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Weierstrass semi-rational expansion method and new doubly periodic solutions of the generalized Hirota–Satsuma coupled KdV system

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Abstract

In the paper, with the aid of symbolic computation, we investigate the generalized Hirota–Satsuma coupled KdV system via our Weierstrass semi-rational expansion method presented recently using the rational expansion of Weierstrass elliptic function and its first-order derivative. As a consequence, three families of new Weierstrass elliptic function solutions via Weierstrass elliptic function $\wp(\xi; g_2, g_3)$ and its first-order derivative $\wp'(\xi; g_2, g_3)$. Moreover, the corresponding new Jacobi elliptic function solutions and solitary wave solutions are also presented, and when $\xi \to \infty$, these solitary wave solutions approach to some constants.

1. Introduction

With the rapid development of nonlinear sciences, a lot of nonlinear wave equations were presented. To explain some corresponding physical phenomena, it is very important to investigate their exact solutions [1]. Starting from the four-reduction of the KP hierarchy, Satsuma and Hirota [2] presented the following system (called generalized Hirota–Satsuma coupled KdV system):

$$u_t = \alpha u_{xxx} + \beta u u_x + \gamma (w - v^2)_x, \tag{1.1}$$
$$v_t = -2\alpha v_{xx} - \beta u v_x, \tag{1.2}$$
$$w_t = -2\alpha w_{xx} - \beta u w_x, \tag{1.3}$$

where α, β, γ are parameters. When $w = 0$, (1.1)–(1.3) become to be the well-known Hirota–Satsuma coupled KdV equation [3]. Recently, Tam et al. [4] got some exact solutions in terms of the Hirota's bilinear method. Fan [12,13] also obtained its solutions. More recently, we [5,14] obtained new solitary wave solutions and

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Jacobi elliptic function solutions via some powerful methods [6]. To our knowledge, Weierstrass elliptic function solutions of (1.1)–(1.3) were not reported before.

Recently, we developed the new Weierstrass semi-rational expansion method [7] which is more powerful than the known methods [8,9] related to Weierstrass function [10,11]. Moreover we [7] have chosen the *single* soliton equations: mKdV equation, $(2+1)$-dimensional KP equation and $(3+1)$-dimensional Jimbo-Miwa equation, to illustrate the powerful method such that many types of new doubly periodic solutions were derived.

In this paper, with the aid of symbolic computation, we plan to improve our method to illustrate the generalized Hirota–Satsuma coupled KdV system (1.1)–(1.3) such that many types of new doubly periodic solutions and the corresponding solitary wave solutions are derived.

2. Our Weierstrass semi-rational expansion method

In the following we introduce our Weierstrass semi-rational expansion method, which was presented via the rational expansion of the Weierstrass elliptic function and its first-order derivative.

Step (i). For a given nonlinear differential equation with a physical field u and two independent variables x, t

$$F(u, u_t, u_x, u_{xx}, u_{xt}, u_{tt}, \ldots) = 0, \tag{2.1}$$

we make the travelling wave transformation $u(x,t) = u(\xi)$, $\xi = k(x - \lambda t)$ such that (2.1) reduces to nonlinear ordinary differential equation with constant coefficients

$$G(u, u', u'', u''', \ldots) = 0, \tag{2.2}$$

where the prime denotes derivative with respect to ξ.

Note that we here require that the function G be a polynomial in ξ with constant coefficients. If the function G is not a polynomial in ξ, then we may choose new variables to change it into some polynomial. Otherwise the method needs to be improved.

Step (ii). We assume that (2.2) possesses the power series solution in terms of the Weierstrass elliptic function and its first-order derivative

$$u(\xi) = u(\wp(\xi; g_2, g_3)) = A_0 + \sum_{i=1}^{n} \left[\frac{[A_i\wp(\xi; g_2, g_3) + B_i\wp'(\xi; g_2, g_3)]}{R + P\wp(\xi; g_2, g_3) + Q\wp'(\xi; g_2, g_3)}\right]^i, \tag{2.3}$$

where $\wp'(\xi; g_2, g_3)$ denotes the first-order derivative, and $n, A_i, B_i, A_0, R, P, Q$ are parameters to be determined later, and $\wp(\xi; g_2, g_3)$ is the Weierstrass elliptic function satisfying

$$\wp'^2(\xi) = 4\wp^3(\xi) - g_2\wp(\xi) - g_3, \tag{2.4}$$

from which we have the second-order derivative form

$$\wp''(\xi) = 6\wp^2(\xi) - \frac{1}{2}g_2, \tag{2.5}$$

with g_2, g_3 being real parameters and called invariants [10,11].

According to Eqs. (2.3)–(2.5), to determine the parameter n, we define a polynomial degree function as $D(u(\wp)) = n$, thus we have $D\left(u^p(\wp)\left(\frac{d^s u(\wp)}{d\xi^s}\right)^q\right) = np + q(n+s)$. Therefore we can determine n in (2.3) by balancing the highest degree linear term and nonlinear terms in (2.1) or (2.2).

Note that if $n = 0$, then the method does not work. If $n \neq 0$ is not a positive integers, then the transformation (2.3) takes

$$u(\xi) = u(\wp(\xi; g_2, g_3)) = \left[A_0 + \frac{[A_i\wp(\xi; g_2, g_3) + B_i\wp'(\xi; g_2, g_3)]}{R + P\wp(\xi; g_2, g_3) + Q\wp'(\xi; g_2, g_3)}\right]^n. \tag{2.6}$$

Step (iii). The substitution of (2.3) into (2.2) along with (2.4) and (2.5) leads to a polynomial of $\wp'^i\wp^j$ $(i = 0, 1; j = 0, 1, 2, 3, \ldots)$. Setting their coefficients to zero yields a set of algebraic equations with respect to the unknowns $k, \lambda, A_i, B_i, g_2, g_3$.

Step (iv). With the aid of symbolic computation, we solve the set of algebraic equations obtained, which may not be consistent. Finally we derive the doubly periodic solutions of the given nonlinear wave equations in terms of Weierstrass elliptic function and its first-order derivative.

3. New Weierstrass elliptic function solutions

Here we consider the generalized Hirota–Satsuma coupled KdV equation (1.1)–(1.3). According to Step (i), we make a travelling wave transformation in the form

$$u(x,t)=u(\xi),\quad v(x,y,t)=v(\xi),\quad w(x,t)=w(\xi),\quad \xi=k(x-\lambda t), \tag{3.1}$$

where k is the wave numbers and λ wave speed. Therefore system (1.1)–(1.3) reduces to

$$\lambda u'+k^2\alpha u'''+\beta uu'+\gamma(w-v^2)'=0, \tag{3.2}$$

$$\lambda v'-2\alpha k^2v'''-\beta uv'=0, \tag{3.3}$$

$$\lambda w'-2\alpha k^2w'''-\beta uw'=0. \tag{3.4}$$

According to Step (ii), we assume that (3.2)–(3.4) have the solutions

$$u(\xi)=a_0+\frac{a_1\wp(\xi;g_2,g_3)+b_1\wp'(\xi;g_2,g_3)}{R+P\wp(\xi;g_2,g_3)+Q\wp'(\xi;g_2,g_3)}+\left[\frac{a_2\wp(\xi;g_2,g_3)+b_2\wp'(\xi;g_2,g_3)}{R+P\wp(\xi;g_2,g_3)+Q\wp'(\xi;g_2,g_3)}\right]^2, \tag{3.5}$$

$$v(\xi)=A_0+\frac{A_1\wp(\xi;g_2,g_3)+B_1\wp'(\xi;g_2,g_3)}{R+P\wp(\xi;g_2,g_3)+Q\wp'(\xi;g_2,g_3)}, \tag{3.6}$$

$$w(\xi)=h_0+\frac{h_1\wp(\xi;g_2,g_3)+r_1\wp'(\xi;g_2,g_3)}{R+P\wp(\xi;g_2,g_3)+Q\wp'(\xi;g_2,g_3)}, \tag{3.7}$$

where $\wp(\xi;g_2,g_3)$ satisfies (2.4) and (2.5), and $A_0,A_1,B_1,a_0,a_1,a_2,b_1,b_2,h_0,h_1,r_1,R,P,Q$ are constants to be determined later.

With the aid of Maple, by substitution of (3.5)–(3.7) into (3.2)–(3.4) along with (2.4) and (2.5), and balancing the coefficients of these terms $\wp'^i\wp^j$ $(i=0,1;\ j=0,1,2,\ldots)$, we can obtain a system of algebraic equations with respect to unknowns $k,\lambda,A_0,A_1,B_1,a_0,a_1,a_2,b_1,b_2,h_1,h_0,r_1,R,P,Q$, which is very complicated to solve. To simplify the set of equations, we do not directly use (3.5)–(3.7). We further take the transformations

$$u(\xi)=av^2(\xi)+bv(\xi)+c,\qquad w(\xi)=Av(\xi)+B, \tag{3.8}$$

where a,b,c,A,B are parameters to be determined later. Therefore we know that (3.3) and (3.4) lead to the same equation

$$2\alpha k^2v'''=-\beta av^2v'-\beta bvv'+(\lambda-\beta c)v'. \tag{3.9}$$

Integrating (3.9) once yields

$$2\alpha k^2v''=-\frac{1}{3}\beta av^3-\frac{1}{2}\beta bv^2+(\lambda-\beta c)v+c_0, \tag{3.10}$$

where c_0 is an integration constant.

The substitution of (3.8)–(3.10) into (3.2) yields a polynomial in $v(\xi)$ and $v'(\xi)$. Setting their coefficients to zero leads to a set of equations

$$\begin{cases}\frac{3}{4}\lambda b+\frac{1}{2}\gamma A+\frac{3}{2}ac_0+\frac{1}{4}\beta cb=0,\\[4pt] 3\lambda a+\frac{1}{4}\beta b^2-\beta ca-\gamma=0,\end{cases} \tag{3.11}$$

which leads to these conditions

$$c_0=-\frac{2}{3}\,\frac{-9\lambda^2b-6\gamma A\lambda+2\gamma A\beta c+\beta^2c^2b}{\beta b^2-4\gamma},\qquad a=\frac{\beta b^2-4\gamma}{4(-3\lambda+\beta c)}. \tag{3.12}$$

Therefore we only need to investigate the solution $v(\xi)$ of the one nonlinear ordinary differential Eq. (3.9), and then under the condition (3.12) we can derive the corresponding solutions $u(\xi)$ and $w(\xi)$ via (3.8).

We substitute (3.6) into (3.9) along with (2.4) and (2.5), and equating the coefficients of these terms $\wp'^i\wp^j$ $(i=0,1;\ j=0,1,2,\ldots)$, we can obtain a set of algebraic equations with respect to unknowns $k,\lambda,A_0,A_1,B_1,R,P,Q$. By solving the set of algebraic equations, we have the following cases:

Case 1. $B_1=0,\ Q=0$,

$$b=\frac{-12P^4\alpha k^2g_3+96\alpha k^2R^3P-3\beta aA_1^2R^2-4\beta aA_0PA_1R^2}{2\beta A_1PR^2},$$
$$g_2=\frac{12P^4\alpha k^2g_3+48\alpha k^2R^3P-\beta aA_1^2R^2}{12P^3\alpha k^2R},$$
$$\lambda=\frac{1}{2P^2R^2A_1}(-12\alpha k^2P^4A_1g_3+2\beta cR^2P^2A_1+48A_1\alpha k^2R^3P-\beta aA_1^3R^2-2\beta aA_0^2R^2P^2A_1-12A_0P^5\alpha k^2g_3$$
$$+96A_0P^2\alpha k^2R^3-3A_0P\beta aA_1^2R^2).$$

Case 2. $Q=0$,

$$A_1=-\frac{P(2aA_0+b)}{2a},\quad \lambda=-\frac{\beta(P^3b^2-4P^3ac+16a^2B_1^2R)}{4P^3a},\quad k=\frac{B_1}{P}\sqrt{-\frac{\beta a}{3\alpha}},$$
$$g_2=\frac{P^3R^2b^2+4P^3a^2A_0^2R^2+4P^3bA_0R^2a+4a^2B_1^2P^3g_3+16a^2B_1^2R^3}{4P^2a^2B_1^2R}.$$

Case 3

$$\lambda=-\frac{1}{Q^2(-A_1Q+B_1P)^2}(-4A_1Q^2\beta aA_0b_1^2P-3A_1^2Q^2\alpha k^2P^2+2\beta aA_0B_1Q^3A_1^2$$
$$-36\alpha k^2A_1Q^3RB_1+B_1^2A_1^2Q^2\beta a-2A_1Q\beta aB_1^3P+60\alpha k^2Q^2RB_1^2P+2\beta aA_0B_1^3P^2Q$$
$$-2\beta aA_0^2Q^3PA_1B_1+\beta aA_0^2Q^4A_1^2+\beta aA_0^2Q^2P^2B1^2+2\beta cPQ^3A_1B_1-\beta cQ^4A_1^3$$
$$-\beta cP^2Q^2B_1^2-6A_0P^3Q^2\alpha k^2A_1+72A_0PQ^3\alpha k^2B-1R-48A_0Q^4\alpha k^2A_1R$$
$$+6A_0P^4Q\alpha k^2B_1+3B_1^2P^4\alpha k^2+B_1^4P^2\beta a),$$
$$b=-\frac{2}{\beta Q(-A_1Q+B_1P)^2}(\beta aA_0QB_1^2P^2-3\alpha k^2A_1QP^3-2\beta aA_0Q^2A_1B_1P$$
$$+36\alpha k^2B_1Q^2PR-2\beta aB_1^2A_1QP+\beta aA_0Q^3A_1^2-24\alpha k^2A_1Q^3R$$
$$+3\alpha k^2B_1P^4+\beta aB_1^3P^2+\beta aA_1^2B_1Q^2),$$
$$g_2=\frac{1}{48\alpha k^2Q^4(-A_1Q+B_1P)^2}(-96RQ^4A_1^2\alpha k^2P+6B_1P^5\alpha k^2A_1Q-3P^4\alpha k^2A_1^2Q^2$$
$$+4B_1^3P^3A_1Q\beta a-6B_1^2P^2A_1^2Q^2\beta a-B_1^4P^4\beta a-\beta aA_1^4Q^4-3B_1^2P^6\alpha k^2$$
$$+4B_1PA_1^3Q^3\beta\alpha+168RB_1P^2\alpha k^2Q^3A_1+144R^2B_1^2\alpha k^2Q^4-72RB_1^2P^3\alpha k^2Q^2),$$
$$g_3=-\frac{R}{48\alpha k^2Q^4(-A_1Q+B_1P)^3}(3\alpha k^2B_1^3P^6-6\alpha k^2A_1QP^5B_1^2+\beta aB_1^5P^4$$
$$+3\alpha k^2A_1^2Q^2P^4B_1-4\beta aB_1^4A_1QP^3+72\alpha k^2B_1^3Q^2RP^3+6\beta aB_1^3A_1^2Q^2P^2$$
$$-168\alpha k^2B_1^2Q^3RP^2A_1-4\beta aB_1^2A_1^3Q^3P+144\alpha k^2B_1Q^4RPA_1^2$$
$$+48Q^4R^2\alpha k^2B_1^3+\beta aA_1^4B_1Q^4-48\alpha k^2A_1^3Q^5R).$$

Therefore according to (3.6) and (3.8) and Cases 1–3, we can get three families of new Weierstrass elliptic function solutions of (1.1)–(1.3):

Family 1

$$u_1(x,y,t) = a\left[A_0 + \frac{A_1\wp(\xi;g_2,g_3)}{R+P\wp(\xi;g_2,g_3)}\right]^2 + \frac{bA_1\wp(\xi;g_2,g_3)}{R+P\wp(\xi;g_2,g_3)} + bA_0 + c, \tag{3.7a}$$

$$v_1(x,y,t) = A_0 + \frac{A_1\wp(\xi;g_2,g_3)}{R+P\wp(\xi;g_2,g_3)}, \tag{3.7b}$$

$$w_1(x,y,t) = AA_0 + \frac{AA_1\wp(\xi;g_2,g_3)}{R+P\wp(\xi;g_2,g_3)} + B, \tag{3.7c}$$

where $\xi = k(x-\lambda t)$, and λ, g_2, b are given by Case 1.

Family 2

$$u_2(x,y,t) = a\left[A_0 + \frac{A_1\wp(\xi;g_2,g_3)+B_1\wp'(\xi;g_2,g_3)}{R+P\wp(\xi;g_2,g_3)}\right]^2 + \frac{bA_1\wp(\xi;g_2,g_3)+bB_1\wp'(\xi;g_2,g_3)}{R+P\wp(\xi;g_2,g_3)} + bA_0 + c, \tag{3.8a}$$

$$v_2(x,y,t) = A_0 + \frac{A_1\wp(\xi;g_2,g_3)+B_1\wp'(\xi;g_2,g_3)}{R+P\wp(\xi;g_2,g_3)}, \tag{3.8b}$$

$$w_2(x,y,t) = AA_0 + \frac{AA_1\wp(\xi;g_2,g_3)+AB_1\wp'(\xi;g_2,g_3)}{R+P\wp(\xi;g_2,g_3)} + B, \tag{3.8c}$$

where $\xi = k(x-\lambda t)$, and A_1, λ, k, g_2 are given by Case 2.

Family 3

$$u_3(x,y,t) = a\left[A_0 + \frac{A_1\wp(\xi;g_2,g_3)+B_1\wp'(\xi;g_2,g_3)}{R+P\wp(\xi;g_2,g_3)+Q\wp'(\xi;g_2,g_3)}\right]^2 + \frac{b[A_1\wp(\xi;g_2,g_3)+B_1\wp'(\xi;g_2,g_3)]}{R+P\wp(\xi;g_2,g_3)+Q\wp'(\xi;g_2,g_3)} + bA_0 + c, \tag{3.9a}$$

$$v_3(x,y,t) = A_0 + \frac{A_1\wp(\xi;g_2,g_3)+B_1\wp'(\xi;g_2,g_3)}{R+P\wp(\xi;g_2,g_3)+Q\wp'(\xi;g_2,g_3)}, \tag{3.9b}$$

$$w_3(x,y,t) = AA_0 + \frac{A[A_1\wp(\xi;g_2,g_3)+B_1\wp'(\xi;g_2,g_3)]}{R+P\wp(\xi;g_2,g_3)+Q\wp'(\xi;g_2,g_3)} + B, \tag{3.9c}$$

$\xi = k(x-\lambda t)$, and λ, b, g_2, g_3 are given by Case 3.

In particular, we know that the Weierstrass elliptic function can be written in the form

$$\wp(\xi;g_2,g_3) = e_2 - (e_2-e_3)\mathrm{cn}^2(\sqrt{e_1-e_3}\,\xi;m), \tag{3.10}$$

in terms of the Jacobi elliptic cosine function, where $m^2 = (e_2-e_3)/(e_1-e_3)$ is the modulus of the Jacobi elliptic function, e_i ($i = 1,2,3$; $e_1 \geqslant e_2 \geqslant e_3$) are three roots of the cubic equation $4y^3 - g_2y - g_3 = 0$. Since when $m \to 1$, i.e., $e_2 \to e_1$, we have $\mathrm{cn}(\xi;m) \to \mathrm{sech}(\xi)$, $\mathrm{sn}(\xi;m) \to \tanh(\xi)$, $\mathrm{dn}(\xi;m) \to \mathrm{sech}(\xi)$, thus we can rewrite the solutions (3.7a,b,c)–(3.9a,b,c) via Jacobi elliptic functions and hyperbolic functions:

Type (i). From (3.7a)–(3.7c) we have

$$u_1'(\xi) = a\left[A_0 + \frac{A_1[e_2 - M\mathrm{cn}^2(\sqrt{N}\xi;m)]}{R+P[e_2 - M\mathrm{cn}^2(\sqrt{N}\xi;m)]}\right]^2 + \frac{bA_1[e_2 - M\mathrm{cn}^2(\sqrt{N}\xi;m)]}{R+P[e_2 - M\mathrm{cn}^2(\sqrt{N}\xi;m)]} + bA_0 + c, \tag{3.11a}$$

$$v_1'(\xi) = A_0 + \frac{A_1[e_2 - M\mathrm{cn}^2(\sqrt{N}\xi;m)]}{R+P[e_2 - M\mathrm{cn}^2(\sqrt{N}\xi;m)]}, \tag{3.11b}$$

$$w_1'(\xi) = AA_0 + B + \frac{AA_1[e_2 - M\mathrm{cn}^2(\sqrt{N}\xi;m)]}{R+P[e_2 - M\mathrm{cn}^2(\sqrt{N}\xi;m)]}, \tag{3.11c}$$

where $M = e_2 - e_3$, $N = e_1 - e_3$. When $m \to 1$, these solutions (3.11a)–(3.11c) become solitary wave solutions

$$u_1''(\xi) = a\left[A_0 + \frac{A_1[e_2 - M\,\mathrm{sech}^2(\sqrt{N}\xi)]}{R + P[e_2 - M\,\mathrm{sech}^2(\sqrt{N}\xi)]}\right]^2 + \frac{bA_1[e_2 - M\,\mathrm{sech}^2(\sqrt{N}\xi)]}{R + P[e_2 - M\,\mathrm{sech}^2(\sqrt{N}\xi)]} + bA_0 + c, \tag{3.12a}$$

$$v_1''(\xi) = A_0 + \frac{A_1[e_2 - M\,\mathrm{sech}^2(\sqrt{N}\xi)]}{R + P[e_2 - M\,\mathrm{sech}^2(\sqrt{N}\xi)]}, \tag{3.12b}$$

$$w_1''(\xi) = AA_0 + B + \frac{AA_1[e_2 - M\,\mathrm{sech}^2(\sqrt{N}\xi)]}{R + P[e_2 - M\,\mathrm{sech}^2(\sqrt{N}\xi)]}. \tag{3.12c}$$

Note that when $\xi \to \infty$, we have $u_1''(\xi) \to a\left[A_0 + \frac{A_1e_2}{R+Pe_2}\right]^2 + \frac{bA_1e_2}{R+Pe_2} + bA_0 + c$; $v_1''(\xi) \to A_0 + \frac{A_1e_2}{R+Pe_2}$; $w_1''(\xi) \to AA_0 + \frac{AA_1e_2}{R+Pe_2} + B$.

Type (ii). From (3.8a)–(3.8c), we get

$$u_2'(\xi) = a\left[A_0 + \frac{A_1[e_2 - M\mathrm{cn}^2(\sqrt{N}\xi;m)] + 2B_1M\sqrt{N}\mathrm{sn}(\sqrt{N}\xi;m)\mathrm{cn}(\sqrt{N}\xi;m)\mathrm{dn}(\sqrt{N}\xi;m)}{R + P[e_2 - M\mathrm{cn}^2(\sqrt{N}\xi;m)]}\right]^2 + \frac{bA_1[e_2 - M\mathrm{cn}^2(\sqrt{N}\xi;m)] + 2bB_1M\sqrt{N}\mathrm{sn}(\sqrt{N}\xi;m)\mathrm{cn}(\sqrt{N}\xi;m)\mathrm{dn}(\sqrt{N}\xi;m)}{R + P[e_2 - M\mathrm{cn}^2(\sqrt{N}\xi;m)]} + bA_0 + c, \tag{3.13a}$$

$$v_2'(\xi) = A_0 + \frac{A_1[e_2 - M\mathrm{cn}^2(\sqrt{N}\xi;m)] + 2B_1M\sqrt{N}\mathrm{sn}(\sqrt{N}\xi;m)\mathrm{cn}(\sqrt{N}\xi;m)\mathrm{dn}(\sqrt{N}\xi;m)}{R + P[e_2 - M\mathrm{cn}^2(\sqrt{N}\xi;m)]}, \tag{3.13b}$$

$$w_2'(\xi) = AA_0 + \frac{AA_1[e_2 - M\mathrm{cn}^2(\sqrt{N}\xi;m)] + 2B_1M\sqrt{N}\mathrm{sn}(\sqrt{N}\xi;m)\mathrm{cn}(\sqrt{N}\xi;m)\mathrm{dn}(\sqrt{N}\xi;m)}{R + P[e_2 - M\mathrm{cn}^2(\sqrt{N}\xi;m)]} + B, \tag{3.13c}$$

when $m \to 1$, solutions (3.13a)–(3.13c) become solitary wave solutions

$$u_2''(\xi) = a\left[A_0 + \frac{A_1[e_2 - M\,\mathrm{sech}^2(\sqrt{N}\xi)] + 2B_1M\sqrt{N}\tanh(\sqrt{N}\xi)\mathrm{sech}^2(\sqrt{N}\xi)}{R + P[e_2 - M\,\mathrm{sech}^2(\sqrt{N}\xi)]}\right]^2 + \frac{bA_1[e_2 - M\,\mathrm{sech}^2(\sqrt{N}\xi)] + 2bB_1M\sqrt{N}\tanh(\sqrt{N}\xi)\mathrm{sech}^2(\sqrt{N}\xi)}{R + P[e_2 - M\,\mathrm{sech}^2(\sqrt{N}\xi)]} + bA_0 + c, \tag{3.14a}$$

$$v_2''(\xi) = A_0 + \frac{A_1[e_2 - M\,\mathrm{sech}^2(\sqrt{N}\xi)] + 2B_1M\sqrt{N}\tanh(\sqrt{N}\xi)\mathrm{sech}^2(\sqrt{N}\xi)}{R + P[e_2 - M\,\mathrm{sech}^2(\sqrt{N}\xi)]}, \tag{3.14b}$$

$$w_2''(\xi) = AA_0 + \frac{AA_1[e_2 - M\,\mathrm{sech}^2(\sqrt{N}\xi)] + 2AB_1M\sqrt{N}\tanh(\sqrt{N}\xi)\mathrm{sech}^2(\sqrt{N}\xi)}{R + P[e_2 - M\,\mathrm{sech}^2(\sqrt{N}\xi)]} + B. \tag{3.14c}$$

Note that when $\xi \to \infty$, we have $u_1''(\xi) \to a\left[A_0 + \frac{A_1e_2}{R+Pe_2}\right]^2 + \frac{bA_1e_2}{R+Pe_2} + bA_0 + c$; $v_1''(\xi) \to A_0 + \frac{A_1e_2}{R+Pe_2}$; $w_1''(\xi) \to AA_0 + \frac{AA_1e_2}{R+Pe_2} + B$.

Type (iii)

$$u_3'(\xi) = a\left[A_0 + \frac{A_1[e_2 - M\mathrm{cn}^2(\sqrt{N}\xi;m)] + 2B_1M\sqrt{N}\mathrm{sn}(\sqrt{N}\xi;m)\mathrm{cn}(\sqrt{N}\xi;m)\mathrm{dn}(\sqrt{N}\xi;m)}{R + P[e_2 - M\mathrm{cn}^2(\sqrt{N}\xi;m)] + 2QM\sqrt{N}\mathrm{sn}(\sqrt{N}\xi;m)\mathrm{cn}(\sqrt{N}\xi;m)\mathrm{dn}(\sqrt{N}\xi;m)}\right]^2 + \frac{bA_1[e_2 - M\mathrm{cn}^2(\sqrt{N}\xi;m)] + 2bB_1M\sqrt{N}\mathrm{sn}(\sqrt{N}\xi;m)\mathrm{cn}(\sqrt{N}\xi;m)\mathrm{dn}(\sqrt{N}\xi;m)}{R + P[e_2 - M\mathrm{cn}^2(\sqrt{N}\xi;m)] + 2QM\sqrt{N}\mathrm{sn}(\sqrt{N}\xi;m)\mathrm{cn}(\sqrt{N}\xi;m)\mathrm{dn}(\sqrt{N}\xi;m)} + bA_0 + c, \tag{3.15a}$$

$$v_3'(\xi) = A_0 + \frac{A_1[e_2 - M\mathrm{cn}^2(\sqrt{N}\xi;m)] + 2B_1M\sqrt{N}\mathrm{sn}(\sqrt{N}\xi;m)\mathrm{cn}(\sqrt{N}\xi;m)\mathrm{dn}(\sqrt{N}\xi;m)}{R + P[e_2 - M\mathrm{cn}^2(\sqrt{N}\xi;m)] + 2QM\sqrt{N}\mathrm{sn}(\sqrt{N}\xi;m)\mathrm{cn}(\sqrt{N}\xi;m)\mathrm{dn}(\sqrt{N}\xi;m)}, \tag{3.15b}$$

$$w_3'(\xi) = AA_0 + \frac{AA_1[e_2 - M\mathrm{cn}^2(\sqrt{N}\xi;m)] + 2AB_1M\sqrt{N}\mathrm{sn}(\sqrt{N}\xi;m)\mathrm{cn}(\sqrt{N}\xi;m)\mathrm{dn}(\sqrt{N}\xi;m)}{R + P[e_2 - M\mathrm{cn}^2(\sqrt{N}\xi;m)] + 2QM\sqrt{N}\mathrm{sn}(\sqrt{N}\xi;m)\mathrm{cn}(\sqrt{N}\xi;m)\mathrm{dn}(\sqrt{N}\xi;m)} + B, \tag{3.15c}$$

when $m \to 1$, solutions (3.15a)–(3.15c) become solitary wave solutions

$$u_3''(\xi) = a\left[A_0 + \frac{A_1[e_2 - M\,\text{sech}^2(\sqrt{N}\xi)] + 2B_1M\sqrt{N}\tanh(\sqrt{N}\xi)\text{sech}^2(\sqrt{N}\xi)}{R + P[e_2 - M\,\text{sech}^2(\sqrt{N}\xi)] + 2QM\sqrt{N}\tanh(\sqrt{N}\xi)\text{sech}^2(\sqrt{N}\xi)}\right]^2 + \frac{bA_1[e_2 - M\,\text{sech}^2(\sqrt{N}\xi)] + 2bB_1M\sqrt{N}\tanh(\sqrt{N}\xi)\text{sech}^2(\sqrt{N}\xi)}{R + P[e_2 - M\,\text{sech}^2(\sqrt{N}\xi)] + 2QM\sqrt{N}\tanh(\sqrt{N}\xi)\text{sech}^2(\sqrt{N}\xi)} + bA_0 + c, \tag{3.16a}$$

$$v_3''(\xi) = \frac{A_1[e_2 - M\,\text{sech}^2(\sqrt{N}\xi)] + 2B_1M\sqrt{N}\tanh(\sqrt{N}\xi)\text{sech}^2(\sqrt{N}\xi)}{R + P[e_2 - M\,\text{sech}^2(\sqrt{N}\xi)] + 2QM\sqrt{N}\tanh(\sqrt{N}\xi)\text{sech}^2(\sqrt{N}\xi)}, \tag{3.16b}$$

$$w_3''(\xi) = \frac{AA_1[e_2 - M\,\text{sech}^2(\sqrt{N}\xi)] + 2AB_1M\sqrt{N}\tanh(\sqrt{N}\xi)\text{sech}^2(\sqrt{N}\xi)}{R + P[e_2 - M\,\text{sech}^2(\sqrt{N}\xi)] + 2QM\sqrt{N}\tanh(\sqrt{N}\xi)\text{sech}^2(\sqrt{N}\xi)} + B. \tag{3.16c}$$

Note that when $\xi \to \infty$, we have $u_1''(\xi) \to a\left[A_0 + \frac{A_1e_2}{R+Pe_2}\right]^2 + \frac{bA_1e_2}{R+Pe_2} + bA_0 + c$; $v_1''(\xi) \to A_0 + \frac{A_1e_2}{R+Pe_2}$; $w_1''(\xi) \to AA_0 + \frac{AA_1e_2}{R+Pe_2} + B$.

Note that when $m \to 0$, we also derive the responding periodic solutions of system (1.1)–(1.3). Here we omit them.

4. Conclusions

In brief, with the aid of symbolic computation, we have obtained three families of new Weierstrass elliptic function solutions of the generalized Hirota–Satsuma coupled KdV system via our Weierstrass semi-rational expansion method. Moreover, the corresponding new Jacobi elliptic function solutions and solitary wave solutions are also presented, and when $\xi \to \infty$, these solitary wave solutions approaches to some constants. In addition, our method can be also extend to a wide of class of nonlinear evolution systems, and can be automatically performed in the computer via Maple.

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