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A Series of Soliton-like and Double-like Periodic Solutions of a (2+1)-Dimensional Asymmetric Nizhnik–Novikov–Vesselov Equation*

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Abstract *We generalize the algebraic method presented by Fan [J. Phys. A: Math. Gen.* **36** *(2003) 7009)] to uniformly construct a series of soliton-like solutions and double-like periodic solutions for nonlinear partial differential equations (NPDE). As an application of the method, we choose a (2+1)-dimensional asymmetric Nizhnik–Novikov–Vesselov equation and successfully construct new and more general solutions including a series of nontraveling wave and coefficient functions' soliton-like solutions, double-like periodic and trigonometric-like function solutions.*

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1 Introduction

The tanh method provides a straightforward and effective algorithm to obtain such particular solutions for a large number of nonlinear partial differential equations (NPDE). Recently, much research work has been concentrated on the various extensions and applications of tanh method,[1] such as the extended tanh-function method by Fan,[2] generalized hyperbolic-function method,[3] the improved tanh-function method by Yan,[4] modified tanh-function method by S.A. Elwakil, *et al.*,[5] generalized extended tanh-function method by Chen and Zheng,[6] generalized Riccati equation expansion method by Chen and Li,[7] further extended tanh-function method by Lv,[8] and so on.

Recently, Fan[9] developed a new algebraic method to seek more new solitary wave solutions of NPDEs that can be expressed as a polynomal in an element which satisfies a more general Riccati equation.[2] Compared with most of the existing tanh methods, the proposed method not only gives a unified formulation to construct various travelling wave solutions, but also provides a guideline to classify the various types of the travelling wave solutions according to the values of some parameters. More recently, by means of a more general ansatz, Chen and Wang[10] further developed this method and used it to construct more solutions in terms of special function of nonlinear evolution equations (NLEEs). On the other hand, E. Yomba[11] improved the method[6−8] and obtained some new soliton-like solutions for the (2+1)-dimensional dispersive long wave equation.

The present work is motivated by the desire to generalize the above work made in Refs. [9] $\sim$ [11] by proposing a more general ansatz so that it can be used to obtain more types and general formal solutions which contain not only the results obtained by using the method in Ref. [9] but also a series of nontraveling wave and coefficient functions' soliton-like solutions, double-like periodic solutions and triangular-like solutions for nonlinear partial differential equations, in which the restriction on $\xi(x,y,t)$ as merely a linear function of x, y, t and the restriction on the coefficients being constants are removed.

For illustration, we apply the generalized method to solve a (2+1)-dimensional asymmetric Nizhnik–Novikov–Vesselov equation and successfully construct new and more general solutions including a series of nontraveling wave and coefficient functions' soliton-like solutions, double-like periodic solutions, and triangular-like solutions.

Our paper is organized as follows. In Sec. 2, the detailed derivation of the generalized algebraic method will be given. The applications of the generalized method to (2+1)-dimensional asymmetric Nizhnik–Novikov–Vesselov equation are illustrated in Sec. 3. The conclusion is then given in Sec. 4.

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2 Summary of Generalized Method

In the following we would like to outline the main steps of our general method.

Step 1 For a given nonlinear partial differential equation (NPDE) system with some physical fields $u_i(x,y,t)$ $(i = 1,\ldots,n)$ in three variables x, y, t,

$$F_i(u_i, u_{it}, u_{ix}, u_{iy}, u_{itt}, u_{ixt}, u_{iyt}, u_{ixx}, u_{iyy}, u_{ixy}, \ldots) = 0\,, \tag{1}$$

we express the solutions of the NPDE by the new more general ansatz

$$u_i(\xi) = a_{i0} + \sum_{j=1}^{m_i}\left(a_{ij}\phi^j + b_{ij}\phi^{j-1}\sqrt{\sum_{\rho=0}^{r} h_\rho \phi^\rho}\,\right), \tag{2}$$

where m_i is an integer to be determined by balancing the highest-order derivative terms with the nonlinear terms in Eq. (1), the new variable $\phi = \phi(\xi)$ satisfies

$$\phi' = \frac{\mathrm{d}\phi}{\mathrm{d}\xi} = \sqrt{\sum_{\rho=0}^{r} h_\rho \phi^\rho}\ , \tag{3}$$

and $a_{i0} = a_{i0}(x,y,t)$, $a_{ij} = a_{ij}(x,y,t)$, $b_{ij} = b_{ij}(x,y,t)$, $(i = 1,2,\ldots;\ j = 1,2,\ldots,m_i)$ and $\xi = \xi(x,y,t)$ are all differentiable functions to be determined later. Here h_0, h_1, h_2, h_3, h_4 are constants.

Step 2 Substitute Eq. (2) into Eq. (1) along with Eq. (3) and then set all coefficients of $\phi^p\big(\sqrt{\sum_{\rho=0}^{r} h_\rho\phi^\rho}\,\big)^q$ $(q = 0,1;$ $p = 0,1,2,\ldots)$ to be zero to get an over-determined partial differential equations with respect to a_{i0}, a_{ij}, b_{ij} , $(i = 1,2,\ldots;\ j = 1,2,\ldots,m_i)$ and ξ.

Step 3 Solve the over-determined partial differential equations by use of *Maple*, we would end up with the explicit expressions for a_{i0}, a_{ij}, b_{ij} $(i = 1,2,\ldots;\ j = 1,2,\ldots,m_i)$ and ξ or the constraints among them.

Step 4 By using the results obtained in the above steps, we can derive a series of fundamental-like solutions such as polynomial-like, exponential-like, solitary-like wave, rational-like, triangular-like periodic, Jacobi and Weierstrass doubly-like periodic solutions. Because we are interested in solitary-like wave, Jacobi and Weierstrass doubly-like periodic solutions and tan-like and cot-like type solutions appearing in pairs with tanh-like and coth-like type solutions respectively, therefore polynomial-like, rational-like, triangular-like periodic solutions are omitted in this paper. By considering the different values of h_0, h_1, h_2, h_3, and h_4, equation (3) has many kinds of solitary-like wave, Jacobi and Weierstrass doubly-like periodic solutions, which are listed as follows.

(i) ***Solitary-like wave solutions***

(a) Bell shaped soliton-like solutions

$$\phi = \sqrt{-\frac{h_2}{h_4}}\ \mathrm{sech}\left(\sqrt{h_2}\,\xi\right), \quad h_0 = h_1 = h_3 = 0\,, \quad h_2 > 0\,, \quad h_4 < 0\,, \tag{4}$$

$$\phi = -\frac{h_2}{h_3}\,\mathrm{sech}^2\Big(\frac{\sqrt{h_2}}{2}\,\xi\Big), \qquad h_0 = h_1 = h_4 = 0\,, \quad h_2 > 0\,. \tag{5}$$

(b) Kink shaped soliton-like solutions

$$\phi = \sqrt{-\frac{h_2}{2h_4}}\ \tanh\Big(\sqrt{-\frac{h_2}{2}}\,\xi\Big), \quad h_0 = \frac{{h_2}^2}{4h_4}\,, \quad h_1 = h_3 = 0\,, \quad h_2 < 0\,, \quad h_4 > 0\,. \tag{6}$$

(c) Soliton-like solutions

$$\phi = \frac{h_2\mathrm{sech}^2(\sqrt{h_2}\,\xi/2)}{2\sqrt{h_2h_4}\tanh(\sqrt{h_2}\,\xi/2) - h_3}\,, \quad h_0 = h_1 = 0\,, \quad h_2 > 0\,. \tag{7}$$

(ii) ***Jacobi and Weierstrass doubly-like periodic solutions***

$$\phi = \sqrt{\frac{-h_2m^2}{h_4(2m^2-1)}}\ \mathrm{cn}\left(\sqrt{\frac{h_2}{2m^2-1}}\,\xi\right), \quad h_4 < 0\,, \quad h_2 > 0\,, \quad h_0 = \frac{h_2^2m^2(1-m^2)}{h_4(2m^2-1)^2}\,, \tag{8}$$

$$\phi = \sqrt{\frac{-m^2}{h_4(2-m^2)}}\ \mathrm{dn}\left(\sqrt{\frac{h_2}{2-m^2}}\,\xi\right), \quad h_4 < 0\,, \quad h_2 > 0\,, \quad h_0 = \frac{h_2^2(1-m^2)}{h_4(2-m^2)^2}\,, \tag{9}$$

$$\phi = \sqrt{\frac{-h_2 m^2}{h_4(m^2+1)}}\mathrm{sn}\Big(\sqrt{-\frac{h_2}{m^2+1}}\,\xi\Big)\,,\quad h_4>0\,,\quad h_2<0\,,\quad h_0=\frac{h_2^2 m^2}{h_4(m^2+1)^2}\,,\tag{10}$$

where m is a modulus.

$$\phi = \wp\Big(\frac{\sqrt{h_3}}{2}\,\xi, g_2, g_3\Big)\,,\quad h_2=0\,,\quad h_3>0\,,\tag{11}$$

where $g_2=-4h_1/h_3$ and $g_3=-4h_0/h_3$ are called invariants of Weierstrass elliptic function. The Jacobi elliptic functions are doubly periodic and possess properties of triangular functions:

$$\mathrm{sn}^2\xi+\mathrm{cn}^2\xi=1\,,\quad \mathrm{dn}^2\xi=1-m^2\,\mathrm{sn}^2\xi\,,$$

$$(\mathrm{sn}\,\xi)'=\mathrm{cn}\,\xi\,\mathrm{dn}\,\xi\,,\qquad (\mathrm{cn}\,\xi)'=-\mathrm{sn}\,\xi\,,$$

$$(\mathrm{dn}\,\xi)'=-m^2\,\mathrm{sn}\,\xi\,\mathrm{cn}\,\xi\,.$$

When $m\to 1$, the Jacobi functions degenerate to the hyperbolic functions, i.e.

$$\mathrm{sn}\,\xi\to\tanh\xi\,,\qquad \mathrm{cn}\,\xi\to\mathrm{sech}\,\xi\,,$$

when $m\to 0$, the Jacobi functions degenerate to the triangular functions, i.e.

$$\mathrm{sn}\,\xi\to\sin\xi\,,\qquad \mathrm{cn}\,\xi\to\cos\xi\,.$$

The more detailed notations for the Weierstrass and Jacobi elliptic functions can be found in Refs. [12] and [13].

Remarks

(i) ***Generalization***

The method proposed here is more general than the method in Ref. [9] by Fan and the improved method[11] by E. Yomba. Firstly, compared with the method in Ref. [9], the restriction on $\xi(x,y,t)$ as merely a linear function of x,y,t and the restriction on the coefficients a_{i0}, a_{ij}, b_{ij} ($i=1,2,\ldots$; $j=1,2,\ldots,m_i$) as constants are removed. Secondly, compared with the improved method[11] by E. Yomba, equation (3), which the new variable $\phi=\phi(\xi)$ satisfies is more general. More importantly, we add terms $b_{ij}\phi^{j-1}\sqrt{\sum_{\rho=0}^{r}h_\rho\phi^\rho}$ in new ansatz (2), so more types of solutions would be expected for some equations.

(ii) ***Feasibility***

For the generalization of the ansatz, naturally more complicated computation is expected than ever before. Even if the availability of computer symbolic systems like *Maple* or *Mathematica* allow us to perform the complicated and tedious algebraic calculation and differentiation on a computer, in general it is very difficult, sometime impossible, to solve the set of over-determined partial differential equations in (step 3). As the calculation goes on, in order to drastically simplify the work or make the work feasible, we often choose special function forms for a_{i0}, a_{ij}, b_{ij} ($i=1,2,\ldots$; $j=1,2,\ldots,m_i$) and ξ, on a trial-and-error basis.

3 Further Extension

In fact, We naturally present a more general ansatz

$$u_i(\xi)=a_{i0}+\sum_{j=1}^{m_i}\left(a_{ij}\phi^j+b_{ij}\phi^{-j}+f_{ij}\phi^{j-1}\sqrt{\sum_{\rho=0}^{r}h_\rho\phi^\rho}+k_{ij}\frac{\sqrt{\sum_{\rho=0}^{r}h_\rho\phi^\rho}}{\varphi^j}\right),\tag{12}$$

where a_{i0}, a_{ij}, b_{ij}, f_{ij}, k_{ij} ($i=1,2,\ldots$; $j=1,2,\ldots,m_i$) and ξ are differentiable function to be determined later. When a_{i0}, a_{ij}, b_{ij}, f_{ij}, k_{ij} ($i=1,2,\ldots$; $j=1,2,\ldots,m_i$) are constants and ξ is a linear function with respect to x, y, and t in the above ansatz, we have studied in Ref. [10]. Therefore, for some nonlinear equations, more types of solutions would be expected.

4 Exact Soliton-like Solutions of (2+1)-Asymmetric Nizhnik–Novikov–Vesselov (ANNV) Equation

Let us consider the ANNV equations,[14]

$$u_t-u_{xxx}-3(uv)_x=0\,,\qquad u_x-v_y=0\,.\tag{13}$$

This equation system is also named as the (2+1)-dimensional KdV equation or BLMP (Boiti–Leon–Manna-Pempinelli) equation by Boiti *et al.*[15] using the idea of the weak Lax pair. The ANNV equation (13) can also be obtained from the inner parameter-dependent symmetry constraint, the KP equation.[16] Lou point out that the ANNV equation (13) is an asymmetric part of the Nizhnik–Novikov–Vesselov (NNV) equation.[17] For more details about the results of this system, the reader is advised to see the remarkable achievements in Refs. [14] ∼ [20].

By balancing the highest-order contributions from both the linear and nonlinear terms in Eq. (13), we suppose that equation (13) has the following formal solutions,

$$u(x,y,t)=a_0+a_1\phi+b_1\sqrt{\sum_{\rho=0}^{4}h_\rho\phi^\rho}+a_2\phi^2+b_2\phi\sqrt{\sum_{\rho=0}^{4}h_\rho\phi^\rho}\,,$$

$$v(x,y,t)=A_0+A_1\phi+B_1\sqrt{\sum_{\rho=0}^{4}h_\rho\phi^\rho}+A_2\phi^2+B_2\phi\sqrt{\sum_{\rho=0}^{4}h_\rho\phi^\rho}\,, \tag{14}$$

where $a_0=a_0(y,t)$, $a_1=a_1(y,t)$, $b_1=b_1(y,t)$, $a_2=a_2(y,t)$, $b_2=b_2(y,t)$, $A_0=A_0(y,t)$, $A_1=A_1(y,t)$, $B_1=B_1(y,t)$, $A_2=A_2(y,t)$, $B_2=B_2(y,t)$, and $\xi=kp+q$ ($k=k(x)$, $p=p(y,t)$, and $q=q(y,t)$) are all differential functions, and $\phi=\phi(\xi)$ satisfies Eq. (3).

With the aid of *Maple*, substituting Eq. (14) along with Eq. (3) into Eq. (13) yields a set of partial differential equations for $\phi^i\big(\sqrt{\sum_{\rho=0}^4 h_\rho\phi^\rho}\,\big)^j$ ($i=0,1,\ldots;j=0,1$). Setting the coefficients of these terms $\phi^i\big(\sqrt{\sum_{\rho=0}^4 h_\rho\phi^\rho}\,\big)^j$ to zero yields a set of over-determined partial differential equations with respect to a_0, a_1, b_1, a_2, b_2, A_0, A_1, B_1, A_2, B_2, k, p, and q.

By use of *Maple*, solving the over-determined partial differential equations, we get the following results

$$\begin{aligned}
&A_0=\frac{\mathrm{d}F_1(t)/\mathrm{d}t+3\,C_4\,C_3\,C_1}{3C_3\,C_1}\,,\qquad k=C_1\,x+C_2\,,\qquad a_0=\frac{(\mathrm{d}F_2(y)/\mathrm{d}t)(-{C_3}^2{C_1}^2h_2\pm 3\,C_4)}{3C_3\,C_1}\,,\\
&p=C_3\,,\qquad b_1=\pm\sqrt{h_4}(\mathrm{d}F_2(y)/\mathrm{d}y)C_1\,C_3\,,\qquad B_1=\pm\sqrt{h_4}{C_3}^2{C_1}^2\,,\\
&q=F_2(y)+F_1(t)\,,\qquad A_2={C_3}^2h_4{C_1}^2\,,\qquad a_2=\Big(\frac{\mathrm{d}}{\mathrm{d}y}F_2(y)\Big)h_4C_3\,C_1\,,\\
&A_1=\frac{1}{2}\,h_3{C_1}^2{C_3}^2\,,\qquad a_1=\frac{1}{2}\,h_3C_1\,C_3\,\frac{\mathrm{d}}{\mathrm{d}y}F_2(y)\,,\qquad b_2=B_2=0\,.
\end{aligned}\tag{15}$$

From Eqs. (14) and (15), we obtain the following solutions for Eqs. (13).

Family 1 From Eqs. (15), when $h_0=h_1=h_3=0$, $h_2>0$, and $h_4<0$, we obtain the following soliton-like solutions for the ANNV equation,

$$\begin{aligned}
u_1=\;&\frac{(\mathrm{d}F_2(y)/\mathrm{d}y)(-{C_3}^2{C_1}^2h_2\pm 3\,C_4)}{3C_3\,C_1}\pm\Big(\frac{\mathrm{d}}{\mathrm{d}y}F_2(y)\Big)C_1\,C_3\sqrt{h_2^2(\mathrm{sech}^4(\sqrt{h_2}\,\xi)-\mathrm{sech}^2(\sqrt{h_2}\,\xi))}\\
&+\frac{1}{2}\,h_3C_1\,C_3\,\frac{\mathrm{d}}{\mathrm{d}y}F_2(y)\sqrt{-\frac{h_2}{h_4}}\,\mathrm{sech}\,(\sqrt{h_2}\,\xi)-\Big(\frac{\mathrm{d}}{\mathrm{d}y}F_2(y)\Big)h_2C_3\,C_1\mathrm{sech}^2(\sqrt{h_2}\,\xi)\,,\\
v_1=\;&\frac{\mathrm{d}F_1(t)/\mathrm{d}t+3\,C_4\,C_3\,C_1}{3C_3\,C_1}\pm{C_3}^2{C_1}^2\sqrt{h_2^2(\mathrm{sech}^4(\sqrt{h_2}\,\xi)-\mathrm{sech}^2(\sqrt{h_2}\,\xi))}\\
&+\frac{1}{2}\,h_3{C_1}^2{C_3}^2\sqrt{-\frac{h_2}{h_4}}\,\mathrm{sech}\,(\sqrt{h_2}\,\xi)-{C_3}^2{C_1}^2h_2\mathrm{sech}^2(\sqrt{h_2}\,\xi)\,,
\end{aligned}\tag{16}$$

where $\xi=kp+q$, k, p, and q are determined by Eq. (16).

Family 2 From Eqs. (16), when $h_1=h_3=0$, $h_0={h_2}^2/4h_4$, $h_2<0$, and $h_4>0$, we obtain the following soliton-like solutions for the ANNV equation,

$$\begin{aligned}
u_2=\;&\frac{(\mathrm{d}F_2(y)/\mathrm{d}y)({C_3}^2{C_1}^2h_2\pm 3\,C_4)}{3C_3C_1}-\frac{1}{2}\Big(\frac{\mathrm{d}}{\mathrm{d}y}F_2(y)\Big)C_3C_1h_2\tanh^2\Big(\frac{\sqrt{-2\,h_2}\,\xi}{2}\Big)\\
&\pm\frac{1}{2}\Big(\frac{\mathrm{d}}{\mathrm{d}y}F_2(y)\Big)C_3C_1\sqrt{{h_2}^2-2\,{h_2}^2\tanh^2\Big(\frac{\sqrt{-2\,h_2}\,\xi}{2}\Big)+{h_2}^2\tanh^4\Big(\frac{\sqrt{-2\,h_2}\,\xi}{2}\Big)}\,,\\
v_2=\;&\frac{\mathrm{d}F_1(t)/\mathrm{d}t+3\,C_4C_3C_1}{3C_3C_1}-\frac{1}{2}\,{C_3}^2{C_1}^2h_2\tanh^2\Big(\frac{\sqrt{-2\,h_2}\,\xi}{2}\Big)\\
&\pm\frac{1}{2}\,{C_3}^2{C_1}^2\sqrt{{h_2}^2-2\,{h_2}^2\tanh^2\Big(\frac{\sqrt{-2\,h_2}\,\xi}{2}\Big)+{h_2}^2\tanh^4\Big(\frac{\sqrt{-2\,h_2}\,\xi}{2}\Big)}\,,
\end{aligned}\tag{17}$$

where $\xi=kp+q$, k, p, and q are determined by Eq. (16).

Family 3 From Eqs. (16), when $h_1=h_3=0$, $h_0=h_2^2m^2(1-m^2)/h_4(2m^2-1)^2$, $h_4<0$ and $h_2>0$, we obtain the following double-like periodic solution for the ANNV equation,

$$u_3=\frac{(\mathrm{d}F_2(y)/\mathrm{d}y)(C_3^2{C_1}^2h_2\pm 3\,C_4)}{3C_3C_1}-\frac{(\mathrm{d}F_2(y)/\mathrm{d}y)C_3C_1h_2m^2\mathrm{cn}^2(\sqrt{h_2/(2\,m^2-1)}\,\xi)}{2\,m^2-1}$$

$$\pm\Big(\frac{\mathrm{d}}{\mathrm{d}y}F_2(y)\Big)C_3C_1\sqrt{\frac{{h_2}^2m^2(1-m^2)}{(2m^2-1)^2}-\frac{{h_2}^2m^2\mathrm{cn}^2(\sqrt{h_2/(2m^2-1)}\,\xi)}{2m^2-1}+\frac{{h_2}^2m^4\mathrm{cn}^4(\sqrt{h_2/(2m^2-1)}\,\xi)}{(2m^2-1)^2}}\,,$$

$$v_3=\frac{\mathrm{d}F_1(t)/\mathrm{d}t+3C_4C_3C_1}{3C_3C_1}-\frac{{C_3}^2{C_1}^2h_2m^2\mathrm{cn}^2(\sqrt{h_2/(2m^2-1)}\,\xi)}{2m^2-1}$$

$$\pm{C_3}^2{C_1}^2\sqrt{\frac{{h_2}^2m^2(1-m^2)}{(2m^2-1)^2}-\frac{{h_2}^2m^2\mathrm{cn}^2(\sqrt{h_2/(2m^2-1)}\,\xi)}{2m^2-1}+\frac{{h_2}^2m^4\mathrm{cn}^4(\sqrt{h_2/(2m^2-1)}\,\xi)}{(2m^2-1)^2}}\,,\qquad(18)$$

where $\xi=kp+q$, k, p, and q are determined by Eq. (16).

Family 4 From Eqs. (16), when $h_1=h_3=0$, $h_0=h_2^2(1-m^2)/h_4(2-m^2)^2$, $h_4<0$, and $h_2>0$, we obtain the following double-like periodic solutions for the ANNV equation,

$$u_4=\frac{(\mathrm{d}F_2(y)/\mathrm{d}y)({C_3}^2{C_1}^2h_2\pm3C_4)}{3C_3C_1}-\frac{(\mathrm{d}F_2(y)/\mathrm{d}y)C_3C_1h_2\mathrm{dn}^2(\sqrt{h_2/(2-m^2)}\,\xi)}{2-m^2}$$

$$\pm\Big(\frac{\mathrm{d}}{\mathrm{d}y}F_2(y)\Big)C_3C_1\sqrt{\frac{{h_2}^2(1-m^2)}{(2-m^2)^2}-\frac{{h_2}^2\mathrm{dn}^2(\sqrt{h_2/2-m^2}\,\xi)}{2-m^2}+\frac{{h_2}^2\mathrm{dn}^4(\sqrt{h_2/(2-m^2)}\,\xi)}{(2-m^2)^2}}\,,$$

$$v_4=\frac{\mathrm{d}F_1(t)/\mathrm{d}t+3C_4C_3C_1}{3C_3C_1}-\frac{{C_3}^2{C_1}^2h_2\mathrm{dn}^2(\sqrt{h_2/(2-m^2)}\,\xi)}{2-m^2}$$

$$-{C_3}^2{C_1}^2\sqrt{\frac{{h_2}^2(1-m^2)}{(2-m^2)^2}-\frac{{h_2}^2\mathrm{dn}^2(\sqrt{h_2/(2-m^2)}\,\xi)}{2-m^2}+\frac{{h_2}^2\mathrm{dn}^4(\sqrt{h_2/(2-m^2)}\,\xi)}{(2-m^2)^2}}\,,\qquad(19)$$

where $\xi=kp+q$, k, p, and q are determined by Eq. (16).

Family 5 From Eqs. (16), when $h_1=h_3=0$, $h_0=h_2^2m^2/h_4(m^2+1)^2$, $h_4>0$, and $h_2<0$, we obtain the following double-like periodic solutions for the ANNV equation,

$$u_5=\frac{(\mathrm{d}F_2(y)/\mathrm{d}y)({C_3}^2{C_1}^2h_2\pm3C_4)}{3C_3C_1}-\frac{(\mathrm{d}F_2(y)/\mathrm{d}y)C_3C_1h_2m^2\mathrm{sn}^2(\sqrt{-h_2/m^2+1}\,\xi)}{m^2+1}$$

$$\pm\Big(\frac{\mathrm{d}}{\mathrm{d}y}F_2(y)\Big)C_3C_1\sqrt{\frac{{h_2}^2m^2}{(m^2+1)^2}-\frac{{h_2}^2m^2\mathrm{sn}^2(\sqrt{-h_2/(m^2+1)}\,\xi)}{(m^2+1)}+\frac{{h_2}^2m^4\mathrm{sn}^4(\sqrt{-h_2/(m^2+1)}\,\xi)}{(m^2+1)^2}}\,,$$

$$v_5=\frac{\mathrm{d}F_1(t)/\mathrm{d}t+3C_4C_3C_1}{3C_3C_1}-\frac{{C_3}^2{C_1}^2h_2m^2\mathrm{sn}^2(\sqrt{-h_2/(m^2+1)}\,\xi)}{m^2+1}$$

$$\pm{C_3}^2{C_1}^2\sqrt{\frac{{h_2}^2m^2}{(m^2+1)^2}-\frac{{h_2}^2m^2\mathrm{sn}^2(\sqrt{-h_2/(m^2+1)}\,\xi)}{m^2+1}+\frac{{h_2}^2m^4\mathrm{sn}^4(\sqrt{-h_2/(m^2+1)}\,\xi)}{(m^2+1)^2}}\,,\qquad(20)$$

where $\xi=kp+q$, k, p, and q are determined by Eq. (16).

Family 6 From Eqs. (16), when $h_0=h_1=0$ and $h_2>0$, we obtain the following soliton-like solutions for the ANNV equation,

$$u_6=\frac{(\mathrm{d}F_2(y)/\mathrm{d}y)({C_3}^2{C_1}^2h_2\pm3C_4)}{3C_3C_1}+\frac{h_3C_1C_3(\mathrm{d}F_2(y)/\mathrm{d}y)h_2\mathrm{sech}^2(\sqrt{h_2}\,\xi/2)}{2(2\sqrt{h_2h_4}\tanh(\sqrt{h_2}\,\xi/2)-h_3)}$$

$$+\frac{(\mathrm{d}F_2(y)/\mathrm{d}y)h_4C_3C_1{h_2}^2\mathrm{sech}^4(\sqrt{h_2}\,\xi/2)}{(2\sqrt{h_2h_4}\tanh(\sqrt{h_2}\,\xi/2)-h_3)^2}\pm\frac{(\mathrm{d}F_2(y)/\mathrm{d}y)C_3C_1}{(2\sqrt{h_2h_4}\tanh(\sqrt{h_2}\,\xi/2)-h_3)}$$

$$\times\sqrt{h_4{h_2}^3\mathrm{sech}^4\Big(\frac{1}{2}\sqrt{h_2}\,\xi\Big)+\frac{h_4h_3{h_2}^3\mathrm{sech}^6(\sqrt{h_2}\,\xi/2)}{2\sqrt{h_2h_4}\tanh(\sqrt{h_2}\,\xi/2)-h_3}+\frac{h_4^2{h_2}^4\mathrm{sech}^8(\sqrt{h_2}\,\xi/2)}{(2\sqrt{h_2h_4}\tanh(\sqrt{h_2}\,\xi/2)-h_3)^2}}\,,$$

$$v_6=\frac{\mathrm{d}F_1(t)/\mathrm{d}t+3C_4C_3C_1}{3C_3C_1}+\frac{h_3{C_1}^2{C_3}^2h_2\mathrm{sech}^2(\sqrt{h_2}\,\xi/2)}{2(2\sqrt{h_2h_4}\tanh(\sqrt{h_2}\,\xi/2)-h_3)}$$

$$+\frac{{C_3}^2h_4{C_1}^2{h_2}^2\mathrm{sech}^4(\sqrt{h_2}\,\xi/2)}{(2\sqrt{h_2h_4}\tanh(\sqrt{h_2}\,\xi/2)-h_3)^2}\pm\frac{{C_3}^2{C_1}^2}{(2\sqrt{h_2h_4}\tanh(\sqrt{h_2}\,\xi/2)-h_3)}$$

$$\times\sqrt{h_4{h_2}^3\mathrm{sech}^4\Big(\frac{1}{2}\sqrt{h_2}\,\xi\Big)+\frac{h_4h_3{h_2}^3\mathrm{sech}^6(\sqrt{h_2}\,\xi)/2}{2\sqrt{h_2h_4}\tanh(\sqrt{h_2}\,\xi/2)-h_3}+\frac{h_4^2{h_2}^4\mathrm{sech}^8(\sqrt{h_2}\,\xi/2)}{(2\sqrt{h_2h_4}\tanh(\sqrt{h_2}\,\xi/2)-h_3)^2}}\,, \tag{21}$$

where $\xi = kp + q$, k, p, and q are determined by Eq. (1b).

5 Summary and Conclusions

In summary, based on the symbolic computation and the method in Ref. [9], by introducing a new and more general ansates than the method in Ref. [9] and the improved method in Ref. [11], we have proposed a generalized method for searching for more types and general exact solutions for NPDEs. The (2+1)-dimensional asymmetric Nizhnik–Novikov–Vesselov equation is chosen to illustrate this algorithm such that we can successfully obtain the solutions found by the method presented by Fan[9] and by the improved method by Yomba[11] and find other new and more general solutions at the same time. Some kinds of solutions derived by the generalized transformation are a series of nontraveling wave and coefficient functions' soliton-like solution, double-like periodic and trigonometric-like function solutions. It is helpful to study new and more general solutions of partial differential equations that model nonlinear physical systems. The method can easily be extended to other NPDEs and is sufficient to seek more new formal solutions of NPDEs.

References

[1] E.J. Parkes and B.R. Duffy, Comput. Phys. Commun. **98** (1996) 288; E.J. Parkes and B.R. Duffy, Phys. Lett. **A229** (1997) 217.

[2] E. Fan, Phys. Lett. **A277** (2000) 212; E. Fan, J. Zhang, and Benny Y.C. Hon, Phys. Lett. **A291** (2001) 376.

[3] Y.T. Gao and B. Tian, Comput. Phys. Commun. **133** (2001) 158; B. Tian and Y.T. Gao, J. Phys. A: Math. Gen. **29** (1996) 2895.

[4] Z.Y. Yan, Phys. Lett. **A292** (2001) 100; Z.Y. Yan and H. Q. Zhang, Phys. Lett. **A285** (2001) 355.

[5] S.A. Elwakil, S.K. El-labany, M.A. Zahran, and R. Sabry, Phys. Lett. **A299** (2002) 179.

[6] Y. Chen and Y. Zheng, Int. J. Mod. Phys. **C14(5)** (2003) 601; X.D. Zheng, Y. Chen, and H.Q. Zhang, Phys. Lett. **A311** (2003) 145.

[7] Y. Chen, B. Li, and H.Q. Zhang, Commun. Theor. Phys. (Beijing, China) **40** (2003) 137; Y. Chen, B. Li, and H.Q. Zhang, Chin. Phys. **12(9)** (2003) 940.

[8] Z.S. Lü and H.Q. Zhang, Phys. Lett. **A307** (2003) 269.

[9] E. Fan, Comput. Phys. Commun. **153** (2003) 17; E. Fan, Chaos, Solitons, and Fractals **15** (2003) 559; E. Fan, Phys. Lett. **A300** (2002) 243; E. Fan, J. Phys. A: Math. Gen. **36** (2003) 7009.

[10] Y. Chen and Q. Wang, Int. J. Mod. Phys. **C15** (2004) 595; Y. Chen, Q. Wang, and B. Li, Commun. Theor. Phys. (Beijing, China) **42** (2004) 329.

[11] Emmanuel Yomba, Chaos, Solitons and Fractals **20** (2003) 1135.

[12] K. Chandrasekharan, *Elliptic Function*, Springer, Berlin (1978).

[13] Patrick Du Val, *Elliptic Function and Elliptic Curves*, Cambridge University Press, Cambridge (1973).

[14] Z.X. Wang and X.J. Xia, *Special Function*, World Scientific, Singapore (1989).

[15] S.Y. Lou and H.Y. Ruan, J. Phys. A: Math. Gen. **34** (2001) 305.

[16] M. Boiti, J.J.P. Leon, M. Manna, and F. Pempinelli, Inverse Problems **2** (1986) 271 .

[17] S.Y. Lou and X.B. Hu, J. Math. Phys. **38** (1997) 6401.

[18] L.P. Nizhnik, Sov. Phys. CDokl. **25** (1980) 706; A.P. Veselov and S.P. Novikov, Sov. Math. CDokl. **30** (1984) 588; S.P. Novikov and A.P. Veselov, Physica **D18** (1986) 267.

[19] S.Y. Lou and H.Y. Ruan, J. Phys. A: Math. Gen. **34** (2001) 305.

[20] X.Y. Tang, S.Y. Lou, and Y. Zhang. Phys. Rev. **E66** (2002) 046601.