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Binary Darboux Transformation for the Modified Kadomtsev–Petviashvili Equation *

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Via the elementary Darboux transformation (DT) of the modified Kadomtsev–Petviashvili (mKP) equation, a binary Darboux transformation (BDT) of the mKP equation is constructed.

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The Darboux transformation (DT) was first introduced by Darboux in 1882 as a transformation which generates a new Sturm–Liouville differential equation from an old one giving a relation between two solutions.[1] DT has been proved to be one of the most effective methods to construct soliton solutions for the nonlinear evolution equations.[2−6] In 1990, Matveev and Salle [2] first investigated the DT in integral form and presented binary Darboux transformation (BDT). Nimmo[3,4] has carried out a lot of excellent work about BDT: in Ref. [3], the general construction of BDT for KP hierarchy preserving certain properties of the operator, such as self-adjointness, is given; the BDT of two-dimensional Zakharov–Shabati/AKNs spectral problem[4] is obtained by composing the elementary transformation, for one solution matrix, with its inverse for another solution matrix. Recently, Yang and Liu[5] construct a BDT for the coupled Kadomtsev–Petviashvili equation (CKP), which is the consequence of Nimmo's general result.

In this Letter, based on the framework of Nimmo,[4] we construct a binary Darboux transformation (BDT) for the modified Kadomtsev–Petviashvili (mKP) equation and deduce the corresponding N-step iteration.

The mKP equation

$$4u_t+u_{xxx}-6u^2u_x+6u_x(\partial^{-1}u_y)+3\partial^{-1}u_{yy}=0, \quad (1)$$

describes water waves in (x,y)-plane when the nonlinearity is higher than for the KP equation. This equation has been introduced in different approaches.[7] In Ref. [8], its N-times DT via Painlevé analysis was obtained.

The Lax pair for the mKP equation is

$$\phi_y=\phi_{xx}+2u\phi_x, \quad (2)$$

$$4\phi_t=-4\phi_{xxx}-12u\phi_{xx}-6u_x\phi_x-6u^2\phi_x -6\int u_y dx\phi_x. \quad (3)$$

Assume that u be the solution of mKP equation (1) and ϕ_1 be the fixed solution of Eqs. (2) and (3). The elementary DT for the mKP equation is defined by

$$u[1]=u+\left(\ln\frac{\phi_{1,x}}{\phi_1}\right)_x,\quad \phi[1]=\phi-\frac{\phi_1}{\phi_{1,x}}\phi_x. \quad (4)$$

You can directly prove that $\phi[1]$ and $u[1]$ satisfy the equations

$$\phi[1]_y=\phi[1]_{xx}+2u[1]\phi[1]_x, \quad (5)$$

$$4\phi[1]_t=-4\phi[1]_{xxx}-12u[1]\phi[1]_{xx}-6u[1]_x\phi[1]_x -6u[1]^2\phi[1]_x-6\int u[1]_y dx\phi[1]_x. \quad (6)$$

where $u[1]$ defined by Eq. (4) is a new solution of mKP equation.

In Ref. [9], N-times repeated DT for the mKP equation is deduced. Assume that u be the solution of mKP equation (1) and ϕ_1, ϕ_2, $\ldots$, ϕ_N be the solutions of Eqs. (2) and (3), then the N-times repeated DT (4) and (5) is given by

$$u[N]=u+\left(\ln\frac{W_2(\phi_1,\phi_2,\ldots,\phi_N)}{W_1(\phi_1,\phi_2,\ldots,\phi_N)}\right)_x, \quad (7)$$

$$\phi[N]=\frac{W_2(\phi,\phi_1,\phi_2,\ldots,\phi_N)}{W_1(\phi_1,\phi_2,\ldots,\phi_N)}, \quad (8)$$

where

$$W_1(\phi_1,\phi_2,\ldots,\phi_N)=\det(A),$$
$$A_{ij}=\frac{d^{j-1}}{dx^{j-1}}\phi_i, 1\le i,j\le N,$$

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$$W_2(\phi_1, \phi_2, \ldots, \phi_N) = \det(B),$$
$$B_{ij} = \frac{d^j}{dx^j}\phi_i, 1 \le i, j \le N.$$

One can see that the N-step iteration of the elementary DT for mKP equation is in the form of Wronski determinant.

In the following, we will make use of the elementary DT (4) to construct the BDT of mKP equation. First, we investigate the t-independent Lax pair Eq. (2). Assuming that θ be the non-zero fixed solution of Eq. (2), from Eqs. (4), we know that via

$$u_\theta = u + \Big(\ln \frac{\theta_x}{\theta}\Big)_x, \quad \phi[1] = \phi - \frac{\theta}{\theta_x}\phi_x, \tag{9}$$

Eq. (2) is preserved form invariant:

$$\phi[1]_y = \phi[1]_{xx} + 2u_\theta \phi[1]_x. \tag{10}$$

For the arbitrary solution ϕ of Eq. (2), since $\theta \neq 0$, we may write $\phi = \theta V$.
Substituting $\phi = \theta V$ into Eq. (2), we will see that V should satisfy $2\theta_x V_x + \theta V_{xx} - \theta V_y = 0$, which we solve by introducing ψ such that

$$V_x = (\theta^{-1})_x\psi, \quad V_y = (\theta^{-1})_y\psi + (\theta^{-1})_x\psi_x. \tag{11}$$

The integrability condition $V_{xy} = V_{yx}$ for Eqs. (11) implies that

$$\psi_y = \psi_{xx} + 2\Big(u + \Big(\ln \frac{\theta_x}{\theta}\Big)_x\Big)\psi_x, \tag{12}$$

i.e.ψ is a solution of Eq. (10).
The adjoint of Eqs. (2) and (10) are

$$-\phi^*_y = \phi^*_{xx} - 2u\phi^*_x, \tag{13}$$
$$-\phi[1]^*_y = \phi[1]^*_{xx} - 2u_\theta \phi[1]^*_x. \tag{14}$$

For convenience, we define S, S_θ, S^*, and S^*_θ to be the sets of all solutions of Eqs. (2), (10), (13) and (14). One can directly calculate that once $\phi \in S$, there is $\phi^{-1} \in S^*_\theta$, i.e. ϕ^{-1} satisfies Eqs. (14):

$$-(\phi^{-1})_y = (\phi^{-1})_{xx} - 2u_\theta(\phi^{-1})_x.$$

If we define an integral operator Δ by

$$\Delta(\psi, \psi^*) = c + \int_l \psi\psi^*_x dx + (\psi\psi^*_y + \psi_x\psi^*_x)dy, \tag{15}$$

then Eqs. (11) show that every $\phi \in S$ is $\phi = \theta\Delta(\psi, \theta^{-1})$, for some $\psi \in S_\theta$.
Assume that $\hat\phi$ and $\hat u$ also satisfy Eq. (2), i.e.

$$\hat\phi_y = \hat\phi_{xx} + 2\hat u \hat\phi_x, \tag{16}$$

and $\hat\theta$ is a fixed non-zero solution of Eq. (16), requiring after the similar transformation to Eqs. (9), i.e.

$$u_\theta = \hat u + \Big(\ln \frac{\hat\theta_x}{\hat\theta}\Big)_x, \quad \phi[1] = \hat\phi - \frac{\hat\theta}{\hat\theta_x}\hat\phi_x, \tag{17}$$

then one can obtain the same equation (10).

From above, it is seen that u and $\hat u$ have the same property. What we want to do is to find the transformation between Eqs. (2) and (16), which is called the BDT. In the same way, we define $\hat S$ to be the set of all solutions of Eq. (16) and the mappings between S and S_θ, $\hat S$ and S_θ to be G_θ and $G_{\hat\theta}$, respectively, i.e.

$$\phi_y = \phi_{xx} + 2u\phi_x \ (S) \to \phi[1]_y = \phi[1]_{xx} + 2u_\theta\phi[1]_x(S_\theta), \ \phi[1] = G_\theta(\phi), \tag{18}$$
$$\hat\phi_y = \hat\phi_{xx} + 2\hat u\hat\phi_x \ (\hat S) \to \phi[1]_y = \phi[1]_{xx} + 2u_\theta\phi[1]_x, (S_\theta), \ \phi[1] = G_{\hat\theta}(\hat\phi), \tag{19}$$
$$S \xrightarrow{G_\theta} S_\theta \xleftarrow{G_{\hat\theta}} \hat S. \tag{20}$$

Via (18) and (19) or (20), one can easily get

$$\hat\phi = G^{-1}_{\hat\theta} G_\theta(\phi). \tag{21}$$

Then the most important thing to construct the BDT is to find the inverse of G_θ and the relationship between θ and $\hat\theta$.

It is seen that for every $\psi \in S_\theta$, there will be $\phi = \theta\Delta(\psi, \theta^{-1}) \in S$. Therefore, one can obtain

$$G^{-1}_\theta = \theta\Delta(\cdot, \theta^{-1}). \tag{22}$$

Via $\phi[1] = \phi - \dfrac{\theta}{\theta_x}\phi_x$, it is known that $G_\theta = 1 - \dfrac{\theta}{\theta_x}\partial_x(\theta \in S)$ maps S into S_θ. Since $\theta^{-1} \in S^*_\theta$, the operator $\Big(1 - \dfrac{\theta^{-1}}{(\theta^{-1})_x}\partial_x\Big)$ acts in the opposite direction to G_θ, mapping S^*_θ into S^*, i.e. $S^*_\theta \xrightarrow{G_{\theta^{-1}}} S^*$.

In the same way to the above, for any $\psi^* \in S^*_\theta$, we may write $\psi^* = \theta^{-1}p$, where p satisfies

$$p_x = \theta_x\phi^*, \quad p_y = \theta_y\phi^* - \theta_x\phi^*_x. \tag{23}$$

From the integrability condition for Eqs. (23), one can see $\phi^* \in S^*$. We introduce another integral operator Γ on $S \times S^*$

$$\Gamma(\phi, \phi^*) = c + \int_l \phi_x\phi^* dx + (\phi_y\phi^* - \phi_x\phi^*_x)dy, \tag{24}$$

for some constant c and a curve l in the plane, then Eqs. (23) show that every $\psi^* \in S^*_\theta$ is

$$\psi^* = \theta^{-1}\Gamma(\theta, \phi^*) \tag{25}$$

for some $\phi^* \in S^*$. Because $\hat\theta$ has the same property with θ, there is $\hat\theta^{-1} \in S^*_\theta$. Due to Eq. (25), it is seen that

$$\hat\theta^{-1} = \theta^{-1}\Gamma(\theta, \theta^*), \tag{26}$$

Due to Eqs. (22) and (26), we can construct the BDT for Eq. (2)

$$\begin{aligned}\hat{\phi} &= G_{\hat{\theta}}^{-1}G_\theta(\phi) = \hat{\theta}\Delta(G_\theta(\phi),\hat{\theta}^{-1})\\ &= \frac{\theta}{\Gamma(\theta,\theta^*)}\Big[c + \int_l G_\theta(\phi)\Big(\frac{\theta}{\Gamma(\theta,\theta^*)}\Big)_x dx + \Big(G_\theta(\phi)\\ &\quad\cdot\Big(\frac{\theta}{\Gamma(\theta,\theta^*)}\Big)_y + (G_\theta(\phi))_x\Big(\frac{\theta}{\Gamma(\theta,\theta^*)}\Big)_x\Big)dy\Big].\end{aligned}\tag{27}$$

Substituting $G_\theta(\phi) = \phi - \dfrac{\theta}{\theta_x}\phi_x$ and $\Gamma(\theta,\theta^*) = c + \int_l \theta_x\theta^* dx + (\theta_y\theta^* - \theta_x\theta^*_x)dy$ into Eq. (27), after an integration by parts, we can obtain

$$\hat{\phi} = \phi - \frac{\theta\Gamma(\phi,\theta^*)}{\Gamma(\theta,\theta^*)}.\tag{28}$$

On the other hand, from Eqs. (9), (17) and (26), one can easily obtain

$$\begin{aligned}\hat{u} &= u + \Big(\ln\frac{\theta_x}{\theta}\Big)_x - \Big(\ln\frac{\hat{\theta}_x}{\hat{\theta}}\Big)_x = u + \Big[\ln\Big(\frac{\theta_x}{\theta}\frac{\hat{\theta}}{\hat{\theta}_x}\Big)\Big]_x\\ &= u + \Big[\ln\Big(\frac{\Gamma(\theta,\theta^*)}{\Gamma(\theta,\theta^*) - \theta\theta^*}\Big)\Big]_x.\end{aligned}\tag{29}$$

Due to Eqs. (15) and (24), there is

$$\Gamma(\theta,\theta^*) + \Delta(\theta,\theta^*) = \theta\theta^*.$$

Thus Eq. (29) can be written as

$$\hat{u} = u + \Big[\ln\Big(-\frac{\Gamma(\theta,\theta^*)}{\Delta(\theta,\theta^*)}\Big)\Big]_x.\tag{30}$$

Therefor via the BDT (28) and (30), Eq. (2) has been transformed into $\hat{\phi}_y = \hat{\phi}_{xx} + 2\hat{u}\hat{\phi}_x$, where θ and θ^* are the fixed solutions of Eq. (2) and its adjoint Eq. (13). Meanwhile, one can verify directly that via

$$\hat{\phi}^* = \phi^* - \frac{\theta^*\Delta(\theta,\phi^*)}{\Delta(\theta,\theta^*)},\tag{31}$$

Eq. (13)is also form invariable $-\hat{\phi}^*_y = \hat{\phi}^*_{xx} - 2\hat{u}\hat{\phi}^*_x$. Using Eqs. (28), (30) and (31), one can construct the N-step iteration BDT of Eq. (2).

However, above we only consider the first one Eq. (2) of the Lax pair of mKP equation, it requires that Eq. (3) should be form invariant, i.e.

$$\begin{aligned}4\hat{\phi}_t =& -4\hat{\phi}_{xxx} - 12\hat{u}\hat{\phi}_{xx} - 6\hat{u}_x\hat{\phi}_x\\ &-6\hat{u}^2\hat{\phi}_x - 6\int\hat{u}_y dx\hat{\phi}_x,\end{aligned}\tag{32}$$

by the transformation (28) and (30). Substituting Eqs. (28) and (30) into Eq. (32) directly, with the help of

$$\phi_t = -\phi_{xxx} - 3u\phi_{xx} - \frac{3}{2}u_x\phi_x - \frac{3}{2}u^2\phi_x - \frac{3}{2}\phi_x\alpha,\tag{33}$$

$$\theta_t = -\theta_{xxx} - 3u\theta_{xx} - \frac{3}{2}u_x\theta_x - \frac{3}{2}u^2\theta_x - \frac{3}{2}\theta_x\alpha,\tag{34}$$

to eliminate ϕ_t and θ_t, one can see that there must be

$$[\Gamma(\theta,\theta^*)]_t = \theta_t\theta^* - \theta_x\theta^*_{xx} + \theta_{xx}\theta^*_x + 3u\theta_x\theta^*_x,$$

$$[\Delta(\theta,\theta^*)]_t = \theta\theta^*_t + \theta_x\theta^*_{xx} - \theta_{xx}\theta^*_x - 3u\theta_x\theta^*_x.$$

Here in Eqs. (33) and (34), we define $\int u_y dx$ as α for convenience. Therefore we should amend the operators Γ and Δ and rewrite them as

$$\begin{aligned}\tilde{\Gamma}(\psi,\psi^*) =& c + \int\psi_x\psi^* dx + (\psi_y\psi^* - \psi_x\psi^*_x)dy + (\psi_t\psi^*\\ &-\psi_x\psi^*_{xx} + \psi_{xx}\psi^*_x + 3u\psi_x\psi^*_x)dt,\end{aligned}\tag{35}$$

$$\begin{aligned}\tilde{\Delta}(\psi,\psi^*) =& c + \int\psi\psi^*_x dx + (\psi\psi^*_y + \psi_x\psi^*_x)dy + (\psi\psi^*_t\\ &+\psi_x\psi^*_{xx} - \psi_{xx}\psi^*_x - 3u\psi_x\psi^*_x)dt.\end{aligned}\tag{36}$$

It is seen that there is also

$$\tilde{\Gamma}(\psi,\psi^*) + \tilde{\Delta}(\psi,\psi^*) = \psi\psi^*.$$

From the above discussion, we have

Conclusion 1. The Lax pair of mKP equation (2) and (3) and the corresponding adjoint Lax pair

$$\begin{aligned}-\phi^*_y &= \phi^*_{xx} - 2u\phi^*_x,\\ 4\phi^*_t &= -4\phi^*_{xxx} + 12u\phi^*_{xx} + 6u_x\phi^*_x\\ &\quad -6u^2\phi^*_x - 6\int u_y dx\phi^*_x.\end{aligned}\tag{37}$$

are form invariant, via the BDT

$$\hat{\phi} = \phi - \frac{\theta\tilde{\Gamma}(\phi,\theta^*)}{\tilde{\Gamma}(\theta,\theta^*)},\tag{38}$$

$$\hat{\phi}^* = \phi^* - \frac{\theta^*\tilde{\Delta}(\theta,\phi^*)}{\tilde{\Delta}(\theta,\theta^*)},\tag{39}$$

$$\hat{u} = u + \Big[\ln\Big(-\frac{\tilde{\Gamma}(\theta,\theta^*)}{\tilde{\Delta}(\theta,\theta^*)}\Big)\Big]_x.\tag{40}$$

Here θ and θ^* are the fixed solutions of Eqs. (2), (3) and (13), (37), respectively and the integral operators $\tilde{\Gamma}$, $\tilde{\Delta}$ are defined by Eqs. (35) and (36). Thus from Eq. (40), one can obtain new solutions of mKP equation from an old one.

The BDT (38), (39) and (40) can be iterated step by step. In fact, it is seen that

$$\begin{aligned}\hat{u}[2] &= \hat{u} + \Big[\ln\Big(-\frac{\tilde{\Gamma}(\hat{\theta},\hat{\theta}^*)}{\tilde{\Delta}(\hat{\theta},\hat{\theta}^*)}\Big)\Big]_x\\ &= u + \Big[\ln\Big(\frac{\tilde{\Gamma}(\theta_1,\theta_1^*)}{\tilde{\Delta}(\theta_1,\theta_1^*)}\cdot\frac{\tilde{\Gamma}(\hat{\theta},\hat{\theta}^*)}{\tilde{\Delta}(\hat{\theta},\hat{\theta}^*)}\Big)\Big]_x,\end{aligned}\tag{41}$$

where we denote θ and θ^* as θ_1 and θ_1^* for convenience.

By Eqs. (38) and (39), we have

$$\hat{\theta} = \theta_2 - \frac{\theta_1 \tilde{\Gamma}(\theta_2, \theta_1^*)}{\tilde{\Gamma}(\theta_1, \theta_1^*)}, \tag{42}$$

$$\hat{\theta}^* = \theta_2^* - \frac{\theta_1^* \tilde{\Delta}(\theta_1, \theta_2^*)}{\tilde{\Delta}(\theta_1, \theta_1^*)}, \tag{43}$$

where θ_2 and θ_2^* are another fixed solutions of Eqs. (2), (3), (13), and (37). After direct calculation, one can obtain

$$\tilde{\Gamma}(\theta_1, \theta_1^*) \cdot \tilde{\Gamma}(\hat{\theta}, \hat{\theta}^*) = \begin{vmatrix} \tilde{\Gamma}(\theta_1, \theta_1^*) & \tilde{\Gamma}(\theta_1, \theta_2^*) \\ \tilde{\Gamma}(\theta_2, \theta_1^*) & \tilde{\Gamma}(\theta_2, \theta_2^*) \end{vmatrix}, \tag{44}$$

$$\tilde{\Delta}(\theta_1, \theta_1^*) \cdot \tilde{\Delta}(\hat{\theta}, \hat{\theta}^*) = \begin{vmatrix} \tilde{\Delta}(\theta_1, \theta_1^*) & \tilde{\Delta}(\theta_1, \theta_2^*) \\ \tilde{\Delta}(\theta_2, \theta_1^*) & \tilde{\Delta}(\theta_2, \theta_2^*) \end{vmatrix}. \tag{45}$$

Then (41) can be rewritten as

$$\hat{u}[2] = u + \left[\ln \frac{\begin{vmatrix} \tilde{\Gamma}(\theta_1, \theta_1^*) & \tilde{\Gamma}(\theta_1, \theta_2^*) \\ \tilde{\Gamma}(\theta_2, \theta_1^*) & \tilde{\Gamma}(\theta_2, \theta_2^*) \end{vmatrix}}{\begin{vmatrix} \tilde{\Delta}(\theta_1, \theta_1^*) & \tilde{\Delta}(\theta_1, \theta_2^*) \\ \tilde{\Delta}(\theta_2, \theta_1^*) & \tilde{\Delta}(\theta_2, \theta_2^*) \end{vmatrix}} \right]_x. \tag{46}$$

In the same way, one can obtain

$$\hat{\phi}[2] = \frac{\begin{vmatrix} \phi & \theta_1 & \theta_2 \\ \tilde{\Gamma}(\phi, \theta_2^*) & \tilde{\Gamma}(\theta_1, \theta_2^*) & \tilde{\Gamma}(\theta_2, \theta_2^*) \\ \tilde{\Gamma}(\phi, \theta_1^*) & \tilde{\Gamma}(\theta_1, \theta_1^*) & \tilde{\Gamma}(\theta_2, \theta_1^*) \end{vmatrix}}{\begin{vmatrix} \tilde{\Gamma}(\theta_1, \theta_2^*) & \tilde{\Gamma}(\theta_2, \theta_2^*) \\ \tilde{\Gamma}(\theta_1, \theta_1^*) & \tilde{\Gamma}(\theta_2, \theta_1^*) \end{vmatrix}}, \tag{47}$$

$$\hat{\phi}[2]^* = \frac{\begin{vmatrix} \phi^* & \theta_1^* & \theta_2^* \\ \tilde{\Gamma}(\theta_2, \phi^*) & \tilde{\Gamma}(\theta_2, \theta_1^*) & \tilde{\Gamma}(\theta_2, \theta_2^*) \\ \tilde{\Gamma}(\theta_1, \phi^*) & \tilde{\Gamma}(\theta_1, \theta_1^*) & \tilde{\Gamma}(\theta_1, \theta_2^*) \end{vmatrix}}{\begin{vmatrix} \tilde{\Gamma}(\theta_2, \theta_1^*) & \tilde{\Gamma}(\theta_2, \theta_2^*) \\ \tilde{\Gamma}(\theta_1, \theta_1^*) & \tilde{\Gamma}(\theta_1, \theta_2^*) \end{vmatrix}}. \tag{48}$$

Step by step, the N-times BDT of the mKP equation can be derived in form of the determinant of Grammian.

Conclusion 2. Taking N solutions of the linear system (2), (3) and (13), (37) (θ_k, θ_k^*) $(k = 1, 2, \ldots, N)$, we can obtain new solution

$$u[N] = u + \left[\ln \left((-1)^N \frac{\begin{vmatrix} \tilde{\Gamma}(\theta_1, \theta_1^*) & \tilde{\Gamma}(\theta_1, \theta_2^*) & \ldots & \tilde{\Gamma}(\theta_1, \theta_N^*) \\ \tilde{\Gamma}(\theta_2, \theta_1^*) & \tilde{\Gamma}(\theta_2, \theta_2^*) & \ldots & \tilde{\Gamma}(\theta_2, \theta_N^*) \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\Gamma}(\theta_N, \theta_1^*) & \tilde{\Gamma}(\theta_N, \theta_2^*) & \ldots & \tilde{\Gamma}(\theta_N, \theta_N^*) \end{vmatrix}}{\begin{vmatrix} \tilde{\Delta}(\theta_1, \theta_1^*) & \tilde{\Delta}(\theta_1, \theta_2^*) & \ldots & \tilde{\Delta}(\theta_1, \theta_N^*) \\ \tilde{\Delta}(\theta_2, \theta_1^*) & \tilde{\Delta}(\theta_2, \theta_2^*) & \ldots & \tilde{\Delta}(\theta_2, \theta_N^*) \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\Delta}(\theta_N, \theta_1^*) & \tilde{\Delta}(\theta_N, \theta_2^*) & \ldots & \tilde{\Delta}(\theta_N, \theta_N^*) \end{vmatrix}} \right) \right]_x, \tag{49}$$

where the integral operators $\tilde{\Gamma}$ and $\tilde{\Delta}$ are defined by Eqs. (35) and (36).

In summary, the BDT for the mKP equation is constructed and its corresponding N-step iteration is deduced. By introducing two integral operators $\tilde{\Gamma}$ and $\tilde{\Delta}$, the N-step iterations of the elementary DT and BDT for mKP are in form of the determinants of Wronski and Grammian, respectively.

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