A Generalized Sub-Equation Expansion Method and Some Analytical Solutions to the Inhomogeneous Higher-Order Nonlinear Schrödinger Equation

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On the basis of symbolic computation a generalized sub-equation expansion method is presented for constructing some exact analytical solutions of nonlinear partial differential equations. To illustrate the validity of the method, we investigate the exact analytical solutions of the inhomogeneous high-order nonlinear Schrödinger equation (IHNLSE) including not only the group velocity dispersion, self-phase-modulation, but also various high-order effects, such as the third-order dispersion, self-steepening and self-frequency shift. As a result, a broad class of exact analytical solutions of the IHNLSE are obtained. From our results, many previous solutions of some nonlinear Schrödinger-type equations can be recovered by means of suitable selections of the arbitrary functions and arbitrary constants. With the aid of computer simulation, the abundant structure of bright and dark solitary wave solutions, combined-type solitary wave solutions, dispersion-managed solitary wave solutions, Jacobi elliptic function solutions and Weierstrass elliptic function solutions are shown by some figures.

Key words: Inhomogeneous High-Order NLS Equation; Elliptic Function Solutions; Solitary Wave Solutions.
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