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On exact solutions of the nonlinear Schrödinger equations in optical fiber

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Abstract

In this paper, with the help of symbolic computation, the projective Riccati equations method is extended to find some new exact solutions of the nonlinear Schrödinger model with varying dispersion, nonlinearity, and gain or absorption. As a result, four families of Soliton-like solutions in these models are obtained.

1. Introduction

The construction of the exact solutions of nonlinear partial differential equations (PDEs) is one of the most important and essential tasks in nonlinear science. In the past few decades, many authors had mainly studied solitary wave solutions of nonlinear PDEs by using various methods, such as the inverse scattering method [1,2], Backlund transformation [3], Hirota bilinear method [4], the tanh method [5,6], various extended tanh methods [7–11], generalized hyperbolic-function method [12] and generalized Riccati equation expansion method [13] and so on.

In Ref. [14], Conte and Musette presented an indirect method to seek some solitary wave solutions of nonlinear PDEs that can be expressed as a polynomial in two elementary functions which satisfy a project Riccati system [15]. By use of this method, some solitary wave solutions of many nonlinear PDEs have been obtained [14,16]. Recently, Yan [17] and Chen–Li [18] further developed Conte and Musette's method by introducing a more general projective Riccati equations and obtained many exact travelling wave solutions of some nonlinear PDEs.

In this paper, we would like to further extended the general projective Riccati equations method [14–18] to find some soliton-like solutions for some nonlinear PDEs. Then we choose the nonlinear Schrödinger equations (NLSE) with varying coefficients in optical fiber [2,19–22] to illustrate the extended method. Today, NLSE optical solitons are regarded as the natural data bits and as an important alternative for the next generation of ultrahigh speed optical telecommunication systems (see, e.g., Refs. [19–22] for detail).

The rest of this paper is organized as follows. In Section 2, we establish the extended projective Riccati equations method. In Section 3, we apply the extended method to the NLSE in optical fiber and obtain four families of exact soliton-like solutions for these models. Section 4 is a short summary and discussion.

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2. Extended projective Riccati equations method

The key idea of the extended projective Riccati equations method is to take full advantages of the following projective equations [14–18]:

$$\sigma'(\xi) = \epsilon\sigma(\xi)\tau(\xi), \quad \tau'(\xi) = R + \epsilon\tau^2(\xi) - \mu\sigma(\xi), \quad \epsilon = \pm 1, \tag{1}$$

$$\tau^2(\xi) = -\epsilon\left[R - 2\mu\sigma(\xi) + \frac{\mu^2 - 1}{R}\sigma^2(\xi)\right], \quad R \neq 0. \tag{2}$$

where R, μ are constants and $' = d/\xi$. We know that Eqs. (1) and (2) have the following solutions:

Case 1. When $\epsilon = -1$,

$$\begin{cases} \tau_1(\xi) = \dfrac{\sqrt{R}\tanh(\sqrt{R}\xi)}{\mu\,\mathrm{sech}(\sqrt{R}\xi) + 1}, & \sigma_1(\xi) = \dfrac{R\,\mathrm{sech}(\sqrt{R}\xi)}{\mu\,\mathrm{sech}(\sqrt{R}\xi) + 1}, \\ \tau_2(\xi) = \dfrac{\sqrt{R}\coth(\sqrt{R}\xi)}{\mu\,\mathrm{csch}(\sqrt{R}\xi) + 1}, & \sigma_2(\xi) = \dfrac{R\,\mathrm{csch}(\sqrt{R}\xi)}{\mu\,\mathrm{csch}(\sqrt{R}\xi) + 1}. \end{cases} \tag{3}$$

Case 2. When $\epsilon = 1$,

$$\begin{cases} \tau_3(\xi) = \dfrac{\sqrt{R}\tan(\sqrt{R}\xi)}{\mu\sec(\sqrt{R}\xi) + 1}, & \sigma_3(\xi) = \dfrac{R\sec(\sqrt{R}\xi)}{\mu\sec(\sqrt{R}\xi) + 1}, \\ \tau_4(\xi) = -\dfrac{\sqrt{R}\cot(\sqrt{R}\xi)}{\mu\csc(\sqrt{R}\xi) + 1}, & \sigma_4(\xi) = \dfrac{R\csc(\sqrt{R}\xi)}{\mu\csc(\sqrt{R}\xi) + 1}. \end{cases} \tag{4}$$

Now by use of the above results, we establish the extended projective Riccati equations method as follows.

Given a nonlinear PDE with, say, two variables: $\{x, t\}$,

$$p(u_t, u_x, u_{xt}, u_{tt}, u_{xx}, \ldots) = 0, \tag{5}$$

Step 1. We assume that (5) has the following solutions:

$$u(x,t) = a_0 + \sum_{i=1}^{m} \sigma^{i-1}(\xi)[a_i\tau(\xi) + b_i\sigma(\xi)], \tag{6}$$

where $a_0 = a_0(x,t)$, $a_i = a_i(x,t)$, $b_i = b_i(x,t)$, $(i = 1, \ldots, m)$, $\xi = \xi(x,t)$ are all unknown functions of $\{x,t\}$, $\sigma(\xi)$ and $\tau(\xi)$ satisfy Eqs. (1) and (2).

The parameter m can be found by balancing the highest order derivative term and the nonlinear terms in (5) (m is usually a positive integer). If m is a fraction or a negative integer, we first make the transformation

$$u(x,t) = \varphi^m(x,t). \tag{7}$$

Then substitute (7) into (5) and return to determine balance constant m again.

The transformation (6) is more general than the transformations in tanh method [5,6], various extended tanh-function methods [7–11], generalized hyperbolic-function method [12], generalized Riccati equation expansion method [13] and projective Riccati equations method [14,16–18]. Firstly, compared with tanh method, various extended tanh-function methods as well as projective Riccati equations method, the restriction on $\xi(x,t)$ as merely a linear function $\{x,t\}$ and the restriction on the coefficients a_0, a_i, b_i $(i = 1, \ldots, m)$ as constants are removed. Secondly, when $\mu = 0$ in Eqs. (1) and (2), generalized Riccati equation expansion method and generalized hyperbolic-function method can be recovered. When $\mu \neq 0$, some new and more general solutions would be expected for some nonlinear PDEs.

Step 2. Substituting (6) along with (1) and (2) into (5), extracting the numerator of the resulting system, we can obtain a set of algebraic polynomials for $\tau^i(\xi)\sigma^j(\xi)$ $(i = 0, 1; j = 0, 1, \ldots)$. Setting the coefficients of these terms $\tau^i(\xi)\sigma^j(\xi)$ to zero, we get a system of over-determined PDEs with respect to unknown functions $\{a_0, a_i, b_i\ (i = 1, \ldots, m), \xi\}$.

Step 3. Solving the above system by use of symbolic computation system—*Maple*, we would end up with the explicit expressions for μ, a_0, a_i, b_i $(i = 1, \ldots, m)$ and ξ or the constraints among them.

In order that the equations derived in *Step* 2 can be solved easily, we may choose special forms of a_i, b_i and ξ on a trial-and-error basis. (As we do in Section 3.)

Thus according to {(3), (4), (6)} and the conclusions in *Step 3*, we can obtain many families of exact solutions for Eq. (5).

3. Exact solutions of NLSE in the optical fiber

In the real communication system of optical soliton, the transmission of soliton is described by the nonlinear Schrödinger model with varying coefficients [19–22]

$$\mathrm{i}\frac{\partial u}{\partial z}+\frac{1}{2}\beta(z)\frac{\partial^2 u}{\partial t^2}+\delta(z)|u|^2u=\mathrm{i}\alpha(z)u, \tag{8}$$

where $\beta(z)$ and $\delta(z)$ are slowly increasing dispersion coefficient and nonlinear coefficient, respectively; $\alpha(z)$ represents the heat-insulating amplify or lose. Because it is very difficult to solve variable coefficient nonlinear equation, this equation was studied only by means of numerical analysis or approximate methods [19–21]. Recently, Ruan and Chen [22] studied Eq. (8) by the symmetry approach and find some exact solutions of it.

In order to obtain some exact solutions of Eq. (8), firstly we make the transformation

$$u(t,z)=V(t,z)\exp[\mathrm{i}\Theta(t,z)]. \tag{9}$$

Then substituting (9) into (8) and setting the real and imaginary parts of the resulting equation equal to zero, we obtain the following sets of partial differential equations

$$-V\Theta_z+\frac{1}{2}\beta(z)(V_{tt}-V\Theta_t^2)+\delta(z)V^3=0, \tag{10}$$

$$V_z+\frac{1}{2}\beta(z)(2V_t\Theta_t+V\Theta_{tt})-\alpha(z)V=0. \tag{11}$$

Now we use the extended projective Riccati equations method to investigate the Eqs. (10) and (11).

By balancing V_{tt} and V^3 in (10), we obtain $m=1$ in (6). Therefore we assume the solutions of Eqs. (10) and (11) in the following special forms

$$V(t,z)=a_0(z)+a_1(z)\tau(\xi)+b_1(z)\sigma(\xi),\quad \xi=tp(z)+q(z), \tag{12}$$

$$\Theta(t,z)=t\Gamma(z)+\Omega(z), \tag{13}$$

where $a_0(z)$, $a_1(z)$, $b_1(z)$, $p(z)$, $q(z)$, $\Gamma(z)$ and $\Omega(z)$ are functions of z to be determined, $\tau(\xi)$ and $\sigma(\xi)$ satisfy Eqs. (1) and (2).

Substituting (1), (2), (12) and (13) into (10) and (11), collecting coefficients of monomials of $\tau(\xi)$, $\sigma(\xi)$ and t of the resulting system's numerator (Notice that $\beta(z)$, $\delta(z)$, $\alpha(z)$, $a_0(z)$, $a_1(z)$, $b_1(z)$, $p(z)$, $q(z)$, $\Gamma(z)$ and $\Omega(z)$ are independent of t.), then setting each coefficients to zero, we obtain the following over-determined PDEs system with respect to differentiable functions $\{\beta(z),\delta(z),\alpha(z),a_0(z),a_1(z),b_1(z),p(z),q(z),\Gamma(z),\Omega(z)\}$ of $\epsilon=-1$.

$$-R\left(-\frac{\partial a_1}{\partial z}+\alpha a_1\right)=0, \tag{14}$$

$$-Rb_1\frac{\partial p}{\partial z}=0, \tag{15}$$

$$2b_1(-3\delta a_1^2+b_1^2\delta R+\beta p^2\mu^2-\beta p^2+3\delta a_1^2\mu^2)=0, \tag{16}$$

$$2a_1(3b_1^2\delta R+\delta a_1^2\mu^2-\delta a_1^2+\beta p^2\mu^2-\beta p^2)=0, \tag{17}$$

$$-6\delta a_0a_1^2+6\delta a_0a_1^2\mu^2+6\delta a_0b_1^2R-12\delta a_1^2b_1\mu R-3\beta p^2b_1\mu R=0, \tag{18}$$

$$-R\left(-\frac{\partial a_0}{\partial z}+\alpha a_0\right)=0, \tag{19}$$

$$-b_1R\left(\frac{\partial q}{\partial z}+\beta\Gamma p\right)=0, \tag{20}$$

$$-2\frac{\partial\Gamma}{\partial z}a_0R = 0, \tag{21}$$

$$-a_0R\left(-2\delta a_0^2 - 6\delta a_1^2R + 2\frac{\partial\Omega}{\partial z} + \Gamma^2\beta\right) = 0, \tag{22}$$

$$-a_1R\left(-2\delta a_1^2R + \Gamma^2\beta - 6\delta a_0^2 + 2\frac{\partial\Omega}{\partial z}\right) = 0, \tag{23}$$

$$a_1R(-4\delta a_1^2\mu - \mu p^2\beta + 12\delta a_0b_1) = 0, \tag{24}$$

$$-a_1\frac{\partial p}{\partial z}(\mu-1)(\mu+1) = 0, \tag{25}$$

$$a_1\mu\frac{\partial p}{\partial z}R = 0, \tag{26}$$

$$-a_1(\mu-1)(\mu+1)\left(\frac{\partial q}{\partial z} + \beta\Gamma p\right) = 0, \tag{27}$$

$$R\left(a_1\mu\frac{\partial q}{\partial z} + \frac{\partial b_1}{\partial z} + \beta\Gamma pa_1\mu - b_1\alpha\right) = 0, \tag{28}$$

$$-R\left(-6R\delta a_1^2b_1 - R\beta p^2b_1 + \beta\Gamma^2b_1 + 12\mu a_1^2a_0\delta - 6\delta a_0^2b_1 + 2\frac{\partial\Omega}{\partial z}b_1\right) = 0, \tag{29}$$

$$-2a_1\frac{\partial\Gamma}{\partial z}R = 0, \tag{30}$$

$$-2\frac{\partial\Gamma}{\partial z}b_1R = 0. \tag{31}$$

In the above system of PDEs, a_0 and b_0 denote $a_0(z)$ and $b_0(z)$, respectively, and so on.
Solving Eqs. (14)–(31) by means of *Maple*, we obtained the following results.

Case 1.

$$\mu = a_0 = 0, \quad \Gamma = C_1, \quad b_1 = b_1, \quad p = C_3, \quad \delta = \delta,$$
$$\beta = \frac{4\delta b_1^2R}{C_3^2}, \quad a_1 = \pm\sqrt{-R}b_1, \quad \alpha = \frac{\partial b_1}{\partial z}\frac{1}{b_1},$$
$$q = \frac{-4C_1R\int\delta b_1^2\,\mathrm{d}z + C_6C_3}{C_3}, \quad \Omega = \frac{-R^2\int\delta b_1^2\,\mathrm{d}z\,C_3^2 - 2R\int\delta b_1^2\,\mathrm{d}z\,C_1^2 + C_5C_3^2}{C_3^2}, \tag{32}$$

where C_1, C_3, C_5 and C_6 are arbitrary constants; $b_1 = b_1$ denotes b_1 being an arbitrary function of z, $\delta = \delta$ denotes δ being also an arbitrary function of z. (Note: in the rest of this paper, C_i denotes an arbitrary constant.)

Case 2.

$$\mu = a_0 = b_1 = 0, \quad \Gamma = C_1, \quad a_1 = a_1, \quad \Omega = \frac{1}{2}\frac{2\int\delta a_1^2\,\mathrm{d}z\,C_2^2R + \int\delta a_1^2\,\mathrm{d}z\,C_1^2 + 2C_3C_2^2}{C_2^2},$$
$$\delta = \delta, \quad q = \frac{C_1\int\delta a_1^2\,\mathrm{d}z + C_4C_2}{C_2}, \quad \beta = -\frac{\delta a_1^2}{C_2^2}, \quad p = C_2, \quad \alpha = \frac{\partial a_1}{\partial z}\frac{1}{a_1}. \tag{33}$$

Case 3.

$$\mu = a_0 = a_1 = 0, \quad \Gamma = C_1, \quad \Omega = \frac{1}{2}\frac{R^2\int\delta b_1^2\,\mathrm{d}z\,C_2^2 - R\int\delta b_1^2\,\mathrm{d}z\,C_1^2 + 2C_3C_2^2}{C_2^2},$$
$$b_1 = b_1, \quad \delta = \delta, \quad p = C_2, \quad \beta = \frac{\delta b_1^2R}{C_2^2}, \quad \alpha = \frac{\partial b_1}{\partial z}\frac{1}{b_1}, \quad q = \frac{-C_1R\int\delta b_1^2\,\mathrm{d}z + C_4C_2}{C_2}. \tag{34}$$

Case 4.

$$\mu = \pm 1, \quad a_0 = b_1 = 0, \quad \delta = \delta, \quad \Omega = \frac{\int \delta a_1^2 \, \mathrm{d}z \, C_2^2 R + 2 \int \delta a_1^2 \, \mathrm{d}z \, C_1^2 + C_3 C_2^2}{C_2^2},$$

$$\Gamma = C_1, \quad \beta = -\frac{4a_1^2 \delta}{C_2^2}, \quad a_1 = a_1, \quad p = C_2, \quad q = \frac{4C_1 \int \delta a_1^2 \, \mathrm{d}z + C_4 C_2}{C_2}, \quad \alpha = \frac{\partial a_1}{\partial z} \frac{1}{a_1}. \tag{35}$$

Therefore from (3), (9), (12), (13) and (32)–(34), we obtain four families of exact solutions for Eq. (8) as follows:

Family 1.

$$u_{11} = cR \exp\left[\int \alpha(z) \, \mathrm{d}z\right] \left[\pm \mathrm{i} \tanh(\sqrt{R}\xi) + \mathrm{sech}(\sqrt{R}\xi)\right] \exp[\mathrm{i}(C_1 t + \Omega(z))], \tag{36}$$

$$u_{12} = cR \exp\left[\int \alpha(z) \, \mathrm{d}z\right] \left[\pm \mathrm{i} \coth(\sqrt{R}\xi) + \mathrm{csch}(\sqrt{R}\xi)\right] \exp[\mathrm{i}(C_1 t + \Omega(z))], \tag{37}$$

where c, R are arbitrary constants and

$$\xi = C_3 t - C_1 C_3 \int \beta(z) \, \mathrm{d}z, \quad \Omega(z) = -\frac{(RC_3^2 + 2C_1^2) \int \beta(z) \, \mathrm{d}z}{4} + C_5, \quad \beta(z) = \frac{4c^2 R \delta(z) \exp[2 \int \alpha(z) \, \mathrm{d}z]}{C_3^2}.$$

Family 2.

$$u_{21} = c\sqrt{R} \exp\left[\int \alpha(z) \, \mathrm{d}z\right] \tanh\left\{\sqrt{R}\left[C_2 t - C_1 C_2 \int \beta(z) \, \mathrm{d}z + C_4\right]\right\} \exp\{\mathrm{i}[C_1 t + \Omega(z)]\}, \tag{38}$$

$$u_{22} = c\sqrt{R} \exp\left[\int \alpha(z) \, \mathrm{d}z\right] \coth\left\{\sqrt{R}\left[C_2 t - C_1 C_2 \int \beta(z) \, \mathrm{d}z + C_4\right]\right\} \exp\{\mathrm{i}[C_1 t + \Omega(z)]\}, \tag{39}$$

where

$$\Omega(z) = -\frac{(2C_2^2 R + C_1^2) \int \beta(z) \, \mathrm{d}z}{2} + C_3, \quad \beta(z) = -\frac{\delta(z) c^2 \exp[2 \int \alpha(z) \, \mathrm{d}z]}{C_2^2}.$$

Family 3.

$$u_{31} = cR \exp\left[\int \alpha(z) \, \mathrm{d}z\right] \mathrm{sech}\left\{\sqrt{R}\left[C_2 t - C_1 C_2 \int \beta(z) \, \mathrm{d}z + C_4\right]\right\} \exp\{\mathrm{i}[C_1 t + \Omega(z)]\}, \tag{40}$$

$$u_{32} = cR \exp\left[\int \alpha(z) \, \mathrm{d}z\right] \mathrm{csch}\left\{\sqrt{R}\left[C_2 t - C_1 C_2 \int \beta(z) \, \mathrm{d}z + C_4\right]\right\} \exp\{\mathrm{i}[C_1 t + \Omega(z)]\}, \tag{41}$$

where

$$\Omega(z) = \frac{(RC_2^2 - C_1^2) \int \beta(z) \, \mathrm{d}z}{2} + C_3, \quad \beta(z) = \frac{\delta(z) c^2 \exp[2 \int \alpha(z) \, \mathrm{d}z]}{C_2^2}.$$

Family 4.

$$u_{41} = \frac{c\sqrt{R} \tanh\{\sqrt{R}[C_2 t - C_1 C_2 \int \beta(z) \, \mathrm{d}z + C_4]\}}{\pm \mathrm{sech}\{\sqrt{R}[C_2 t - C_1 C_2 \int \beta(z) \, \mathrm{d}z + C_4]\} + 1} \exp[\mathrm{i}(C_1 t + \Omega(z))], \tag{42}$$

$$u_{42} = \frac{c\sqrt{R} \coth\{\sqrt{R}[C_2 t - C_1 C_2 \int \beta(z) \, \mathrm{d}z + C_4]\}}{\pm \mathrm{csch}\{\sqrt{R}[C_2 t - C_1 C_2 \int \beta(z) \, \mathrm{d}z + C_4]\} + 1} \exp[\mathrm{i}(C_1 t + \Omega(z))], \tag{43}$$

where

$$\beta(z) = -\frac{4\delta(z) \exp[\int \alpha(z) \, \mathrm{d}z]}{C_2^2}, \quad \Omega(z) = -\frac{(RC_2^2 + 2C_1^2) \int \beta(z) \, \mathrm{d}z}{4} + C_3.$$

Because some arbitrary functions of z are included in Eq. (8), we can obtain some significant equations in optical fiber if we select arbitrary function suitably. Some exact solutions of corresponding equations can be obtained through the solutions (36)–(43) easily. We will give some significant equations from Eq. (8) and predict some new exact solutions in the following discussion. For simplicity, here we only discuss the solution u_{31} of Eq. (8) under some selections of $\beta(z)$, $\delta(z)$ and $\alpha(z)$ suitably.

(1) Taking $\beta(z) = 2$, $\delta(z) = (\theta+1)\exp(-2\theta z)$, $\alpha(z) = \theta =$constant in Eq. (8), we have

$$\mathrm{i}u_z + |u|^2 u + (\theta+1)\exp(-2\theta z)|u|^2 u = \mathrm{i}\theta u, \tag{44}$$

Eq. (44) represents phenomenon in optical fiber whose nonlinearity changes exponentially and possesses constant gain [22].

From (40), we deduce a exact solution of Eq. (44) as follows.

$$u(t,z) = \tilde{c}R\exp(\theta z)\mathrm{sech}\left[\sqrt{R}(C_2 t - 2C_1C_2 z + \tilde{C}_4)\right]\exp\left[\mathrm{i}(C_1 t + (RC_2^2 - C_1^2)z + \tilde{C}_3)\right], \tag{45}$$

where $\tilde{c}$, $\tilde{C}_3$, $\tilde{C}_4$ are arbitrary constants and $2C_2^2 = (\theta+1)\tilde{c}^2$.

Due to arbitrariness of $\tilde{c}$, $\tilde{C}_3$, $\tilde{C}_4$, R, C_1, C_2 and the constraints $2C_2^2 = (\theta+1)\tilde{c}^2$, we can verify that the solution (45) reproduce the solution (27) obtained by Ruan and Chen in [22]. At the same time, from (45), we can conclude that the amplitude of the soliton solution increases in exponent but its width of pulse keeps unchanged.

(2) Now we consider the exact solutions of the following equation

$$\mathrm{i}u_z + u_{tt} + \frac{\theta(\sin z + C)^2}{[\sin(\theta z) + C]^2}|u|^2 u = \mathrm{i}\left[\frac{\theta\cos(\theta z)}{\sin(\theta z) + C} - \frac{\cos z}{\sin z + c}\right]u, \tag{46}$$

where C, θ are arbitrary constants.

From (40), we find an exact solution of (46) as follows.

$$u(t,z) = \tilde{c}R\frac{\sin(\theta z) + C}{\sin z + C}\mathrm{sech}\left[\sqrt{R}\left(C_2 t - 2C_1C_2 z + \tilde{C}_4\right)\right]\exp\left[\mathrm{i}(C_1 t + (RC_2^2 - C_1^2)z + \tilde{C}_3)\right], \tag{47}$$

where $\tilde{c}$, $\widetilde{C}_3$, $\widetilde{C}_4$ are arbitrary constants and $2C_2^2 = \theta\tilde{c}^2$.

Due to arbitrariness of $\tilde{c}$, $\tilde{C}_3$, $\tilde{C}_4$, R, C_1, C_2 and the constraints $2C_2^2 = \theta\tilde{c}^2$, from the solution (47), the solutions (28) obtained in [22] can be recovered.

4. Summary and discussion

In this paper, by introducing a more general transformation (6), we proposed the extended projective Riccati equations method. The proposed method is more powerful than tanh method, various extended tanh methods, generalized hyperbolic function method and generalized Riccati equation expansion method. Making use of the proposed method, we have obtained four families of exact solutions of the NLSE in optical fiber with the help of symbolic computation system—*Maple*. Because three arbitrary functions of z are included in the Eq. (8), we can find some variable coefficient equations with real physical significance by means of some suitable selections of the arbitrary functions. Only if we know three arbitrary functions: $\beta(z)$, $\delta(z)$ and $\alpha(z)$, from (36)–(43), the exact solutions of corresponding equations can be obtained easily. Though we only discuss the exact solutions of two Eqs. (44) and (46) from the results (36)–(43), one can obtain exact solutions of other type of optical fibers expressed by coefficient varying nonlinear Schrödinger equation after selecting arbitrary functions suitably. The method proposed here can be applied to other PDEs and coupled ones. We hope the method given here is useful to study the soliton phenomena in soliton theory.

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