1}{2}+\mathrm{i}$, $p_2=\frac{3}{2}+\frac{1}{2}\mathrm{i}$, $q_1=\frac{1}{2}+\mathrm{i}$, $q_2=1+\frac{1}{3}\mathrm{i}$, $\mu_1=\mu_2=\nu_1=-\nu_2=\sigma_3=\rho_1=\alpha_1=1$ and $\xi_{10}=\xi_{20}=\eta_{10}^{(1)}=\eta_{10}^{(2)}=\eta_{20}^{(1)}=\eta_{20}^{(2)}=0$. It is obvious that the bright solitoff interaction takes place in $A^{(1)}$ and $A^{(2)}$, the V-type interaction of dark solitons appears in $A^{(3)}$, while the V-type interaction of bright solitons takes place in L.

2.2 One-Bright-Two-Dark Soliton in the SW Components

For this case, assuming $A^{(1)}$ is of bright type, $A^{(2)}$ and $A^{(3)}$ are of dark type, the dependent variable transformations

$$A^{(1)}=\frac{g^{(1)}}{f},\quad A^{(2)}=\rho_1 \mathrm{e}^{\mathrm{i}(\alpha_1 x-\alpha_1^2 t)}\frac{h^{(1)}}{f},$$
$$A^{(3)}=\rho_2 \mathrm{e}^{\mathrm{i}(\alpha_2 x-\alpha_2^2 t)}\frac{h^{(2)}}{f},\quad L=2(\log f)_{xx}, \tag{36}$$

convert the (3+1)-component Maccari system (6) and (7) into the bilinear form

$$(D_x^2+\mathrm{i}D_t)g^{(1)}\cdot f=0, \tag{37}$$

$$(D_x^2+2\mathrm{i}\alpha_k D_x+\mathrm{i}D_t)h^{(k)}\cdot f=0,\quad k=1, 2, \tag{38}$$

$$D_x D_y f\cdot f=\sigma_1 g^{(1)}g^{(1)*}-\sum_{k=1}^{2}\sigma_{k+1}\rho_k^2(f^2-h^{(k)}h^{(k)*}), \tag{39}$$

where $g^{(1)}(x, y, t)$, $h^{(1)}(x, y, t)$, and $h^{(2)}(x, y, t)$ are complex, $f(x, y, t)$ is real, α_k and ρ_k, $(k=1, 2)$ are real constants.

To obtain one-bright-two-dark mixed soliton solution, we consider the following τ function solution to the two-component KP hierarchy

$$\tau_0(k_1, k_2) = \begin{vmatrix} \mathbf{A} & I \\ -I & \mathbf{B} \end{vmatrix}, \tag{40}$$

$$\tau_1(k_1, k_2) = \begin{vmatrix} \mathbf{A} & I & \Omega^T \\ -I & \mathbf{B} & \mathbf{0}^T \\ \mathbf{0} & -\bar{\Psi} & 0 \end{vmatrix}, \quad \tau_{-1}(k_1, k_2) = \begin{vmatrix} \mathbf{A} & I & \mathbf{0}^T \\ -I & \mathbf{B} & \Psi^T \\ -\bar{\Omega} & \mathbf{0} & 0 \end{vmatrix}, \tag{41}$$

where $\Omega, \Psi, \bar{\Omega}, \bar{\Psi}$ are the N-component row vectors defined previously, the entries of the matrices $\mathbf{A}$ and $\mathbf{B}$ are given by

$$a_{ij}(k_1, k_2) = \frac{1}{p_i + \bar{p}_j}\left(-\frac{p_i - c_1}{\bar{p}_j + c_1}\right)^{k_1}\left(-\frac{p_i - c_2}{\bar{p}_j + c_2}\right)^{k_2} e^{\xi_i + \bar{\xi}_j},$$
$$b_{ij} = \frac{\mu_1}{q_i + \bar{q}_j} e^{\eta_i + \bar{\eta}_j},$$

with

$$\xi_i = \frac{1}{p_i - c_1} x_{-1}^{(1)} + \frac{1}{p_i - c_2} x_{-1}^{(2)} + p_i x_1 + p_i^2 x_2 + \xi_{i0},$$
$$\bar{\xi}_j = \frac{1}{\bar{p}_j + c_1} x_{-1}^{(1)} + \frac{1}{\bar{p}_j + c_2} x_{-1}^{(2)} + \bar{p}_j x_1 - \bar{p}_j^2 x_2 + \bar{\xi}_{j0},$$
$$\eta_i = q_i y_1^{(1)} + \eta_{i0}, \quad \bar{\eta}_j = \bar{q}_j y_1^{(1)} + \bar{\eta}_{j0},$$

where $p_i, \bar{p}_j, q_i, \bar{q}_j, \xi_{i0}, \bar{\xi}_{j0}, \eta_{i0}, \bar{\eta}_{j0}, c_1$ and c_2 are complex constants. Based on the Sato theory [30], the τ functions (40)–(41) satisfy the bilinear equations

$$(D_{x_1}^2 - D_{x_2})\tau_1(k_1, k_2)\cdot\tau_0(k_1, k_2) = 0, \tag{42}$$

$$(D_{x_1}^2 - D_{x_2} + 2c_1 D_{x_1})\tau_0(k_1+1, k_2)\cdot\tau_0(k_1, k_2) = 0, \tag{43}$$

$$(D_{x_1}^2 - D_{x_2} + 2c_2 D_{x_1})\tau_0(k_1, k_2+1)\cdot\tau_0(k_1, k_2) = 0, \tag{44}$$

$$D_{x_1} D_{y_1^{(1)}} \tau_0(k_1, k_2)\cdot\tau_0(k_1, k_2) = -2\mu_1\tau_1(k_1, k_2)\tau_{-1}(k_1, k_2), \tag{45}$$

$$(D_{x_1} D_{x_{-1}^{(1)}} - 2)\tau_0(k_1, k_2)\cdot\tau_0(k_1, k_2) = -2\tau_0(k_1+1, k_2)\tau_0(k_1-1, k_2), \tag{46}$$

$$(D_{x_1} D_{x_{-1}^{(2)}} - 2)\tau_0(k_1, k_2)\cdot\tau_0(k_1, k_2) = -2\tau_0(k_1, k_2+1)\tau_0(k_1, k_2-1). \tag{47}$$

Similarly, we first consider the complex conjugate reduction by assuming $x_1, x_{-1}^{(1)}, x_{-1}^{(2)}, y_1^{(1)}$, and μ_1 are real; x_2, c_1, and c_2 are pure imaginary and letting $p_j^* = \bar{p}_j$, $q_j^* = \bar{q}_j$, $\xi_{j0}^* = \bar{\xi}_{j0}$ and $\eta_{j0}^* = \bar{\eta}_{j0}$. Then, it is easy to know that

$$a_{ij}(k_1, k_2) = a_{ji}^*(-k_1, -k_2), \quad b_{ij} = b_{ji}^*.$$

Moreover, we let

$$f = \tau_0(0, 0), \quad g^{(1)} = \tau_1(0, 0), \quad h^{(1)} = \tau_0(1, 0), \quad h^{(2)} = \tau_0(0, 1),$$

thus, f is real and

$$g^{(1)*} = -\tau_{-1}(0, 0), \quad h^{(1)*} = \tau_0(-1, 0), \quad h^{(2)*} = \tau_0(0, -1).$$

Hence, the bilinear equations (42)–(47) become to

$$(D_{x_1}^2 - D_{x_2})g^{(1)}\cdot f = 0, \tag{48}$$

$$(D_{x_1}^2 - D_{x_2} + 2c_k D_{x_1})h^{(k)}\cdot f = 0, \quad k = 1, 2, \tag{49}$$

$$D_{x_1} D_{y_1^{(1)}} f\cdot f = 2\mu_1 g^{(1)} g^{(1)*}, \tag{50}$$

$$(D_{x_1} D_{x_{-1}^{(k)}} - 2)f\cdot f = -2h^{(k)}h^{(k)*}, \quad k = 1, 2. \tag{51}$$

By using the independent variable transformations

$$x_1 = x, \quad x_2 = \mathrm{i}t, \quad x_{-1}^{(k)} = -\frac{1}{2}\sigma_{k+1}\rho_k^2 y,$$
$$y_1^{(1)} = \frac{1}{2}\nu_1 y, \quad k = 1, 2, \tag{52}$$

then, the bilinear equations (48)–(51) are recast to the bilinear equations (37)–(39) with $\sigma_1 = \mu_1\nu_1$, $c_1 = i\alpha_1$, and $c_2 = i\alpha_2$.

In conclude, we have obtained the one-bright-two-dark mixed soliton solution to the (3+1)-component Maccari system

$$f = \begin{vmatrix} \mathbf{A} & I \\ -I & \mathbf{B} \end{vmatrix}, \quad g^{(1)} = \begin{vmatrix} \mathbf{A} & I & \Omega^T \\ -I & \mathbf{B} & \mathbf{0}^T \\ \mathbf{0} & \Psi^{(1)} & 0 \end{vmatrix}, \quad h^{(k)} = \begin{vmatrix} \mathbf{A}^{(k)} & I \\ -I & \mathbf{B} \end{vmatrix}, \tag{53}$$

with the elements in $\mathbf{A}$, $\mathbf{A}^{(k)}$, $\mathbf{B}$, Ω, and $\Psi^{(1)}$ are given by

$$a_{ij} = \frac{1}{p_i + p_j^*} e^{\xi_i + \xi_j^*}, \quad a_{ij}^{(k)} = \frac{1}{p_i + p_j^*}\left(-\frac{p_i - i\alpha_k}{p_j^* + i\alpha_k}\right) e^{\xi_i + \xi_j^*},$$
$$b_{ij} = \frac{\mu_1}{q_i + q_j^*} e^{\eta_i^{(1)} + \eta_j^{(1)*}}, \tag{54}$$

$$\Omega = (e^{\xi_1}, e^{\xi_2}, \cdots, e^{\xi_N}), \quad \Psi^{(1)} = -(e^{\eta_1^{(1)*}}, e^{\eta_2^{(1)*}}, \cdots, e^{\eta_N^{(1)*}}), \tag{55}$$

where $\xi_i = p_i x + i p_i^2 t - \frac{1}{2}\sum_{k=1}^{2}\frac{\sigma_{k+1}\rho_k^2}{p_i - i\alpha_k} y + \xi_{i0}$, $\eta_i^{(1)} = \frac{1}{2}\nu_1 q_i y + \eta_{i0}^{(1)}$, p_i, q_i, ξ_{i0} and $\eta_{i0}^{(1)}$, $(i = 1, 2, \cdots, N)$ are complex constants.

2.2.1 One-Soliton Solution

To get one-soliton solution, we take $N = 1$ in the formula (53). For this case, the τ functions read

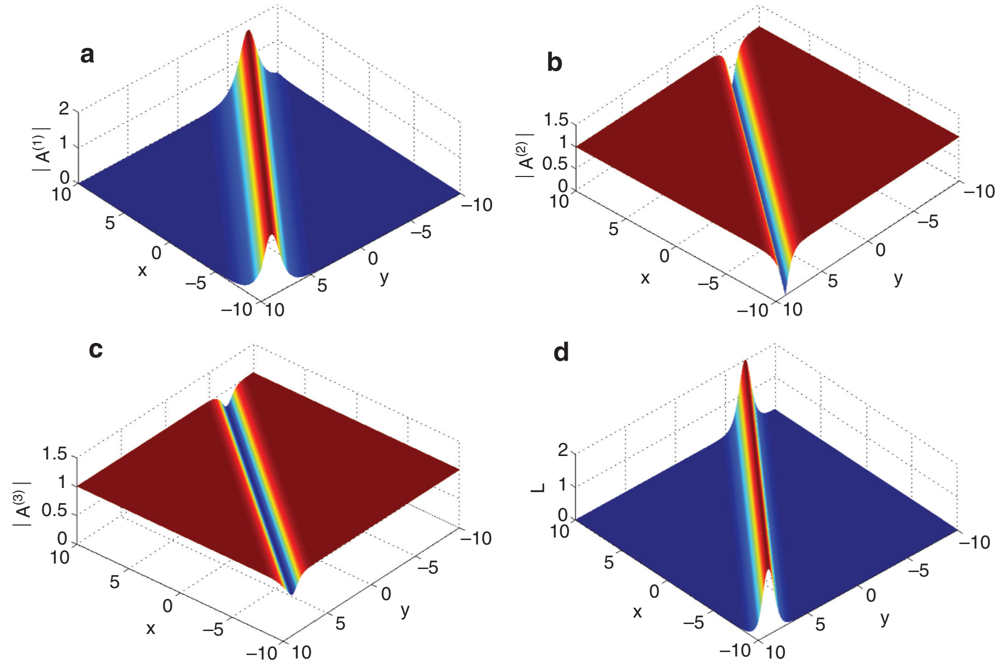

Figure 3: One-bright-two-dark mixed one-soliton of the (3+1)-component Maccari system at time $t=0$. (a) The $A^{(1)}$ component, (b) the $A^{(2)}$ component, (c) the $A^{(3)}$ component, and (d) the L component.

$$f=\begin{vmatrix} a_{11} & 1 \\ -1 & b_{11} \end{vmatrix},\quad g^{(1)}=\begin{vmatrix} a_{11} & 1 & e^{\xi_1} \\ -1 & b_{11} & 0 \\ 0 & -e^{\eta_1^{(1)*}} & 0 \end{vmatrix},\quad h^{(k)}=\begin{vmatrix} a_{11}^{(k)} & 1 \\ -1 & b_{11} \end{vmatrix}, \tag{56}$$

where a_{11}, $a_{11}^{(k)}$, $(k=1,\ 2)$, and b_{11} can be obtained from (54), and $\xi_1=p_1x+\mathrm{i}p_1^2t-\frac{1}{2}\sum_{k=1}^{2}\frac{\sigma_{k+1}\rho_k^2}{p_1-\mathrm{i}\alpha_k}y+\xi_{10}$, $\eta_1^{(1)}=\frac{1}{2}\nu_1q_1y+\eta_{10}^{(1)}$. The τ function f in (56) can be rewritten as

$$f=1+\frac{\mu_1}{4p_{1R}q_{1R}}e^{2\xi_{1R}+2\eta_{1R}^{(1)}}. \tag{57}$$

To get nonsingular solution, we take $\mu_1=1$ and $p_{1R}q_{1R}>0$. This kind of mixed one-soliton solution is displayed in Figure 3 with the parametric choice $p_1=1+i$, $q_1=3+2i$, $\mu_1=\nu_1=\sigma_2=-\sigma_3=\rho_1=\rho_2=\alpha_1=1$ and $\alpha_2=\xi_{10}=\eta_{10}^{(1)}=0$. The parameters α_1 and α_2 affect the depths of dark solitons in the mixed one-soliton solution, which can be evidenced from the Figure 3(b) and (c) where $A^{(2)}$ is a dark soliton while $A^{(3)}$ is a gray one.

2.2.2 Two-Soliton Solution

By taking $N=2$ in the formula (53), we get the τ functions for two-soliton solution

$$f=\begin{vmatrix} a_{11} & a_{12} & 1 & 0 \\ a_{21} & a_{22} & 0 & 1 \\ -1 & 0 & b_{11} & b_{12} \\ 0 & -1 & b_{21} & b_{22} \end{vmatrix},\quad g^{(1)}=\begin{vmatrix} a_{11} & a_{12} & 1 & 0 & e^{\xi_1} \\ a_{21} & a_{22} & 0 & 1 & e^{\xi_2} \\ -1 & 0 & b_{11} & b_{12} & 0 \\ 0 & -1 & b_{21} & b_{22} & 0 \\ 0 & 0 & -e^{\eta_1^{(1)*}} & -e^{\eta_2^{(1)*}} & 0 \end{vmatrix},$$

$$h^{(k)}=\begin{vmatrix} a_{11}^{(k)} & a_{12}^{(k)} & 1 & 0 \\ a_{21}^{(k)} & a_{22}^{(k)} & 0 & 1 \\ -1 & 0 & b_{11} & b_{12} \\ 0 & -1 & b_{21} & b_{22} \end{vmatrix}, \tag{58}$$

where a_{ij}, $a_{ij}^{(k)}$, $(k=1,\ 2)$ and b_{ij} can be obtained from (54), and $\xi_i=p_ix+\mathrm{i}p_i^2t-\frac{1}{2}\sum_{k=1}^{2}\frac{\sigma_{k+1}\rho_k^2}{p_i-i\alpha_k}y+\xi_{i0}$, $\eta_i^{(1)}=\frac{1}{2}\nu_1q_iy+\eta_{i0}^{(1)}$ for $i=1,\ 2$.

As a matter of fact, soliton bound state is a fascinating class of multi-soliton moving with a common speed. Especially, the mixed two-soliton bound state can be obtained from the mixed two-soliton solution by restricting two solitons moving with the same velocity. To this end, we rewrite $\xi_{iR}+\eta_{iR}^{(1)*}=p_{iR}x+\Pi_{iR}y-2p_{iR}p_{iI}t$ and $\xi_{iI}+\eta_{iI}^{(1)*}=p_{iI}x+\Pi_{iI}y+(p_{iR}^2-p_{iI}^2)t$ with $\Pi_{iR}=\frac{1}{2}\left(\sum_{k=1}^{2}\frac{-\sigma_{k+1}\rho_k^2p_{iR}}{(p_{iI}-\alpha_k)^2+p_{iR}^2}+\nu_1q_{iR}\right)$ and $\Pi_{iI}=\frac{1}{2}\left(\sum_{k=1}^{2}\frac{\sigma_{k+1}\rho_k^2(p_{iI}-\alpha_k)}{(p_{iI}-\alpha_k)^2+p_{iR}^2}-\nu_1q_{iI}\right)$. The conditions $p_{1I}=p_{2I}$ and $\frac{\Pi_{1R}}{p_{1R}}=\frac{\Pi_{2R}}{p_{2R}}$

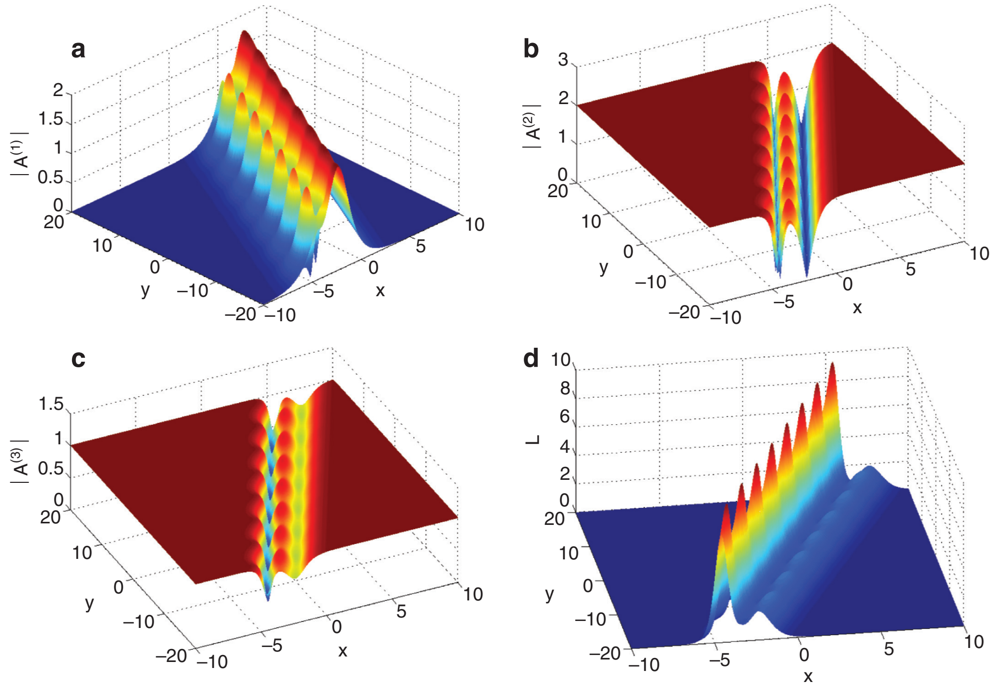

Figure 4: One-bright-two-dark mixed two-soliton bound state of the (3+1)-component Maccari system at time $t=0$. (a) The $A^{(1)}$ component, (b) the $A^{(2)}$ component, (c) the $A^{(3)}$ component, and (d) the L component.

lead to a two-soliton bound state, which is displayed in Figure 4 with the parameters chosen as $p_1=1+\mathrm{i}$, $p_2=2+\mathrm{i}$, $q_1=3+2\mathrm{i}$, $q_2=\frac{3}{5}$, $\mu_1=\nu_1=\sigma_2=-\sigma_3=\rho_2=\alpha_1=1$, $\rho_1=2$ and $\alpha_2=\xi_{10}=\xi_{20}=\eta_{10}^{(1)}=\eta_{20}^{(1)}=0$.

3 General Mixed *N*-Soliton Solution of the 2D Multicomponent Maccari System

In this section, we consider the general mixed N-soliton solution consisting of m bright solitons and $M-m$ dark solitons in the SW components to the $(M+1)$-component Maccari system (4) and (5). For this purpose, we introduce the dependent variable transformations

$$A^{(k)}=\frac{g^{(k)}}{f},\quad A^{(l)}=\rho_l e^{i(\alpha_l x-\alpha_l^2 t)}\frac{h^{(l)}}{f},\quad u=2(\log f)_{xx}, \tag{59}$$

where $k=1, 2,\cdots, m$, and $l=1, 2,\cdots, M-m$, which convert equations (4) and (5) into

$$(D_x^2+\mathrm{i}D_t)g^{(k)}\cdot f=0,\quad k=1, 2, \cdots, m, \tag{60}$$

$$(D_x^2+2\mathrm{i}\alpha_l D_x+\mathrm{i}D_t)h^{(l)}\cdot f=0,\quad l=1, 2, \cdots, M-m, \tag{61}$$

$$D_x D_y f\cdot f=\sum_{k=1}^{m}\sigma_k g^{(k)}g^{(k)*}-\sum_{l=1}^{M-m}\sigma_{l+m}\rho_l^2(f^2-h^{(l)}h^{(l)*}). \tag{62}$$

In the same spirit as the (3+1)-component Maccari system, it is easy to show that the following τ functions satisfy the bilinear equations (60)–(62) and hence provide the bright-dark mixed N-soliton solution to the $(M+1)$-component Maccari system

$$f=\begin{vmatrix}\mathbf{A} & I\\ -I & \mathbf{B}\end{vmatrix},\quad g^{(k)}=\begin{vmatrix}\mathbf{A} & I & \Omega^T\\ -I & \mathbf{B} & \mathbf{0}^T\\ \mathbf{0} & \Psi^{(k)} & 0\end{vmatrix},\quad h^{(l)}=\begin{vmatrix}\mathbf{A}^{(l)} & I\\ -I & \mathbf{B}\end{vmatrix}, \tag{63}$$

where $\mathbf{A}$, $\mathbf{A}^{(l)}$, and $\mathbf{B}$ are $N\times N$ matrices with entries defined as

$$a_{ij}=\frac{1}{p_i+p_j^*}e^{\xi_i+\xi_j^*},\quad a_{ij}^{(l)}=\frac{1}{p_i+p_j^*}\left(-\frac{p_i-\mathrm{i}\alpha_l}{p_j^*+\mathrm{i}\alpha_l}\right)e^{\xi_i+\xi_j^*},$$
$$b_{ij}=\frac{1}{q_i+q_j^*}\sum_{k=1}^{m}\mu_k e^{\eta_i^{(k)}+\eta_j^{(k)*}}, \tag{64}$$

meanwhile, Ω and $\Psi^{(k)}$ are N-component row vectors

$$\Omega=(e^{\xi_1}, e^{\xi_2}, \cdots, e^{\xi_N}),\quad \Psi^{(k)}=-(e^{\eta_1^{(k)*}}, e^{\eta_2^{(k)*}}, \cdots, e^{\eta_N^{(k)*}}), \tag{65}$$

with $\xi_i=p_i x+\mathrm{i}p_i^2 t-\frac{1}{2}\sum_{l=1}^{M-m}\frac{\sigma_{l+m}\rho_l^2}{p_i-\mathrm{i}\alpha_l}y+\xi_{i0}$, $\eta_i^{(k)}=\frac{1}{2}\nu_k q_i y+\eta_{i0}^{(k)}$,

where $\sigma_k=\mu_k\nu_k$, p_i, q_i, ξ_{i0}, and $\eta_{i0}^{(k)}$, $(k=1, 2,\cdots, m;\ l=1, 2,\cdots, M-m;\ i=1, 2,\cdots, N)$ are complex constants.

In parallel with the vector NLS equation [8] and the multicomponent YO system [17], the formula of the general bright-dark mixed N-soliton solution also includes the all-bright and the all-dark N-soliton solutions as special cases. For instance, the all-bright N-soliton solution can be directly obtained from the mixed N-soliton sloution (63) with taking $m=M$, therefore, it shares the same determinant form as the mixed N-soliton sloution. Whereas, the formula of the all-dark N-soliton solution is different from the one of the mixed N-soliton sloution. As pointed in the literatures [8] and [17], it is known that the all-dark N-soliton solution can alternatively take the same form as (63) except redefining the matrix $\mathbf{B}$ to be an identity matrix, i.e. $b_{ij}=\delta_{ij}$, and δ_{ij} is the Kronecker symbol.

Remark 1: For the standard 2D Maccari system (1)–(3), the mixed-type soliton solutions in its two SW components consist of only one case: one-bright-one-dark soliton, which can be directly obtained from the formulas (59) and (63) by taking $M=2$ and $\mu_1=\nu_1=\sigma_2=m=1$. In this case, compared with the multicomponent Maccari system (4) and (5), we can not get the solitoff excitation nor the V-type interaction solutions duo to loss of mixed-type nonlinearity coefficients. Additionally, in the [28], various rational solutions of the standard 2D Maccari system (1)–(3) are obtained through a modified KP hierarchy reduction method. Similarly, the various rational solutions of the multi-component Maccari system (4) and (5) can also be constructed by using the method in the [28]. And the rational solutions of the multicomponent system may bring some novel interesting dynamical properties of rogue waves. As this topic is not the object of our concern in the present paper, we do not discuss it in detail here.

4 Conclusion

In summary, the general bright-dark mixed N-soliton solution of the 2D multicomponent Maccari system with all possible combinations of nonlinearity coefficients including all-positive, all-negative, and mixed types are obtained based on the KP hierarchy reduction technique. Taking the (3+1)-component Maccari system as an example, its two kinds of mixed N-soliton solution (two-bright-one-dark soliton and one-bright-two-dark soliton in the SW components) are constructed in detail. In Section 2.1, the two-bright-one-dark mixed N-soliton solution is derived. It is worth noting that the derivation starts from a (2+1)-component KP hierarchy with one copy of shifted singular point (c_1). On the other hand, for the construction of one-bright-two-dark mixed N-soliton solution in Section 2.2, we begin with a (1+1)-component KP hierarchy with two copies of shifted singular points (c_1 and c_2). Therefore, it is not difficult to conclude that the number of components in the KP hierarchy equals to the number of SW components supporting bright solitons while the number of the copies of shifted singular points matches the number of SW components supporting dark solitons. Then, the similar analysis is extended to the (M+1)-component Maccari system to obtain its m-bright-(M–m)-dark mixed N-soliton solution. It is obvious that the mixed N-soliton solution of the (M+1)-component Maccari system can be derived from the reduction of a (m+1)-component KP hierarchy with M–m copies of shifted singular points. The mixed N-soliton solution obtained also includes the general all-bright and all-dark N-soliton solutions as special cases. For the two-bright-one-dark mixed soliton solution, dynamical analysis shows that solioff excitation and solioff interaction take place in the SW components supporting bright solitons, and V-type solitary and interaction appear in the SW component supporting dark solitons and the LW component. In addition, for the one-bright-two-dark mixed soliton solution, the two-soliton bound state is also discussed and exhibited graphically.

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