



Numerical complexiton solutions for the complex KdV equation by the homotopy perturbation method

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ARTICLE INFO

Keywords:

Homotopy perturbation method
Complex KdV equation
Numerical solution
Numerical complexiton solutions

ABSTRACT

In this paper, the homotopy perturbation method is extended to investigate the numerical complexiton solutions of the complex KdV equation. By constructing special forms of initial conditions, three new types of realistic numerical solutions are obtained: numerical positon solution expressed by the trigonometric functions, numerical negaton solution expressed by the hyperbolic functions and particularly the numerical analytical complexiton solutions expressed by combinations of the two kinds of functions. All these numerical solutions obtained can rapidly converge to the exact solutions obtained by Lou et al. Illustrative numerical figures are exhibited the efficiency of the proposed method.

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1. Introduction

Many powerful methods [1–16] have been used to find the explicit solutions of nonlinear differential equations. Especially recent years, the applications of the homotopy perturbation method (HPM) [17–20] have appeared in many works [21–25], which show the method is a powerful technique to study numerical solutions. Compared with other classical methods [2–16], the HPM [1,17–25] always deforms the difficult problem into a simple and solvable one, which is a coupling of the traditional perturbation method and homotopy in topology. With this method, the rapidly convergent series solutions with easily computable components can be obtained.

Recently, many experts have paid great attention to construct the complexiton solutions by different methods and some works have been done [26–29]. By using the bilinear form, Ma [26] has found the complexiton solutions to the KdV equation. By implementing some pure algebraic mapping relations, Lou et al. [27] have obtained many types of complexiton solutions of the $(n + 1)$ -dimensional sine-Gordon equation. Chen and Wang [28] have also derived the complexiton solutions of the Whitham–Broer–Kaup equation through the multiple Riccati equations rational expansion method. In Ref. [29], Hu et al. have investigated the complexiton solutions of the complex KdV equation by the Darboux transformation technique. However, to our knowledge, no work has been done to obtain the numerical complexiton solutions by the HPM till now; not much work has been done for the coupled equations by the HPM [1,21–25], either.

Motivated by the above works [26–29], we would like to extend the applications of the HPM from the single equation to the coupled systems to construct the numerical complexiton solutions for the complex KdV equation. It should be pointed that in order to obtain the complexiton solutions we must choose the initial conditions as two different kinds of wave functions: the trigonometric functions and hyperbolic functions.

The complex KdV equation is written as:

$$U_t - 6UU_x + U_{xxx} = 0, \quad (1)$$

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where

$$U = u + Iv, \quad I = \sqrt{-1}.$$

In fact, the study to the complex KdV equation is very interesting and important that is because when substituting $U = u + Iv$ into Eq. (1), collecting the real and imaginary parts, a coupled system

$$\begin{cases} u_t + 6vv_x - 6uu_x + u_{xxx} = 0, \\ v_t - 6uv_x - 6vu_x + v_{xxx} = 0, \end{cases} \quad (2)$$

is derived. That is to say the two Eqs. (1) and (2) are equal. From this, we predict the complex KdV equation (1) owns all the integrable properties that the real KdV equation has, such as possesses infinitely many symmetries and infinitely many conservation laws [30], can be solved by the inverse scattering transformation [2], Hirota bilinear method [5] and Darboux transformation [3] and pass the P-test [31], etc. In addition, the coupled system (2) is always considered as a special case of the general coupled KdV systems derived from the two layer fluid dynamical systems and two-component Bose–Einstein condensates [32–34], which are very useful in dynamics and physics.

The paper is organized as follows: In Section 2, some descriptions are given on the HPM to the coupled systems. In Section 3, by choosing special forms of initial conditions, the proposed method is applied to study Eq. (1) and three new types of numerical solutions especially the numerical complexiton solution are obtained. Numerical simulation figures are used to verify the accuracy of the proposed method. Finally, conclusions are followed.

2. The homotopy perturbation method to the coupled systems

The HPM to the coupled systems is described as follows. Consider the nonlinear coupled differential equations in this form:

$$\begin{cases} L_1(u) + R_1(u, v) + f_1(r) = 0, \\ L_2(v) + R_2(u, v) + f_2(r) = 0, \quad r \in \Omega \end{cases} \quad (3)$$

with the boundary conditions of

$$B_1\left(u, \frac{\partial u}{\partial n}\right) = 0, \quad B_2\left(v, \frac{\partial v}{\partial n}\right) = 0, \quad r \in \Gamma,$$

where L_i ($i = 1, 2$) are the linear operators and their inverse operator can be easily solved; R_i are the operators for the remainder parts of the functions u, v ; $f_i(r)$ are the known analytical functions, B_i are the boundary operators, Γ is the boundary of the domain Ω and n is the unit outward normal of Ω .

We construct the following homotopy $\Omega \times [0, 1] \rightarrow R$ for the extended HPM, which satisfies

$$\begin{cases} H_1(u, v, p) = (1 - p)[L_1(u) - L_1(u0)] + p[L_1(u) + R_1(u, v) + f_1(r)] = 0, \\ H_2(u, v, p) = (1 - p)[L_2(v) - L_2(v0)] + p[L_2(v) + R_2(u, v) + f_2(r)] = 0, \end{cases} \quad (4)$$

or

$$\begin{cases} H_1(u, v, p) = L_1(u) - L_1(u0) + p[L_1(u0) + R_1(u, v) + f_1(r)] = 0, \\ H_2(u, v, p) = L_2(v) - L_2(v0) + p[L_2(v0) + R_2(u, v) + f_2(r)] = 0, \end{cases} \quad (5)$$

where $p \in [0, 1]$ is an embedding parameter; $u0$ and $v0$ are the known initial approximations of Eq. (3) that satisfy the boundary conditions above.

It is clear that

$$\begin{cases} H_1(u, v, 0) = L_1(u) - L_1(u0) = 0, \\ H_2(u, v, 0) = L_2(v) - L_2(v0) = 0, \\ H_1(u, v, 1) = L_1(u) + R_1(u, v) + f_1(r) = 0, \\ H_2(u, v, 1) = L_2(v) + R_2(u, v) + f_2(r) = 0. \end{cases} \quad (6)$$

This shows that $H_1(u, v, p)$ and $H_2(u, v, p)$ continuously deform from the trivial problem $L_1(u) - L_1(u0) = 0$ and $L_2(v) - L_2(v0) = 0$ to the original problem investigated (3), respectively. In topology, we call the deformations: from $L_1(u) - L_1(u0)$ to $L_1(u) + R_1(u, v) + f_1(r)$ and from $L_2(v) - L_2(v0)$ to $L_2(v) + R_2(u, v) + f_2(r)$, are homotopic.

For the coupled system (3), we assume the solutions $u(x, t)$ and $v(x, t)$ are given by infinity series of the homotopy parameter p in this form

$$\begin{cases} u = \sum_{i=0}^{+\infty} p^i u_i = u_0 + pu_1 + p^2 u_2 + p^3 u_3 + \dots \\ v = \sum_{i=0}^{+\infty} p^i v_i = v_0 + pv_1 + p^2 v_2 + p^3 v_3 + \dots \end{cases} \quad (7)$$

The approximate solutions of Eq. (3) can be obtained when $p \rightarrow 1$:

$$\begin{cases} u = \lim_{p \rightarrow 1} \sum_{i=0}^{+\infty} p^i u_i = u_0 + u_1 + u_2 + u_3 + \dots \\ v = \lim_{p \rightarrow 1} \sum_{i=0}^{+\infty} p^i v_i = v_0 + v_1 + v_2 + v_3 + \dots \end{cases} \quad (8)$$

The convergence must depend on the choices for L_i , R_i and f_i ($i = 1, 2$) and the initial conditions. The series (7), generally speaking, is convergent for most cases. The convergence rate of the series (7) mainly depends on three factors [1]:

1. The second derivative of $R_i(u, v)$ ($i = 1, 2$) with respect to u and v , respectively, must be small, for the parameter p may be relatively large, i.e. $p \rightarrow 1$.
2. The norm of $L^{-1} \frac{\partial R_i}{\partial u}$ and $L^{-1} \frac{\partial R_i}{\partial v}$ ($i = 1, 2$) must be smaller than one, in order that the series converges.
3. In process of computation, due to choice of initial conditions and solving the series of ODE, there appear some arbitrary functions (see next section in u_i, v_i ($i = 0, 1, \dots$)), It is a crucial problem how to design f_i and g_i , i.e., we need to choose the proper functions f_i, g_i in u_i and v_i to guarantee the high convergence rate of the series (7). Generally, f_i and g_i chosen depend on the initial condition.

Remark 1. For the linear operators L_i , one can choose them arbitrarily. However, we usually choose the ones whose inverse operators are easily solvable and can make the complicate calculation easy. When one of $L_i = R_i = f_i = 0$ ($i = 1, 2$), the traditional HPM [1,21–25] will appear, which shows that the our method is more general.

Remark 2. It should be pointed that many kinds of numerical solutions can be obtained by using the HPM directly if suitable initial values are chosen. In order to obtain the numerical complexiton solutions, we must take the initial values as two different wave functions: the trigonometric functions and hyperbolic functions.

In the following, we will apply the HPM to the complex KdV equation to derive the numerical complexiton solutions. Numerical simulation figures are used to verify whether the proposed method leads to the high accuracy.

3. Numerical complexiton solutions for the complex KdV equation

3.1. Numerical positon solution

Consider the operator form of the real and imaginary parts of the complex KdV Eq. (1)

$$\begin{cases} L(u) + 6vv_x - 6uu_x + u_{xxx} = 0, \\ L(v) - 6uv_x - 6vu_x + v_{xxx} = 0, \end{cases}$$

where the operator $L(\cdot) = \partial/\partial t$.

In order to obtain the numerical positon solution, we assume the initial condition in the form of trigonometric functions:

$$\begin{cases} u(x, 0) = G(x)/F(x), \\ v(x, 0) = H(x)/F(x), \end{cases} \quad (9)$$

where

$$\begin{aligned} F(x) &= (a^2 \cos^2 \xi_1 + b^2 \cos^2 \xi_2)^2, \quad \xi_1 = mx + \delta_1, \quad \xi_2 = mx + \delta_2, \\ G(x) &= 2m^2[(a^2 - b^2)(a^2 \cos^2 \xi_1 - b^2 \cos^2 \xi_2) + 4a^2 b^2 \cos(\delta_1 - \delta_2) \cos \xi_1 \cos \xi_2], \\ H(x) &= 4abm^2[(a^2 - b^2)(a^2 \cos^2 \xi_1 - b^2 \cos^2 \xi_2) \cos(\delta_1 - \delta_2) - (a^2 - b^2) \cos^2 \xi_1 \cos^2 \xi_2]. \end{aligned}$$

According to the extended HPM, we construct the following homotopy:

$$\begin{cases} (1 - p)L(u) + p(L(u) + 6vv_x - 6uu_x + u_{xxx}) = 0, \\ (1 - p)L(v) + p(L(v) - 6uv_x - 6vu_x + v_{xxx}) = 0, \end{cases} \quad (10)$$

or

$$\begin{cases} L(u) + p(6vv_x - 6uu_x + u_{xxx}) = 0, \\ L(v) - p(6uv_x + 6vu_x - v_{xxx}) = 0. \end{cases} \quad (11)$$

Assuming that the solutions are given in series form of the homotopy parameter p

$$\begin{cases} u(x, t) = \sum_{p=0}^{+\infty} p^i u_i = u_0 + pu_1 + p^2 u_2 + p^3 u_3 + \dots \\ v(x, t) = \sum_{p=0}^{+\infty} p^i v_i = v_0 + pv_1 + p^2 v_2 + p^3 v_3 + \dots \end{cases} \quad (12)$$

Substituting (12) into (11), yields a system of algebraic equations of p . Collecting all the same power of p and setting all the coefficients of the terms p^i to zero, gets a over-determined system of differential equations with unknown variables u_i and v_i ($i = 0, 1, \dots$). For the convenience of the readers, we only write the first few terms of equations:

$$\begin{aligned}
 p^0 : \quad & u_{0,t} = 0, \\
 & v_{0,t} = 0, \\
 p^1 : \quad & u_{1,t} + 6(v_0 v_{0,x} - u_0 u_{0,x}) + u_{0,xxx} = 0, \\
 & v_{1,t} - 6(u_0 v_{0,x} + v_0 u_{0,x}) + v_{0,xxx} = 0, \\
 p^2 : \quad & u_{2,t} + 6(v_0 v_{1,x} + v_1 v_{0,x} - u_0 u_{1,x} - u_1 u_{0,x}) + u_{1,xxx} = 0, \\
 & v_{2,t} - (6u_0 v_{1,x} + u_1 v_{0,x} + v_0 u_{1,x} + v_1 u_{0,x}) + v_{1,xxx} = 0, \\
 p^3 : \quad & u_{3,t} + 6(v_0 v_{2,x} + v_1 v_{1,x} + v_2 v_{0,x} - u_0 u_{2,x} - u_1 u_{1,x} - u_2 u_{0,x}) + u_{2,xxx} = 0, \\
 & v_{3,t} - 6(v_0 u_{2,x} + v_1 u_{1,x} + v_2 u_{0,x} + u_0 v_{2,x} + u_1 v_{1,x} + u_2 v_{0,x}) + v_{2,xxx} = 0.
 \end{aligned} \tag{13}$$

With the aid of the symbolic computation system *Maple*, we can easily obtain the solution:

$$\begin{aligned}
 u_0 &= f_0(x), \quad v_0 = g_0(x), \\
 u_1 &= f_1(x) + f^1(x)t, \quad v_1 = g_1(x) + g^1(x)t, \\
 u_2 &= f_2(x) + f^2(x)t + \frac{1}{2}f^3(x)t^2, \quad v_2 = g_2(x) + g^2(x)t + \frac{1}{2}g^3(x)t^2, \\
 u_3 &= f_3(x) + f^4(x)t + \frac{1}{2}f^5(x)t^2 + \frac{1}{3}f^6(x)t^3, \quad v_3 = g_3(x) + g^4(x)t + \frac{1}{2}g^5(x)t^2 + \frac{1}{3}g^6(x)t^3,
 \end{aligned} \tag{14}$$

where

$$\begin{aligned}
 f_i(x) &= u_i(x, 0), \quad \sum_{i=0}^{\infty} f_i(x) = u(x, 0) = G(x)/F(x), \quad f^1(x) = 6f_0 f_{0,x} - 6g_0 g_{0,x} \\
 g_i(x) &= v_i(x, 0), \quad \sum_{i=0}^{\infty} g_i(x) = v(x, 0) = H(x)/F(x), \quad g^1(x) = 6f_0 g_{0,x} + 6g_0 f_{0,x} \\
 f^2(x) &= 6(f_0 f_1)_x - 6(g_0 g_1)_x - f_{1,xxx}, \quad f^3 = 6(f_0 f^1)_x - 6(g_0 g^1)_x - f^1_{xxx}, \\
 g^2(x) &= 6(f_0 g_1)_x + 6(f_1 g_0)_x - g_{1,xxx}, \quad g^3 = 6(f_0 g^1)_x + 6(g_0 f^1)_x - g^1_{xxx}, \\
 f^4(x) &= 6(f_0 f_2 - g_0 g_2)_x + 6(f_1 f_{1,x} - g_1 g_{1,x}) - f_{2,xxx}, \\
 g^4(x) &= 6(f_0 g_2 + g_0 f_2)_x + 6(f_1 g_1)_x - g_{2,xxx}, \\
 f^5(x) &= 6(f_0 f^2 + f_1 f^1)_x - 6(g_0 g^2 + g_1 g^1)_x - f^2_{xxx}, \\
 g^5(x) &= 6(f_0 g^2 + f_1 g^1)_x + 6(g_0 f^2 + g_1 f^1)_x - g^2_{xxx}, \\
 f^6(x) &= 3(f_0 f^3 - g_0 g^3)_x + 6f^1 f^1_x - 6g^1 g^1_x - \frac{1}{2}f^3_{xxx}, \\
 g^6(x) &= 3(f_0 g^3 + g_0 f^3)_x + 6f^1 g^1_x + 6f^1_x g^1 - \frac{1}{2}g^3_{xxx}.
 \end{aligned} \tag{15}$$

Here f_i, g_i, f^i and g^i are the abbreviations of $f_i(x), g_i(x), f^i(x)$ and $g^i(x)$. $f_0(x)$ and $g_0(x)$ are the arbitrary functions on x . So we obtain the numerical positon solution of the complex KdV equation in a finite series as

$$\begin{aligned}
 u(x, t) &= u_0 + u_1 + u_2 + u_3 + \dots \\
 &= G(x)/F(x) + (f^1(x) + f^2(x) + f^4(x))t + \frac{1}{2}(f^3(x) + f^5(x))t^2 + \frac{1}{3}f^6(x)t^3 + \dots \\
 v(x, t) &= v_0 + v_1 + v_2 + v_3 + \dots \\
 &= H(x)/F(x) + (g^1(x) + g^2(x) + g^4(x))t + \frac{1}{2}(g^3(x) + g^5(x))t^2 + \frac{1}{3}g^6(x)t^3 + \dots
 \end{aligned} \tag{16}$$

where $G(x)/F(x), H(x)/F(x)$ are the initial conditions (9), f^i and g^i ($i = 0, 6$) are defined in (15).

As we know the exact solution of the complex KdV equation given by Hu et al. [29] through the Darboux transformation is

$$\begin{aligned}
 u &= \frac{2m^2[(a^2 - b^2)(a^2 \cos^2 \zeta'_1 - b^2 \cos^2 \zeta'_2) + 4a^2 b^2 \cos(\delta_1 - \delta_2) \cos \zeta'_1 \cos \zeta'_2]}{(a^2 \cos^2 \zeta'_1 + b^2 \cos^2 \zeta'_2)^2}, \\
 v &= \frac{4abm^2[(a^2 - b^2)(a^2 \cos^2 \zeta'_1 - b^2 \cos^2 \zeta'_2) \cos(\delta_1 - \delta_2) - (a^2 - b^2) \cos^2 \zeta'_1 \cos^2 \zeta'_2]}{(a^2 \cos^2 \zeta'_1 + b^2 \cos^2 \zeta'_2)^2},
 \end{aligned} \tag{17}$$

where

$$\zeta'_1 = mx + 4m^3 t + \delta_1, \quad \zeta'_2 = mx + 4m^3 t + \delta_2.$$

Remark 3. It is obvious that the numerical positon solution (16) and the exact solution (17) are nonsingular for $v \neq 0$. When the constants δ_1 and δ_2 satisfy $\delta_2 = \delta_1 + n\pi$ ($n = 0, \pm 1, \pm 2, \dots$), we know the two kinds of solutions are singular. This case coincides with the fact that the positon of the real KdV equation is singular.

In order to guarantee the high accuracy of the numerical positon solution by the proposed method, we depict the numerical simulation figures. Here we choose $f_0 = G(x)/F(x)$, $g_0 = H(x)/F(x)$ and $f_1 = g_1 = f_2 = g_2 = \dots = 0$.

Fig. 1 stands for the numerical positon solution (16) by using the N-term approximation and Fig. 2 shows the exact one (17) at time $t = 0$ with the parameters selected as

$$a = 2, \quad b = 4, \quad \delta_1 = 0, \quad \delta_2 = 1 \quad \text{and} \quad m = 2. \tag{18}$$

The comparison between the two different kinds of solutions can be seen in Fig. 3, from which we know the approximate numerical solution (16) can rapidly converge to the exact solution (17). That is to say a good result is achieved.

3.2. Numerical negaton solution

We choose the initial condition in the hyperbolic functions form:

$$\begin{cases} u(x, 0) = G(x)/F(x), \\ v(x, 0) = H(x)/F(x), \end{cases} \tag{19}$$

to construct the numerical negaton solution for Eq. (2). Here

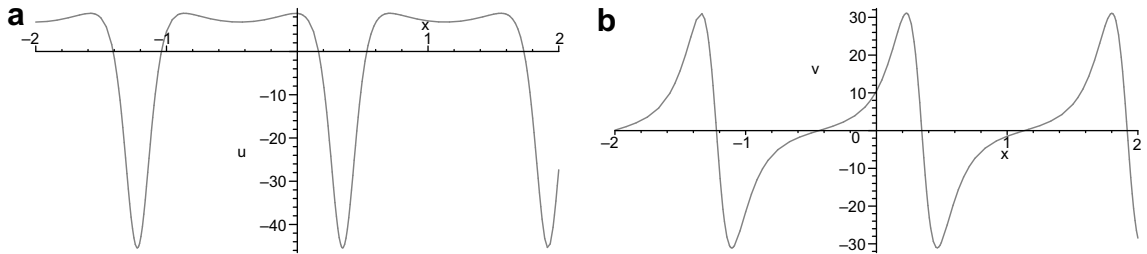


Fig. 1. Numerical positon solution (16) for the complex KdV equation: (a) $u(x,t)$, (b) $v(x,t)$ with the parameters selected in (18) at $t = 0$.

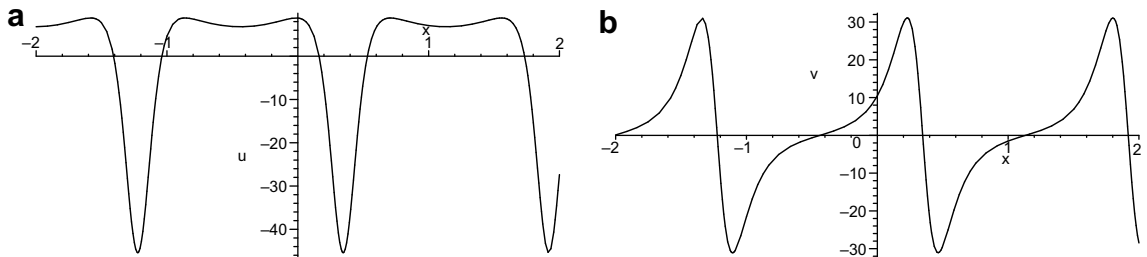


Fig. 2. Exact positon solution (17) for the complex KdV system: (a) $u(x,t)$, (b) $v(x,t)$ with the constants chosen by (18) at $t = 0$.

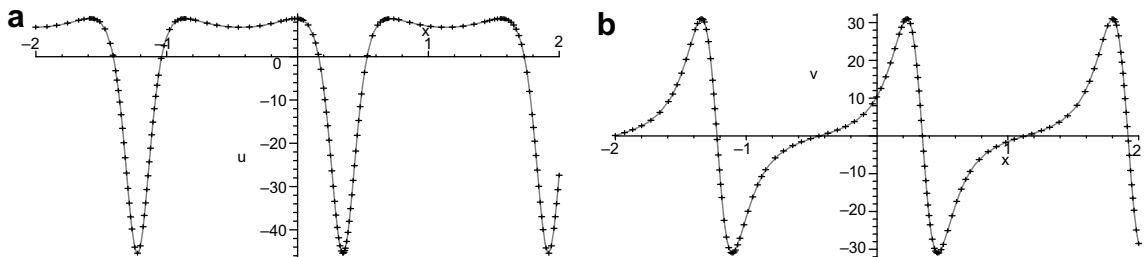


Fig. 3. The comparison between numerical positon solution (16) and exact one (17) at $t = 0$: (a) $u(x,t)$, (b) $v(x,t)$. Line stands for the figures of the numerical solution and point for the exact solution.

$$\begin{aligned}
 F(x) &= (a^2 \sinh^2 \eta_1 + b^2 \sinh^2 \eta_2)^2, \quad \eta_1 = mx + \delta_1, \quad \eta_2 = mx + \delta_2, \\
 G(x) &= 2m^2 [(a^2 - b^2)(a^2 \sinh^2 \eta_1 - b^2 \sinh^2 \eta_2) + 4a^2 b^2 \cosh(\delta_1 - \delta_2) \sinh \eta_1 \sinh \eta_2], \\
 H(x) &= 2abm^2 (a^2 \sinh 2\eta_1 + b^2 \sinh 2\eta_2) \sinh(\delta_1 - \delta_2).
 \end{aligned}
 \tag{20}$$

The procedure is just similar to the above: substituting the series solutions (12) into the homotopy expression (11), collecting the equations by p , setting the coefficients to zero, solving these differential equations and then getting the solutions. Here we just give the final numerical negaton results:

$$\begin{aligned}
 u(x, t) &= u_0 + u_1 + u_2 + u_3 + \dots \\
 &= G(x)/F(x) + (f^1(x) + f^2(x) + f^4(x))t + \frac{1}{2}(f^3(x) + f^5(x))t^2 + \frac{1}{3}f^6(x)t^3 + \dots \\
 v(x, t) &= v_0 + v_1 + v_2 + v_3 + \dots \\
 &= H(x)/F(x) + (g^1(x) + g^2(x) + g^4(x))t + \frac{1}{2}(g^3(x) + g^5(x))t^2 + \frac{1}{3}g^6(x)t^3 + \dots
 \end{aligned}
 \tag{21}$$

where $F(x)$, $G(x)$ and $H(x)$ are defined in (20), f^i and g^i ($i = 0, 6$) are defined in (15). The exact solution in the form of hyperbolic functions given in Ref. [25] is

$$\begin{aligned}
 u &= \frac{2m^2 [(a^2 - b^2)(a^2 \sinh^2 \eta'_1 - b^2 \sinh^2 \eta'_2) + 4a^2 b^2 \cosh(\delta_1 - \delta_2) \sinh \eta'_1 \sinh \eta'_2]}{(a^2 \sinh^2 \eta'_1 + b^2 \sinh^2 \eta'_2)^2}, \\
 v &= \frac{2abm^2 (a^2 \sinh 2\eta'_1 + b^2 \sinh 2\eta'_2) \sinh(\delta_1 - \delta_2)}{(a^2 \sinh^2 \eta'_1 + b^2 \sinh^2 \eta'_2)^2},
 \end{aligned}
 \tag{22}$$

where

$$\eta'_1 = mx - 4m^3 t + \delta_1, \quad \eta'_2 = mx - 4m^3 t + \delta_2.$$

Analysis the two kinds of negaton solutions (21) and (22) for the complex KdV system, we find that when $\delta_2 = \delta_1 + n\pi l$ ($n = 0, \pm 1, \pm 2, \dots$) these solutions are singular. That is to say, under this condition, the complex KdV equation reduces to the real KdV equation. In other cases, the two kinds of solutions are nonsingular.

Figs. 4 and 5 show the numerical negaton solution (21) and the exact solution (22) at $t = 0$ with the parameters chosen as

$$a = 1, \quad b = 5, \quad \delta_1 = 0, \quad \delta_2 = 2 \quad \text{and} \quad m = 2.
 \tag{23}$$

Fig. 6 shows that there is almost no difference between the two solutions, which predicts a very high level of accuracy is achieved by extended HPM.

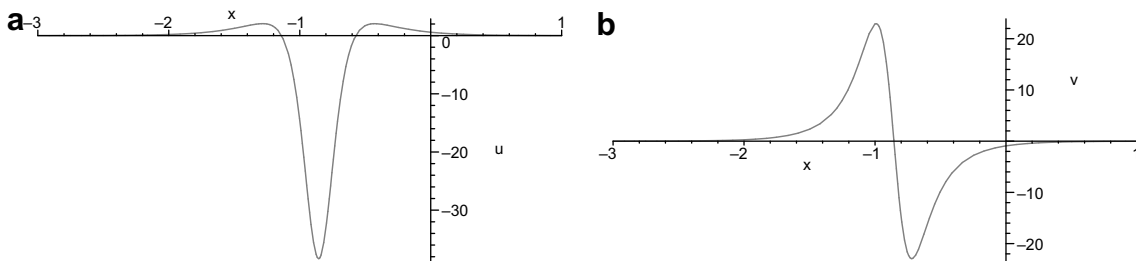


Fig. 4. Numerical negaton solution (21) with the initial value (19): (a) $u(x, t)$, (b) $v(x, t)$ under the case (23) at $t = 0$ for the complex KdV model.

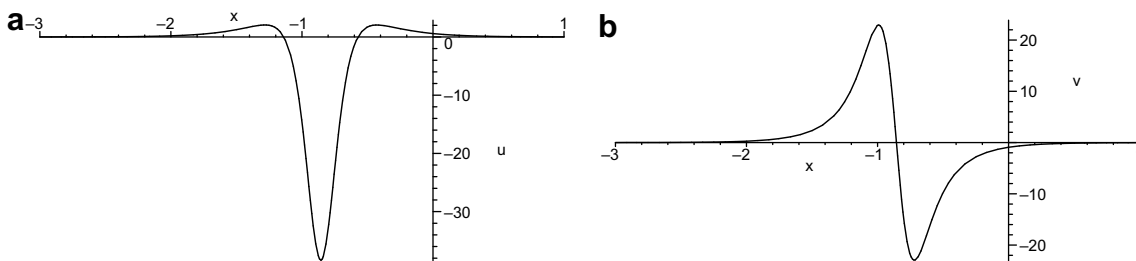


Fig. 5. Exact negaton solution (22): (a) for $u(x, t)$, (b) for $v(x, t)$ with the same constants as in Fig. 4.

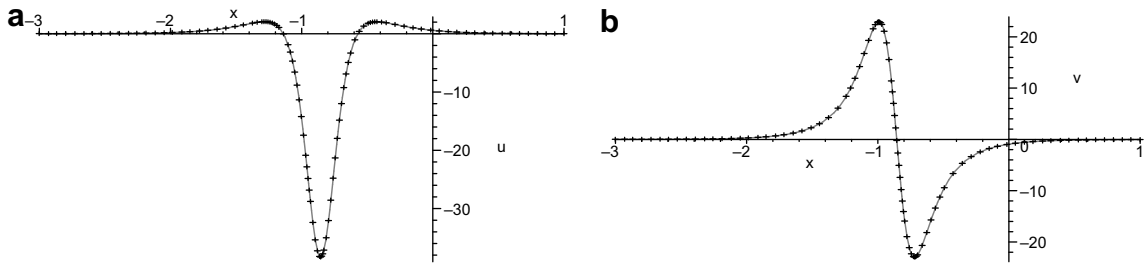


Fig. 6. The comparison between numerical and exact solution (a) $u(x,t)$; (b) $v(x,t)$. Line show the figures for the numerical solution and point for the exact.

3.3. Numerical analytical complexiton solution

In this section, we would like to construct the numerical complexiton solution for the complex KdV equation. In order to obtain the complexiton solution, we choose the initial value in the form of two different wave functions: the hyperbolic functions and trigonometric functions. We take the initial condition as (see Figs. 7–9):

$$u(x, 0) = -[\ln(F^2 + G^2)]_{xx}, \quad v(x, 0) = -2 \left[\arctan \frac{G}{F} \right]_{xx}, \quad (24)$$

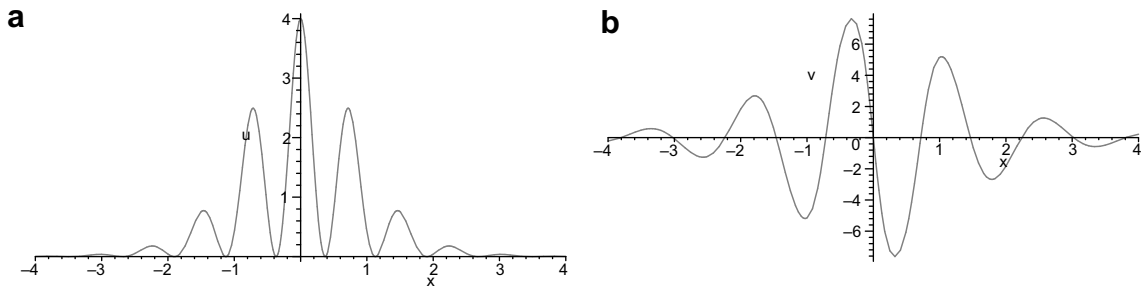


Fig. 7. Numerical analytical complexiton solution (30) with the initial value (26): (a) for $u(x,t)$ and (b) for $v(x,t)$ at $t = 0$.

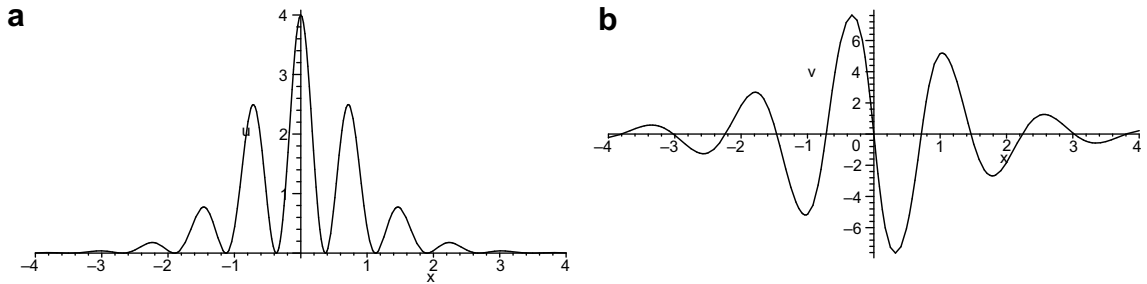


Fig. 8. Exact analytical solution given by (28) (a) for $u(x,t)$ and (b) for $v(x,t)$ at $t = 0$.

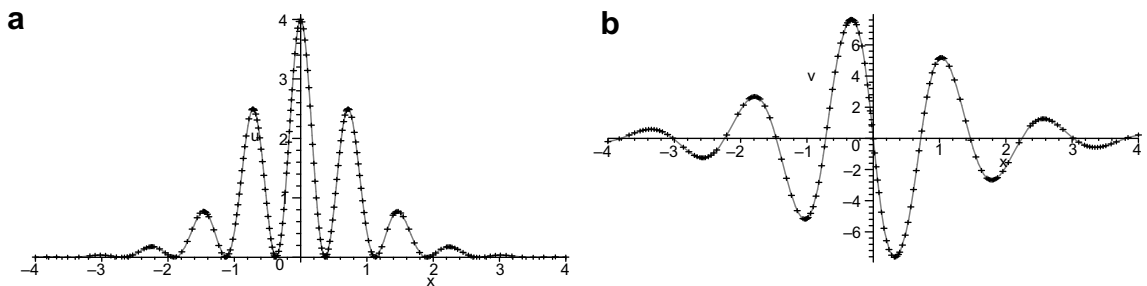


Fig. 9. The comparison between the two different solutions (28) and (30): (a) $u(x,t)$ and (b) $v(x,t)$. Line depicts the figures for the numerical solution and point for the exact.

where

$$\begin{aligned} F &= 4\lambda_1(c_4C_3 - c_2C_1) \cosh 2\lambda_2x + 4\lambda_2(c_1C_1 - c_3C_3) \sin 2\lambda_1x, \\ G &= 4\lambda_1(c_2C_3 + c_4C_1) \cosh 2\lambda_2x - 4\lambda_2(c_3C_1 + c_1C_3) \sin 2\lambda_1x. \end{aligned} \quad (25)$$

For the convenience of seeing a clear structure of the complexiton, we set these parameters as $(c_1, c_2, c_3, c_4, C_1, C_2, C_3, C_4) = (5, 0, 0, 5, -1, 2, 2, 1)$ and $(\lambda_1, \lambda_2) = (2, 0.5)$. Substituting F, G and these parameters into (24), we can obtain the following special initial condition:

$$\begin{cases} u(x, 0) = A(x)/B(x), \\ v(x, 0) = C(x)/B(x), \end{cases} \quad (26)$$

where

$$\begin{aligned} A(x) &= -[272 \cos 8x + 240 \cos 8x \cosh 2x + 272 \cosh 2x - 128 \sin 8x \sinh 2x + 240], \\ B(x) &= \frac{835}{8} - \frac{17}{2} \cos 8x + \frac{1}{8} \cos 16x + 136 \cosh 2x - 8 \cos 8x \cosh 2x + 32 \cosh 4x, \\ C(x) &= -[1862 \sin 4x \cosh x + 30 \cosh x \sin 12x + 240 \cos 4x \sinh x \\ &\quad + 16 \sinh x \cos 12x + 480 \sin 4x \cosh 3x + 256 \cos 4x \sinh 3x]. \end{aligned} \quad (27)$$

One of the special case for the exact solutions in Ref. [25] is given by

$$\begin{cases} u = A(x, t)/B(x, t), \\ v = C(x, t)/B(x, t), \end{cases} \quad (28)$$

where

$$\begin{aligned} A(x, t) &= -[272 \cos(8x + 104t) + 240 \cos(8x + 104t) \cosh(2x + 94t) \\ &\quad + 272 \cosh(2x + 94t) - 128 \sin(8x + 104t) \sinh(2x + 94t) + 240], \\ B(x, t) &= \frac{835}{8} - \frac{17}{2} \cos(8x + 104t) + \frac{1}{8} \cos(16x + 208t) + 136 \cosh(2x + 94t) \\ &\quad - 8 \cos(8x + 104t) \cosh(2x + 94t) + 32 \cosh(4x + 188t), \\ C(x, t) &= -[1862 \sin(4x + 52t) \cosh(x + 47t) + 30 \cosh(x + 47t) \sin(12x + 156t) \\ &\quad + 240 \cos(4x + 52t) \sinh(x + 47t) + 16 \sinh(x + 47t) \cos(12x + 156t) \\ &\quad + 480 \sin(4x + 52t) \cosh(3x + 141t) + 256 \cos(4x + 52t) \sinh(3x + 141t)]. \end{aligned} \quad (29)$$

Repeating the above procedure and with the aid of *Maple*, we can obtain the final numerical results:

$$\begin{aligned} u(x, t) &= u_0 + u_1 + u_2 + u_3 + \dots \\ &= A(x)/B(x) + (f^1(x) + f^2(x) + f^4(x))t + \frac{1}{2}(f^3(x) + f^5(x))t^2 + \frac{1}{3}f^6(x)t^3 + \dots \\ v(x, t) &= v_0 + v_1 + v_2 + v_3 + \dots \\ &= C(x)/B(x) + (g^1(x) + g^2(x) + g^4(x))t + \frac{1}{2}(g^3(x) + g^5(x))t^2 + \frac{1}{3}g^6(x)t^3 + \dots \end{aligned} \quad (30)$$

where $A(x), B(x), C(x)$ and $f^i, g^i (i = 1, \dots, 6)$ are expressed by (27) and (15), respectively.

Remark 4. For many integrable systems, the known complexiton solutions generally have singularities [26]. However, by analysis (30) we know that the numerical complexiton solution is analytical and has no singularities. It is the first time to find the this type of numerical complexiton solution for the integrable system by the HPM up to now.

Remark 5. When we construct suitable forms of initial conditions, other types of numerical solutions such as numerical negaton interaction solution, positon–negaton interaction solution, Jacobi elliptic function solution, soliton, rational solution, etc. can also be obtained.

4. Conclusions

In this paper, we extend the HPM from the single equation to the coupled system to derive the numerical complexiton solution for the complex KdV equation. By choosing the initial conditions as the forms of trigonometric functions, hyperbolic functions and the combination of the two, three new types of numerical solutions: numerical positon solution, numerical negaton solution and particularly numerical analytical complexiton solution are obtained, which can rapidly converge to the exact solutions obtained by Hu et al. [29]. So, we predict the extended HPM is a very powerful tool to solve nonlinear coupled problems, especially for the complex KdV equation. Whether we can introduce other new algorithms to solve the system and whether we can adopt other techniques to accelerate the convergent speed, these questions will be further studied. The complex KdV model can be considered as a special case of the general coupled KdV system derived from the two

layer fluid dynamical systems and the two-component Bose–Einstein condensates [32–34], which are very useful in dynamics and physics. So the numerical solutions obtained in this paper are useful in these fields.

Acknowledgements

The author would like to thank the anonymous referees very much for their careful reading of the manuscript and many valuable suggestions. The work is supported by the National Natural Science Foundation of China (Grant No. 10735030), Shanghai Leading Academic Discipline Project (No. B412), Program for Changjiang Scholars and Innovative Research Team in University (IRT0734), Zhejiang Provincial Natural Science Foundations of China (Grant No. Y604056) and Doctoral Foundation of Ningbo City (No. 2005A61030).

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