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Numerical Solutions of a Class of Nonlinear Evolution Equations with Nonlinear Term of Any Order*

AN Hong-Li[1,3] and CHEN Yong[1,2,3,†]

[1]Nonlinear Science Center and Department of Mathematics, Ningbo University, Ningbo 315211, China

[2]Institute of Theoretical Computing, East China Normal University, Shanghai 200062, China

[3]Key Laboratory of Mathematics Mechanization, the Chinese Academy of Sciences, Beijing 100080, China

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Abstract *In this paper, the Adomian decomposition method is developed for the numerical solutions of a class of nonlinear evolution equations with nonlinear term of any order,* $u_{tt}+au_{xx}+bu+cu^p+du^{2p-1}=0$*, which contains some important famous equations. When setting the initial conditions in different forms, some new generalized numerical solutions: numerical hyperbolic solutions, numerical doubly periodic solutions are obtained. The numerical solutions are compared with exact solutions. The scheme is tested by choosing different values of p, positive and negative, integer and fraction, to illustrate the efficiency of the ADM method and the generalization of the solutions.*

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Key words: Adomian decomposition method, nonlinear evolution equations, Jacobi elliptic function, numerical solution

1 Introduction

In the past decades, great effort has been devoted to studying the explicit and numerical solutions to nonlinear evolution equation. A number of powerful methods have been proposed, such as the Adomian decomposition method[1−6] (in short, ADM), Bäcklund transformation, Darboux transformation, Cole–Hopf transformation, various tanh methods, variable separation approach, Painlevé method, rational expansion method, and so on.[7−14] Among them, the Adomian decomposition method provides an effective procedure to investigate approximate solutions, or even closed-form analytical solutions of nonlinear differential equations. It provides more realistic solutions by solving the nonlinear problem without simplification and series solutions which generally converge very rapidly in real physics models. More recently, some papers[4−6] focus on improving the ADM to investigate many nonlinear differential equations (even the fractional differential equations) with different initial conditions such that solitary wave solutions, rational solutions, compacton solutions and other types of solutions were found. In this paper, by the ADM, we discuss a class of nonlinear evolution equations with nonlinear term of any order,[14]

$$u_{tt}+au_{xx}+bu+cu^p+du^{2p-1}=0\,, \qquad (1)$$

where a, b, c, d, and $p\neq 1$ are arbitrary constants. A different equation can be got when p takes a different constant. It is easily to see that equation (1) contains some important nonlinear mathematical physics equations such as Duffing equation,[7] Klein–Gordon equation,[15] Landau–Ginburg–Higgs equation,[16] sin-Gordon equation[15,16] and ϕ^4 equation.[7] In Ref. [14], Chen *et al.* presented and investigated Eq. (1), and some exact solutions are obtained by improved tanh method. The goal of the paper is, without any transformation and just using the ADM with different initial conditions, to investigate some new numerical solutions regarding to different p. The numerical solutions are compared with the theoretical exact solutions by analyzing the absolute error and relative error. It is organized as follows. In Sec. 2, some necessary details on the ADM are given. In Sec. 3, we extend the ADM to investigate the numerical solutions of the nonlinear evolution Eq. (1) and obtain some new results. Moreover we make the error analysis using the tables and graphs to the efficiency of the ADM method. Finally, conclusions are followed.

2 Description of the ADM

The Adomian method[1,2] is described like this: considering the differential equation

$$Lu+Ru+Nu=g\,, \qquad (2)$$

where L is the highest order derivative and invertible, R is a linear differential operator and its order is less than L, Nu are the nonlinear terms and g is the source term. Setting Lu the single left term, we have

$$Lu=g-Ru-Nu\,. \qquad (3)$$

Applying the inverse operator L^{-1} on both sides of Eq. (3) and using the initial conditions yield

$$u=f(x)+L^{-1}g-L^{-1}(Ru)-L^{-1}(Nu)\,, \qquad (4)$$

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†E-mail: chenyong@nbu.edu.cn

where the function $f(x)$ is the term arising from the given conditions. It must be noted that if L is a second-order operator, L^{-1} will be a twofold integration operator and $L^{-1}Lu = u(x,0) + tu_t(x,0)$. So $f = u(x,0) + tu_t(x,0)$ and $u_0 = f + L^{-1}g$. For nonlinear equation, according to the ADM,[1,2] the unknown solution $u(x,t)$ is given in an infinity series form,

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t)\,, \tag{5}$$

and the nonlinear terms Nu are usually decomposed into another infinity series form

$$Nu = \sum_{n=0}^{\infty} B_n\,, \tag{6}$$

where B_n is the so-called Adomian polynomials. These Adomian polynomials can be calculated for all forms of nonlinear terms according to specific algorithms constructed by Adomian. Usually, the general form of the Adomian polynomial B_n is defined as

$$B_n = \frac{1}{n!}\frac{\mathrm{d}^n}{\mathrm{d}\lambda^n}\Big[N\Big(\sum_{k=0}^{\infty}\lambda^k u_k\Big)\Big]_{\lambda=0}\,, \quad n = 0,1,2,\ldots \tag{7}$$

We note that the nonlinear terms of the nonlinear evolution (1) to be discussed later all have the form $Nu = F(u) = u^m$. The first few expressions for the Adomian polynomials, defined as C_n for the nonlinear term u^m, are[2]

$$\begin{aligned}
&C_0 = F(u_0))\,,\\
&C_1 = u_1F'(u_0)\,,\\
&C_2 = u_2F'(u_0) + \frac{u_1^2}{2!}F''(u_0)\,,\\
&C_3 = u_3F'(u_0) + u_1u_2F''(u_0) + \frac{u_1^3}{3!}F'''(u_0)\,.
\end{aligned} \tag{8}$$

It is important to note that C_n only depends on u_i $(i = 0,\ldots,n-1)$ and the sum of the subscripts in each term of C_n is n. Substituting the initial condition and the Adomian polynomials (6) into Eq. (4) yields the following recursive relations:

$$\begin{aligned}
&u_0 = f(x) + L^{-1}g\,,\\
&u_{n+1} = -L^{-1}(Ru_n) - L^{-1}(B_n), n \geq 0\,.
\end{aligned} \tag{9}$$

3 Applications of the ADM for Numerical Solutions of Nonlinear Evolution Equations with Nonlinear Term of Any Order

For the class of nonlinear evolution equations with nonlinear term of any order[14]

$$u_{tt} + au_{xx} + bu + cu^p + du^{2p-1} = 0\,,$$

where a, b, c, d, and $p \neq 1$ are arbitrary constants, first we write it in the operator form

$$Lu + au_{xx} + bu + cu^p + du^{2p-1} = 0\,,$$

where $L = \partial^2/\partial t^2$. The inverse operator L^{-1} is a twofold integral operator and defined as

$$L^{-1}(\cdot) = \int_0^t\int_0^t (\cdot)\,\mathrm{d}t\mathrm{d}t\,.$$

Applying the inverse operator L^{-1} on the above equation and solving u yields

$$u = f - L^{-1}(au_{xx} + bu + cu^p + du^{2p-1})\,, \tag{10}$$

where $f = u(x,0) + tu_t(x,0)$ arises from the given conditions. For convenience, we assume the Adomian polynomials C_n and D_n for the two nonlinear terms u^p and u^{2p-1} have the forms of

$$u^p = \sum_{n=0}^{\infty} C_n\,, \quad u^{2p-1} = \sum_{n=0}^{\infty} D_n\,.$$

Because the nonlinear terms u^p and u^{2p-1} can be written in the form u^m, the Adomian polynomials can be easily calculated by the expressions (8). As a result, the recursive relations for Eq. (10) become

$$\begin{aligned}
u_0 &= f(x)\,,\\
u_{n+1} &= -L^{-1}(au_{n,xx} + bu_n)\\
&\quad - L^{-1}(cC_n + dD_n)\,, \quad n \geq 0\,.
\end{aligned} \tag{11}$$

In the following, we will discuss the ADM for the numerical solutions of the nonlinear evolution Eq. (1) with different particular initial conditions.

3.1 *Initial Condition Is in Jacobi Elliptic Function* sn *Form*

Considering the operator form of the nonlinear evolution (1)

$$u = f - L^{-1}(au_{xx} + bu + cu^p + du^{2p-1})\,,$$

where $f = u(x,0) + tu_t(x,0)$ with the initial condition

$$u(x,0) = \Big[\sqrt{-\frac{2m^2k^2(\lambda^2+a)}{d}}\,\mathrm{sn}\,(kx,m)\Big]^{2/p}\,.$$

Substituting the initial value and Adomian polynomials into the recursive relations (11) yields

$$\begin{aligned}
u_0 &= u(x,0) + tu_t(x,0)\\
&= \Big[\sqrt{-\frac{2m^2k^2(\lambda^2+a)}{d}}\,\mathrm{sn}\,(kx,m)\Big]^{2/p}\,,\\
u_{n+1} &= -L^{-1}(au_{n,xx} + bu_n) - L^{-1}(cC_n + dD_n)\,.
\end{aligned}$$

With the aid of symbolic computation system *Maple*, the first few terms of the decomposition series are

$$\begin{aligned}
u_0 &= A\,,\\
u_1 &= -L^{-1}(au_{0,xx} + bu_0) - L^{-1}(cC_0 + dD_0)\\
&= \frac{t^2}{2p^2}\mathrm{sn}^{-2}(kx,m)(a_0 + a_1\mathrm{sn}^2(kx,m) + a_2\mathrm{sn}^4(kx,m))\,,\\
u_2 &= -L^{-1}(au_{1,xx} + bu_1) - L^{-1}(cC_1 + dD_1)\\
&= \frac{t^4}{24p^2A^2}\mathrm{sn}^{-4}(kx,m)\\
&\quad\times[a_3 + a_4\mathrm{sn}^2(kx,m) + a_5\mathrm{sn}^4(kx,m)\\
&\quad + a_6\mathrm{sn}^6(kx,m) + a_7\mathrm{sn}^8(kx,m)]\,,
\end{aligned} \tag{12}$$

where

$$A = \left[\sqrt{-\frac{2m^2k^2(\lambda^2+a)}{d}}\,\mathrm{sn}(kx,m)\right]^{2/p},$$

$$\begin{aligned}
a_0 &= 2aAk^2(p-2)\,,\\
a_1 &= bAp^2 + cA^pp^2 - 4aAk^2 - 4aAk^2m^2 + dA^{2p-1}p^2\,,\\
a_2 &= 2aAk^2m^2(2+p)\,, \quad a_3 = -12a^2k^4A^3(p-2)\,,\\
a_4 &= 2ak^2(p-2)(4ak^2A^3 + 4am^2k^2A^3 - bA^3\\
&\quad + dA^{2p+1} - cpA^{p+2} - 2dA^{2p+1})\,,\\
a_5 &= A^3(b^2p^2 - 4abk^2 - 4abk^2m^2 + 16a^2k^4m^2)\\
&\quad + cpA^{p+2}(bp^2 + bp - 4ak^2m^2 - 4ak^2)\\
&\quad + A^{2p+1}(c^2p^3 - 8apdk^2 + 4adk^2\\
&\quad + 4adk^2m^2 + 2bdp^3 - 8adpk^2m^2)\\
&\quad + cdp^2A^{3p}(3p-1) + d^2p^2A^{4p-1}(2p-1)\,,\\
a_6 &= -2am^2k^2A^3(p+2)(4am^2k^2 + 4ak^2 - b)\\
&\quad + 2am^2k^2A^{p+2}(p+2)(cp - dA^{p-1} + 2dA^{p-1})\,,\\
a_7 &= 12a^2k^4m^4A^3(p+2)\,.
\end{aligned}$$

So we have the generalized numerical solution of Eq. (10) in a series form

$$\begin{aligned}
u(x,t) &= A + \frac{t^2}{2p^2}\mathrm{sn}^{-2}(kx,m)(a_0 + a_1\mathrm{sn}^2(kx,m)\\
&\quad + a_2\mathrm{sn}^4(kx,m)) + \frac{t^4}{24p^2A^2}\mathrm{sn}^{-4}(kx,m)\\
&\quad \times [a_3 + a_4\mathrm{sn}^2(kx,m) + a_5\mathrm{sn}^4(kx,m)\\
&\quad + a_6\mathrm{sn}^6(kx,m) + a_7\mathrm{sn}^8(kx,m)] + \cdots. \qquad (13)
\end{aligned}$$

Remark 1 p in the solution (12) is arbitrary. When $a<0$, $p=2$, and $c=0$, the solution (12) becomes the corresponding solution for the well-known Klein–Gordon equation $u + au_{xx} + bu + du^3 = 0$. It admits the doubly periodic exact solution

$$u(x,t) = \sqrt{-\frac{2m^2k^2(\lambda^2+a)}{d}} \times \mathrm{sn}\left[\sqrt{\frac{b}{(\lambda^2+a)(1+m^2)}}(x-\lambda t), m\right], \qquad (14)$$

obtained by Jacobi elliptic function expansion method by Liu *et al.*[18] However, except this, we also discuss the other cases of the values for p and get the corresponding solution. In order to verify numerically whether the proposed methodology leads to accurate solutions, we will evaluate the ADM solutions using the N-term approximation for the well-known Klein–Gordon equation to compare the exact solution (13). The results show that we achieved a very good approximation to the actual solution of the equations by using only few terms of the decomposition series solution derived above.

Table 1 The numerical solution ϕ_n, exact solution $u(x,t)$, absolute error and relative error when $p=2$, $(a,b,c,d) = (-2, 26/25, 0, -2/25)$, $(k,m,\lambda) = (1, 1/5, -\sqrt{3})$.

x_i	t_i	ϕ_n	$u(x,t)$	$\lvert u(x,t)-\phi_n\rvert$	$\frac{\lvert u(x,t)-\phi_n\rvert}{\lvert u(x,t)\rvert}$
2	0.001	0.919 009 995 6	0.918 338 800 7	$0.671\,194\,9\times10^{-3}$	$0.730\,879\,387\,3\times10^{-3}$
2	0.002	0.919 005 974 0	0.917 663 586 0	$0.134\,238\,80\times10^{-2}$	$0.146\,283\,237\,2\times10^{-2}$
2	0.003	0.918 999 271 3	0.916 985 693 9	$0.201\,357\,74\times10^{-2}$	$0.219\,586\,566\,4\times10^{-2}$
5	0.001	−0.972 594 761 9	−0.972 199 750 7	$0.395\,011\,2\times10^{-3}$	$0.406\,306\,625\,5\times10^{-3}$
5	0.002	−0.972 590 541 1	−0.971 800 520 0	$0.790\,021\,1\times10^{-3}$	$0.812\,945\,747\,3\times10^{-3}$
5	0.003	−0.972 583 506 8	−0.971 398 477 7	$0.118\,502\,91\times10^{-2}$	$0.121\,992\,068\,9\times10^{-2}$
10	0.001	−0.460 087 258 3	−0.461 618 574 2	$0.153\,131\,59\times10^{-2}$	$0.331\,727\,531\,3\times10^{-2}$
10	0.002	−0.460 085 140 2	−0.463 147 767 3	$0.306\,262\,71\times10^{-2}$	$0.661\,263\,492\,2\times10^{-2}$
10	0.003	−0.460 081 609 8	−0.464 675 539 2	$0.459\,392\,94\times10^{-2}$	$0.988\,631\,639\,2\times10^{-2}$

From Table 1, we can obviously see the approximate numerical solution ϕ_n in Eq. (12), the exact solution (13), as well as the absolute and relative errors between them, when $(a,b,c,d) = (-2, 26/25, 0, -2/25)$, $(k,m,\lambda) = (1, 1/5, -\sqrt{3})$. Figure 1 shows the generalized Jacobi elliptic function numerical solution (12) for $\phi_2 = u_0+u_1+u_2$ when $p=1/2$. Figure 2(a) and 2(b) are the figures for the Klein–Gordon equation $u + au_{xx} + bu + du^3 = 0$ of the approximate solution (12) $\phi_2 = u_0+u_1+u_2$ and the exact solution (13) with $p=2$, $(a,b,c,d) = (-2, 26/25, 0, -2/25)$, $(k,m,\lambda) = (1, 1/5, -\sqrt{3})$, respectively. Figure 3 is the comparison of them at $t = 0.05$. From the comparison figure, it shows that the solution obtained by us rapidly converges to the solution obtained by Liu *et al.*

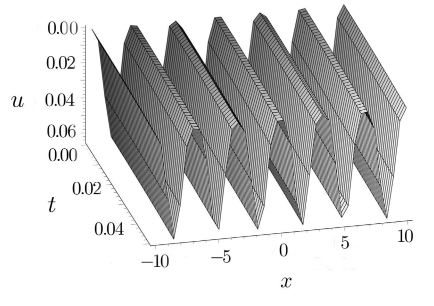

Fig. 1 The figure of the generalized approximate numerical solution (12) with $p = 1/2$, $(a,b,c,d) = (-1, 1/3, 1, -2)$, $(m,\lambda,k) = (1/2, -\sqrt{2}, 1)$.

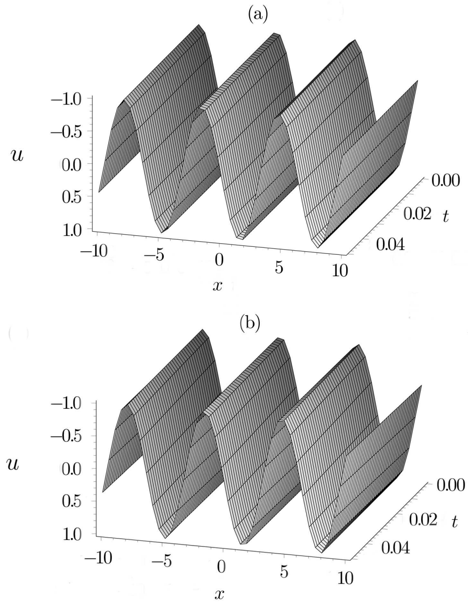

Fig. 2 (a) The figure of the approximate numerical solution (12) for ϕ_2; (b) The exact solution (13), when $p=2$, $(a,b,c,d)=(-2,26/25,0,-2/25)$, $(k,m,\lambda)=(1,1/5,-\sqrt{3})$.

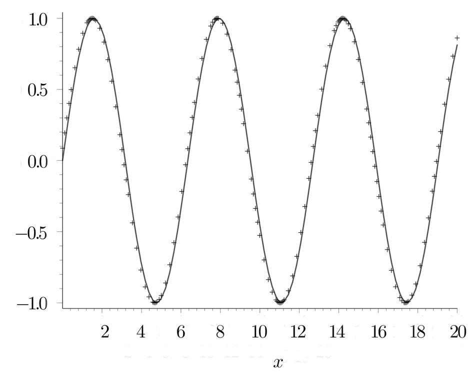

Fig. 3 The comparison of numerical solution (12) for ϕ_2 and exact solutions (13) at $t=0.05$. Line stands for the numerical solution figure and point the exact one.

3.2 *Initial Condition Is in the* $m-k\tanh(\alpha x)$ *Form*

Rewrite the operator form of the nonlinear evolution Eq. (1)

$$u=f-L^{-1}(au_{xx}+bu+cu^p+du^{2p-1})\,.$$

Here we assume the initial condition

$$u(x,0)=[m-k\tanh(\alpha x)]^{1/(p-1)}\,.$$

Substituting the initial value and corresponding Adomian polynomials into the recursive relations (11), with the aid of symbolic computation system *Maple*, the first few terms of the decomposition series are

$$\begin{aligned}
u_0&=A\,,\\
u_1&=-L^{-1}(au_{0,xx}+bu_0)-L^{-1}(cC_0+dD_0)\\
&=\frac{t^2}{2A(p-1)^2}(a_0+a_1\tanh(\alpha x)+a_2\tanh^2(\alpha x)\\
&\quad+a_3\tanh^3(\alpha x)+a_4\tanh^4(\alpha x))\,,\\
u_2&=-L^{-1}(au_{1,xx}+bu_1)-L^{-1}(cC_1+dD_1)\\
&=-\frac{t^4}{24A(p-1)^2}(a_5+a_6\tanh(\alpha x)\\
&\quad+a_7\tanh^2(\alpha x)+\cdots+a_{11}\tanh^6(\alpha x))\,,
\end{aligned}$$

where

$$\begin{aligned}
&A=(m-k\tanh(\alpha x))^{1/(p-1)}\,,\quad a_0=-dk^2A^2(p-1)^2\,,\\
&a_1=-2km(p-1)[dA^2(1-p)+cA^{3-p}(1-p)\\
&\qquad+A^{4-2p}(b-bp+a\alpha^2)]\,,\\
&a_2=m^2[dA^2(2p-d-p^2)-cA^{3-p}(p-1)^2\\
&\qquad+2a\alpha^2A^{4-2p}-bA^{4-2p}(p-1)^2]\,,\\
&a_3=2amk\alpha^2A^{4-2p}(p-1)\,,\quad a_4=-apm^2\alpha^2A^{4-2p}\,,\\
&\ldots\\
&a_{10}=24mka^2\alpha^4A^{4-2p}(p-1)\,,\\
&a_{11}=-20pm^2a^2\alpha^4A^{4-2p}\,.
\end{aligned}$$

So the generalized numerical solution of Eq. (10) with the initial condition $u(x,0)=(m-k\tanh(\alpha x))^{1/(p-1)}$ in a series form is

$$\begin{aligned}
u(x,t)&=A+\frac{t^2}{2A(p-1)^2}[a_0+a_1\tanh(\alpha x)+a_2\tanh^2(\alpha x)\\
&\quad+a_3\tanh^3(\alpha x)+a_4\tanh^4(\alpha x)]\\
&\quad-\frac{t^4}{24A(p-1)^2}[a_5+a_6\tanh(\alpha x)+\cdots\\
&\quad+a_7\tanh^2(\alpha x)+a_{11}\tanh^6(\alpha x))+\cdots\,. \qquad (15)
\end{aligned}$$

Remark 2 Because of the arbitrariness of p, we can choose $p=5/2$. In this case, equation (1) has the exact solution

$$\begin{aligned}
u&=\Big[-\frac{5c}{14d}\mp\sqrt{\frac{5b}{8d}}\\
&\quad\times\tanh\Big(\sqrt{-R}\Big(x\mp t\sqrt{-a+\frac{9b}{16R}}+\xi_0\Big)\Big)\Big]^{2/3}\,, \qquad (16)
\end{aligned}$$

obtained by Chen *et al.*[14] using the improved method through a proper transformation in Case 3 and this solution is the convergent solution (14) obtained by ADM. In order to verify numerically whether the proposed methodology leads to accurate solutions, we will evaluate the ADM solutions using the N-term approximation for the Eq. (10) ($p=5/2$) to compare the exact solution (15). The results show that we achieved a very good approximation to the actual solution of the equation by using only few terms of the decomposition series solution derived above.

Figure 4 shows the generalized numerical solution (14) with $p = -6$, $(a, b, c, d) = (-1/2, 1/5, 2/3, 1/8)$, $(m, \alpha, k) = (1, 1, 2)$.

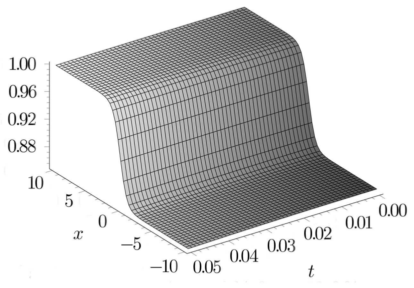

Fig. 4 The figure of the generalized approximate numerical solution (14) with $p = -6$, $(a, b, c, d) = (-1/2, 1/5, 2/3, 1/8)$, $(m, \alpha, k) = (1, 1, 2)$.

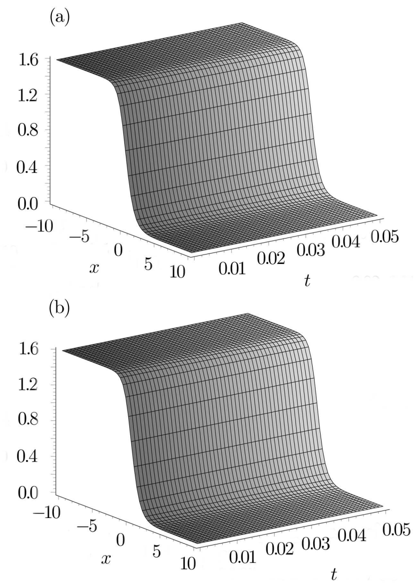

Fig. 5 (a) The figure of the approximate numerical solution (14) for ϕ_2; (b) The exact solution (15), when $p = 5/2$, $(a, b, c, d) = (-29/80, 1/5, 7/20, 1/8)$, $(k, m, \lambda, R) = (1, 1, 1, -1)$.

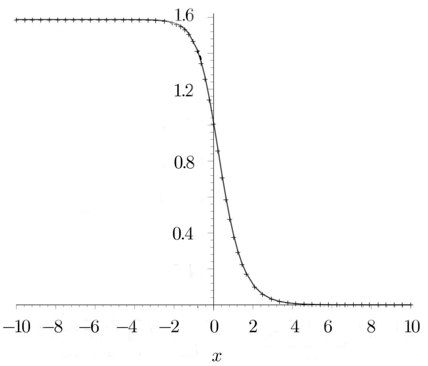

Fig. 6 The comparison of numerical solution (14) for ϕ_2 and exact solution (15) at $t = 0.05$. Line is the numerical solution figure and point the exact one.

Figure 5(a) and 5(b) are the figures for the approximate solution $\phi_2 = u_0 + u_1 + u_2$ and the exact solution (15) with $p = 5/2$, $(a, b, c, d) = (-29/80, 1/5, 7/20, 1/8)$, $(k, m, \lambda, R) = (1, 1, 1, -1)$, respectively.

Figure 6 is the comparison of them at $t = 0.05$. The comparison figure shows that the solution obtained by us rapidly converges to the solution obtained by Chen *et al.*

3.3 *Initial Condition Is in the* sech(kx) *Form*

Setting the initial condition as

$$u(x, 0) = [\mathrm{sech}(kx)]^{1/(p-1)},$$

for the nonlinear evolution Eq. (1) operator form

$$u = f - L^{-1}(au_{xx} + bu + cu^p + du^{2p-1}).$$

The steps are similar to the above. Substitute the initial value and corresponding Adomian polynomials into the recursive relations and then get the solution. With the help of *Maple* and omitting the heavy calculation, we just give the result

$$u = A - \frac{t^2}{2A(p-1)^2}[(p-1)^2(cA^{p+1} + dA^{2p-1}) - A^2(p-1)(b+ak^2-bp)+apk^2A^2\tanh^2(kx))+\cdots, \quad (17)$$

where $A = [\mathrm{sech}(kx)]^{1/(p-1)}$.

Remark 3 When we give the arbitrary value of p and $c = 0$, for example $p = 3/2$, the numerical solution (16) and the exact solution obtained by Chen[14] in Case 4

$$u = \Big\{\pm\sqrt{\frac{bp}{d}} \times \mathrm{sech}\Big[-\sqrt{R}\Big(x \mp t\sqrt{\frac{b(p-1)^2 - aR}{R}}\Big)\Big]\Big\}^{1/(p-1)}, \quad (18)$$

can be considered as the same. However the exact solution given by Chen holds when $c = 0$, he does not discuss the case of $c \neq 0$. Our solution (16) considers this case, so we predict that the solution (16) converges to the solution (17) in special case and it is a generalized solution.

Figure 7 shows the generalized numerical solution (16) with $p = 5$, $(a, b, c, d) = (-3/4, 2, 1, 3)$, $(p, k) = (5, 1)$.

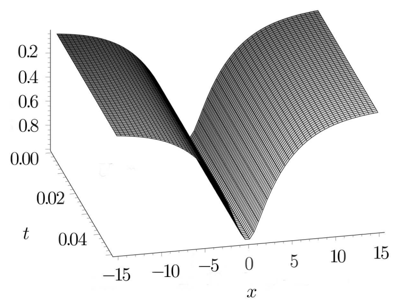

Fig. 7 The figure of the generalized approximate numerical solution (16) with $(a, b, c, d) = (-3/4, 2, 1, 3)$, $(p, k) = (5, 1)$.

Figure 8(a) and 8(b) are the figures for the approximate solution ϕ_2 and the exact solution (17) with $p = 3/2$, $(a, b, c, d) = (-3/4, 2, 0, 3)$, $(k, R) = (1, -1)$, respectively.

Figure 9 is the comparison of them at $t = 0.05$. The comparison figure shows the two solutions obtained by the ADM and the improved method are nearly the same.

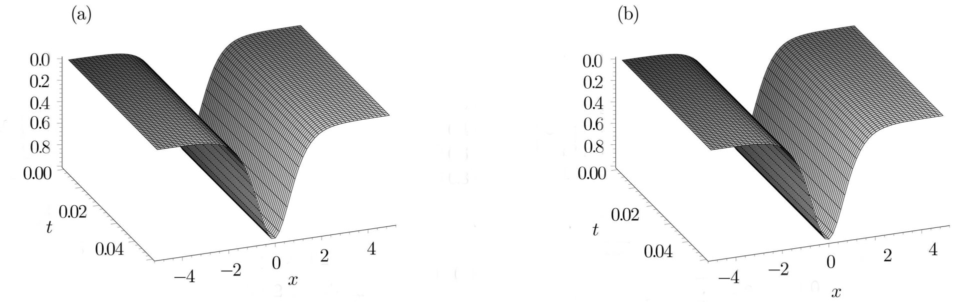

Fig. 8 (a) The figure of the approximate numerical solution for ϕ_2; (b) The exact solution (17) with $p = 3/2$, $(a, b, c, d) = (-3/4, 2, 0, 3)$, $(k, R) = (1, -1)$.

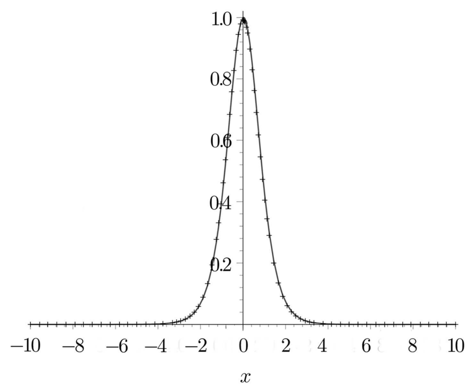

Fig. 9 The comparison of numerical solution ϕ_2 in Eq. (16) and exact solution (17) at $t = 0.05$. Line is the numerical solution figure and point the exact.

4 Conclusion

In summary, by using the ADM and choosing different forms of the initial condition values, we have obtained some new generalized numerical solutions of the nonlinear evolution equations $u_{tt} + au_{xx} + bu + cu^p + du^{2p-1} = 0$ with nonlinear term of any order. This equation includes many important nonlinear equations of mathematical physics, so we think that the study to Eq. (1) is very significant. The numerical solutions obtained by us are more realistic series solution, resulting computation shows that the numerical solutions generally converge very rapidly. In particular, by choosing different forms of the initial condition values, the ADM solutions converge to the previous solutions derived in Refs. [14] and [18], but also the rich ADM solutions corresponding to different p are obtained without other efforts. In addition, we compare the approximate numerical solutions obtained with the previous known exact solutions to predict the effectiveness of the ADM and the solutions are more accurate solutions. Whether existing another new operators and algorithms to solve this kind of nonlinear evolution equation with the initial conditions, we will further study these questions.

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