Commun. Theor. Phys. (Beijing, China) **40** (2003) pp. 137–142
© International Academic Publishers
Vol. 40, No. 2, August 15, 2003

# Symbolic Computation and Construction of Soliton-Like Solutions to the (2+1)-Dimensional Breaking Soliton Equation*

CHEN Yong,† LI Biao, and ZHANG Hong-Qing

Department of Applied Mathematics, Dalian University of Technology, Dalian 116024, China

(Received October 14, 2002; Revised December 17, 2002)

***Abstract*** *Based on the computerized symbolic system **Maple**, a new generalized expansion method of Riccati equation for constructing non-travelling wave and coefficient functions' soliton-like solutions is presented by a new general ansätz. Making use of the method, we consider the (2+1)-dimensional breaking soliton equation, $u_t + bu_{xxy} + 4buv_x + 4bu_xv = 0$, $u_y = v_x$, and obtain rich new families of the exact solutions of the breaking soliton equation, including the non-travelling wave and constant function soliton-like solutions, singular soliton-like solutions, and triangular function solutions.*

**PACS numbers:** 03.40.Kf
**Key words:** generalized expansion method of Riccati equation, symbolic computation, breaking soliton equation, soliton-like solutions, solitons

## 1 Introduction

In recent years, much attention has been paid on the study of (2+1)-dimensional soliton systems particularly after the advent of dromions which are exponentially localized in the structures driven by some straight-line ghost solitons unearthed by Boiti and his co-workes[1] and Forkas and Santini.[2] Some significant models such as the Kadomtsev–Petviashvili (KP) equation,[3] Davey–Stewartson (DS) equation,[4,5] Nizhnik–Noviko–Vesselov (NNV) equation,[6,7] the asymmetric NNV (ANNV) equation,[6,8] and breaking soliton equation,[9,10] etc. have also been established in nonlinear physics.

To find some exact physically significant coherent soliton solutions (which are localized in all directions) in (2+1) dimensions is much more difficult than in (1+1) dimensions. Recently, two kinds of "variable separating" procedures have been established for some types of (2+1)-dimensional integrable models (see Refs. [5], [7], [8], and [10]–[13]). The second type of variable separation method had been established for some types of (2+1)-dimensional integrable model like the DS equation,[5] NNV and ANNV equations,[7,8] and some special types of exact solutions of these equations can be obtained by selecting the arbitrary functions appropriately.

In this paper, we would present a new method named generalized expansion method of Riccati equation, which is mainly stemmed from the tanh method,[14,15] extended tanh-function method,[16−21] modified extended tanh-function method,[22] and generalized hyperbolic-function method,[23] and use it to consider the soliton-like solutions for the (2+1)-dimensional breaking soliton equation,[9,10]

$$u_t + bu_{xxy} + 4buv_x + 4bu_xv = 0\,, \quad u_y = v_x\,. \tag{1}$$

Equations (1) describes the (2+1)-dimensional interaction of a Riemann wave propagating along the $y$-axis with a long wave along the $x$-axis, and it seems to have been generated.[24] Recently, by use of a variable separation approach, Ruan[10] studied the coherent structures of Eq. (1) and obtained some special types of the driomion solutions, lumps, ring solutions, curved solitons, and breathers by selecting the arbitrary functions appropriately.

In this paper, using our method, we obtain rich new families of the exact solutions of the (2+1)-dimensional breaking soliton equation, including the non-travelling wave and constant function soliton-like solutions, singular soliton-like solutions, triangular function solutions.

The plan of this paper is as follows. In Sec. 2, we describe briefly the generalized expansion method of Riccati equation. In Sec. 3, we apply the method to (2+1)-dimensional breaking soliton equation and bring out rich soliton-like solutions. Conclusions will be presented finally.

## 2 Generalized Expansion Method of Riccati Equation

Let us simply describe the generalized expansion method of Riccati equation.

For a given system of nonlinear evolution equations (NEES) in three variables $x$, $y$, and $t$,

$$\begin{aligned} &E_1(u, v, u_t, v_t, u_x, v_x, u_y, v_y, u_{xx}, v_{xx}, u_{xt}, v_{xt}, u_{xy}, v_{xy}, u_{yt}, v_{yt}, \cdots,) = 0\,, \\ &E_2(u, v, u_t, v_t, u_x, v_x, u_y, v_y, u_{xx}, v_{xx}, u_{xt}, v_{xt}, u_{xy}, v_{xy}, u_{yt}, v_{yt}, \cdots,) = 0\,, \end{aligned} \tag{2}$$

we seek the following formal solutions of the given system by the new more general ansätz,

$$u(x, y, t) = a_0 + \sum_{i=1}^{m} [a_i\phi^i(\xi) + b_i\phi^{i-1}(\xi)\sqrt{R + \phi^2(\xi)} + k_i\phi^{-i}(\xi)]\,,$$

*The project supported by National Natural Science Foundation of China under Grant No. 1007201 and the National Key Basic Research Development Project Program under Grant No. G1998030600

†Email: chenyong@dlut.edu.cn

$$v(x,y,t)=A_0+\sum_{j=1}^{n}[A_j\phi^i(\xi)+B_j\phi^{j-1}(\xi)\sqrt{R+\phi^2(\xi)}+K_j\phi^{-j}(\xi)]\,,\tag{3}$$

where $m$, $n$ are integers to be determined by balancing the highest order derivative terms with the nonlinear terms in Eq. (2), $R$ is a real constant, while $a_0=a_0(x,y,t)$, $A_0=A_0(x,y,t)$, $a_i=g_i(x,y,t)$, $b_i=b_i(x,y,t)$, $k_i=k_i(x,y,t)$, $A_j=A_i(x,y,t)$, $B_j=B_j(x,y,t)$, $K_j=K_j(x,y,t)$ $(i=1,\ldots,m;j=1,\ldots,n)$, and $\xi=\xi(x,y,t)$ are all differentiable functions, and $\phi(\xi)$ satisfies

$$\frac{\mathrm{d}\phi(\xi)}{\mathrm{d}\xi}=R+\phi^2(\xi)\,.\tag{4}$$

It is easy to see that the ansätz (3) is more general than the ansätz in the generalized hyperbolic-function method,[23] tanh method,[14,15] extended tanh-function method,[16−21] modified extended tanh-function method.[22] Firstly, compared with the tanh method, extended tanh-function, as well as the modified extended tanh-function method, the restriction on $\xi(x,y,t)$ as merely a linear function $x,y,t$ and the restriction on the coefficients $a_i$, $b_i$, $k_i$, $A_j$, $B_j$, $K_j$ $(i=0,\ldots,m;j=0,\ldots,n)$ and $\xi$ as constants are removed. Secondly, compared with the generalized hyperbolic-function method, we can not only recover the exact solutions for a given NEEs which are the superposition of different powers of the sech $\xi$ function, tanh $\xi$ function or their combinations, but also we can, with no extra efforts, find other new and more general types of solutions, such as singular soliton-like solutions, coth-type solutions, triangular periodic-like solutions, tan-type solutions, and these formal functions' combination, even rational solutions, *et al.* More importantly, we add terms $k_i\phi^{-i}(\xi)$ in new ansätz (3), so more types of solutions would be expected for some equations.

There exist the following steps to be considered further:

***Step 1*** Determining the values of $m$ and $n$ of system (3) by respectively balancing the highest-order partial derivative terms and the nonlinear terms in system (2).

***Step 2*** Substituting Eqs. (3) along with Eq. (4) into Eq. (2), multiplying the most simplifying common denominator in the obtained system, setting the coefficients of $\phi^r(\xi)(\sqrt{R+\phi^2(\xi)}\,)^s$ $(r=0,1,\ldots;\ s=0,1)$. (***Note*** here $\phi^r(\xi)$ denotes $r$ power of $\phi(\xi)$ and $(\sqrt{R+\phi^2(\xi)})^s$ denotes $s$ power of $\sqrt{R+\phi^2(\xi)}$ ) to zero, we obtain a set of over-determined partial differential equations with regard to differential functions $a_i$, $b_i$, $k_i$, $A_j$, $B_j$, $K_j$ $(i=0,\ldots,m;j=0,\ldots,n)$ and $\xi$.

***Step 3*** Solving the over-determined partial differential equations by use of the PDE tools package of *Maple*, we would end up with the explicit expressions for $a_i$, $b_i$, $k_i$, $A_j$, $B_j$, $K_j$ $(i=0,\ldots,m;j=0,\ldots,n)$ and $\xi$, or the constrains among them.

***Step 4*** It is well known that the general solutions of Riccati equation (4) are

$$\phi(\xi)=\begin{cases}-\sqrt{-R}\tanh(\sqrt{-R}\,\xi)\,, & R<0\,,\\ -\sqrt{-R}\coth(\sqrt{-R}\,\xi)\,, & R<0\,,\\ \sqrt{R}\tan(\sqrt{R}\,\xi)\,, & R>0\,,\\ -\sqrt{R}\cot(\sqrt{R}\,\xi)\,, & R>0\,,\\ -1/\xi\,, & R=0\,.\end{cases}\tag{5}$$

Thus according to Eqs. (3), (5) and the conclusions in ***Step 3***, the soliton-like solutions of Eqs. (2) can be obtained.

For the generalization of the ansätz, naturally more complicated computation is expected than ever before. Even if the availability of computer symbolic systems like ***Maple*** or ***Mathematica*** allows us to perform the complicated and tedious algebraic calculation and differential calculation on a computer, in general, it is very difficult, sometime impossible, to solve the set of over-determined partial differential equations in ***Step 3***. As the calculation goes on, in order to drastically simplify the work or make the work feasible, we often choose special function forms for $a_i$, $b_i$, $k_i$, $A_j$, $B_j$, $K_j$ $(i=0,\ldots,m;j=0,\ldots,n)$ and $\xi$ on a trial-and-error basis.

## 3 The (2+1)-Dimensional Breaking Soliton Equation

In this section, by use the generalized expansion method of Riccati equation, we consider the (2+1)-dimensional breaking soliton equation, i.e., Eqs. (1).[9,10] By balancing the highest-order contributions from both the linear and nonlinear terms in Eqs. (1), we obtain $m=2$, $n=2$ in Eq. (3). Therefore we assume the solutions of Eqs. (1) in the form

$$u(x,y,t)=a_0+a_1\phi(\xi)+a_2\phi(\xi)+b_1\sqrt{R+\phi^2(\xi)}+b_2\phi(\xi)\sqrt{R+\phi^2(\xi)}+k_1\phi^{-1}(\xi)+k_2\phi^{-2}(\xi)\,,\tag{6}$$

$$v(x,y,t)=A_0+A_1\phi(\xi)+A_2\phi^2(\xi)+B_1\sqrt{R+\phi^2(\xi)}+B_2\phi(\xi)\sqrt{R+\phi^2(\xi)}+K_1\phi^{-1}(\xi)+K_2\phi^{-2}(\xi)\,,\tag{7}$$

where $a_i=a_i(y,t)$ $(i=0,1,2)$, $b_i=b_i(y,t)$ $(i=1,2)$, $k_i=k_i(y,t)$ $(i=1,2)$, $A_i=A_i(y,t)$ $(i=0,1,2)$, $B_i=B_i(y,t)$ $(i=1,2)$, $K_i=K_i(y,t)$ $(i=1,2)$ and $\xi=xp+q$ ($p$ is a constant and $q=q(y,t)$) are all differential functions, and $\phi(\xi)$ satisfies Eq. (4).

Substituting Eqs. (6) and (7) along with Eq. (4) into Eq. (1), multiplying $\phi^5(\xi)\sqrt{R+\phi(\xi)^2}$ and $\phi^3(\xi)$ in the first equation and the second equation, respectively, then setting the coefficients of $\phi^s(\xi)(R+\phi(\xi)^2)^{r/2}$ $(r=0,1;\ s=0,1,2,\ldots)$ to zero in the obtained system of partial differential equation, we can deduce the following set of over-determined partial differentiable equations with respect to the unknown differentiable functions $a_0$, $a_1$, $a_2$, $b_1$, $b_2$, $k_1$, $k_2$, $A_0$, $A_1$, $A_2$, $B_1$, $B_2$, $K_1$, $K_2$, and $q$ (***Note*** in this paper, $a_{iy}=\partial a_i(y,t)/\partial y$, and so on.)

$$-4bpk_1A_0-4bpa_0K_1-4bpk_2A_1+4bpb_1B_2R^2+4bpa_2K_1R+4bpb_2B_1R^2+a_{0t}+4bpa_1RA_0+4bpa_0A_1R$$

$$+2bp^2k_{2y} - q_tk_1 + 4bpk_1A_2R + 2bp^2q_ya_1R^2 - 2bp^2q_yk_1R + 2bp^2a_{2y}R^2 + q_ta_1R - 4bpa_1K_2 = 0\,, \tag{8}$$

$$q_ya_1 - pA_1 + a_{2y} = 0\,, \tag{9}$$

$$2pK_2 + k_{1y} - 2q_yk_2 = 0\,, \tag{10}$$

$$pK_1R + k_{2y} - q_yk_1R = 0\,, \tag{11}$$

$$-6bpR(pq_yk_1R^2 - pk_{2y}R + 2K_1k_2 + 2k_1K_2) = 0\,, \tag{12}$$

$$-2pA_2R + a_{1y} + 2q_ya_2R = 0\,, \tag{13}$$

$$-12bpR(b_1K_2 + B_1k_2) = 0\,, \tag{14}$$

$$-pB_2R + b_{1y} + q_yb_2R = 0\,, \tag{15}$$

$$-2pB_2 + 2q_yb_2 = 0\,, \tag{16}$$

$$4bpb_1A_0R - 4bpk_2B_1 - 4bpb_1K_2 + 5bp^2b_{2y}R^2 + 8bpb_1A_2R^2 + 8bpa_1B_2R^2 + 5bp^2q_yb_1R^2$$
$$+8bpa_2R^2B_1 + q_tb_1R + 8bpb_2A_1R^2 + b_{2t}R + 4bpb_2K_1R + 4bpk_1B_2R + 4bpa_0B_1R = 0\,, \tag{17}$$

$$6bp(pb_1q_y + pb_{2y} + 2a_1B_2 + 2B_1a_2 + 2A_2b_1 + 2A_1b_2) = 0\,, \tag{18}$$

$$12bpa_1B_1R + b_{1t} + 3bp^2b_{1y}R + 12bpb_2A_2R^2 + 33bp^2q_yb_2R^2$$
$$+12bpb_2A_0R + 12bpb_1A_1R + 12bpa_0B_2R + 12bpa_2B_2R^2 + 3q_tb_2R = 0\,, \tag{19}$$

$$-4bpR^2(B_1k_1 + b_1K_1 + K_2b_2 + B_2k_2) = 0\,, \tag{20}$$

$$R(5bp^2q_yb_2R^2 + 4bpa_1B_1R + q_tb_2R + 4bpa_0B_2R + 4bpb_2A_0R$$
$$+bp^2b_{1y}R + 4bpb_1A_1R - 4bpb_2K_2 + b_{1t} - 4bpk_2B_2 - 4bpb_1K_1 - 4bpk_1B_1) = 0\,, \tag{21}$$

$$4bpa_0B_1 + 4bpb_1A_0 + q_tb_1 + 20bpa_1B_2R + 11bp^2b_{2y}R + b_{2t}$$
$$+20bpa_2B_1R + 4bpb_2K_1 + 20bpb_1A_2R + 4bpk_1B_2 + 20bpb_2A_1R + 11bp^2q_yb_1R = 0\,, \tag{22}$$

$$8bpa_1B_1 + 2bp^2b_{1y} + 8bpb_2A_0 + 28bpa_2B_2R + 52bp^2q_yb_2R + 8bpb_1A_1 + 28bpb_2A_2R + 2q_tb_2 + 8bpa_0B_2 = 0\,, \tag{23}$$

$$8bp(3b_2q_yp + 2B_2a_2 + 2A_2b_2) = 0\,, \tag{24}$$

$$-4bpa_1K_2R - 8bp^2q_yk_1R^2 - q_tk_1R - 12bpk_2K_1 - 4bpa_0K_1R$$
$$+8bp^2k_{2y}R + k_{2t} - 12bpk_1K_2 - 4bpk_2A_1R - 4bpk_1RA_0 = 0\,, \tag{25}$$

$$8bpb_1B_1R + 16bp^2q_ya_2R^2 + a_{1t} + 8bpa_0A_2R + 8bpb_2B_2R^2 + 8bpa_2RA_0 + 8bpa_1A_1R + 2q_ta_2R + 2bp^2a_{1y}R = 0\,, \tag{26}$$

$$4bpa_2K_1 + diff(a_2,t) + 12bpa_1A_2R + 4bpa_1A_0 + 16bpb_1B_2R + 16bpb_2B_1R$$
$$+8bp^2q_ya_1R + 12bpa_2A_1R + 8bp^2a_{2y}R + 4bpk_1A_2 + 4bpa_0A_1 + q_ta_1 = 0\,, \tag{27}$$

$$8bpb_1B_1 + 24bpb_2B_2R + 8bpa_0A_2 + 2q_ta_2 + 16bpa_2A_2R + 2bp^2a_{1y} + 8bpa_1A_1 + 40bp^2q_ya_2R + 8bpa_2A_0 = 0\,, \tag{28}$$

$$6bp(pa_{2y} + pq_ya_1 + 2B_1b_2 + 2B_2b_1 + 2A_1a_2 + 2a_1A_2) = 0\,, \tag{29}$$

$$-8bpa_0K_2R - 8bpk_2RA_0 - 40bp^2q_yk_2R^2 - 8bpk_1K_1R - 2q_tk_2R + 2bp^2k_{1y}R^2 - 16bpk_2K_2 = 0\,, \tag{30}$$

$$2bp^2k_{1y}R - 16bp^2q_yk_2R - 8bpk_2A_0 - 8bpk_1K_1 - 2q_tk_2 - 8bpa_0K_2 + k_{1t} = 0\,, \tag{31}$$

$$-8bpR^2(b_1K_2 + B_1k_2) = 0\,, \tag{32}$$

$$8bp(3q_ya_2p + 2a_2A_2 + 2B_2b_2) = 0\,, \tag{33}$$

$$-8bpk_2R(3q_yR^2p + 2K_2) = 0\,, \tag{34}$$

$$a_{0y} - q_yk_1 + q_ya_1R - pA_1R + pK_1 = 0\,, \tag{35}$$

$$-2pA_2 + 2q_ya_2 = 0\,, \tag{36}$$

$$-2R(q_yk_2 - pK_2) = 0\,, \tag{37}$$

$$-pB_1 + b_{2y} + q_yb_1 = 0\,. \tag{38}$$

Using the powerful PDE tools package of ***Maple***, solving the set of partial differential equations (8) $\sim$ (38), we can obtain the following results. (***Note*** in the rest of this paper, $q(y,t)$ denotes arbitrary function with respect to $y$, $t$, and $C_1$, $C_2$ are arbitrary constants).

***Case 1*** $a_1 = b_1 = k_1 = k_2 = b_2 = A_1 = B_1 = B_2 = K_1 = K_2 = 0\,,$

$$a_0 = -2Rp^2\,, \quad a_2 = -\frac{3}{2}p^2\,, \quad A_0 = -\frac{q_t}{4bp}\,, \quad A_2 = -\frac{3}{2}pq_y\,, \quad q = q(y,t)\,. \tag{39}$$

***Case 2*** $a_1 = b_1 = b_2 = k_1 = k_2 = A_1 = B_1 = B_2 = K_1 = K_2 = 0\,,$

$$a_0=-\frac{11}{10}Rp^2\,,\quad a_2=-\frac{3}{2}p^2\,,\quad A_0=-\frac{18bp^2q_yR+5q_t}{20bp}\,,\quad A_2=-\frac{3}{2}pq_y\,,\quad q=q(y,t)\,. \tag{40}$$

***Case 3*** $a_1=b_1=b_2=k_1=k_2=A_1=B_1=B_2=K_1=K_2=0\,,$

$$a_0=C_1\,,\quad a_2=-\frac{3}{2}p^2\,,\quad A_0=-\frac{8bp^2q_yR+q_t+4bC_1q_y}{4bp}\,,\quad A_2=-\frac{3}{2}pq_y\,,\quad q=q(y,t)\,. \tag{41}$$

***Case 4*** $a_1=a_2=b_1=b_2=k_1=A_1=A_2=B_1=B_2=K_1=0\,,$

$$a_0=C_1\,,\quad k_2=-\frac{3}{2}p^2R^2\,,\quad A_0=-\frac{8bp^2q_yR+q_t+4bC_1q_y}{4bp}\,,\quad K_2=-\frac{3}{2}q_yR^2p\,,\quad q=q(y,t)\,. \tag{42}$$

***Case 5*** $a_1=b_1=k_1=k_2=A_1=B_1=K_1=K_2=0\,,\quad a_0=-\frac{5}{4}Rp^2\,,\quad a_2=-\frac{3}{4}p^2\,,\quad b_2=\frac{3}{4}p^2\,,$

$$A_0=-\frac{q_t}{4bp}\,,\quad A_2=-\frac{3}{4}pq_y\,,\quad B_2=\frac{3}{4}pq_y\,,\quad q=q(y,t)\,. \tag{43}$$

***Case 6*** $a_1=b_1=k_1=k_2=A_1=B_1=K_1=K_2=0\,,\quad a_0=-\frac{5}{4}Rp^2\,,\quad a_2=-\frac{3}{4}p^2\,,\quad b_2=-\frac{3}{4}p^2\,,$

$$A_0=-\frac{q_t}{4bp}\,,\quad A_2=-\frac{3}{4}pq_y\,,\quad B_2=-\frac{3}{4}pq_y\,,\quad q=q(y,t)\,. \tag{44}$$

***Case 7*** $a_1=b_1=k_1=k_2=A_1=B_1=K_1=K_2=0\,,\quad a_0=C_2\,,\quad a_2=-\frac{3}{4}p^2\,,\quad b_2=\frac{3}{4}p^2\,,$

$$A_0=-\frac{5bp^2q_yR+q_t+4bC_2q_y}{4bp}\,,\quad A_2=-\frac{3}{4}pq_y\,,\quad B_2=\frac{3}{4}pq_y\,,\quad q=q(y,t)\,. \tag{45}$$

***Case 8*** $a_1=b_1=k_1=k_2=A_1=B_1=K_1=K_2=0\,,\quad a_0=C_1\,,\quad a_2=-\frac{3}{4}p^2\,,\quad b_2=-\frac{3}{4}p^2\,,$

$$A_0=-\frac{5bp^2q_yR+q_t+4bC_1q_y}{4bp}\,,\quad A_2=-\frac{3}{4}pq_y\,,\quad B_2=-\frac{3}{4}pq_y\,,\quad q=q(y,t)\,. \tag{46}$$

***Case 9*** $a_1=b_1=b_2=k_1=A_1=B_1=B_2=K_1=0\,,a_0=C_1\,,\quad a_2=-\frac{3}{2}p^2\,,\quad k_2=-\frac{3}{2}p^2R^2\,,$

$$A_0=-\frac{8bp^2q_yR+q_t+4bC_1q_y}{4bp}\,,\quad A_2=-\frac{3}{2}pq_y\,,\quad K_2=-\frac{3}{2}q_yR^2p\,,\quad q=q(y,t)\,. \tag{47}$$

From Eqs. (5) $\sim$ (7) and (39) $\sim$ (47), we can obtain the following solutions for the (2+1)-dimensional breaking soliton equation.

***Type 1*** From Case 1, we can obtain the following solutions:

$$u_{11}=-2Rp^2+\frac{3}{2}p^2R\tanh^2[\sqrt{-R}\,(xp+q(y,t))]\,,\qquad v_{11}=-\frac{q_t}{4bp}+\frac{3}{2}pq_yR\tanh^2[\sqrt{-R}\,(xp+q(y,t))]\,,\quad R<0\,, \tag{48}$$

$$u_{12}=-2Rp^2+\frac{3}{2}p^2R\coth^2[\sqrt{-R}\,(xp+q(y,t))]\,,\qquad v_{12}=-\frac{q_t}{4bp}+\frac{3}{2}pq_yR\coth^2[\sqrt{-R}\,(xp+q(y,t))]\,,\quad R<0\,, \tag{49}$$

$$u_{13}=-2Rp^2-\frac{3}{2}p^2R\tan^2[\sqrt{R}\,(xp+q(y,t))]\,,\qquad v_{13}=-\frac{q_t}{4bp}-\frac{3}{2}pq_yR\tan^2[\sqrt{R}\,(xp+q(y,t))]\,,\quad R>0\,, \tag{50}$$

$$u_{14}=-2Rp^2-\frac{3}{2}p^2R\cot^2[\sqrt{R}\,(xp+q(y,t))]\,,\qquad v_{14}=-\frac{q_t}{4bp}-\frac{3}{2}pq_yR\cot^2[\sqrt{R}\,(xp+q(y,t))]\,,\quad R>0\,. \tag{51}$$

***Type 2*** From Case 2, we can obtain the following solutions:

$$\begin{aligned}u_{21}&=-\frac{11}{10}Rp^2+\frac{3}{2}p^2R\tanh^2[\sqrt{-R}\,(xp+q(y,t))]\,,\\ v_{21}&=-\frac{18bp^2q_yR+5q_t}{20bp}+\frac{3}{2}pq_yR\tanh^2[\sqrt{-R}\,(xp+q(y,t))]\,,\quad R<0\,,\end{aligned} \tag{52}$$

$$\begin{aligned}u_{22}&=-\frac{11}{10}Rp^2+\frac{3}{2}p^2R\coth^2[\sqrt{-R}\,(xp+q(y,t))]\,,\\ v_{22}&=-\frac{18bp^2q_yR+5q_t}{20bp}+\frac{3}{2}pq_yR\coth^2[\sqrt{-R}\,(xp+q(y,t))]\,,\quad R<0\,,\end{aligned} \tag{53}$$

$$\begin{aligned}u_{23}&=-\frac{11}{10}Rp^2-\frac{3}{2}p^2R\tan^2[\sqrt{R}\,(xp+q(y,t))]\,,\\ v_{23}&=-\frac{18bp^2q_yR+5q_t}{20bp}-\frac{3}{2}pq_yR\tan^2[\sqrt{R}\,(xp+q(y,t))]\,,\quad R>0\,,\end{aligned} \tag{54}$$

$$u_{24} = -\frac{11}{10}Rp^2 - \frac{3}{2}p^2 R\cot^2[\sqrt{R}\,(xp + q(y,t))]\,,$$
$$v_{24} = -\frac{18bp^2 q_y R + 5q_t}{20bp} - \frac{3}{2}pq_y R\cot^2[\sqrt{R}\,(xp + q(y,t))]\,, \quad R > 0\,. \tag{55}$$

***Type 3*** From Cases 3 and 4, we can obtain the following solutions:

$$u_{21} = C_1 + \frac{3}{2}p^2 R\tanh^2[\sqrt{-R}\,(xp + q(y,t))]\,,$$
$$v_{21} = -\frac{8bp^2 q_y R + q_t + 4bC_1 q_y}{4bp} + \frac{3}{2}pq_y R\tanh^2[\sqrt{-R}\,(xp + q(y,t))]\,, \quad R < 0\,, \tag{56}$$
$$u_{22} = C_1 + \frac{3}{2}p^2 R\coth^2[\sqrt{-R}\,(xp + q(y,t))]\,,$$
$$v_{22} = -\frac{8bp^2 q_y R + q_t + 4bC_1 q_y}{4bp} + \frac{3}{2}pq_y R\coth^2[\sqrt{-R}\,(xp + q(y,t))]\,, \quad R < 0\,, \tag{57}$$
$$u_{23} = C_1 - \frac{3}{2}p^2 R\tan^2[\sqrt{R}\,(xp + q(y,t))]\,,$$
$$v_{23} = -\frac{8bp^2 q_y R + q_t + 4bC_1 q_y}{4bp} - \frac{3}{2}pq_y R\tan^2[\sqrt{R}\,(xp + q(y,t))]\,, \quad R > 0\,, \tag{58}$$
$$u_{24} = C_1 - \frac{3}{2}p^2 R\cot^2[\sqrt{R}\,(xp + q(y,t))]\,,$$
$$v_{24} = -\frac{8bp^2 q_y R + q_t + 4bC_1 q_y}{4bp} - \frac{3}{2}pq_y R\cot^2[\sqrt{R}\,(xp + q(y,t))]\,, \quad R > 0\,. \tag{59}$$

***Type 4*** From Cases 5 and 6, we can obtain the following solutions:

$$u_{41} = -\frac{5}{4}Rp^2 + \frac{3}{4}p^2 R\big[\tanh^2(\sqrt{-R}\,\xi) \pm \mathrm{i}\tanh(\sqrt{-R}\,\xi)\,\mathrm{sech}\,(\sqrt{-R}\,\xi)\big]\,,$$
$$v_{41} = -\frac{q_t}{4bp} + \frac{3}{4}pq_y R\big[\tanh^2(\sqrt{-R}\,\xi) \pm \mathrm{i}\tanh(\sqrt{-R}\,\xi)\,\mathrm{sech}\,(\sqrt{-R}\,\xi)\big]\,, \quad R < 0\,, \tag{60}$$
$$u_{42} = -\frac{5}{4}Rp^2 + \frac{3}{4}p^2 R\big[\coth^2(\sqrt{-R}\,\xi) \pm \coth(\sqrt{-R}\,\xi)\mathrm{csch}(\sqrt{-R}\,\xi)\big]\,,$$
$$v_{42} = -\frac{q_t}{4bp} + \frac{3}{4}pq_y R\big[\coth^2(\sqrt{-R}\,\xi) \pm \coth(\sqrt{-R}\,\xi)\mathrm{csch}(\sqrt{-R}\,\xi)\big]\,, \quad R < 0\,, \tag{61}$$
$$u_{43} = -\frac{5}{4}Rp^2 - \frac{3}{4}p^2 R\big[\tan^2(\sqrt{R}\,\xi) \pm \tan(\sqrt{R}\,\xi)\sec(\sqrt{R}\,\xi)\big]\,,$$
$$v_{43} = -\frac{q_t}{4bp} - \frac{3}{4}pq_y R\big[\tan^2(\sqrt{R}\,\xi) \pm \tan\sqrt{R}\,\xi)\sec(\sqrt{R}\,\xi)\big]\,, \quad R > 0\,, \tag{62}$$
$$u_{44} = -\frac{5}{4}Rp^2 - \frac{3}{4}p^2 R\big[\cot^2(\sqrt{R}\,\xi) \pm \cot(\sqrt{R}\,\xi)\csc(\sqrt{R}\,\xi)\big]\,,$$
$$v_{44} = -\frac{q_t}{4bp} - \frac{3}{4}pq_y R\big[\cot^2(\sqrt{R}\,\xi) \pm \cot(\sqrt{R}\,\xi)\csc(\sqrt{R}\,\xi)\big]\,, \quad R > 0\,, \tag{63}$$

where $\xi = xp + q(y,t)$.

***Type 5*** From Cases 7 and 8, we can obtain the following solutions:

$$u_{51} = C_2 + \frac{3}{4}p^2 R\big[\tanh^2(\sqrt{-R}\,\xi) \pm \mathrm{i}\tanh(\sqrt{-R}\,\xi)\,\mathrm{sech}\,(\sqrt{-R}\,\xi)\big]\,,$$
$$v_{51} = -\frac{5bp^2 q_y R + q_t + 4bC_2 q_y}{4bp} + \frac{3}{4}pq_y R\big[\tanh^2(\sqrt{-R}\,\xi) \pm \mathrm{i}\tanh(\sqrt{-R}\,\xi)\,\mathrm{sech}\,(\sqrt{-R}\,\xi)\big]\,, \quad R < 0\,, \tag{64}$$
$$u_{52} = C_2 + \frac{3}{4}p^2 R\big[\coth^2(\sqrt{-R}\,\xi) \pm \coth(\sqrt{-R}\,\xi)\mathrm{csch}(\sqrt{-R}\,\xi)\big]\,,$$
$$v_{52} = -\frac{5bp^2 q_y R + q_t + 4bC_2 q_y}{4bp} - \frac{3}{4}pq_y R\big[\coth^2(\sqrt{-R}\,\xi) \pm \coth(\sqrt{-R}\,\xi)\mathrm{csch}(\sqrt{-R}\,\xi)\big]\,, \quad R < 0\,, \tag{65}$$
$$u_{53} = C_2 - \frac{3}{4}p^2 R\big[\tan^2(\sqrt{R}\,\xi) \pm \tan(\sqrt{R}\,\xi)\sec(\sqrt{R}\,\xi)\big]\,,$$
$$v_{53} = -\frac{5bp^2 q_y R + q_t + 4bC_2 q_y}{4bp} - \frac{3}{4}pq_y R\big[\tan^2(\sqrt{R}\,\xi) \pm \tan\sqrt{R}\,\xi)\sec(\sqrt{R}\,\xi)\big]\,, \quad R > 0\,, \tag{66}$$

$$u_{54} = C_2 - \frac{3}{4}p^2 R\big[\cot^2(\sqrt{R}\,\xi) \pm \cot(\sqrt{R}\,\xi)\csc(\sqrt{R}\,\xi)\big]\,,$$
$$v_{54} = -\frac{5bp^2 q_y R + q_t + 4bC_2 q_y}{4bp} - \frac{3}{4}pq_y R\big[\cot^2(\sqrt{R}\,\xi) \pm \cot(\sqrt{R}\,\xi)\csc(\sqrt{R}\,\xi)\big]\,, \quad R > 0\,, \tag{67}$$

where $\xi = xp + q(y,t)$. When setting $C_2 = C_1$ in the above solutions, we can obtain another set of solutions for Eq. (1).

***Type 6*** From Case 9, we can obtain the following solutions:

$$u_{61} = C_1 + \frac{3}{2}p^2\big[\tanh^2(\sqrt{-R}\,\xi) \pm \coth^2(\sqrt{-R}\,\xi)\big]\,,$$
$$v_{61} = -\frac{8bp^2 q_y R + q_t + 4bC_1 q_y}{4bp} + \frac{3}{2}pq_y R\big[\tanh^2(\sqrt{-R}\,\xi) \pm \coth^2(\sqrt{-R}\,\xi)\big]\,, \quad R < 0\,, \tag{68}$$
$$u_{62} = C_1 - \frac{3}{2}p^2\big[\tan^2(\sqrt{R}\,\xi) \pm \cot^2(\sqrt{R}\,\xi)\big]\,,$$
$$v_{62} = -\frac{8bp^2 q_y R + q_t + 4bC_1 q_y}{4bp} - \frac{3}{2}pq_y R\big[\tan^2(\sqrt{R}\,\xi) \pm \cot^2(\sqrt{R}\,\xi)\big]\,, \quad R < 0\,, \tag{69}$$

where $\xi = xp + q(y,t)$.

## 4 Conclusions

In summary, based on the computerized symbolic computation, by introducing a new more general ansätz than the ansätz in the extended tanh-function method, modified extended tanh-function method, and generalized hyperbolic-function method, we have proposed a generalized expansion method of Riccati equation for searching for exact solutions of NEEs and implemented in computerized symbolic system *Maple*. Making use of our method and with the aid of *Maple*, we study the (2+1)-dimensional breaking soliton equation and obtain new families of the exact solutions. In our obtained exact solutions the restriction on $\xi(x,y,t)$ as merely a linear function $x,y,t$ and the restriction on the coefficients $a_i$, $b_i$, $k_i$, $A_j$, $B_j$, $K_j$ $(i = 0,\ldots,m; j = 0,\ldots,n)$ and $\xi$ as constants are removed, and with no extra efforts, the singular solitonic solution and triangular function solutions could be obtained. To make the work feasible, how to choose the forms for $a_i$, $b_i$, $k_i$, $A_j$, $B_j$, $K_j$ $(i = 0,\ldots,m; j = 0,\ldots,n)$ and $\xi$ in the ansätz would be the key step in the computation of our method. The method proposed in this paper for the (2+1)-dimensional breaking soliton equation may be extended to find exact soliton-like solutions of other NEEs.

## Acknowledgment

The authors would like to express their sincere thanks to the referees for their helpful suggestion.

## References

[1] M. Boiti, J.J.P. Leon, L. Martina, and F. Pempinelli, Phys. Lett. **A132** (1998) 432.
[2] A.S. Fokas and P.M. Santini, Physica **D44** (1990) 99.
[3] B.B. Kadomtsov and V.I. Petviashivili, Sov. Phys. Dokl. **15** (1970) 539.
[4] A. Davey and K. Stewartson, Proc. R. Soc. (London) A **338** (1974) 17.
[5] S.Y. Lou and J.Z. Lu, J. Phys. **A29** (1996) 4029.
[6] S.P. Novikov and A.P. Veselov, Physica **D18** (1986) 267.
[7] S.Y. Lou, Phys. Lett. **A277** (2000) 94.
[8] S.Y. Lou and H.Y. Ruan, J. Phys. **A34** (2001) 305.
[9] R. Radha and M. Lakshamanan, Phys. Lett. **A197** (1995) 7.
[10] H.Y. Ruan, J. Phys. Soc. Jpn. **71** (2002) 453.
[11] S.Y. Lou, Phys. Scr. **65** (2000) 7.
[12] X.Y. Tang, S.Y. Lou, and Y. Zhang, Phys. Rev. **E66** (2002) 046601.
[13] S.Y. Lou, X.Y. Tang, and J. Lin, Commun. Theor. Phys. (Beijing, China) **36** (2001) 145.
[14] E.J. Parkes and B.R. Duffy, Phys. Lett. **A229** (1997) 217.
[15] E.J. Parkes and B.R. Duffy, Comput. Phys. Commun. **98** (1996) 288.
[16] E. Fan, J. Zhang, and Benny Y.C. Hon, Phys. Lett. **A291** (2001) 376.
[17] Z.Y. Yan, Phys. Lett. **A292** (2001) 100.
[18] Y. Chen, B. Li, and H.Q. Zhang, Commun. Theor. Phys. (Beijing, China) **38** (2002) 261.
[19] B. Li, Y. Chen, and H.Q. Zhang, J. Phys. A: Math. Gen. **35** (2002) 8253.
[20] B. Li, Y. Chen, and H.Q. Zhang, Chaos, Solitons and Fractals **15** (2003) 647.
[21] E. Fan, Phys. Lett. **A277** (2000) 212.
[22] S.A. Elwakil, S.K. El-labany, M.A. Zahran, and R. Sabry, Phys. Lett. **A299** (2002) 179.
[23] Y.T. Gao and B. Tian, Comput. Phys. Commun. **133** (2001) 158.
[24] O.I. Bogoyavlensky, Izv. Akad. Nauk SSSR, Ser. Mat. **53** (1989) 243; ibid. **54** (1990) 123.

# Symbolic Computation and Construction of Soliton-Like Solutions to the(2+1)-Dimensional Breaking Soliton Equation

作者： CHEN Yong, LI Biao, ZHANG Hong-Qing
作者单位： Department of Applied Mathematics, Dalian University of Technology, Dalian 116024, China
刊名： 理论物理通讯(英文版) ISTIC SCI
英文刊名： COMMUNICATIONS IN THEORETICAL PHYSICS
年，卷(期)： 2003,40(8)


## 参考文献(25条)

1. S P Novikov;A P Veselov 查看详情 1986

2. S Y Lou;J Z Lu 查看详情 1996

3. A Davey;K Stewartson 查看详情 1974

4. M Boiti;J J P Leon;L Martina;F Pempinelli 查看详情 1998

5. R Radha;M Lakshamanan DROMION LIKE STRUCTURES IN THE (2+1)-DIMENSIONAL BREAKING SOLITON EQUATION[外文期刊] 1995(1)

6. S Y Lou;H Y Ruan 查看详情 2001

7. S Y Lou 查看详情 2000

8. B·B· Kadomtsov;V.I. Petviashivili 查看详情 1970(15)

9. 查看详情 1990(54)

10. O I Bogoyavlensky;Izv Akad 查看详情 1989

11. Y T Gao;B Tian 查看详情 2001(133)

12. S A Elwakil;S K El-labany;M A Zahran;R Sabry Modified extended tanh-function method for solving nonlinear partial differential equations[外文期刊] 2002(2/3)

13. E Fan 查看详情 2000

14. B Li;Y Chen;H Q Zhang 查看详情 2003(15)

15. A S Fokas;P M Santini 查看详情[外文期刊] 1990

16. B Li;Y Chen;H Q Zhang 查看详情 2002(35)

17. Y CHEN;B Li;H Q Zhang New Explicit Solitary Wave Solutions and Periodic Wave Solutions for the Generalized Coupled Hirota-Satsuma KdV System[期刊论文]-Communications in Theoretical Physics 2002(38)

18. Z Y YAN 查看详情 2001

19. E Fan;J Zhang;Benny Y C Hon 查看详情 2001

20. E J Parkes;B R Duffy AN AUTOMATED TANH-FUNCTION METHOD FOR FINDING SOLITARY WAVE SOLUTIONS TO NON-LINEAR EVOLUTION EQUATIONS[外文期刊] 1996(98)

21. E J Parkes;B R Duffy 查看详情 1997

22. S Y Lou;X Y Tang;J Lin Exact Solutions of the Coupled KdV System via a Formally Variable Separation Approach[期刊论文]-Commun. Theor. Phys.(Beijing China) 2001(36)

23. X Y Tang;S Y Lou;Y Zhang 查看详情 2002

24. S Y Lou 查看详情 2000(65)

25. H Y Ruan 查看详情 2002(71)

## 本文读者也读过(10条)

Soliton Equation[期刊论文]-理论物理通讯（英文版）2006,45(3)

6. ZHAO Hong.BAI Cheng-Lin Abundant Multisoliton Structure of (3+1)-Dimensional Breaking Soliton Equation[期刊论文]-理论物理通讯(英文版)2004,42(10)

7. XU Gui-Qiong.LI Zhi-Bin Travelling Wave Solutions to a Special Type of Nonlinear Evolution Equation[期刊论文]-理论物理通讯(英文版)2003,39(1)

8. 欢迎订阅《地球物理学报》(中、英文版)[期刊论文]-地球物理学报2003,46(6)

9. LI Biao.CHEN Yong A Generalized Method and Exact Solutions in Bose-Einstein Condensates in an Expulsive Parabolic Potential[期刊论文]-理论物理通讯（英文版）2007,48(9)

10. ZHENG Chun-Long.ZHANG Jie-Fang Exact Excitation and Abundant Localized Coherent Soliton Structures of (2+1)-Dimensional Perturbed AKNS System[期刊论文]-理论物理通讯(英文版)2003,39(1)

## 引证文献(17条)

1.郭鹏云.陈向华 (2+1)维Boussinesq方程的精确解[期刊论文]-阴山学刊（自然科学版） 2010(1)

2.套格图桑.斯仁道尔吉 (2+1)维破裂孤子方程新的精确孤立波解[期刊论文]-内蒙古大学学报（自然科学版） 2008(2)

3.CAO Li-Na.WANG Deng-Shan.CHEN Lan-Xin Symbolic Computation and q-Deformed Function Solutions of (2+1)-Dimensional Breaking Soliton Equation[期刊论文]-理论物理通讯（英文版） 2007(2)

4.CHEN Yong.LI Biao.ZHENG Yu Exact Analytical Solutions in Bose-Einstein Condensates with Time-Dependent Atomic Scattering Length[期刊论文]-理论物理通讯（英文版） 2007(1)

5.徐传友 (2+1)维破裂孤子方程组的精确解[期刊论文]-阜阳师范学院学报(自然科学版) 2007(4)

6.扎其劳.斯仁道尔吉 (2+1)维破裂孤子方程组的准确周期解[期刊论文]-内蒙古师范大学学报（自然科学汉文版） 2006(2)

7.ZHANG Yuan-Yuan.WANG Qi.ZHANG Hong-Qing Further Extended Jacobi Elliptic Function Rational Expansion Method and New Families of Jacobi Elliptic Function Solutions to (2+1)-Dimensional Dispersive Long Wave Equation[期刊论文]-理论物理通讯（英文版） 2006(2)

8.ZHANG Sheng.XIA Tie-Cheng Symbolic Computation and New Families of Exact Non-travelling Wave Solutions of (2+1)-dimensional Broer-Kaup Equations[期刊论文]-理论物理通讯（英文版） 2006(6)

9.GE Jian-Ya.WANG Rui-Min.DAI Chao-Qing.ZHANG Jie-Fang Variable-Coefficient Mapping Method Based on Elliptical Equation and Exact Solutions to Nonlinear Schr(o)dinger Equations with Variable Coefficient[期刊论文]-理论物理通讯（英文版） 2006(10)

10.ZHANG Yuan-Yuan.ZHENG Ying.ZHANG Hong-Qing New Complexiton Solutions of (2+1)-Dimensional Nizhnik-Novikov-Veselov Equations[期刊论文]-理论物理通讯（英文版） 2006(9)

11.ZHU Jia-Min.MA Zheng-Yi New Exact Solutions to (2+1)-Dimensional Variable Coefficients Broer-Kaup Equations[期刊论文]-理论物理通讯（英文版） 2006(9)

12.JIAO Xiao-Yu.WANG Jin-Huan.ZHANG Hong-Qing An Extended Method for Constructing Travelling Wave Solutions to Nonlinear Partial Differential Equations[期刊论文]-理论物理通讯（英文版） 2005(9)

13.郭鹏云.斯仁道尔吉 破裂孤子方程和浅水波方程的新类孤子解[期刊论文]-内蒙古师范大学学报（自然科学汉文版） 2005(2)

14.套格图桑.斯仁道尔吉 KdV方程和KP方程的新的精确孤立波解[期刊论文]-内蒙古师范大学学报（自然科学汉文版） 2005(2)

15.ZHI Hong-Yan.UZhuo-Sheng.ZHANG Hong-Qing New Exact Solutions to (2+1)-Dimensional Dispersive Long Wave Equations[期刊论文]-理论物理通讯(英文版) 2004(12)

16.CHEN Yong.WANG Qi.LI Biao A Series of Soliton-like and Double-like Periodic Solutions of a (2+1)-Dimensional Asymmetric Nizhnik-Novikov-Vesselov Equation[期刊论文]-理论物理通讯（英文版） 2004(11)

17.李德生 若干非线性演化方程精确求解法的研究[学位论文]博士 2004