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Several reverse-time integrable nonlocal nonlinear equations: Rogue-wave solutions

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A study of rogue-wave solutions in the reverse-time nonlocal nonlinear Schrödinger (NLS) and nonlocal Davey-Stewartson (DS) equations is presented. By using Darboux transformation (DT) method, several types of rogue-wave solutions are constructed. Dynamics of these rogue-wave solutions are further explored. It is shown that the $(1+1)$-dimensional fundamental rogue-wave solutions in the reverse-time NLS equation can be globally bounded or have finite-time blowing-ups. It is also shown that the $(2+1)$-dimensional line rogue waves in the reverse-time nonlocal DS equations can be bounded for all space and time or develop singularities in critical time. In addition, the multi- and higher-order rogue waves exhibit richer structures, most of which have no counterparts in the corresponding local nonlinear equations. *Published by AIP Publishing.* https://doi.org/10.1063/1.5019754

Recently, a number of reverse-space, reverse-time, and reverse space-time nonlocal nonlinear integrable equations were found and triggered renewed interest in integrable systems. These deformations of local integrable equations are introduced with different space and/or time coupling. As a largely unexplored subject, rogue waves in the nonlocal integrable systems have been received much attention. To investigate the connections between solutions at reverse-time points t and –t, we need to consider the reverse-time reduction. By using Darboux transformation method, we derive some interesting results from several recently proposed reverse-time nonlocal integrable equations. It is shown that these rogue-wave solutions exhibit richer structures, which generalize rogue-wave solutions of local NLS and DS equations into the nonlocal models.

I. INTRODUCTION

The integrable nonlinear equations are exactly solvable models which play an important role in the field of nonlinear science, especially in the study of nonlinear physical systems, including nonlinear optics, Bose-Einstein condensates, plasma physics, and ocean water waves. Most of these integrable equations are local equations, that is, the solutions' evolution depends only on the local solution value. In recent years, the integrable nonlocal nonlinear equations were proposed and studied. The first such nonlocal equation was the $\mathcal{PT}$-symmetric nonlocal nonlinear Schrödinger (NLS) equation[1,2]

$$\mathrm{i}q_t(x,t) = q_{xx}(x,t) + 2\sigma q^2(x,t)q^*(-x,t). \tag{1}$$

Here, $\sigma = \pm 1$ is the sign of nonlinearity, and the asterisk $*$ represents complex conjugation. It is noted that $\mathcal{PT}$-symmetric systems have attracted a lot of attention in optics and other physical fields in recent years.[3]

Following this nonlocal $\mathcal{PT}$-symmetric NLS equation, some new reverse space-time and reverse-time type nonlocal nonlinear integrable equations were also introduced and quickly reported.[4] They are integrable infinite dimensional Hamiltonian dynamical systems, which arise from remarkably simple symmetry reductions of general ZS-AKNS scattering problems where the nonlocality appears in both space and time or time alone.

Rogue waves have attracted a lot of attention in recent years due to their dramatic and often damaging effects, such as in the ocean and optical fibers.[5–9] As an unexplored and interesting subject, rogue waves in the nonlocal integrable systems have received much attention.

In this article, we study rogue-wave solutions in several reverse-time integrable nonlocal nonlinear equations via using Darboux transformation method. As typically concrete examples, we focus on the reverse-time nonlocal NLS equation

$$\mathrm{i}q_t(x,t) = q_{xx}(x,t) + 2q^2(x,t)q(x,-t), \tag{2}$$

and the reverse-time nonlocal DS equations

$$iq_t + \frac{1}{2}\gamma^2 q_{xx} + \frac{1}{2}q_{yy} + (qr - \phi)q = 0, \tag{3}$$

$$\phi_{xx} - \gamma^2\phi_{yy} - 2(qr)_{xx} = 0, \tag{4}$$

where $r(x,y,t) = \sigma q(x,y,-t)$; q, r, and ϕ are functions of x, y, and t, respectively; and $\gamma^2 = \pm 1$ is the equation-type parameter (with $\gamma^2 = 1$ being the DS-I and $\gamma^2 = -1$ being DS-II).

We find that rogue-wave solutions in these two nonlocal equations can either be bounded for all space and time or develop finite-time collapsing singularities. In addition, multi- and higher-order rogue-wave solutions exhibit more interesting dynamic patterns, most of which have not been found before in the integrable nonlocal nonlinear equations.

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II. ROGUE-WAVE SOLUTIONS IN THE REVERSE-TIME NONLOCAL NLS EQUATION

The reverse-time nonlocal NLS equation (2) is an integrable Hamilton evolution equation that admits an infinite number of conservation laws.[4] In addition, this equation is invariant under the action of the $\mathcal{PT}$ operator, i.e., the joint transformations $x \to -x$, $t \to -t$, and complex conjugation.[10] That is, if $u(x, t)$ is a solution, so is $u^*(-x, -t)$. For potential applications, this might relate to the concept of $\mathcal{PT}$-symmetry, which is a hot research area in contemporary physics.[3]

For Eq. (2), it can be obtained from the following coupled system:[11,12]

$$\mathrm{i}q_t = q_{xx} - 2q^2 r, \quad \mathrm{i}r_t = -r_{xx} + 2r^2 q, \tag{5}$$

under the *time-reversal* reduction

$$r(x,t) = -q(x,-t). \tag{6}$$

It is well known that the integration of system (5) is based on the fact that it is the condition of simultaneous solvability of the linear system

$$\Phi_x = U\Phi, \quad \Phi_t = V\Phi, \tag{7}$$

where Φ is a column-vector, and

$$U = \begin{pmatrix} -i\lambda & q \\ r & i\lambda \end{pmatrix}, \quad V = \begin{pmatrix} 2i\lambda^2 + iqr & -2\lambda q - iq_x \\ -2\lambda r + ir_x & -2i\lambda^2 - iqr \end{pmatrix}.$$

A. Time-reversal reduction of Darboux transformation

Next, the Darboux transformation makes it possible to construct solutions of the system (5) from the known potential functions $[q(x,t), r(x,t)]$, which can be written as ratios of determinants.[13–16] The reduction of system (5) and its solutions to the reverse-time NLS equation (2) reduces to take the following reductions:

Proposition 1. *For any spectral* $\lambda_1, \zeta_1 \in \mathbb{C}$, *if*

$$\zeta_1 = -\lambda_1, \quad \Psi_1(x,t,\zeta_1) = \alpha\Phi_1^T(x,-t,\lambda_1), \tag{8}$$

where α is a complex constant. Here, Φ_1 solves the spectral equation (7) at $\lambda = \lambda_1$, and Ψ_1 solves the corresponding adjoint spectral equation of (7) at $\zeta = \zeta_1$. Then, the N-fold Darboux formula[15,16] *for coupled system (5) accords with reduction (6), that is, the Darboux transformation for the reverse-time NLS equation (2).*

This proposition can be readily proved via a direct calculation. Next, to construct high-order rogue waves for Eq. (2), we choose a plane wave solution $e^{-2\,\mathrm{i}t}$ to be the seed solution, then the general wave function for the linear system (7) has been constructed,[17] with a constant normalization, it can be further simplified into the following form:[16]

$$\Phi(x,t) = \mathcal{D}\phi(x,t), \tag{9}$$

where $\mathcal{D} = \mathrm{diag}(e^{-\mathrm{i}t}, e^{\mathrm{i}t})$, and

$$\phi(x,t) = \frac{1}{\sqrt{h-1}} \begin{pmatrix} \sinh\left[A + \frac{1}{2}\ln\left(h + \sqrt{h^2-1}\right)\right] \\ \sinh\left[-A + \frac{1}{2}\ln\left(h + \sqrt{h^2-1}\right)\right] \end{pmatrix},$$

$$A = \sqrt{-\lambda^2 - 1}(x - 2\lambda t + \theta).$$

Here, imposing the conditions of λ being purely imaginary, i.e., $\lambda = ih$, $|\lambda| > 1$, and θ is complex constant which can be taken as $\theta = \sum_{j=0}^{N-1} s_j \epsilon^{2j}$, $s_j \in \mathbb{C}$.

Furthermore, utilizing symmetry condition (8), the adjoint wave function at $\zeta = -\lambda$ can be obtained as

$$\Psi(x,t) = \psi(x,t)\mathcal{D}^*, \quad \psi(x,t) = \phi^T(x,-t). \tag{10}$$

Thus, setting $\lambda = \mathrm{i}(1+\epsilon^2), \zeta = -\mathrm{i}(1+\tilde{\epsilon}^2)$ in functions (9) and (10), applying the generalized Darboux transformation scheme,[15–17] we can construct the N-th order rogue-wave solution for the focusing reverse-time NLS equation.

B. Dynamics of rogue-wave solutions

In this section, we give an analysis on the rogue-wave solutions for the reverse-time NLS equation (2).

The expression for the first-order (fundamental) rogue-wave solution can be obtained and simplified as

$$q_1(\hat{x},t) = e^{-2\mathrm{i}t}\left[1 + \frac{4(4\mathrm{i}t-1)}{16t^2 + 4(\hat{x} + \mathrm{i}y_0)^2 + 1}\right], \tag{11}$$

where $\hat{x} = x + x_0$ with $x_0 = \mathrm{Re}(s_0)$ and $y_0 = \mathrm{Im}(s_0)$. Hence, this solution has one non-reducible real parameter y_0. Moreover, it is indeed surprising to find that solution (11) accords with the fundamental rogue wave recently reported in the $\mathcal{PT}$-symmetric NLS equation.[16]

For this rogue wave, when $0 \le y_0^2 < 1/4$, this solution is nonsingular and resemble those features in the Peregrine soliton (corresponds to $y_0 = 0$). The peak amplitude accurately attains at $\left|\frac{4y_0^2+3}{4y_0^2-1}\right|$, which is higher than 3 when $y_0 \neq 0$. Moreover, location of zeros of $u_1(x, t)$ is (x_c, t_c), where $x_c = \pm\sqrt{\frac{4y_0^2+3}{1-4y_0^2}}$, $t_c = -\frac{y_0}{2}x_c$. However, once $y_0^2 \ge 1/4$, this solution will blow-up at location $x_c = 0$ with $t_c = \pm\sqrt{(4y_0^2-1)/16}$. Graphs of solution (11) are qualitatively similar to those of the fundamental rogue waves in the $\mathcal{PT}$-symmetric NLS equation.[16]

Next, we consider the second-order rogue waves. In this case, we get rational solution with two free complex parameters s_0 and s_1. This solution can be either globally bounded or blowing-up in finite-times with certain spatial locations. One such nonsingular triangular pattern rogue-wave solution is observed and displayed in Fig. 1(a). This pattern features the double temporal bumps with a single temporal bump, which resemble those reported in the local NLS equation.[8,9,17,18] Besides, the exploding rogue-wave solutions exhibit more interesting patterns, which have not been observed before. One of those is displayed in Fig. 1(b). This solution contains

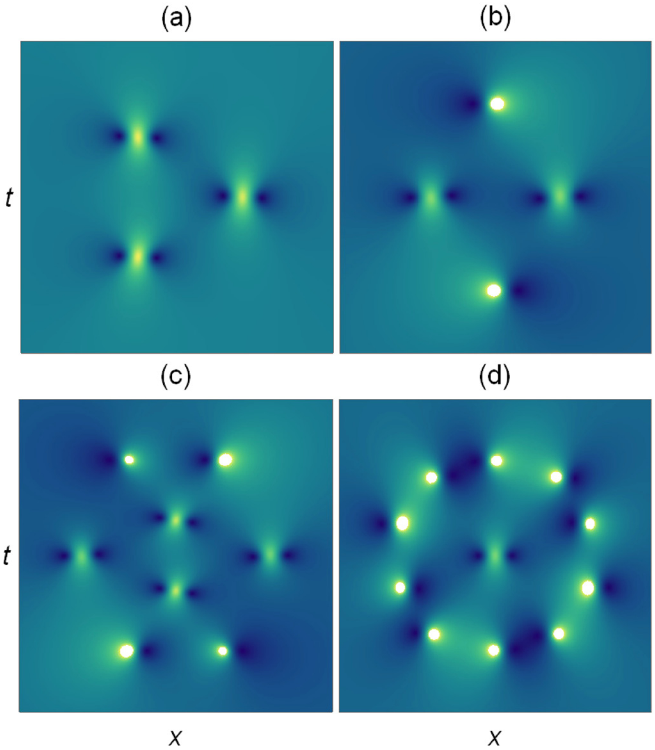

FIG. 1. The upper row displays two second-order rogue waves with parameters: (a) $s_0 = 0.05i; s_1 = 20$ and (b) $s_0 = 2i; s_1 = 30i$. The lower row exhibits two third-order rogue waves with parameters: (c) $s_0 = 2i; s_1 = 10i; s_2 = -180i$ and (d) $s_0 = s_1 = 0; s_2 = 300i$.

two blowing-up peaks on the vertical t axis, and two nonsingular "Peregrine-like" humps on the horizontal x axis. Moreover, the maximum number of singular peaks in these solutions is six.

The third-order rogue waves would exhibit a wider variety of patterns. Apart from the triangular and pentagon pattern, "Peregrine-like" rogue waves, that resembles those patterns reported in the local NLS equation,[18] there are other interesting patterns which are quite different from the known results in other models. Two of them are chosen and displayed in Fig. 1. In panel (c), there is a hybrid pattern rogue-wave solution consisted of four "Peregrine-like" nonsingular humps along with four singular peaks. In panel (d), there is one "Peregrine-like" nonsingular hump surrounded by ten singular peaks. In this case, the maximum number of singular peaks is found to be twelve. These results can be apparently extended to the Nth order rogue waves. By special choices of the free parameters s_k $(k \in \mathbb{N}^+)$, we could derive even richer spatial-temporal patterns, in the form of nonsingular humps, singular peaks, or their hybrid patterns, but with more intensity.

In addition, as to why these rogue-wave solutions are so similar to those of other models, for instance, the local NLS model, we hold the opinion that this could be connected with modulation instability (MI), which gives a physical explanation for the inception of rogue waves. For the local NLS equation and the reverse-time NLS equation (2), their simplest plane wave solutions are in the same form, which is $q_0 = re^{-2ir^2t}$, where r is the background amplitude. considering introducing disturbance quantities $\tilde{q}$ as multiplicative perturbations to the plane wave, i.e., $q = (r + \tilde{q})e^{-2ir^2t}$, and $\tilde{q}$ can be conveniently expressed as linear combinations of pure Fourier modes $\tilde{q} = f_1(x,t)e^{i\kappa(x+\Omega t)} + f_2(x,-t)e^{i\kappa(x-\Omega t)}$, where κ and Ω are the real wave number of the disturbance and the complex phase velocity, respectively. Then, imposing the modulation instability analysis for this constant background solution, we find that when $\kappa^2 > r^2$, this constant background is modulationally unstable with the growth rate $\kappa|\mathrm{Im}(\Omega)| = \frac{\kappa\sqrt{4r^4-(2r^2-4\kappa^2)^2}}{2|\kappa|}$. Moreover, when we perform this modulation stability analysis on the local NLS equation, we get the same growth rate which is in accordance with that in the reverse-time NLS model. Despite the similarity for these rogue-wave solutions, there also exist some different features. For example, the peak amplitude as well as the locations of zeros in the fundamental rogue-wave solution (11) is different from the Peregrine soliton, depending on the parameter y_0.

III. ROGUE-WAVE SOLUTIONS IN THE REVERSE-TIME NONLOCAL DS SYSTEM

In this section, we construct the rogue-wave solution in the reverse-time nonlocal DS systems (3) and (4). Considering the following auxiliary linear system:

$$L\Phi = 0, \quad L = \partial_y - J\partial_x - P, \tag{12}$$

$$M\Phi = 0, \quad M = \partial_t - \sum_{j=0}^{2} V_{2-j}\partial_x^j, \tag{13}$$

where

$$V_0 = \mathrm{i}\gamma^{-1}J, \quad V_1 = \mathrm{i}\gamma^{-1}P, \quad V_2 = \frac{\mathrm{i}}{2\gamma}\left(P_x + \gamma^2 JP_y + Q\right),$$
$$J = \gamma^{-1}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad P = \begin{pmatrix} 0 & q \\ -r & 0 \end{pmatrix}, \quad Q = \begin{pmatrix} \phi_1 & 0 \\ 0 & \phi_2 \end{pmatrix},$$
$$\phi = qr - \frac{1}{2\gamma}(\phi_1 - \phi_2).$$

The compatibility condition $[L, M] = 0$ yields to the reverse-time nonlocal DS equations (3) and (4) under the reverse-time reduction

$$r(x,y,t) = \sigma q(x,y,-t). \tag{14}$$

A. Unified binary Darboux transformation with reduction

The standard scheme for the construction of binary DT was first introduced by Matveev and Salle.[19] Especially, for operators L and M given in (12) and (13), a binary DT has been constructed.[19,20] Generally, the n-fold Datrboux transformation has been reported.[21] Moreover, if we introduce special parameters in the wave-functions as "spectral" parameters, we can apply the generalized Darboux transformation to construct high-order solutions, and the explicit formulas have been given,[21] which can be directly used for our purposed here.

Next, to reduce the binary DT for the reverse-time nonlocal DS equations (3) and (4), it can be verified via a direct calculation that the eigenfunction $\theta(x,y,t)$ and the adjoint eigenfunction $\rho(x,y,t)$ are restricted to satisfy the following reduction:

$$\rho(x,y,t) = \tau_\sigma \theta^*(x,y,-t), \quad \tau_\sigma = \mathrm{diag}(1,\sigma), \tag{15}$$

where θ satisfy $L(\theta)=0$, and ρ admits the adjoint operator: $L^\dagger(\rho)=0$. Here, † stands the (formal) adjoint on an operator.

In the following discussions, we choose a constant $q_{[0]}=c$, $\phi_{[0]}=\sigma c^2$ ($c\in\mathbb{R}$, and it can be further normalized to the unit background) as the seeding solution and perform the Darboux transformation. Then, the eigenfunction was solved from Eqs. (12) and (13)

$$\theta(x,y,t) := \{f_i + \partial_{\varphi_i}\}(\xi_i,\eta_i)^T, \quad f_i\in\mathbb{C}, \tag{16}$$

where $\lambda_i = r_i \exp(\mathrm{i}\varphi_i)$,

$$\xi_i(x,y,t) = c_i e^{\omega_i(x,y,t)}, \quad \eta_i(x,y,t) = \frac{\lambda_i c_i}{c} e^{\omega_i(x,y,t)},$$
$$\omega_i(x,y,t) = \alpha_i x + \beta_i y + \gamma_i t, \quad \gamma_i = \mathrm{i}\gamma^{-1}\alpha_i\beta_i,$$
$$\alpha_i = -\frac{1}{2}\gamma\left(\lambda_i + \sigma c^2\lambda_i^{-1}\right), \quad \beta_i = \frac{1}{2}\left(\lambda_i - \sigma c^2\lambda_i^{-1}\right).$$

Here, r_i and φ_i are real parameters and c_i is set to be one without loss of generality. Moreover, the form of the adjoint wave-functions $\rho(x,y,t)$ can be obtained from reduction (15).

B. Multi- and high-order Rogue-wave solutions

The first-order rational solution for the reverse-time nonlocal DS equations (3) and (4) is given as

$$q_1(x,y,t) = 1 - \frac{2F_1(t)}{F(x,y,t)}, \tag{17}$$

$$\phi_1(x,y,t) = \sigma + 2\gamma^2[\ln(F(x,y,t))]_{xx}, \tag{18}$$

where $\lambda_1 = r_1 e^{\mathrm{i}\varphi_1}$, with

$$F_1(t) = 4i\gamma^2(\lambda_1^{-2}+\lambda_1^2)t + 2,$$
$$F(x,y,t) = [H(x,y)]^2 + 4\left(\lambda_1^{-2}+\lambda_1^2\right)^2 t^2 + 1,$$
$$H(x,y) = \sigma\lambda_1^{-1}(\gamma x + y) - \lambda_1(\gamma x - y) - 2\mathrm{i}f_1 + 1.$$

In the case, the reverse-time nonlocal DSI corresponds to $\gamma^2=1$. If $[\mathrm{Re}\,(f_1)]^2 < 0.25$ with $\varphi_1 = k\pi$ (i.e.,, λ_1 is a real number). According to the definition of line rogue waves,[22,23] solution (17) describes the globally bounded line rogue-wave solution. Similarly, for the reverse-time nonlocal DSII equation (corresponds to $\gamma^2=-1$). If $[\mathrm{Re}(f_1)]^2<0.25$ and $\sigma=1$, $r_1=1$, i.e., $|\lambda_1|=1$. We can also derive the fundamental line rogue-wave solution. As $t\to\pm\infty$, both of these two solutions uniformly approach the constant background 1 everywhere in the spatial plane. But in the intermediate times, $|q|$ reaches maximum amplitude $\frac{4[\mathrm{Re}(f_1)]^2+3}{4[\mathrm{Re}(f_1)]^2-1}$ at the center of the line wave at time $t=0$. Moreover, since (17) is a $(2+1)$-dimensional solution, if we fix a finite time point t_c in (17), this rational solution could behave like a $(1+1)$-dimensional soliton-type solution in the spatial plane. However, if $[\mathrm{Re}(f_1)]^2 \geq 0.25$, the fundamental rational solution (17) becomes singular at critical time.

To describe the interaction between n individual fundamental rogue waves, we normally need the multi-rogue waves. For the nonlocal DSI equation, its multi rogue-wave

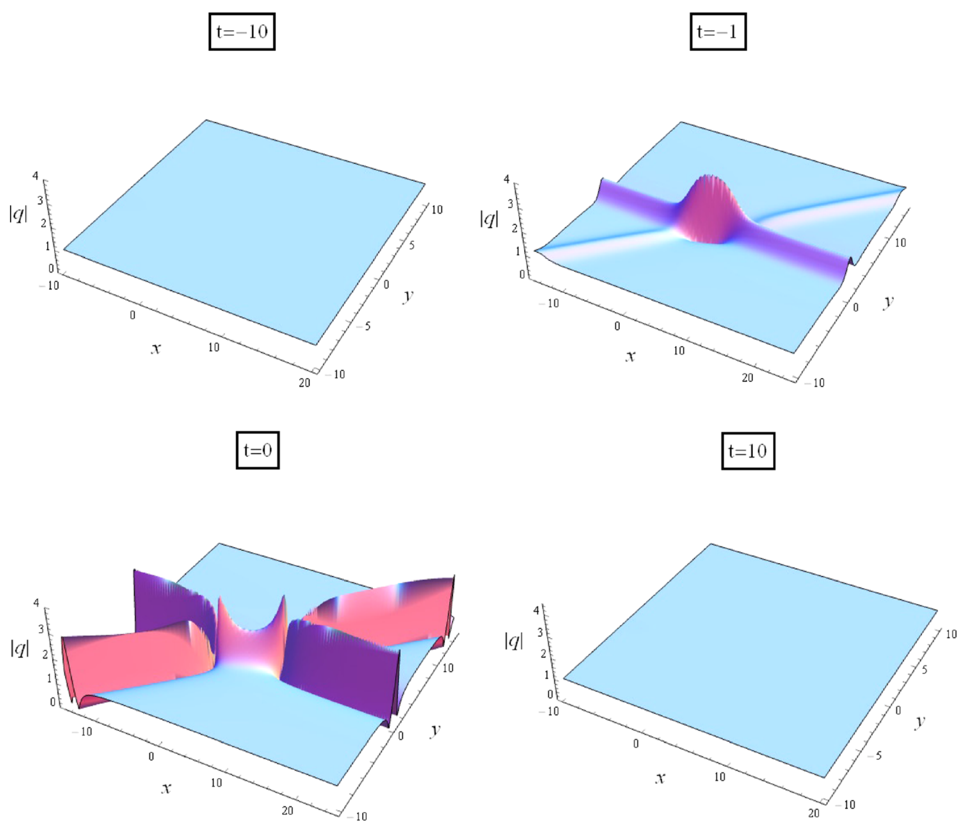

FIG. 2. A two-rogue waves in the reverse-time nonlocal DS-I equation.

solution consists of n separate line rogue waves in the far field of the spatial plane. However, in the near field, the wavefronts of the rogue wave solution are no longer lines, and there would be some interesting curvy wave patterns. For instance, a two-rogue-wave solution is shown in Fig. 2 with parameters given as $\gamma = 1$, $\sigma = 1$, $\lambda_1 = 1$, $\lambda_2 = 2$, $f_1 = 0.05$, and $f_2 = -0.01$.

These two line rogue waves, arising from the constant background, possess higher amplitude in the intersection region at $t = -1$. Afterwards, these higher amplitudes in the intersection region fade, then in the far field, two line rogue solutions rise to higher amplitude at $t = 0$. Afterwards, the solution goes back to the constant background again at larger times (see the $t = 10$ panel). During this process, the maximum value of solution $|q|$ does not exceed 4 for all times. However, if we choose the value of real parameter r_1 not to be one. For example, if we set $\lambda_1 = 1/2$, $\lambda_2 = 2$, $f_1 = 0.05$, and $f_2 = -0.01$, the maximum value of $|q|$ becomes higher and exceeds the value of 4, which is different from the previous pattern.

However, when $\gamma = i$, we find some exploding rogue-wave solutions for the reverse-time nonlocal DSII equation. These exploding rogue-wave solutions go to a constant background 1 as $t \to -\infty$, then blow up to infinity in a finite time interval at isolated spatial locations under certain parameter conditions. Moreover, One can see that the maximum amplitude for this rogue wave solution becomes extremely high near the collapsing point.

The higher-order rogue-wave solutions can be reduced from the higher-order rational solutions. To demonstrate the evolution behaviour of solutions, we consider the second order rogue-wave solution with parameters given as $\gamma = 1$, $\sigma = 1$, $\lambda_1 = 1$, and $f_1 = 0.05$.

An interesting behaviour for this solution can be observed in Fig. 3, these higher-order rogue waves do not uniformly approach the constant background as $t \to \pm\infty$. Instead, only parts of their wave structures approaches background as $t \to \pm\infty$. However, when $|t| \gg 1$, this solution becomes a localized lump sitting on the constant background 1 (see the $t = \pm 6$ panels). And this lump disappears as $t \to 0$. At the same time, a parabola-shaped rogue wave generates from the background. Moreover, when $t = 0$, this parabola is approximately located at the curve $x + y^2 + y + \frac{1}{2} = 0$ in the spatial plane.

Visually, the solution displayed in Fig. 3 can be described as an incoming lump being reflected back by the appearance of a parabola-shaped rogue wave. This interesting pattern is first obtained in the local DSI equation.[22] It is indeed surprising that a similar pattern can be produced in this reverse-time nonlocal DSI equation, although the expression for this solution is different. However, when $\gamma^2 = -1$, we only derive some higher-order rational solutions with almost full-time singularities for the reverse-time nonlocal DS-II equation.

IV. SUMMARY AND DISCUSSION

In this article, rogue-wave solution have been derived for the reverse-time nonlocal NLS equation (2) and the reverse-time nonlocal DSI and DSII equations (3) and (4) using Darboux transformation method under certain reductions. New dynamics patterns in these rogue-wave solutions are further analysed. It is shown that the $(1+1)$-dimensional fundamental rogue waves can be bounded for all x and t or have finite-time collapse. The $(2+1)$-dimensional fundamental line rogue waves are shown to be globally bounded or develop finite-time singularities. The multi- and higher-order rogue waves exhibit richer structures. For example, the $(1+1)$-dimensional higher-order rogue waves exhibit the hybrid of collapsing and non-collapsing peaks, arranged in triangular, pentagon, circular,

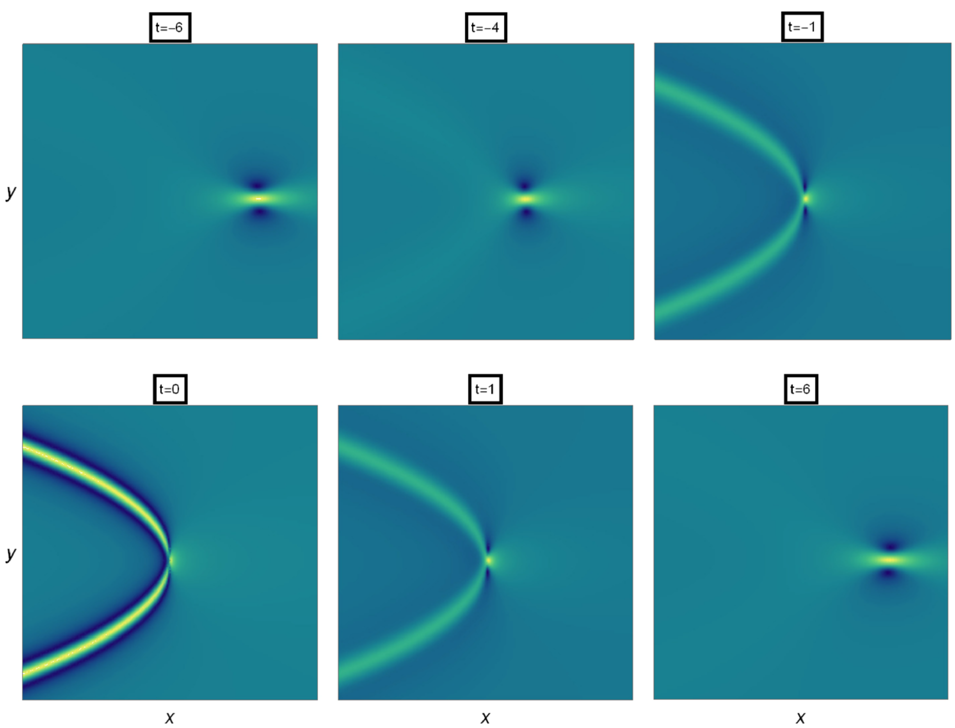

FIG. 3. The second order rogue waves in reverse-time nonlocal DS-I equation.

and other exotic patterns. The multi-line rogue waves describe the interactions between several fundamental line rogue waves, and some curvy wave patterns with higher amplitudes appear due to the interaction.

Interestingly, for the $(2+1)$-dimensional higher-order rogue waves, only part of the wave structure rises from the constant background and then retreats back to it, which possesses the parabola-like shapes. The other part of the wave structure comes from far distance as a localized lump, which interacts with the rogue waves in the near field and then reflects back to the large distance again. The above results reveal more abundant dynamic patterns for rogue-wave solutions in the reverse-time integrable nonlinear equations and further generalize the concept of rogue waves from the local integrable equation into the nonlocal satiation.

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