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Numerical Complexiton Solutions of Complex KdV Equation*

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Abstract *In this paper, we directly extend the applications of the Adomian decomposition method to investigate the complex KdV equation. By choosing different forms of wave functions as the initial values, three new types of realistic numerical solutions: numerical positon, negaton solution, and particularly the numerical analytical complexiton solution are obtained, which can rapidly converge to the exact ones obtained by Lou et al. Numerical simulation figures are used to illustrate the efficiency and accuracy of the proposed method.*

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1 Introduction

As we all know, in the soliton theory, the Korteweg-de Vries (KdV) equation always provides resources for studying integrability of nonlinear differential and difference equations. It is also a very typical and classical integrable model, which is used to describe the weakly shallow long waves. Therefore, many researchers have devoted great energy to study the KdV equation for a long time. As a result, many important properties have been discovered. For example it can be solved by inverse scattering transformation (IST),[1,2] Bäcklund transformation (BT),[3] and Darboux transformation (DT),[4] and it owns infinitely many symmetries, infinitely many conservation laws,[5] Painleve properties,[6] and the Hirota's bilinear form,[7] etc.

During recent years, more and more experts have paid great attention to the exact complexiton solutions of equations,[8−11] which are usually expressed in two different wave functions: the trigonometric functions and hyperbolic functions. In Ref. [11], Hu, Tong and Lou have investigated the complexiton solutions for the complex KdV equation through the Darboux transformation technique. However, to our knowledge, no work has been done to construct the numerical complexiton solutions till now; not much work has been done for the coupled equations by the ADM[12−15] either. Compared with other traditional methods,[1−7,16−21] the obvious advantage of the ADM is that it usually provides more realistic series solutions while no special techniques or assumptions are required.

Motivated by the above works.[8−11] we would like to construct the numerical complexiton solutions of the complex KdV equation by extending the applications of the ADM to the coupled systems. In order to obtain the complexiton solutions, we need to choose two different kinds of wave functions: the trigonometric functions and hyperbolic functions as the initial conditions.

The complex KdV equation studied in this paper is written as

$$U_t - 6UU_x + U_{xxx} = 0\,, \tag{1}$$

where

$$U = u + \mathrm{i}v\,, \quad \mathrm{i} = \sqrt{-1}\,.$$

In fact, the study to the complex KdV equation is very interesting and important because when substituting $U = u + \mathrm{i}v$ into Eq. (1), collecting the real and imaginary parts, a coupled KdV system

$$\begin{aligned} &u_t + 6vv_x - 6uu_x + u_{xxx} = 0\,,\\ &v_t - 6uv_x - 6vu_x + v_{xxx} = 0\,, \end{aligned} \tag{2}$$

can be derived. That is to say, equations (1) and (2) are equal. So we predict that the complex KdV Eq. (1) owns all the integrable properties that the real KdV equation has[1−7] and can be tested with P-test *Maple* package written by Xu and Li.[22] In addition, the coupled systems (2) are always considered as a special case of the general coupled KdV systems derived from the two-layer fluid dynamical systems and two-component Bose–Einstein condensates,[23−25] which are very useful in dynamics and physics.

The paper is organized as follows. In Sec. 2, some necessary descriptions and analysis on the ADM are given. In Sec. 3, by choosing different forms of initial conditions, the proposed method is applied to study Eq. (1) and three new types of numerical solutions especially the numerical complexiton solution are obtained. Numerical figures are

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used to verify whether the proposed method leads to high accuracy. Finally, conclusions are followed.

2 Description and Analysis on ADM to Complex KdV Equation

The Adomian method[12,13] is described as follows. Consider the operator form of the complex KdV Eq. (2):

$$L_t u+L_x u+N_1(u,v)=0\,,\quad L_t v+L_x v+N_2(u,v)=0 \tag{3}$$

with the initial conditions:

$$u(x,0)=f(x)\,,\quad v(x,0)=g(x)\,, \tag{4}$$

where the operators $L_t=\partial/\partial t$ and $L_x=\partial^3/\partial x^3$ stand for the linear differential operators, N_1 and N_2 symbolize the nonlinear terms and $N_1=6vv_x-6uu_x$, $N_2=-6uv_x-6vu_x$. It is assumed that L_t^{-1} is an integral operator given by

$$L_t^{-1}(\cdot)=\int_0^t(\cdot)\,\mathrm{d}s\,. \tag{5}$$

Applying the operator L_t^{-1} on both of the sub-equations of Eq. (3) and using the given initial conditions (4) yields

$$\begin{aligned}u&=f(x)-L_t^{-1}[L_x u+N_1(u,v)]\,,\\ v&=g(x)-L_t^{-1}[L_x v+N_2(u,v)]\,.\end{aligned} \tag{6}$$

According to the decomposition method,[12,13] the solutions are represented as infinite series like

$$u(x,t)=\sum_{n=0}^{\infty}u_n(x,t)\,,\quad v(x,t)=\sum_{n=0}^{\infty}v_n(x,t)\,. \tag{7}$$

The nonlinear terms $N_1(u,v)$ and $N_2(u,v)$ are decomposed as

$$N_1(u,v)=\sum_{n=0}^{\infty}A_n\,,\quad N_2(u,v)=\sum_{n=0}^{\infty}B_n\,, \tag{8}$$

where A_n and B_n are the so-called Adomian polynomials of u_i, v_i $(i=0,\ldots,n)$ and defined by

$$\begin{aligned}A_n&=\frac{1}{n!}\Big[\frac{\mathrm{d}^n}{\mathrm{d}\lambda^n}N_1\Big(\sum_{k=0}^{\infty}\lambda^k u_k,\sum_{k=0}^{\infty}\lambda^k v_k\Big)\Big]\Big|_{\lambda=0}\\ &=6\sum_{k=0}^{n}\Big(v_k\frac{\partial}{\partial x}v_{n-k}-u_k\frac{\partial}{\partial x}u_{n-k}\Big)\,,\\ B_n&=\frac{1}{n!}\Big[\frac{\mathrm{d}^n}{\mathrm{d}\lambda^n}N_2\Big(\sum_{k=0}^{\infty}\lambda^k u_k,\sum_{k=0}^{\infty}\lambda^k v_k\Big)\Big]\Big|_{\lambda=0}\\ &=-6\sum_{k=0}^{n}\Big(u_k\frac{\partial}{\partial x}v_{n-k}+v_k\frac{\partial}{\partial x}u_{n-k}\Big)\,.\end{aligned} \tag{9}$$

It is important to note that A_n and B_n only depend on u_k and v_k $(k=0,\ldots,n)$ and the sum of the subscripts in each term is equal to n. However, these formulas are easy to compute with the aid of *Maple*. For the convenience of the readers, we just give the first few terms of A_n and B_n:

$$\begin{aligned}A_0&=6(v_0v_{0x}-u_0u_{0x})\,,\\ B_0&=-6(u_0v_{0x}+v_0u_{0x})\,,\\ A_1&=6[v_0v_{1x}+v_1v_{0x}-u_0u_{1x}-u_1u_{0x}]\,,\\ B_1&=-6[v_0u_{1x}+v_1u_{0x}+u_0v_{1x}+u_1v_{0x}]\,,\\ A_2&=6[v_0v_{2x}+v_1v_{1x}+v_2v_{0x}-u_0u_{2x}-u_1u_{1x}-u_2u_{0x}]\,,\\ B_2&=-6[v_0u_{2x}+v_1u_{1x}+v_2u_{0x}+u_0v_{2x}+u_1v_{1x}+u_2v_{0x}]\,.\end{aligned}$$

Substituting the decomposition series (7) and (8) into Eq. (6), we can obtain the following iteration recursive relations:

$$\begin{aligned}&u_0=f(x)\,,\quad u_{n+1}=-L_t^{-1}[L_x u_n+A_n]\,,\quad (n\geq 0)\,,\\ &v_0=g(x)\,,\quad v_{n+1}=-L_t^{-1}[L_x v_n+B_n]\,,\quad (n\geq 0)\,.\end{aligned} \tag{10}$$

From Eqs. (9) and (10), we know all of the components u_n and v_n can be calculated easily. Moreover, the decomposition series solutions $u=\sum_{n=0}^{\infty}u_n$, $v=\sum_{n=0}^{\infty}v_n$ usually converge very rapidly in real physical problems.[12]

In the following, by directly extending the ADM to the complex KdV equation, we would like to construct the numerical complexiton solution of the complex KdV equation.

Remark It should be noticed that many kinds of numerical solutions can be obtained by using the ADM if suitable initial values are chosen. In order to obtain the numerical complexiton solutions, we must take the initial values as two different wave functions: the trigonometric functions and hyperbolic functions.

3 Numerical Complexiton Solutions for Complex KdV Equation

3.1 *Numerical Positon Solution*

Considering the complex KdV Eq. (2) with the following initial values,

$$\begin{aligned}&L_t u+6vv_x-6uu_x+u_{xxx}=0\,,\\ &L_t v-6uv_x-6vu_x+v_{xxx}=0\,,\\ &u(x,0)=\frac{G(x)}{F(x)}\,,\quad v(x,0)=\frac{H(x)}{F(x)}\,,\end{aligned} \tag{11}$$

where the initial conditions are chosen in the form of trigonometric functions,

$$\begin{aligned}F(x)&=(a^2\cos^2\xi_1+b^2\cos^2\xi_2)^2\,,\quad \xi_1=mx+\delta_1\,,\quad \xi_2=mx+\delta_2\,,\\ G(x)&=2m^2[(a^2-b^2)(a^2\cos^2\xi_1-b^2\cos^2\xi_2)+4a^2b^2\cos(\delta_1-\delta_2)\cos\xi_1\cos\xi_2]\,,\\ H(x)&=4abm^2[(a^2-b^2)(a^2\cos^2\xi_1-b^2\cos^2\xi_2)\cos(\delta_1-\delta_2)-(a^2-b^2)\cos^2\xi_1\cos^2\xi_2]\,.\end{aligned}$$

Applying the inverse operator L_t^{-1} to the first two sub-equations of Eq. (11) and using the initial condition, according to the expression (10), yields the following recursive formula:

$$u_0(x,t)=u(x,0)\,,\quad u_{n+1}=-L_t^{-1}[u_{n,xxx}+A_n]\,,\quad v_0(x,t)=v(x,0)\,,\quad v_{n+1}=-L_t^{-1}[v_{n,xxx}+B_n]\,. \tag{12}$$

With the aid of *Maple*, the first four terms of the decomposition series are calculated as follows:

$$u_0 = f(x)\,, \quad v_0 = g(x)\,,$$
$$u_1 = -L_t^{-1}[u_{0,xxx} + A_0] = -L_t^{-1}[u_{0,xxx}] - 6L_t^{-1}[v_0 v_{0x} - u_0 u_{0x}] = f_1(x)t\,,$$
$$v_1 = -L_t^{-1}[v_{0,xxx} + B_0] = -L_t^{-1}[v_{0,xxx}] + 6L_t^{-1}[u_0 v_{0x} + v_0 u_{0x}] = g_1(x)t\,,$$
$$u_2 = -L_t^{-1}[u_{1,xxx} + A_1] = -L_t^{-1}[u_{1,xxx}] - 6L_t^{-1}[v_0 v_{1x} + v_1 v_{0x} - u_0 u_{1x} - u_1 u_{0x}] = \frac{1}{2}t^2 f_2(x)\,,$$
$$v_2 = -L_t^{-1}[v_{1,xxx} + B_1] = -L_t^{-1}[v_{1,xxx}] + 6L_t^{-1}[v_0 u_{1x} + v_1 u_{0x} + u_0 v_{1x} + u_1 v_{0x}] = \frac{1}{2}t^2 g_2(x)\,,$$
$$u_3 = -L_t^{-1}[u_{2,xxx} + A_2] = -L_t^{-1}[u_{2,xxx}] - 6L_t^{-1}[v_0 v_{2x} + v_1 v_{1x} + v_2 v_{0x} - u_0 u_{2x} - u_1 u_{1x} - u_2 u_{0x}] = \frac{1}{6}t^3 f_3(x)\,,$$
$$v_3 = -L_t^{-1}[v_{2,xxx} + B_2] = -L_t^{-1}[v_{2,xxx}] + 6L_t^{-1}[v_0 u_{2x} + v_1 u_{1x} + v_2 u_{0x} + u_0 v_{2x} + u_1 v_{1x} + u_2 v_{0x}] = \frac{1}{6}t^3 g_3(x)\,,$$

where

$$f(x) = \frac{G(x)}{F(x)}\,, \quad g(x) = \frac{H(x)}{F(x)}\,, \quad f_1(x) = 6ff' - 6gg' - f'''\,, \quad g_1(x) = 6fg' + 6f'g - g'''\,,$$
$$f_2(x) = 6ff_1' + 6f'f_1 - 6gg_1' - 6g'g_1 - f_1'''\,, \quad g_2(x) = 6fg_1' + 6f'g_1 + 6f_1g' + 6f_1g' - g_1'''\,,$$
$$f_3(x) = 6ff_2' + 6f'f_2 - 6gg_2' - 6g'g_2 + 12f_1f_1' - 12g_1g_1' - f_2'''\,,$$
$$g_3(x) = 6fg_2' + 6f'g_2 + 6gf_2' + 6g'f_2 + 12f_1g_1' + 12f_1'g_1 - g_2'''\,.$$

So the approximate numerical positon solution for the complex KdV Eq. (11) in series form is

$$u = f(x) + f_1(x)t + \frac{1}{2}t^2 f_2(x) + \frac{1}{6}t^3 f_3(x) + \cdots\,, \quad v = g(x) + g_1(x)t + \frac{1}{2}t^2 g_2(x) + \frac{1}{6}t^3 g_3(x) + \cdots\,. \tag{13}$$

As we know, the exact positon solution obtained by Hu, Tong, and Lou[11] through the Darboux transformation is

$$u = \frac{2m^2[(a^2 - b^2)(a^2\cos^2\xi^1 - b^2\cos^2\xi^2) + 4a^2b^2\cos(\delta_1 - \delta_2)\cos\xi^1\cos\xi^2]}{(a^2\cos^2\xi^1 + b^2\cos^2\xi^2)^2}\,,$$
$$v = \frac{4abm^2[(a^2 - b^2)(a^2\cos^2\xi^1 - b^2\cos^2\xi^2)\cos(\delta_1 - \delta_2) - (a^2 - b^2)\cos^2\xi^1\cos^2\xi^2]}{(a^2\cos^2\xi^1 + b^2\cos^2\xi^2)^2}\,, \tag{14}$$

where $\xi^1 = mx + 4m^3t + \delta_1$, $\xi^2 = mx + 4m^3t + \delta_2$.

Remark It is obvious that the numerical positon solution (13) and the exact solution (14) are nonsingular for $v \neq 0$. When constants δ_1 and δ_2 satisfy $\delta_2 = \delta_1 + n\pi$ $(n = 0, \pm 1, \pm 2, \ldots)$, both the solutions are singular. This case coincides with the fact that the positon for the real KdV equation is singular.

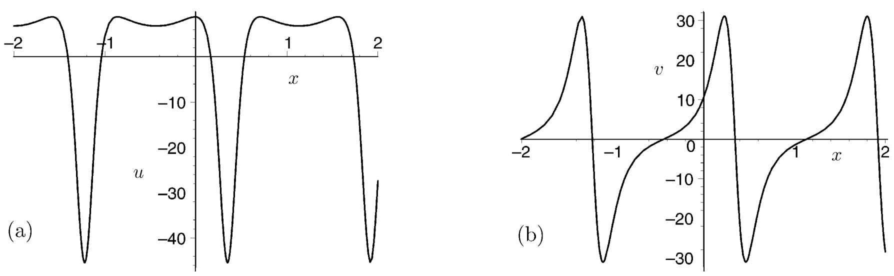

Fig. 1 Numerical positon solution (13) for the complex KdV equation. (a) $u(x,t)$, (b) $v(x,t)$ with the parameters selected as (15) at $t = 0$.

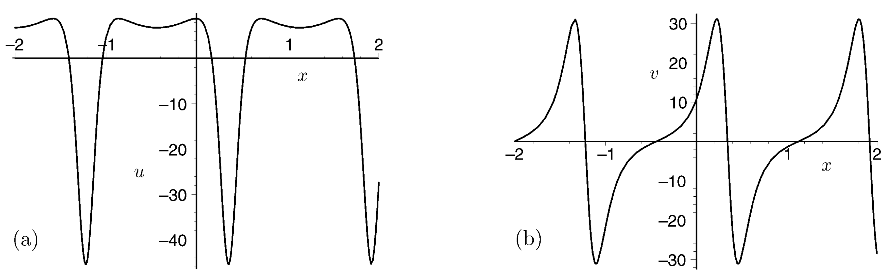

Fig. 2 Exact positon solution for the complex KdV system. (a) $u(x,t)$, (b) $v(x,t)$ expressed by (14) with the constants chosen in (15) at $t = 0$.

In order to illustrate whether the proposed method leads to high accuracy, we depict the numerical figures. Figures 1 and 2 describe the numerical positon solution (13) by using the N-term approximation and the exact one (14) at time $t = 0$ with the parameters selected as

$$a = 2\,, \quad b = 4\,, \quad \delta_1 = 0\,, \quad \delta_2 = 1\,, \quad m = 2\,. \tag{15}$$

From Fig. 3, we know that the approximate numerical solution can rapidly converge to the exact solution, which shows that a good result is achieved.

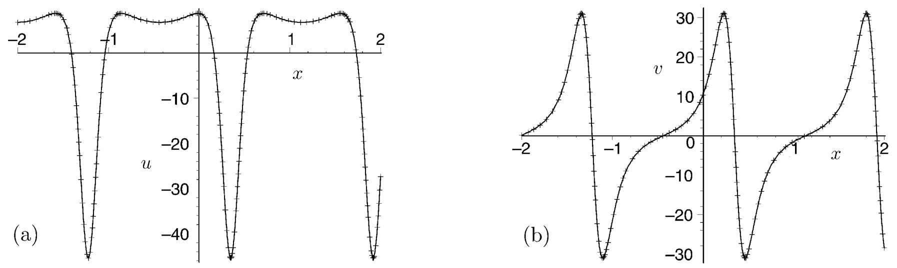

Fig. 3 The comparison between the numerical positon solution (13) and the exact one (14) at $t = 0$. (a) $u(x, t)$; (b) $v(x, t)$. Line stands for figures of the numerical solution and point for the exact one.

3.2 *Numerical Negaton Solution*

Let us take the initial conditions in the hyperbolic function form as

$$u(x, 0) = \frac{G(x)}{F(x)}\,, \quad v(x, 0) = \frac{H(x)}{F(x)} \tag{16}$$

to construct the numerical negaton solution for the complex KdV Eq. (2). Here

$$\begin{aligned}
&F(x) = (a^2 \sinh^2 \eta_1 + b^2 \sinh^2 \eta_2)^2\,, \quad \eta_1 = mx + \delta_1\,, \quad \eta_2 = mx + \delta_2\,,\\
&G(x) = 2m^2[(a^2 - b^2)(a^2 \sinh^2 \eta_1 - b^2 \sinh^2 \eta_2) + 4a^2 b^2 \cosh(\delta_1 - \delta_2) \sinh \eta_1 \sinh \eta_2]\,,\\
&H(x) = 2abm^2(a^2 \sinh 2\eta_1 + b^2 \sinh 2\eta_2) \sinh(\delta_1 - \delta_2)\,.
\end{aligned} \tag{17}$$

As stated above, with the aid of *Maple*, we need to calculate the Adomian polynomials A_n and B_n, substitute the initial value (16) into the (10) and then obtain the recursive expression. Here we omit the main steps and just give the final numerical result:

$$u = f(x) + f_1(x)t + \frac{1}{2}t^2 f_2(x) + \frac{1}{6}t^3 f_3(x) + \cdots\,, \quad v = g(x) + g_1(x)t + \frac{1}{2}t^2 g_2(x) + \frac{1}{6}t^3 g_3(x) + \cdots\,, \tag{18}$$

where

$$\begin{aligned}
&f(x) = \frac{G(x)}{F(x)}\,, \quad g(x) = \frac{H(x)}{F(x)}\,, \quad f_1(x) = 6ff' - 6gg' - f'''\,, \quad g_1(x) = 6fg' + 6f'g - g'''\,,\\
&f_2(x) = 6ff_1' + 6f'f_1 - 6gg_1' - 6g'g_1 - f_1'''\,, \quad g_2(x) = 6fg_1' + 6f'g_1 + 6f_1g' + 6f_1g' - g_1'''\,,\\
&f_3(x) = 6ff_2' + 6f'f_2 - 6gg_2' - 6g'g_2 + 12f_1f_1' - 12g_1g_1' - f_2'''\,,\\
&g_3(x) = 6fg_2' + 6f'g_2 + 6gf_2' + 6g'f_2 + 12f_1g_1' + 12f_1'g_1 - g_2'''\,,\\
&F(x) = (a^2 \sinh^2 \eta_1 + b^2 \sinh^2 \eta_2)^2\,, \quad \eta_1 = mx + \delta_1\,, \quad \eta_2 = mx + \delta_2\,,\\
&G(x) = 2m^2[(a^2 - b^2)(a^2 \sinh^2 \eta_1 - b^2 \sinh^2 \eta_2) + 4a^2 b^2 \cosh(\delta_1 - \delta_2) \sinh \eta_1 \sinh \eta_2]\,,\\
&H(x) = 2abm^2(a^2 \sinh 2\eta_1 + b^2 \sinh 2\eta_2) \sinh(\delta_1 - \delta_2)\,.
\end{aligned}$$

The exact solution obtained by Hu, Tong and Lou[11] through the Darboux transformation is

$$\begin{aligned}
u &= \frac{2m^2[(a^2 - b^2)(a^2 \sinh^2 \eta^1 - b^2 \sinh^2 \eta^2) + 4a^2 b^2 \cosh(\delta_1 - \delta_2) \sinh \eta^1 \sinh \eta^2]}{(a^2 \sinh^2 \eta^1 + b^2 \sinh^2 \eta^2)^2}\,,\\
v &= \frac{2abm^2(a^2 \sinh 2\eta^1 + b^2 \sinh 2\eta^2) \sinh(\delta_1 - \delta_2)}{(a^2 \sinh^2 \eta^1 + b^2 \sinh^2 \eta^2)^2}\,,
\end{aligned} \tag{19}$$

where $\eta^1 = mx - 4m^3 t + \delta_1$, $\eta^2 = mx - 4m^3 t + \delta_2$.

Analyzing the two kinds of negaton solutions (18) and (19), we find that they are singular when $\delta_2 = \delta_1 + n\pi \mathrm{i}$ $(n = 0, \pm 1, \pm 2, \ldots)$, which correspond to the real KdV equation. In other cases, these two solutions are nonsingular.

Figure 4 shows the numerical negaton solution (18) and figure 5 shows the exact solution (19) at $t = 0$ with the parameters chosen as

$$a = 1\,, \quad b = 5\,, \quad \delta_1 = 0\,, \quad \delta_2 = 2\,, \quad m = 2\,. \tag{20}$$

The comparison between the two different kinds of solutions is depicted in Fig. 6. From the figures, we believe that a very high-level accuracy is achieved by the ADM.

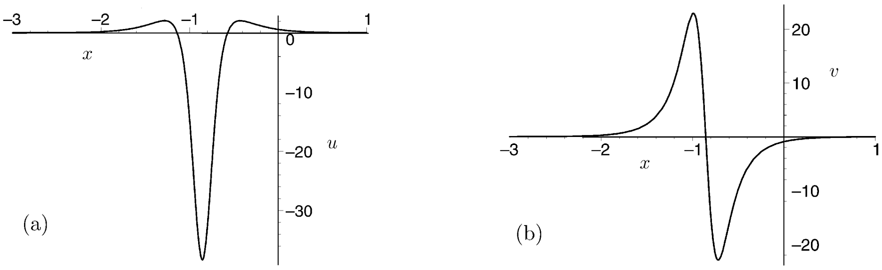

Fig. 4 Numerical negaton solution (18) with the initial value (16). (a) $u(x,t)$; (b) $v(x,t)$, under the case (20) at $t=0$ for the complex KdV model.

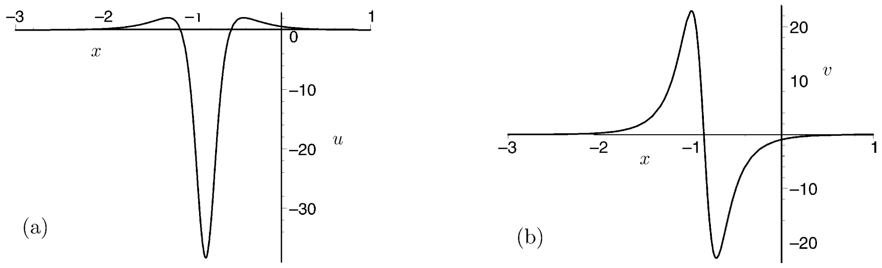

Fig. 5 Exact negaton solution (19). (a) For $u(x,t)$; (b) For $v(x,t)$. The constants are selected as Fig. 4.

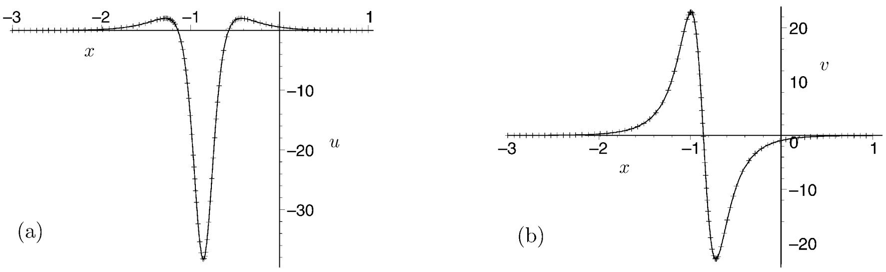

Fig. 6 The comparison between numerical solution (18) and exact solution (19). (a) $u(x,t)$; (b) $v(x,t)$. Line depicts the figures for the numerical solution and point for the exact one.

3.3 *Numerical Analytical Complexiton Solution*

In this subsection, we take the initial conditions in the form of two different wave functions: the hyperbolic functions and trigonometric functions to construct the numerical complexiton solution of the complex KdV equation. The initial values are given by:

$$u(x,0) = -[\ln(F^2+G^2)]_{xx}\,, \quad v(x,0) = -2\Big[\arctan\frac{G}{F}\Big]_{xx}\,, \tag{21}$$

where

$$\begin{aligned} F &= 4\lambda_1(c_4C_3 - c_2C_1)\cosh(2\lambda_2 x) + 4\lambda_2(c_1C_1 - c_3C_3)\sin(2\lambda_1 x)\,,\\ G &= 4\lambda_1(c_2C_3 + c_4C_1)\cosh(2\lambda_2 x) - 4\lambda_2(c_3C_1 + c_1C_3)\sin(2\lambda_1 x)\,. \end{aligned} \tag{22}$$

For the convenience of seeing a clear structure of the complexiton, we set these parameters as $(c_1, c_2, c_3, c_4, C_1, C_2, C_3, C_4) = (5, 0, 0, 5, -1, 2, 2, 1)$ and $(\lambda_1, \lambda_2) = (2, 0.5)$. Substituting these constants into Eq. (21), we can get the following values:

$$u(x,0) = \frac{A(x)}{B(x)}\,, \quad v(x,0) = \frac{C(x)}{B(x)}\,, \tag{23}$$

where

$$\begin{aligned} A(x) &= -\,[272\cos(8x) + 240\cos(8x)\cosh(2x) + 272\cosh(2x) - 128\sin(8x)\sinh(2x) + 240]\,,\\ B(x) &= \frac{835}{8} - \frac{17}{2}\cos(8x) + \frac{1}{8}\cos(16x) + 136\cosh(2x) - 8\cos(8x)\cosh(2x) + 32\cosh(4x)\,, \end{aligned}$$

$$C(x) = -[1862\sin(4x)\cosh(x) + 30\cosh(x)\sin(12x) + 240\cos(4x)\sinh(x) + 16\sinh(x)\cos(12x) + 480\sin(4x)\cosh(3x) + 256\cos(4x)\sinh(3x)]\,.$$

We make the same analysis as above. Using the given initial value (23), substituting the Adomian polynomials (8) into the iteration recursive expressions (10), with the aid of *Maple*, we calculate as many terms as we need. Omitting the complicated expressions, we only give the final numerical solution:

$$u = f(x) + f_1(x)t + \frac{1}{2}t^2 f_2(x) + \frac{1}{6}t^3 f_3(x) + \cdots\,, \quad v = g(x) + g_1(x)t + \frac{1}{2}t^2 g_2(x) + \frac{1}{6}t^3 g_3(x) + \cdots\,, \tag{24}$$

where $f(x) = A(x)/B(x)$, $g(x) = C(x)/B(x)$, and f_i, g_i $(i = 1, 2, 3)$ are defined the same as above in Eqs. (13) and (18).

The corresponding special exact solution given in Ref. [11] through the Darboux transformation is

$$u = \frac{A(x,t)}{B(x,t)}\,, \quad v = \frac{C(x,t)}{B(x,t)}\,, \tag{25}$$

where

$$A(x,t) = -[272\cos(8x+104t) + 240\cos(8x+104t)\cosh(2x+94t) + 272\cosh(2x+94t) - 128\sin(8x+104t)\sinh(2x+94t) + 240]\,,$$

$$B(x,t) = \frac{835}{8} - \frac{17}{2}\cos(8x+104t) + \frac{1}{8}\cos(16x+208t) + 136\cosh(2x+94t) - 8\cos(8x+104t)\cosh(2x+94t) + 32\cosh(4x+188t)\,,$$

$$C(x,t) = -[1862\sin(4x+52t)\cosh(x+47t) + 30\cosh(x+47t)\sin(12x+156t) + 240\cos(4x+52t)\sinh(x+47t) + 16\sinh(x+47t)\cos(12x+156t) + 480\sin(4x+52t)\cosh(3x+141t) + 256\cos(4x+52t)\sinh(3x+141t)]\,.$$

Figures 7 and 8 describe the numerical complexiton solution (23) and the exact one (24). Figure 9 describes the comparison between the numerical solution (23) and the exact one (24).

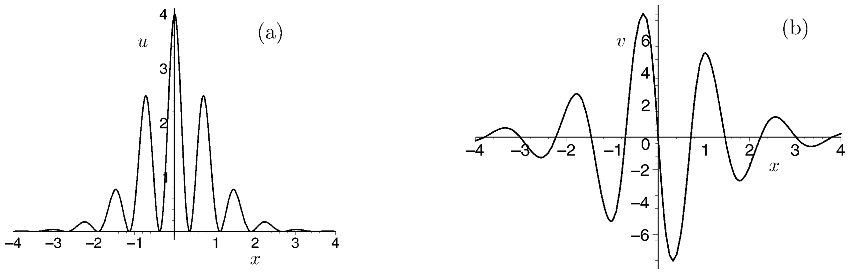

Fig. 7 Numerical analytical complexiton solution (24) with the initial value (23). (a) For $u(x,t)$; (b) For $v(x,t)$ at $t = 0$ for the complex KdV model.

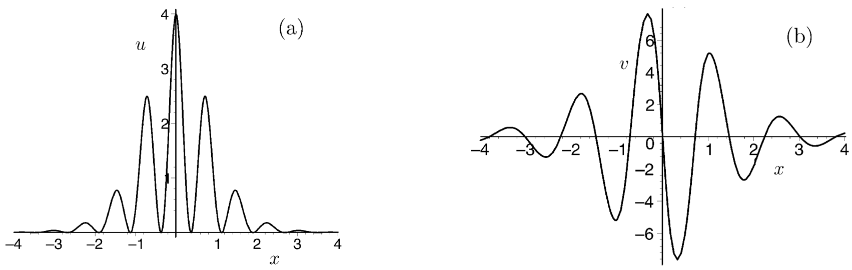

Fig. 8 Exact analytical complexiton solution given by (25) (a) for $u(x,t)$ and (b) for $v(x,t)$ at $t = 0$.

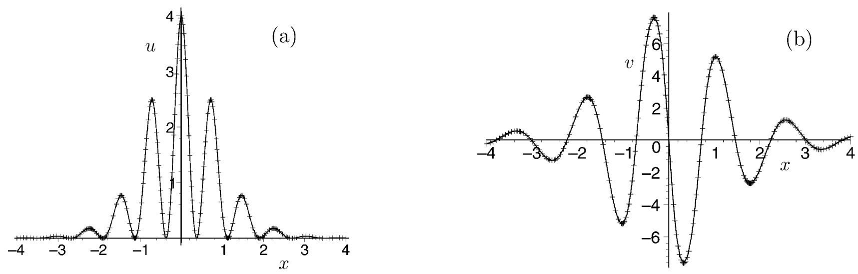

Fig. 9 The comparison between the two different forms of solutions. (a) For $u(x,t)$; (b) For $v(x,t)$ in expressions (24) and (25). Line is the figures for the numerical solution and point for the exact one.

Remark For many integrable systems, the known complexiton solutions generally have singularities.[8] However, by analysis of solution (24), we know that the numerical complexiton solution is analytical and nonsingular. It is the first time to find the numerical complexiton solution for the integrable systems by the ADM up to now.

In addition, other types of numerical solutions such as numerical Jacobi elliptic function solution, numerical soliton solution, numerical rational solution, etc., can be also obtained by the ADM if suitable initial conditions are chosen.

4 Conclusion

Recently, Ma[8] has found the complexiton solutions to the KdV equation by using the bilinear form; Lou *et al.*[9] have obtained many types of complexiton solutions of the $(n+1)$-dimensional sine-Gordon equation by implementing some pure algebraic mapping relations. Chen and Wang[10] have also derived the complexiton solutions of the Whitham–Broer–Kaup equation through the multiple Riccati equations rational expansion method. However, the complexiton solutions obtained in Refs. [8] $\sim$ [11] are exact solutions. Here, by choosing different wave functions as the initial conditions, three new types of numerical solutions: numerical positon solution, numerical negaton solution and particularly the numerical complexiton solution have been obtained, which are expressed by trigonometric functions, hyperbolic functions and the combination of the two functions. All these numerical solutions can rapidly converge to the exact solutions derived by Hu, Tong and Lou.[11] It shows that the ADM is a very useful and effective tool to solve a wide class of differential equations, especially for the integrable complex KdV equation.

For many integrable systems, the known complexiton solutions have singularities.[11] It is very interesting and surprising that the numerical complexiton solution obtained in the paper is nonsingular. Maybe this kind of nonsingular numerical complexiton solution exists in other complex equations. How to find them will be studied further.

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