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A function cascade synchronization method with unknown parameters and applications*

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This paper investigates the function cascade synchronization of chaos system. Combining cascade synchronization scheme, parametric adaptive control and projective synchronization scheme, it proposes a new function cascade synchronization scheme to address a generalized-type synchronization problem of three famous chaotic systems: the Lorenz system, Liu system and Rössler system, the states of two identical chaotic systems with unknown parameters can be asymptotically synchronized by choosing different special suitable error functions. Numerical simulations are used to verify the effectiveness of the proposed synchronization techniques.

Keywords: chaos, projective synchronization, function projective synchronization, cascade synchronization, adaptive control

PACC: 0545, 0200, 0290

1. Introduction

Since Pecora and Carroll[1] introduced a method to synchronize two identical chaotic systems with different initial conditions, which is realized in electronic circuits, chaos control and synchronization has played an important role in nonlinear science due to its many great potential applications in secure communication, chemical reactor, control theory, telecommunications, system identification, artificial neural networks, *et al* (Refs.[2–4] and the references therein). Especially recent decades of years, the synchronization problem in chaotic systems is extensively and intensively studied. Synchronization techniques have been improved in recent years, and many different methods are applied theoretically and experimentally to synchronize the chaotic systems.[5−12] To our knowledge, there exists many kinds of chaos synchronization in dynamical systems, such as complete synchronization, partial synchronization, phase synchronization, anticipated synchronization and lag synchronization, *et al.*[1−24] Accordingly there are many powerful methods applied to synchronize two identical or different chaotic systems.[13−24] The active control theory,[25−28] as a powerful method, has been developed and applied to synchronize both identical chaotic systems and two differential chaotic systems. In particular, various active and nonlinear control methods were used for chaos synchronization of two identical systems. In 1993, Carroll and Pecora[29] demonstrated the cascading of synchronized chaotic systems, that one chaotic system may be synchronized with another by sending a signal from one to the other. Recently, projective synchronization scheme and modified scheme are studied,[24,30−34] that the drive and response systems may synchronize up to a scaling factor but Laypunov exponents and fractal dimensions remain unchanged. Parametric adaptive control for chaos synchronization has been proposed in Refs.[35–39].

Based on above works[30−39] and combined cascade synchronization scheme, parametric adaptive control and projective synchronization scheme, to make the states of two identical chaotic systems asymptotically synchronization by choosing different special suitable error functions, this paper proposes a new function cascade synchronization scheme to address a generalized-type synchronization problem of three famous chaotic systems: the first chaotic attractor model of the Lorenz system[40] was presented by Lorenz in 1963; the Rösler system[41] was found by Rössler in 1976; the Liu system[42] was proposed by Liu *et al* in 2004. Numerical simulations are used to verify the effectiveness of the proposed synchroniza-

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tion techniques.

It is organized as follows. In Section 2, the function cascade synchronization method with unknown parameters for the function projective synchronization is introduced. In Section 3, some numerical simulation examples are given to verify the effectiveness of our method. In Section 4, we give the conclusion of this paper.

2. The function cascade synchronization method with unknown parameters

Firstly, the function projective synchronization[31] is illustrated like this. Consider the following chaotic system:

$$\dot{U} = f(U), \qquad \dot{V} = g(V, \xi(U, V)), \tag{1}$$

where $U = (u_1(t), u_2(t), \cdots u_n(t))^{\mathrm{T}}$, $V = (v_1(t), v_2(t), \cdots v_n(t))^{\mathrm{T}}$ and $\xi(U, V) = (\xi_1(U, V), \xi_2(U, V), \cdots \xi_n(U, V))^{\mathrm{T}}$, which is the controller to be determined later and satisfies $\xi(0, 0) = 0$, $g(V, \xi(0, 0)) = f(V)$. The letters U and V stand for the drive (or master) and response (or slave) systems respectively. If there exist functions $Q(U) = (Q_1(u_1(t)), Q_2(u_2(t)), \cdots Q_n(u_n(t)))^{\mathrm{T}}$, $e = (e_1, e_2, \cdots e_n)^{\mathrm{T}}$ satisfying $e = V - Q(U)U^{\mathrm{T}}$, $\lim\limits_{t\to\infty} ||e|| = 0$, then

$$\lim_{t\to\infty} ||V - Q(U)U^{\mathrm{T}})|| = 0.$$

As for Q, we choose special function, so we term this method 'the function projective synchronization', and call Q a 'scaling function factor'.

Remark 1 When $Q_1 = Q_2 = \cdots = Q_n = 1$, $Q_1 = Q_2 = \cdots = Q_n = \alpha$ and $Q_1 = \alpha_1$, $Q_2 = \alpha_2$, $\cdots$, $Q_n = \alpha_n$, complete synchronization, projective synchronization and modified projective synchronization will appear, respectively.

The 'cascade synchronization method' is expressed as following.[29] Consider the following chaotic dynamical system:

$$\begin{aligned} \dot{x} &= f(x, y, z), \\ \dot{y} &= g(x, y, z), \\ \dot{z} &= h(x, y, z). \end{aligned} \tag{2}$$

Firstly, copy the first two equations of (2), then we get a sub-response system

$$\begin{aligned} \dot{X} &= f(X, q, z), \\ \dot{q} &= g(X, q, z), \end{aligned} \tag{3}$$

where variable z is corresponding signal afforded by the original system. If the conditional Lyapunov exponents of system (3) are negative, we can predict that the synchronization is achieved between the first two equations of Eqs.(2) and (3), so we consider system (3) as a steady response one. Secondly, copy another subsystem as a response system:

$$\begin{aligned} \dot{Y} &= g(X, Y, Z) \\ \dot{Z} &= h(X, Y, Z), \end{aligned} \tag{4}$$

where X is a driver variable corresponding to Eq.(3). If all the Lyapunov exponents of system (4) are negative, then the synchronization is achieved.

Based on the above idea, combined cascade synchronization scheme, parametric adaptive control and projective synchronization scheme, we present the function cascade synchronization of system (2). In the following we will illustrate the method in detail for the systems with uncertain parameters. The drive system and response ones are described as

$$\begin{aligned} \dot{x} &= f(A, x, y, z), \\ \dot{y} &= g(A, x, y, z), \\ \dot{z} &= h(A, x, y, z), \end{aligned} \tag{5}$$

where A is the set of the constant parameters in the drive system. The response system is as following:

$$\begin{aligned} \dot{X} &= f(A^*, X, q, z) + \xi_1, \\ \dot{q} &= g(A^*, X, q, z) + \xi_2. \end{aligned} \tag{6a}$$

And the another copied response system is:

$$\begin{aligned} \dot{Y} &= g(A^*, X, Y, Z) + \xi_3, \\ \dot{Z} &= h(A^*, X, Y, Z) + \xi_4, \end{aligned} \tag{6b}$$

where A^* is the set of the estimate values of the constant parameters, $(\xi_1, \xi_2)^{\mathrm{T}}$ and $(\xi_3, \xi_4)^{\mathrm{T}}$ are desired controllers that make the system (5) synchronize with Eq.(6a) as well as Eq.(6b) with a desired scaling function factor $Q(x)$, i.e., $\lim\limits_{t\to\infty} ||X - xQ_1(x)|| = 0$, $\lim\limits_{t\to\infty} ||q - yQ_2(y)|| = 0$, $\lim\limits_{t\to\infty} ||Y - yQ_3(y)|| = 0$, $\lim\limits_{t\to\infty} ||Z - zQ_4(z)|| = 0$. Therefore we can get

$$\begin{aligned} &\lim_{t\to\infty} ||X - xQ_1(x)|| = 0, \\ &\lim_{t\to\infty} ||Y - yQ_3(y)|| = 0, \\ &\lim_{t\to\infty} ||Z - zQ_4(z)|| = 0. \end{aligned} \tag{6c}$$

Remark 2 The function cascade synchronization method with unknown parameters can be used to study the synchronization of two identical chaotic systems, which the parameters of two chaotic systems are unknown and time varying.

3. Applications of the function cascade synchronization method with unknown parameters

In the following, we will apply the function cascade synchronization method to three typical chaotic systems with uncertain parameters respectively.

3.1. The Lorenz system

First, the Lorenz system is written as this:

$$\begin{aligned} \dot{x} &= \sigma(y-x), \\ \dot{y} &= \delta x - xz - y, \\ \dot{z} &= xy - bz, \end{aligned} \tag{7}$$

where x, y and z are state variables and σ, δ and b are the control parameters of the Lorenz system. When $\sigma = 10$, $\delta = 28$ and $b = \frac{8}{3}$, the Lorenz system (7) has a chaotic attractor. Let us assume that the response system is given by

$$\begin{aligned} \dot{X} &= g(t)(q-X) + \xi_1, \\ \dot{q} &= d(t)X - Xz - q + \xi_2, \end{aligned} \tag{8}$$

where $(\xi_1, \xi_2)^{\mathrm{T}}$ is the control function that is to be designed below and $g(t)$, $d(t)$ are the estimate values of the parameters σ, δ respectively.

Then we define the Lyapunov function of systems (7) and (8) as:

$$V_1(e_1, e_2, \tilde{g}, \tilde{d}) = \frac{1}{2}\left(e_1^2 + e_2^2 + \tilde{g}^2 + \tilde{d}^2\right), \tag{9}$$

where $e_1 = X - \alpha_1 x$, $e_2 = q - y\tanh y$, $\tilde{g} = g(t) - \sigma$ and $\tilde{d} = d(t) - \delta$. The goal of the control is to find a controller $(\xi_1, \xi_2)^{\mathrm{T}}$ such that the states of the Lorenz drive system (7) and the states of the response system (8) are globally asymptotically synchronized by function cascade method.

If the Lyapunov function (9) satisfies the conditions

$$\begin{aligned} V_1 > 0 \ \text{ if } \ (e_1, e_2, \tilde{g}, \tilde{d}) \neq (0,0,0,0). \\ V_1 = 0 \ \text{ if } \ (e_1, e_2, \tilde{g}, \tilde{d}) = (0,0,0,0), \end{aligned}$$

and

$$\begin{aligned} \dot{V}_1 > 0 \ \text{ if } \ (e_1, e_2, \tilde{g}, \tilde{d}) \neq (0,0,0,0), \\ \dot{V}_1 = 0 \ \text{ if } \ (e_1, e_2, \tilde{g}, \tilde{d}) = (0,0,0,0). \end{aligned}$$

Then $e_i (i = 1, 2)$ will asymptotically tend to zero, i.e. $\lim_{t\to\infty} ||X - \alpha_1 x|| = 0$ and $\lim_{t\to\infty} ||q - y\tanh y|| = 0$.

With the aid of the Maple, we get the time derivative of V_1 as

$$\begin{aligned} \dot{V}_1(e_1, e_2, \tilde{g}, \tilde{d}) = & -g(t)e_1^2 + e_1(g(t)q + \sigma\alpha_1 x - \sigma\alpha_1 y - \alpha_1 x g(t) + \xi_1) + e_2(d(t)e_1 \\ & -ze_1 - \delta xy + y^2 - q + xyz + \alpha_1 x d(t) - \alpha_1 xz + y\tanh y + xz\tanh y \\ & +\delta xy\tanh^2 y - xyz\tanh^2 y - y^2\tanh^2 y - \delta x\tanh y + \xi_2) + \dot{g}(t)\tilde{g} + \dot{d}(t)\tilde{d}. \end{aligned}$$

We choose the controller $(\xi_1, \xi_2)^{\mathrm{T}}$ as the follows

$$\begin{aligned} &\xi_1 = (\sigma - 1)e_1 + \sigma(\alpha_1 y - q). \\ &\xi_2 = -e_2 + e_1(z - \delta) + \tanh y(\delta x - y - xz)(1 - y\tanh y) + x(\delta - z)(y - \alpha_1) + q - y^2, \end{aligned}$$

and the parameters estimation update law $\tilde{g}$ and $\tilde{d}$ as

$$\begin{aligned} \dot{g}(t) &= e_1(e_1 - q + \alpha_1 x), \\ \dot{d}(t) &= -e_2(e_1 + \alpha_1 x). \end{aligned}$$

With this choice, the time derivative of $V_1(e_1, e_2, \tilde{g}, \tilde{d})$ is given by

$$\dot{V}_1(e_1, e_2, \tilde{g}, \tilde{d}) = -(e_1^2 + e_2^2).$$

This leads to $\lim_{t\to\infty} ||e_1|| = 0$ and $\lim_{t\to\infty} ||e_2|| = 0$. Therefore the synchronization is achieved between system

(7) and system (8).

Now we copy system (7) and get the response system like:

$$\begin{aligned}\dot{Y} &= \delta X - XZ - Y + \xi_3,\\ \dot{Z} &= XY - B(t)Z + \xi_4,\end{aligned} \tag{10}$$

where ξ_3, ξ_4 are the controllers and $B(t)$ is the estimative value of the unknown parameter b.

We take the Lyapunov function as

$$V_2(e_3, e_4, \tilde{B}) = \frac{1}{2}(e_3^2 + e_3^2 + \tilde{B}^2),$$

where $e_3 = Y - y\tanh y$, $e_4 = Z - \alpha_2 z$ and $\tilde{B} = B(t) - b$. The calculation is the same as before, now we omit the details and just give the results

$$\begin{aligned}\xi_3 &= X(\alpha_2 z - \delta) + y^2(\tanh^2 y - 1)\\ &\quad + x(\delta - z)(y + \tanh y - y\tanh^2 y),\\ \xi_4 &= (b-1)e_4 + \alpha_2 xy - Xy\tanh y,\\ \dot{B}(t) &= e_4^2 + \alpha_2 z e_4.\end{aligned}$$

So we get

$$\dot{V}_2(e_3, e_4, \tilde{B}) = -(e_3^2 + e_4^2).$$

In this way we can predict that the function cascade synchronization is achieved for the Lorenz system. Figure 1 shows the function cascade synchronization for the Lorenz system with uncertain parameters, where $\sigma = 10$, $\delta = 28$, $b = \frac{8}{3}$, $\alpha_1 = -3.5$, $\alpha_2 = -2$. Figure 2(a–c) shows the numerical simulations of the error functions e_x, e_y, e_z. Figure 3(a-c) shows the time evolution of the estimates parameters $g(t)$, $d(t)$, $B(t)$:

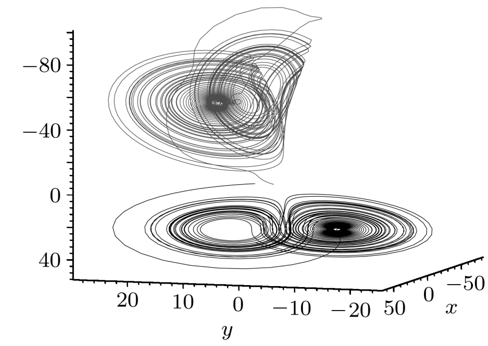

Fig.1. The function cascade synchronization for the parameterized Lorenz system, where $(\sigma, \delta, b) = (10, 28, \frac{8}{3})$, the initial values of (x, y, z) and (X, Y, Z) are $(2, 5, -1)$ and $(-1, 2, -0.5)$. The above figure describes the response system. In comparison with the drive one, the two halves obviously fold toward the same side.

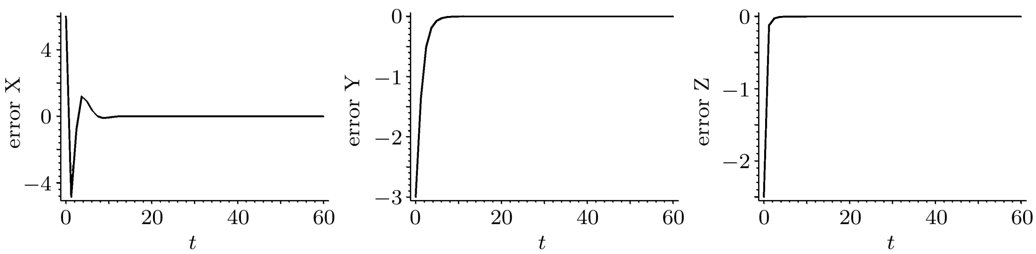

Fig.2. e_x, e_y, e_z.

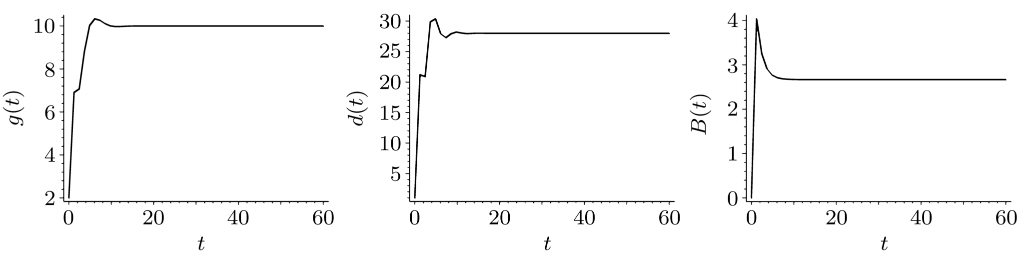

Fig.3. $g(t)$, $d(t)$, $B(t)$.

3.2. The Liu system

The drive system is

$$\begin{aligned}\dot{x} &= a(y - x),\\ \dot{y} &= bx - kxz,\\ \dot{z} &= -cz + hx^2,\end{aligned} \tag{11}$$

and its response system is as follows

$$\begin{aligned}\dot{X} &= A(t)(q - X) + \xi_1,\\ \dot{q} &= bX - kXz + \xi_2,\end{aligned} \tag{12}$$

where $A(t)$ is the estimative value of the parameter a. The step is just like the above, we choose the Lyapunov function as

$$V_1(e_1, e_2, \tilde{A}) = \frac{1}{2}(e_1^2 + e_2^2 + \tilde{A}^2),$$

where $e_1 = X - x^2$,$e_2 = q - \alpha_1 y$ and $\tilde{A} = A(t) - a$. When choosing

$$\begin{aligned} \xi_1 &= (a-1)e_1 - a(q - 2xy + x^2),, \\ \xi_2 &= -e_2 + (kz - b)(e_1 - \alpha_1 x + x^2), \\ \dot{A}(t) &= e_1(e_1 - q + x^2), \end{aligned}$$

we can get the result

$$\dot{V}_1(e_1, e_2, \tilde{A}) = -(e_1^2 + e_2^2).$$

That is to say we realize the synchronization between the systems (11) and (12). Next take system (12) as the drive system, copy the system (11) and we get another response system:

$$\begin{aligned} \dot{Y} &= bX - kXZ + \xi_3 \\ \dot{Z} &= -cZ + hX^2 + \xi_4. \end{aligned} \tag{13}$$

In the same way, we take the Lyapunov function as

$$V_2(e_3, e_4, \tilde{B}) = \frac{1}{2}(e_3^2 + e_4^2 + \tilde{B}^2),$$

where $B(t)$ is the estimate value of the parameter b, $\tilde{B} = B(t) - b$, $e_3 = Y - y^2$ and $e_4 = Z - \alpha_2 z$. Taking

$$\begin{aligned} \xi_3 &= -e_3 + X(kZ - b) - 2xy(kz - b), \\ \xi_4 &= -e_4 + c(Z - \alpha_2 z) - h(X^2 - \alpha_2 x^2), \\ \dot{B}(t) &= -Xe_3, \end{aligned}$$

we will have the results: $\lim_{t\to\infty} ||e_3|| = 0$, $\lim_{t\to\infty} ||e_4|| = 0$, and

$$\dot{V}_2(e_3, e_4, \tilde{B}) = -(e_3^2 + e_4^2).$$

So we predict that the function cascade synchronization is achieved.

Figure 4 shows the function cascade synchronization for the Liu system with uncertain parameters, Fig.5(a–c) shows the behavior of the error functions e_x, e_y and e_z and Fig.6(a–b) shows the time evolution of the estimates of parameters $A(t)$, $B(t)$:

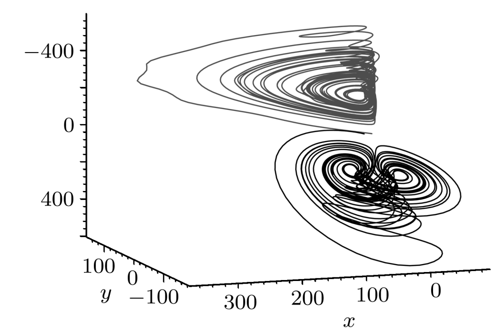

Fig.4. The function cascade synchronization for the Liu system, where $(\alpha_1\alpha_2) = (2, -5)$, $(a, b, c, h, k) = (10, 40, 2.5, 4, 1)$ the initial values of (x, y, z) and (X, Y, Z) are $(2, 3, 2)$ and $(5, 1, -2)$. The above figure depicts the response system. Because the special error functions, the appearance of the response system is very strange, in fact, the two halves fold together.

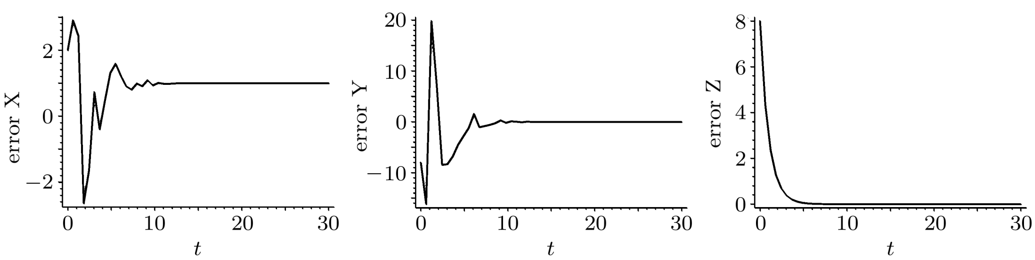

Fig.5. e_x, e_y, e_z.

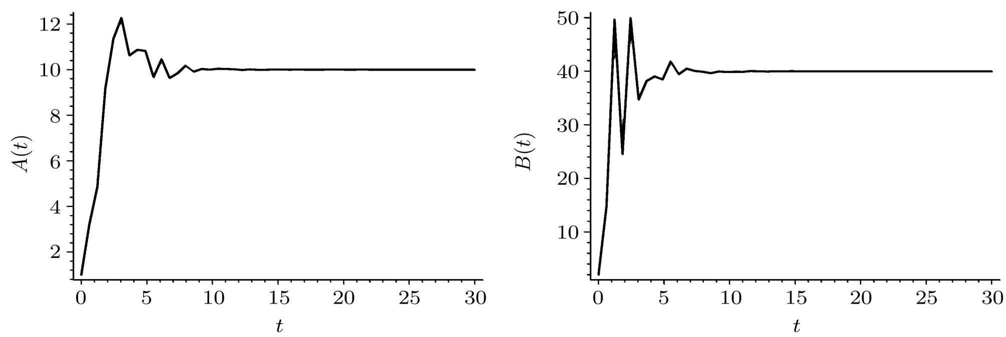

Fig.6. $A(t)$, $B(t)$.

4. The Rössler system

The drive system is

$$\begin{aligned}\dot{x} &= -(y+z),\\ \dot{y} &= x + ay,\\ \dot{z} &= z(x-c)+b,\end{aligned}$$

and the response system is

$$\begin{aligned}\dot{X} &= -(q+z)+\xi_1,\\ \dot{q} &= X + A(t)q + \xi_2,\end{aligned}$$

where $A(t)$ is parameterized by a.

Similarly, we take the Lyapunov functions as:

$$V_1(e_1, e_2, \tilde{A}) = \frac{1}{2}(e_1^2 + e_2^2 + \tilde{A}^2),$$

where $e_1 = X - \alpha_1 x$, $e_2 = q - y\tanh y$ and $\tilde{A} = A(t) - a$. When

$$\begin{aligned}\xi_1 &= q - e_1 + z - \alpha_1 y - \alpha_1 z,\\ \xi_2 &= -X - e_2 + xy + a(y^2 - q\\ &\quad + y\tanh y - y^2\tanh^2 y)\\ &\quad + x\tanh y(1 - y\tanh y),\\ \dot{A}(t) &= -qe_2,\end{aligned}$$

we can get $\lim\limits_{t\to\infty}||e_1|| = 0$ and $\lim\limits_{t\to\infty}||e_2|| = 0$. That's to say the above system is globally asymptotically synchronized.

Next we copy another response system

$$\begin{aligned}\dot{Y} &= X + aY + \xi_3,\\ \dot{Z} &= Z(X - C(t)) + B(t) + \xi_4.\end{aligned}$$

We define its Lyapunov function as:

$$V_2(e_3, e_4, \tilde{B}, \tilde{C}) = \frac{1}{2}(e_3^2 + e_4^2 + \tilde{B}^2 + \tilde{C}^2),$$

where $e_3 = Y - \alpha_2 y$, $e_4 = Z - z(1+\tanh^2 z)$, $\tilde{B} = B(t) - b$ and $\tilde{C} = C(t) - c$. For simplicity, setting

$$\begin{aligned}\xi_3 &= -e_3 - X - ae_3 + \alpha_2 x,\\ \xi_4 &= e_4(c - 1 - X) + b\tanh^2 z\\ &\quad + z(x - X)(1 + \tanh^2 z)\\ &\quad + 2z\tanh z(1 - \tanh^2 z)(xz - cz + b),\end{aligned}$$

and

$$\begin{aligned}\dot{B}(t) &= -e_4,\\ \dot{C}(t) &= e_4(z + e_4 + z\tanh^2 z),\end{aligned}$$

we get

$$\dot{V_2}(e_3, e_4, \tilde{B}, \tilde{C}) = -(e_3^2 + e_4^2),$$

which means that the synchronization is achieved.

Figure 7 shows the simulation result of the parameterized Rössler system, where $(a, b, c) = (0.2, 0.2, 5.7)$, $(\alpha_1, \alpha_2) = (5, 5)$. Figure 8(a-c) describes the time evolution of the estimates of parameters $A(t)$, $B(t)$ and $C(t)$.

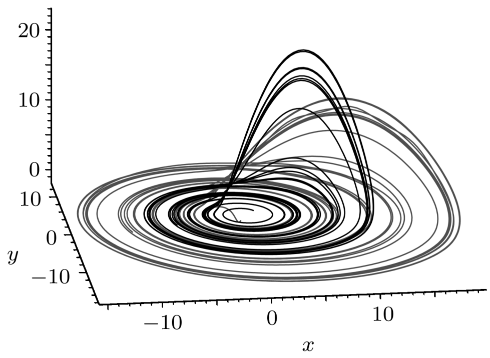

Fig.7. The function cascade synchronization for the Rössler system, $(\alpha_1, \alpha_2) = (5, 5)$, $(a, b, c) = (0.2, 0.2, 5.7)$, the initial values of (x, y, z) and (X, Y, Z) are $(1, 1, 0)$ and $(-1, -1, -2)$. The higher one stands for the response system and the other the drive.

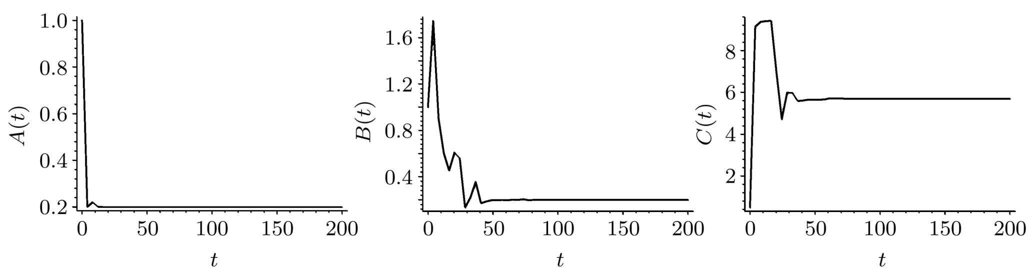

Fig.8. $A(t)$, $B(t)$, $C(t)$.

5. Conclusion

This paper presents a function cascade synchronization scheme and has addressed the function projective synchronization problem of the Lorenz system, the Liu system and the Rössler system with uncertain parameters. Based on Lyapunov stability theory, we can synchronize the states of two identical chaotic systems by choosing different special suitable error functions. In fact, $Q_n(x_n)$ as the error functions has many choices, such as constant α, $\tanh x_n$, x_n^i, *et al* Numeric simulations and interesting plots are used to verify the effectiveness of the proposed synchronization techniques.

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