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Localized waves and interaction solutions to an extended (3+1)-dimensional Jimbo–Miwa equation

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ABSTRACT

Based on Hirota bilinear method, four kinds of localized waves, solitons, breathers, lumps and rogue waves of the extended (3+1)-dimensional Jimbo–Miwa equation are constructed. Breathers are obtained through choosing appropriate parameters on soliton solutions, while lumps and rogue waves are derived via the long wave limit on the soliton solutions. The energy, phase shift, shape, and propagation direction of these localized waves can be influenced and controlled by parameters. Considering mixed cases of the above four types of solutions, we also give many kinds of interaction solutions in the same plane with different parameters or different planes with the same parameters. Dynamical characteristics of these localized waves and interaction solutions are further analyzed and vividly demonstrated through figures.

1. Introduction

Soliton [1], lump [2], breather [3], and rogue wave [4] are four types of nonlinear localized waves with distinct dynamical and physical characteristics in nonlinear systems. Soliton has ionic and stability properties; lump [5] is a rational function solution and localized in all directions in the space; breather [6] is localized in one certain direction with periodic structure; Rogue wave [7] is localized in both time and space. Breathers and rogue waves [8,9] are two typical localized waves with obvious special unstable nonlinear structures. Breathers can demonstrate rogue wave phenomena and have two typical kinds of breathers, Akhmediev breathers and Kuznetsov–Ma breathers, which have different propagation directions and distributions. The high amplitude wave produced during the collision between soliton and breather can be used to elaborate the generation mechanism of rogue wave. The study of nonlinear localized waves and interaction solutions [10] among them is one of the important research subject in recent years.

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The well-known $(3+1)$-dimensional Jimbo–Miwa equation

$$u_{xxxy} + 3u_y u_{xx} + 3u_x u_{xy} + 2u_{yt} - 3u_{xz} = 0, \tag{1}$$

can be applied to demonstrate some interesting phenomena in nonlinear physics. It comes from the second equation of the KP hierarchy [11] and has no Painlevé property, which differs from the KP equation. It also has lump-type solutions [12]. In [13], Wazwaz proposed the following two extended $(3+1)$-dimensional Jimbo–Miwa equations (EJM):

$$u_{xxxy} + 3u_y u_{xx} + 3u_x u_{xy} + 2u_{yt} - 3(u_{xz} + u_{yz} + u_{zz}) = 0, \tag{2}$$

$$u_{xxxy} + 3u_y u_{xx} + 3u_x u_{xy} + 2(u_{xt} + u_{yt} + u_{zt}) - 3u_{xz} = 0. \tag{3}$$

These two forms both hold the dimension and order of the normal Jimbo–Miwa equation. In [13,14], multiple soliton solutions and lump solutions of the three equations were constructed. In [15], lumps and interaction solutions of the reduced EJM equation (2) were researched by bilinear method.

As far as we know, no one has reported two breathers, two lumps and interaction solutions of the EJM equation (2). Applying the method adopted in [16], we discover some new interaction solutions, in addition to some common solutions. The article is organized as follows. In Section 2, we mainly introduce solitons and breathers, and their mixed cases of EJM equation (2). In Section 3, applying long wave limit on the multi-soliton solution, lumps, rogue waves and several cases of interacting with each other are derived. The last section contains a short summary.

2. The soliton solution and breather solution

Eq. (2) can be mapped into bilinear form

$$(D_x^3 D_y + 2D_y D_t - 3D_y D_z - 3D_x D_z - 3D_z^2)(f \cdot f) = 0, \tag{4}$$

via a dependent variable transformation

$$u = 2(\ln f)_x = 2\frac{f_x}{f}, \tag{5}$$

where $f = f(x, y, z, t)$, and the derivatives $D_x^3 D_y, D_t D_x, D_t D_y$, and D_z^2 are all bilinear derivative operators [17] defined by

$$\begin{aligned} D_x^\alpha D_y^\beta D_z^\gamma D_t^\delta (f \cdot g) = {} & (\frac{\partial}{\partial x} - \frac{\partial}{\partial x'})^\alpha (\frac{\partial}{\partial y} - \frac{\partial}{\partial y'})^\beta (\frac{\partial}{\partial z} - \frac{\partial}{\partial z'})^\gamma \\ & \times (\frac{\partial}{\partial t} - \frac{\partial}{\partial t'})^\delta f(x, y, z, t) g(x', y', z', t') \,|_{x'=x, y'=y, z'=z, t'=t} \,. \end{aligned} \tag{6}$$

It is quite evident that $u = u(x, y, z, t)$ is a solution of Eq. (2) under the transformation (5), if and only if f solves Eq. (4).

The N-soliton solutions of the EJM equation are given by substituting

$$f = \sum_{\mu=0,1} \exp\Big(\sum_{i=1}^{N} \mu_i \eta_i + \sum_{1 \le i < j}^{N} \mu_i \mu_j \ln(A_{ij})\Big), \tag{7}$$

into (5) through the Hirota method, with

$$\begin{gathered} \omega_i = -\frac{k_i^2 p_i - 3p_i q_i - 3q_i^2 - 3q_i}{2p_i}, \quad \eta_i = k_i(x + p_i y + q_i z + \omega_i t) + \eta_i^0, \\ A_{ij} = \frac{(k_i(k_i - k_j)p_j + q_j^2 + q_j)p_i^2 - p_j(k_j(k_i - k_j)p_j + (2q_j + 1)q_i + q_j)p_i + p_j^2 q_i(q_i + 1)}{(k_i(k_i + k_j)p_j + q_j^2 + q_j)p_i^2 + p_j(k_j(k_i + k_j)p_j - (2q_j + 1)q_i - q_j)p_i + p_j^2 q_i(q_i + 1)} \\ (1 \le i < j = 2, \ldots, N), \end{gathered} \tag{8}$$

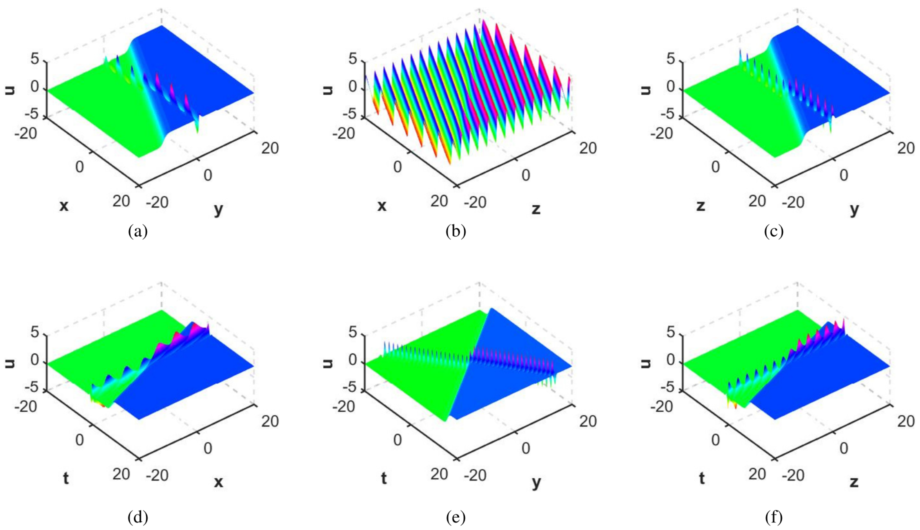


Fig. 1. The interaction solution between soliton and breather of Eq. (2) in different planes with the same parameters.

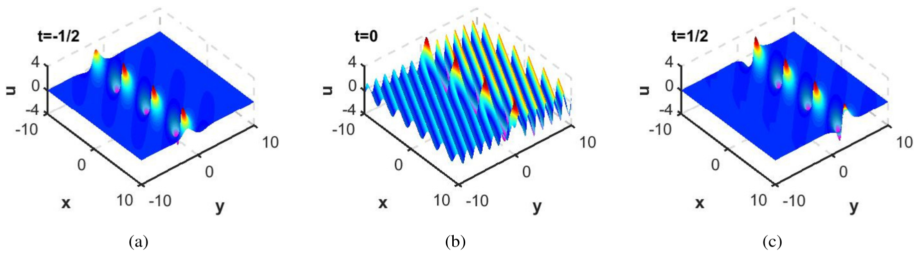


Fig. 2. The interaction solution between two types of breathers for Eq. (2) in the (x, y) plane at $z = 0$.

k_i, p_i, q_i and η_i^0 are arbitrary parameters. $\sum_{\mu=0,1}$ is the summation that covers all possible combinations of $\eta_i, \eta_j \in \{0, 1\}(i, j = 1, 2, \ldots, N)$.

By taking complex conjugate method for arbitrary parameters in Eq. (7), breather solutions of the EJM equation can be constructed in different planes. When $N = 2$, one line breather can be derived in (x, y) plane and general breathers can be obtained in $(x, t), (y, t)$ and (z, t) planes with the parameters constrained as $k_1 = k_2^* = ia, \quad p_1 = p_2 = b, \quad q_1 = q_2^* = c + id$.

For $N = 3$ in Eq. (7), interaction solutions between solitons and breathers can be derived with the appropriate parameters. A kink soliton interacting with a breather can be seen in Fig. 1 with the following parameters $k_1 = k_3^* = i, \ p_1 = p_3^* = 2 + 3i, \ q_1 = q_3 = 2, \ k_2 = 1, \ p_2 = 2, \ q_2 = 1 \ , \eta_1^0 = \eta_2^0 = \eta_3^0 = 0$. Different planes exhibit different physical phenomena and dynamical behavior, especially in (x, z) plane. The breathers have different periods in six different planes.

For $N = 4$ in Eq. (7), by giving appropriate values to arbitrary parameters, we can construct the interaction solutions between line breathers and general breathers in different planes, such as (x, y) and (x, z) planes. The initial state is only the general breather, and as time goes on, the line breather appears to interact with the general breather, and it disappears finally, leaving only the general breather. Their collision process is shown in Fig. 2 with $k_1 = k_2^* = i, \ k_3 = k_4^* = 2i, \ p_1 = p_2^* = 1+i, \ q_3 = q_4^* = 2+2i, \ p_3 = p_4 = q_1 = q_2 = 2$. Cross general breathers also can be constructed in the (y, z) plane, whose collision process is illustrated in Fig. 3. There is a large amplitude at the intersection point.

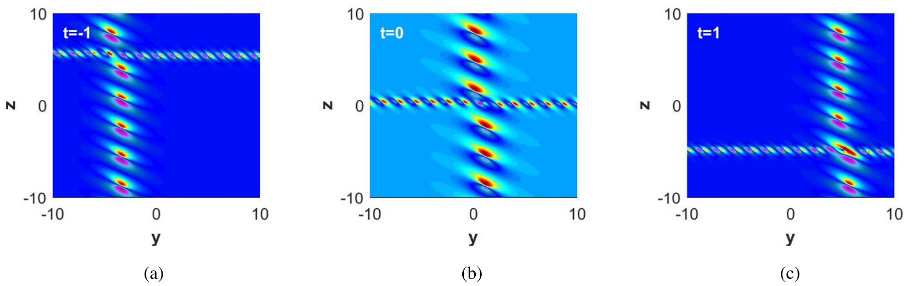


Fig. 3. The interaction solution between two general breathers for Eq. (2) in the (y, z) plane with the same parameters chosen in Fig. 2 at $x = 0$.

3. The lump solution and rogue wave solution

In this section, we will derive the lump solutions and rogue wave solutions by using the long wave limits on the multi-soliton solutions [2]. From the two soliton solutions, first order lump solutions and rogue wave solutions can be obtained. From the three soliton solutions, interaction solutions between kink solitons and lump solutions or rogue wave solutions can be derived by selecting different parameters. From the four soliton solutions, interaction solutions between lump solutions and rogue wave solutions can be constructed based on the parameter selection method.

Case 1. Lump solution

For $N = 2$, lump solutions can be obtained by utilizing the long wave limit method on the two soliton solutions with appropriate parameters. By setting the parameters in Eq. (7) as $k_1 = l_1\epsilon,\quad k_2 = l_2\epsilon,\quad \eta_1^0 = \eta_2^{0*} = i\pi$, and taking the limit as $\epsilon \to 0$, the lump solutions of the EJM equation (2) can be constructed in the following form,

$$u = \frac{2(\theta_1 + \theta_2)}{\theta_1\theta_2 + \theta_0}. \tag{9}$$

with

$$\theta_0 = -\frac{2p_2p_1(p_1 + p_2)}{((q_2 + 1)p_1 - p_2(q_1 + 1))(p_1q_2 - p_2q_1)}, \quad \theta_i = x + p_iy + q_iz + \frac{3t(1 + q_ip_i + q_i^2)}{2p_i} \quad (i = 1, 2). \tag{10}$$

If setting $p_1 = p_2^*,\quad q_1 = q_2^*$, the solution u in Eq. (9) is obvious nonsingular. In order to demonstrate the characteristics of the solution (10), we assume $p_1 = a_1 + ib_1,\quad q_1 = a_2 + ib_2$, and a_1, a_2, b_1, b_2 are all real constants.

When $a_1 \neq 0$, the solution u in Eq. (9) is a constant along the trajectory defined by the path $[x(t), y(t)]$, namely

$$\begin{aligned} x + a_1y + a_2z + \frac{3(a_1^2a_2 + a_1a_2^2 - a_1b_2^2 + a_2b_1^2 + 2a_2b_1b_2 + a_1a_2 + b_1b_2)}{2(a_1^2 + b_1^2)}t \\ - \frac{\sqrt{a_1(a_1^2 + b_1^2)}}{(a_1b_2 - a_2b_1 - b_1)(a_1b_2 - a_2b_1)} = 0, \\ b_1y + b_2z + \frac{3(a_1^2b_2 + 2a_1a_2b_2 - a_2^2b_1 + b_1^2b_2 + b_1b_2^2 + a_1b_2 - a_2b_1)}{2(a_1^2 + b_1^2)}t = 0. \end{aligned} \tag{11}$$

The rational solution keeps permanent lump condition during the moving process in six different planes.

For $N = 3$, interaction solutions between solitons and lumps can be constructed through the long wave limit method on three soliton solutions. With similar parameters constrained in $N = 2$, the corresponding f can be rewritten as

$$f = (\theta_1\theta_2 + a_{12})l_1l_2 + (\theta_1\theta_2 + a_{12} + a_{13}\theta_2 + a_{23}\theta_1 + a_{13}a_{23})l_1l_2e^{\eta_3} \tag{12}$$

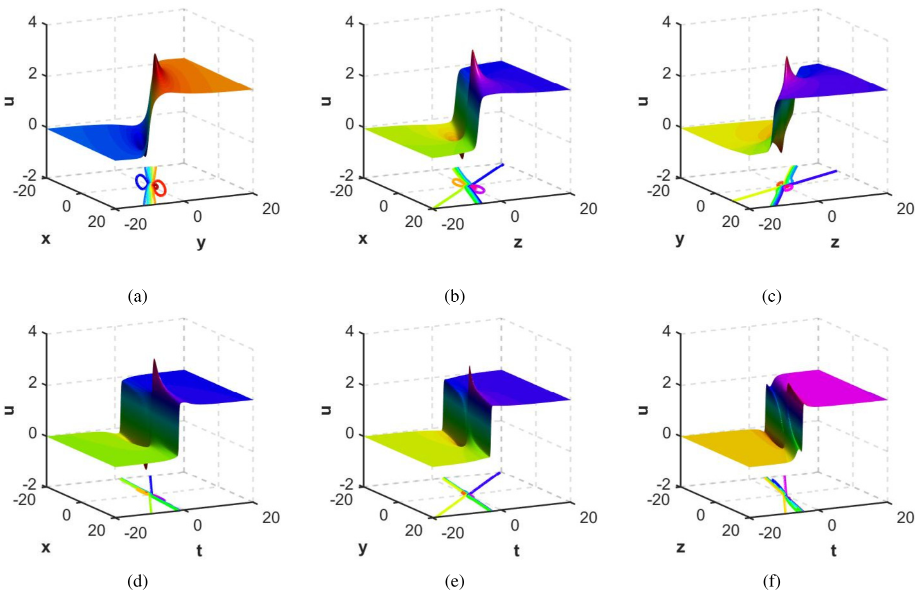


Fig. 4. Interaction solution between kink soliton and lump for Eq. (2) in different planes with $p_1 = 2 + i$, $q_1 = 3 + 2i$, $p_3 = 2$, $q_3 = 1$, $\eta_3^0 = 0$, $k_3 = 1$.

with

$$\theta_i = x + p_i y + q_i z + \frac{3t(1 + q_i p_i + q_i^2)}{2p_i}, \quad a_{ij} = -\frac{2p_2p_1(p_1 + p_2)}{((q_2 + 1)p_1 - p_2(q_1 + 1))(p_1q_2 - p_2q_1)} \quad (i < j),$$
$$a_{i3} = -\frac{2p_3p_i(p_i + p_3)k_3}{(q_3^2 + q_3)p_i^2 + p_3(p_3k_3^2 - (2q_3 + 1)q_i - q_3)p_i + (q_i + 1)p_3^2q_i} \quad (i, j = 1, 2). \tag{13}$$

Under a suitable choice of parameters, interaction solutions between kink solitons and lumps can be derived in different planes, whose dynamical phenomena are exhibited in Fig. 4. Both the three dimensions and the projected images are given out to demonstrate the characteristics. Under the same parameters, the amplitudes of the lumps in different planes are distinct, while the peaks and valleys are divided by the kink solitons. Obviously, the peaks and valleys are located in the high amplitudes and low amplitudes of the kink soliton surfaces, respectively.

For $N = 4$, second order lump solutions can be constructed with four soliton solutions by the above method. Taking

$$k_1 = l_1\epsilon, \quad k_2 = l_2\epsilon, \quad k_3 = l_3\epsilon, \quad k_4 = l_4\epsilon, \quad \eta_1^0 = \eta_2^{0*} = \eta_3^0 = \eta_4^{0*} = i\pi, \tag{14}$$

then the corresponding function f can be rewritten in the following form

$$f = (\theta_1\theta_2\theta_3\theta_4 + a_{12}\theta_3\theta_4 + a_{13}\theta_2\theta_4 + a_{14}\theta_2\theta_3 + a_{23}\theta_1\theta_4 + a_{24}\theta_1\theta_3 + a_{34}\theta_1\theta_2 + a_{12}a_{34}$$
$$+ a_{13}a_{24} + a_{14}a_{23})l_1l_2l_3l_4\epsilon^4 + O(\epsilon^5) \tag{15}$$

where

$$\theta_i = x + p_i y + q_i z + \frac{3t(1 + q_i p_i + q_i^2)}{2p_i},$$
$$a_{ij} = -\frac{2p_jp_i(p_i + p_j)}{((q_j + 1)p_i - p_j(q_i + 1))(p_iq_j - p_jq_i)} \quad (i, j = 1, 2, 3, 4). \tag{16}$$

In the same way, assuming $p_1 = p_2^* = a_1 + ib_1, p_3 = p_4^* = a_2 + ib_2$, $q_1 = q_2^* = c_1 + id_1, q_3 = q_4^* = c_2 + id_2$, and $a_i, b_i, c_i, d_i, (i = 1, 2)$ are all real constants. In fact, different interaction solutions can be constructed

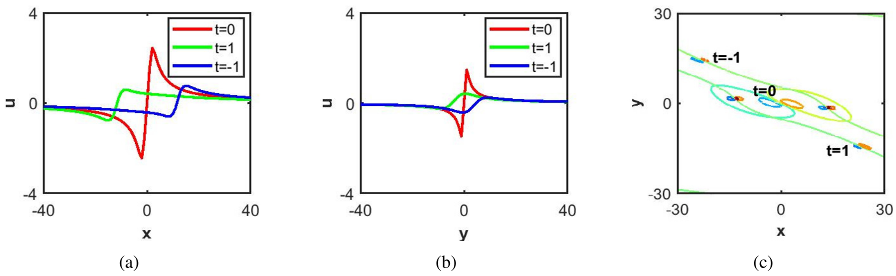


Fig. 5. Overtaking collision between two lumps for Eq. (2) in (x,y) plane with parameters $a_1 = 2, b_1 = 1, a_2 = 1, b_2 = 2, c_1 = 2, d_1 = 3, c_2 = 3, d_3 = 1$.

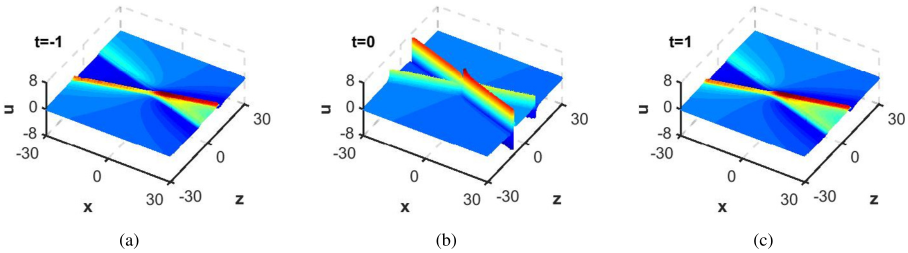


Fig. 6. Two line rogue waves for Eq. (2) in the (x, z) plane with parameters $a_1 = 1, b_1 = 2, a_2 = 3, b_2 = 5, c_1 = 3, c_2 = -4, d_1 = d_2 = 0$ at $y = 0$.

with different parameters. Dynamical behavior of second order lumps in the (x,y) plane can be clearly seen from Fig. 5.

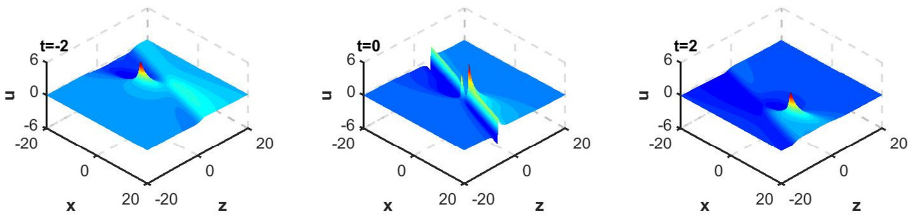
Case 2. Rogue wave solution

Except for lump solutions mentioned in case 1, another kind of dynamical phenomenon is also available. In the following, we will construct rogue waves and interaction solutions for the EJM equation (2). For $N = 2$, when $b_2 = 0$, namely p_1 and p_2 are all real constants, line rogue wave is able to be derived under the same parameters constrained in case 1. The line rogue wave is a rational solution with the process of growth and decay. For $N = 3$, interaction solutions between kink solitons and rogue waves can be derived from the limitation of the three solitons. Taking $p_1 = p_2*$ and $q_1, q_2, q_3, p_3, \eta_3^0, k_3$ are real parameters, thus interaction solutions can be derived with appropriate parameters.

For $N = 4$, second order line rogue waves can be constructed with appropriate parameters. From Fig. 6, dynamical behavior of second order line rogue wave can be clearly seen. There is a downward deformation at the intersection point of two line rogue waves, and the amplitude is up to 8.0063 at $t = 0$. The whole dynamical process come from a constant background and finally return to the constant background again, which are consistent with the first order line rogue wave.

Case 3. Interaction solutions between lumps and rogue waves

In addition, combining the methods mentioned in Case 1 and Case 2, we also obtain the interaction solutions between lumps and line rogue waves. The associated graph for this interaction solution in the (x, z) plane is presented in Fig. 7. These collision processes between line rogue waves and lumps are similar with the collision between line rogue waves and solitons. It should be noted that the amplitude of the lump increases significantly at $t = 0$ and can reach 5.9535. Interestingly, the interaction of these two types of waves implies a downward deformation of the line rogue wave at $t = 0$. Finally, the line rogue wave disappears into the constant background, and the moving lump is preserved eventually.



(a) (b) (c)

Fig. 7. The interaction solution between line rogue wave and lump solution for Eq. (2) in the (x, z) plane with parameters $a_1 = 1, \quad b_1 = 1, \quad a_2 = 2, \quad b_2 = 1, \quad c_1 = -1, \quad d_1 = 2, \quad c_2 = 3, \quad d_2 = 0$ at $y = 0$.

4. Summary and discussions

To conclude, many kinds of localized wave solutions of EJM equation are obtained in this article, such as kink soliton, breather, lump and line rogue wave. Their higher order solutions are also derived. In addition, considering their mixed cases, we also obtain many types of interaction solutions in the same plane with different parameters and the different plane with the same parameters. We analyze their dynamic behavior and vividly demonstrate their evolution process. The methods used in this paper can also be applied in other equations (include Eq. (3)) to obtain localized wave solutions. Next, we look forward to applying numerical methods to simulate these kinds of dynamic phenomena. It is worthy of further exploration that using numerical solutions to verify the above theoretical solutions and their stability in the future.

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References

[1] M.J. Ablowitz, D.J. Kaup, A.C. Newell, H. Segur, The inverse scattering transform-fourier analysis for nonlinear problems, Stud. Appl. Math. 53 (1974) 249–315.
[2] J. Satsuma, M.J. Ablowitz, Two-dimensional lumps in nonlinear dispersive systems, J. Math. Phys. 20 (1979) 1496–1503.
[3] M. Tajiri, T. Arai, Growing-and-decaying mode solution to the Davey–Stewartson equation, Phys. Rev. E 60 (1999) 2297.
[4] J.S. He, H.R. Zhang, L.H. Wang, A.S. Fokas, Generating mechanism for higher-order rogue waves, Phys. Rev. E 87 (2013) 052914.
[5] W.X. Ma, Lump solutions to the Kadomtsev–Petviashvili equation, Phys. Lett. A 379 (2015) 1975–1978.
[6] N. Akhmediev, J.M. Soto-Crespo, A. Ankiewicz, How to excite a rogue wave, Phys. Rev. A 80 (2009) 043818.
[7] Z.Y. Yan, Rogon-like solutions excited in the two-dimensional nonlocal nonlinear Schrödinger equation, J. Math. Anal. Appl. 380 (2011) 689–696.
[8] L.M. Ling, L.C. Zhao, Simple determinent representation for rogue waves of the nonlinear schrödinger equation, Phys. Rev. E 88 (2013) 043201.
[9] X. Wang, C. Liu, L. Wang, Darbxou transformation and rogue wave solutions for the variable-coefficient coupled Hirota equations, J. Math. Anal. Appl. 449 (2017) 1534–1552.
[10] J.C. Chen, Z.Y. Ma, Y.H. Hu, Nonlocal symmetry, Darboux transformation and soliton-cnoidal wave interaction solution for the shallow water wave equation, J. Math. Anal. Appl. 460 (2018) 987–1003.
[11] M. Jimbo, T. Miwa, Solitons and infinite dimensional Lie algebras, Publ. RIMS, Kyoto Univ. 19 (1983) 943–1001.
[12] J.Y. Yang, W.X. Ma, Abundant lump-type solutions of the Jimbo–Miwa equation in (3+1)-dimensions, Comput. Math. Appl. 73 (2017) 220–225.
[13] A.M. Wazwaz, Multiple-soliton solutions for extended (3+1)-dimensional Jimbo–Miwa equations, Appl. Math. Lett. 64 (2017) 21–26.
[14] H.Q. Sun, A.H. Chen, Lump and lump-kink solutions of the (3+1)-dimensional Jimbo–Miwa and two extended Jimbo–Miwa equations, Appl. Math. Lett. 68 (2017) 55–61.

[15] Y.H. Wang, H. Wang, H.H. Dong, H.S. Zhang, C. Temuer, Interaction solutions for a reduced extended (3+1)-dimensional Jimbo–Miwa equation, Nonlinear Dynam. 92 (2018) 487–497.
[16] J.G. Rao, Y. Cheng, J.S. He, Rational and semirational solutions of the nonlocal Davey–Stewartson equations, Stud. Appl. Math. 139 (2017) 568–598.
[17] R. Hirota, The Direct Method in Soliton Theory, Cambridge University Press, 2004.