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Some exact solutions to the inhomogeneous higher-order nonlinear Schrödinger equation by a direct method*

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By symbolic computation and a direct method, this paper presents some exact analytical solutions of the one-dimensional generalized inhomogeneous higher-order nonlinear Schrödinger equation with variable coefficients, which include bright solitons, dark solitons, combined solitary wave solutions, dromions, dispersion-managed solitons, etc. The abundant structure of these solutions are shown by some interesting figures with computer simulation.

Keywords: inhomogeneous high-order nonlinear Schrödinger equation, solitary wave solutions, symbolic computation

PACC: 0340K, 4660D, 5235M

1. Introduction

Optical solitons have become one of the most exciting and extremely active areas of research because of their potential applications in long distance communication. The optical soliton in a dielectric fibre was first proposed by Hasegawa and Tappert,[1] and verified experimentally by Mollenauer *et al.*[2] The well-known nonlinear Schrödinger equation (NLSE) can govern the propagation of picosecond optical solitons in monomode fibres. However, for subpicosecond or femtosecond optical soliton control, the higher order effects influenced by the spatial variations of fibre parameters should be considered. So the problem can be described by the generalized inhomogeneous higher-order NLSE (IHNLSE) with variable coefficients in the form:

$$\begin{aligned}\psi_z =\ & \mathrm{i}(\alpha_1(z)\psi_{tt} + \alpha_2(z)|\psi|^2\psi) + \alpha_3(z)\psi_{ttt} \\ & + \alpha_4(z)(|\psi|^2\psi)_t + \alpha_5(z)\psi(|\psi|^2)_t + \Gamma(z)\psi, \quad (1)\end{aligned}$$

where $\psi(z,t)$ represents the complex envelope of the electrical field, z is the normalized propagation distance, t is the normalized retarded time, and $\alpha_1(z)$, $\alpha_2(z)$, $\alpha_3(z)$, $\alpha_4(z)$, and $\alpha_5(z)$ are the distributed parameters, which are functions of the propagation distance z, related to the group velocity dispersion, self-phase-modulation, third-order dispersion, self-steepening, and the delayed nonlinear response effect, respectively. $\Gamma(z)$ denotes the amplification or absorption coefficient. Equation (1) has been researched by many authors. Li *et al.* obtained some exact analytical solutions by the generalized sub-equation expansion method.[3] Yang *et al.* gave an exact dark soliton solution in explicit form for specified soliton management conditions.[4] Yang *et al.* explicitly presented and analysed exact combined solitary wave solutions that can describe the simultaneous propagation of bright and dark solitary waves in a combined form in inhomogeneous fibre media or in optical communication links with distributed parameters.[5] Equation (1) can include some Schrödinger-type equations, when the distributed parameters are defined. If $a_3(z) = a_4(z) = a_5(z) = \Gamma(z) = 0$, equation (1) becomes the perturbed nonlinear Schrödinger model; Tian *et al.* extended this model by a direct method, performed symbolic computation and obtained two families of exact, analytic bright-solitonic solutions, with and without chirp respectively.[6] If $\Gamma(z) = 0$, Li *et al.* constructed the dark N-soliton solution by using the inverse scattering transform under Hirota parameter conditions.[7] If $a_3(z) = a_4(z) = a_5(z) = 0$, equation (1) becomes an NLS equation with distributed dispersion; Kruglov *et al.* obtained a broad class of ex-

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act self-similar solutions.[8] In recent years, the nonlinear Schrödinger-type equation has been analysed from different points of view and some interesting results have been obtained by many authors.[9−25]

The motivation of this paper lies in the optical and physical importance of Eq. (1) and the need to have some exact analytical solutions. Having some explicit analytical solutions of Eq. (1) may enable one to better understand the optical and physical phenomena which it describes. The exact solutions, which are accurate and explicit, may help physicists and engineers to discuss and examine the sensitivity of the model to several physical parameters. In Ref. [26], Li *et al.* presented an effective direct method, which includes many sub-equation expansion methods,[27−38] such as the tanh function method, the project Riccati equation method,[39] the method proposed in Ref. [5]. By this method, one can obtain abundant exact solutions for Schrödinger-type equations. In this paper, we extend this direct method to construct some exact solutions for Eq. (1) by symbolic computation. As a result, three families of exact solutions for Eq. (1) are derived. Then, based on these exact solutions, soliton propagation and soliton interaction are discussed and simulated by computer.

The paper is organized as follows. In Section 2, we give abundant exact solutions of Eq. (1), which include bright solitons, dark solitons, combined solitary wave solutions, dromions, etc. Then some interesting solutions are obtained and shown by some figures under some special distributed parameters. In Section 3, we give the conclusion of the paper.

2. Exact solutions of the IHNLSE

According to the method of Ref. [23], we can assume the solutions of Eq. (1) as follows:

$$\begin{aligned}\psi &= \left[A_0(z) + A_1(z)\frac{(\delta\cosh(\xi)+\cos(\eta))}{(\cosh(\xi)+\delta\cos(\eta))}\right.\\ &\quad\left.+\,\mathrm{i}B_1(z)\frac{(\alpha\sinh(\xi)+\beta\sin(\eta))}{(\cosh(\xi)+\delta\cos(\eta))}\right]\exp(\varDelta\mathrm{i}),\\ \xi &= p_1(z)t + q_1(z),\\ \eta &= p_2(z)t + q_2(z),\\ \varDelta &= k_2(z)t^2 + k_1(z)t + k_0(z),\end{aligned} \tag{2}$$

where $A_0(z)$, $A_1(z)$, $B_1(z)$, $p_1(z)$, $p_2(z)$, $q_1(z)$, $q_2(z)$, $k_0(z)$, $k_1(z)$, and $k_2(z)$ are real functions of z to be determined, and α, β, δ are real constants. If we set $\eta = \delta = k_2(z) = 0$, then the method in Ref. [5] can be recovered. If $\eta = 0$, the project Riccati equation method[35] can be reproduced.

Substituting Eq. (2) into Eq. (1), at first we remove the exponential terms, and collect coefficients of $(\sinh^i(\xi)\cosh^j(\xi)\sin^m(\eta)\cos^n(\eta))t^k$ $(i = 0,1,2,\ldots; j = 0,1; m = 0,1,2,\ldots; n = 0,1; k = 0,1,\ldots)$ and separate the real part and the imaginary part for each coefficient, thus we can obtain a set of over-determined ordinary differential equations (ODEs). Solving these ODEs, we find three families of solution. Then from Eq. (2) and the solutions of the ODEs, we obtain three families of analytical solutions of Eq. (1) as follows:

Family 1

$$\psi = B_1(z)\beta\left[\frac{-\delta}{2M(1-\delta^2)^{1/2}} + \frac{M}{(1-\delta^2)^{1/2}}\frac{(\delta\cosh(\xi)+\cos(\eta))}{(\cosh(\xi)+\delta\cos(\eta))} + \frac{\mathrm{i}\sin(\eta)}{\cosh(\xi)+\delta\cos(\eta)}\right]\exp(\varDelta\mathrm{i}), \tag{3}$$

$$a_2 = \frac{2C_5^2a_1}{B_1(z)^2\beta^2},\quad a_4 = \frac{6C_5^2a_3}{B_1(z)^2\beta^2},\quad a_5 = -\frac{6C_5^2a_3}{B_1(z)^2\beta^2},\quad \varGamma(z) = (\ln(B_1(z)))_z,$$

where

$$\begin{aligned}\xi &= C_5t + \frac{C_5}{2(1-\delta^2)}\int[4C_4(\delta^2-1)a_1 + a_3(\delta^2(C_5^2+6C_4^2) - 6C_4^2 + 2C_5^2)]\,\mathrm{d}z + C_3,\\ \eta &= \frac{MC_5^2}{(1-\delta^2)^{1/2}}\int(a_1+3a_3C_4)\,\mathrm{d}z + C_1,\\ \varDelta &= C_4t + \frac{1}{2(1-\delta^2)}\int[a_3(2\delta^2C_4^3 + 3\delta^2C_5^2C_4 - 2C_4^3) + a_1(2\delta^2C_4^2 + \delta^2C_5^2 - 2C_4^2)]\,\mathrm{d}z + C_2,\end{aligned}$$

where $M = \pm1$, $\delta \in (-1,1)$, $\beta \neq 0$, C_i, $i = 1,\ldots,5$ are arbitrary constants, a_1, a_3 and $\varGamma(z)$ are arbitrary nonzero functions of the propagation distance z.

If $\delta = 0$, the solution equation (3) reduces to the following bright soliton

$$\psi = M\beta B_1(z)\mathrm{sech}(\xi)\exp(\mathrm{i}(\varDelta + M\eta)), \tag{4}$$

where

$$\xi = C_5 t + C_5 \int (C_5^2 - 3C_4^2)a_3 - 2C_4 a_1 \,\mathrm{d}z + C_3,$$

$$\eta = MC_5^2 \int (a_1 + 3a_3 C_4)\,\mathrm{d}z + C_1,$$

$$\varDelta = C_4 t - C_4^2 \int (a_1 + a_3 C_4)\,\mathrm{d}z + C_2,$$

In order to understand the significance of the solutions expressed by this paper, we investigate the main features of them by using direct computer simulations. For simplicity, we only consider some examples for each solution with some special parameters.

Figures 1(a) and 1(b) show the evolution of solutions (3) with different parameters. As shown in Fig. 1(a), the amplitude of bright solitons (3) increases as the propagation distance increases due to the amplification coefficient $\varGamma(z) = 0.01$, while the time shift and the group velocity of the solitary wave are changing. From Fig. 1(b), the density of the solution changes periodically: firstly it increases, then decreases. Figures 1(c) and 1(d) present the evolution of solutions (4) with different parameters. From Fig. 1(c), the time shift and the group velocity of the solitary wave are changing during the propagation of solitary wave along the fibre. Due to the property of $B_1(z) = 2\mathrm{sech}(0.01z)$, the whole trend of the solitary wave Eq. (4) is that the amplitude first increases then decreases. Figure 1(d) is a dromion solution.

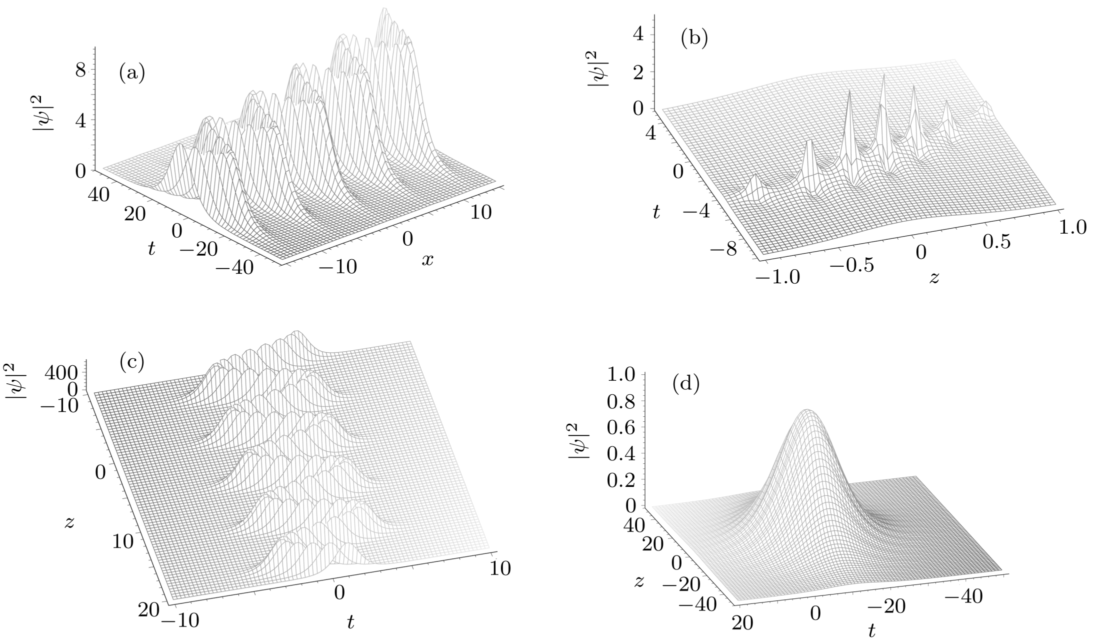

Fig. 1. Evolution of solutions (3) and (4) with the parameters: (a) $C_1 = C_2 = C_3 = 0, C_4 = 2, C_5 = 0.15, M = 1, \delta = 0.3, \beta = 3, a_1 = \sin(z), a_3 = \cos(z), B_1(z) = \exp(0.01z)$; (b) $C_1 = C_2 = 8, C_3 = 0, C_4 = 2, C_5 = 1.5, M = -1, \delta = 0.83, \beta = 10, a_1 = \sin(z), a_3 = \cos(z), B_1(z) = 0.1\,\mathrm{sech}(1.8z)$; (c) $C_1 = C_2 = C_3 = 0, C_4 = C_5 = 1, M = 1, \beta = 13, a_1 = \cos(z), a_3 = \sin(z), B_1(z) = 2\,\mathrm{sech}(0.01z)$; (d) $C_1 = 0, C_2 = C_3 = 1, C_4 = 0.1, C_5 = 0.1, M = 1, \beta = 1, a_1 = \cos(z), a_3 = 1, B_1(z) = \mathrm{sech}(0.05z)$.

Family 2 When $a_1 = a_3 = 0$, we obtain four cases of Eq. (1) below.

Case 1

$$\psi = \left[-A_1(z)\delta + \frac{M\beta B_1(z)}{\delta^2 - 1}\frac{\delta\cosh(\xi) + \cos(\eta)}{\cosh(\xi) + \delta\cos(\eta)} + \frac{\mathrm{i}B_1(z)\beta\sin(\eta)}{\cosh(\xi) + \delta\cos(\eta)}\right]\exp(\mathrm{i}\varDelta), \tag{5}$$

$$a_2 = \frac{a_4[A_1(z)C_5(\delta^2 - 1) - B_1(z)\beta C4]}{A_1(z)(1 - \delta^2)},$$

$$a_5 = -\frac{3}{2}a_4,\ \varGamma(z) = [\ln B_1(z)]_z,$$

where $\xi = C_6 t + C_3$, $\eta = C_4 t + C_1$, $\varDelta = C_5 t + C_2$, $C_i, i = 1, \ldots, 6$ are arbitrary constants, $M = \pm 1$, $\delta \neq \pm 1$, $a_4, A_1(z), B_1(z)$ are arbitrary nonzero functions of z.

From Fig. 2, we can see that when we set the parameters including in solution (5) as some special functions, the solution can present a bright soliton, a dark soliton, a special dromion type of breather and a periodical dispersion-managed soliton.

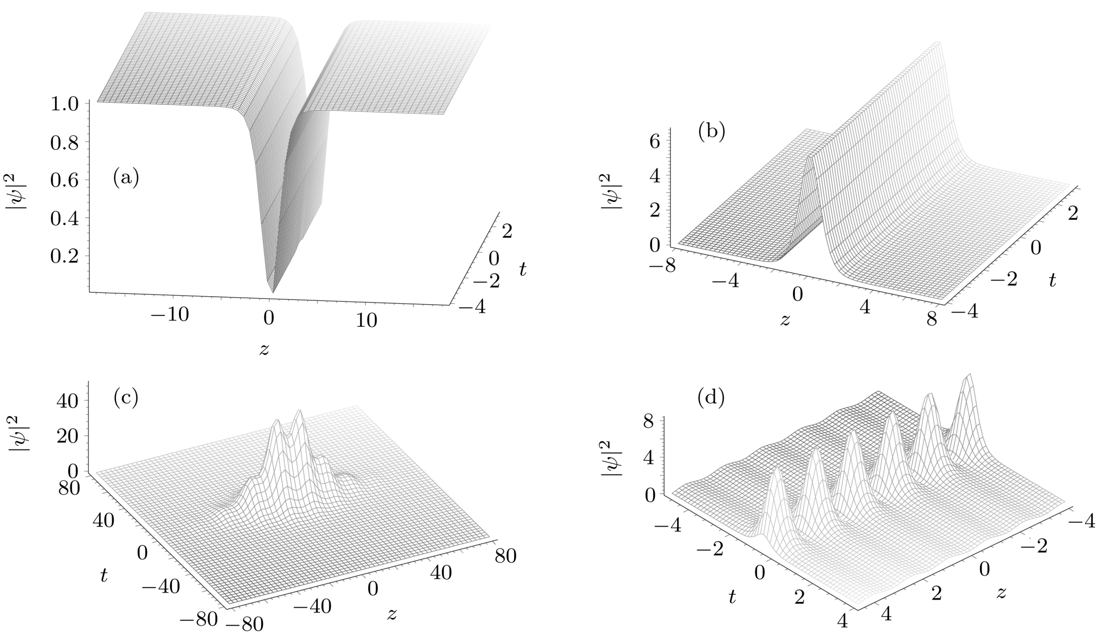

Fig. 2. Evolution of solutions ψ with the parameters: (a) $C_1 = C_2 = C_3 = 1, C_4 = 3, C_5 = 2, C_6 = 5, \delta = 2, M = 1, \beta = 1.5, A_1(z) = 0.5\tanh(0.8z), B_1(z) = 0.5\,\mathrm{sech}(9z)$; (b) $C_1 = C_2 = C_3 = 1, C_4 = 3, C_5 = 2, C_6 = 4, \delta = 2, M = 1, \beta = 0, A_1(z) = 1.3\,\mathrm{sech}(1.2z),\ B_1(z) = \mathrm{sech}(2z)$; (c) $C_1 = 0, C_2 = 1, C_3 = 0.5, C_4 = 0, C_5 = 10, C_6 = -0.1, \delta = 0, A_1(z) = 0, B_1(z) = (2\,\mathrm{sech}(0.05z) + 0.05\sin(0.4z))/(1 - 0.2\sin(0.2z)^2), M = 1, \beta = 3$; (d) $C_1 = 3\pi/2, C_2 = C_3 = 1, C_4 = 0.1, C_5 = 0, C_6 = 2, \delta = 4, B_1(z) = -7\sin(2z)/(1 - 0.07\sin(z)^2), M = 1, \beta = 0.3$.

Case 2

$$\psi = A_1(z)\left[E_7 + \frac{\delta\cosh(\xi) + \cos(\eta)}{\cosh(\xi) + \delta\cos(\eta)}\right]\exp(\mathrm{i}\varDelta), \tag{6}$$
$$a_2 = -E_4 a_4,\quad a_5 = -\frac{3}{2}a_4,\quad \varGamma(z) = [\ln A_1(z)]_z,$$

where $\xi = E_6 t + E_1,\quad \eta = E_5 t + E_2,\quad \varDelta = E_4 t + E_3,\ E_i,\ i = 1,\ldots,7,\ \delta$ are arbitrary constants, a_4 and $A_1(z)$ are arbitrary nonzero functions of the propagation distance z.

As shown in Fig. 3, when $A_1(z)$ is a trigonometric function, the solution equation (6) presents a periodic dispersion-managed soliton. When $A_1(z) = -0.4 + \mathrm{sech}(0.4z)$, we obtain a W-shaped soliton.

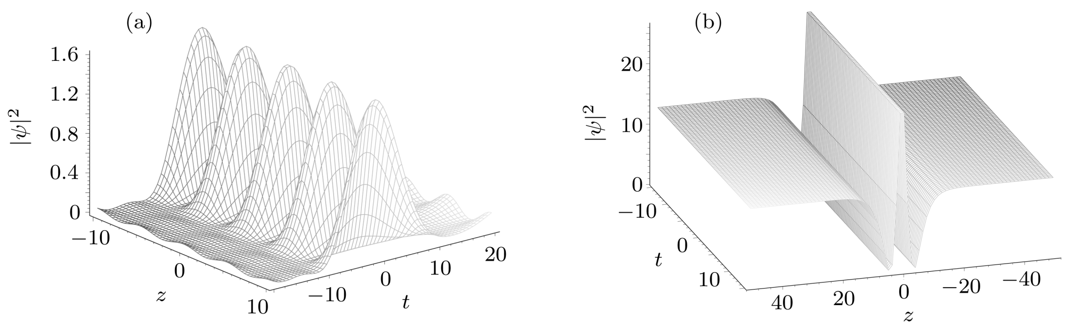

Fig. 3. Evolution of solutions ψ with the parameters: (a) $E_1 = 0.4, E_2 = E_3 = E_6 = 0.1, E_4 = 1, E_5 = 0.2, E_7 = 0.3, \delta = 0.2, A_1(z) = \cos(0.2\pi z)$; (b) $E_1 = 0.004, E_2 = 0.001, E_3 = 0.003, E_4 = 0.006, E_5 = 0.003, E_6 = 0.003, E_7 = 8, \delta = 2, A_1(z) = -0.4 + \mathrm{sech}(0.4z)$.

Case 3

$$\psi = \frac{\mathrm{i}B_1(z)(\alpha\sinh(\xi)+\beta\sin(\eta))}{\cosh(\xi)+\delta\cos(\eta)}\exp(\mathrm{i}\varDelta), \tag{7}$$
$$a_2 = -F_4a_4,\quad a_5 = -\frac{3}{2}a_4,\quad \varGamma(z) = [\ln B_1(z)]_z,$$

where $\xi = (\delta\alpha F_5/\beta)t + F_3$, $\eta = F_5t + F_2$, $\varDelta = F_1t + F_4$, $F_i, i = 1,\ldots,5$, δ, α and $\beta \neq 0$ are arbitrary constants, a_4 and $B_1(z)$ are arbitrary nonzero functions of the propagation distance z.

Case 4

$$\begin{aligned}\psi = {}& r(z)G_1^2\bigg[\frac{(G_1^2-G_3^2)^{1/2}\cosh(\xi)+G_1}{G_3(G_1\cosh(\xi)+(G_1^2-G_3^2)^{1/2})} \\ &+ \frac{\mathrm{i}\sinh(\xi)}{(G_1\cosh(\xi)+(G_1^2-G_3^2)^{1/2})}\bigg]\exp(\mathrm{i}\varDelta), \quad (8)\\ a_2 = {}& G_4a_4,\quad a_5 = -\frac{3}{2}a_4,\quad r(z) = \exp\left(\int \varGamma(z)\,\mathrm{d}z\right),\end{aligned}$$

where

$$\xi = G_5t + \int\left(\frac{r^2(z)a_4G_1^4G_5}{G_3^2}\right)\mathrm{d}z + G_2,$$
$$\varDelta = G_6t + G_2 + \int\left(\frac{a_4r^2(z)G_1^4(G_4+G_6)}{G_3^2}\right)\mathrm{d}z,$$

$G_i,\ i = 1,\ldots,6$, are arbitrary constants, $a_4, \varGamma(z)$ and $B_1(z)$ are arbitrary functions of the propagation distance z.

Family 3 When $a_3 = a_4 = a_5 = 0$, we obtain two sets of solutions for Eq. (1).

Case 1

$$\begin{aligned}\psi = {}& B_1(z)\bigg[\frac{(\alpha^2+\beta^2\delta^2)}{2\delta M[(\alpha^2+\beta^2)(\delta^2-1)]^{1/2}} \\ &+ \frac{M(\alpha^2+\beta^2)^{1/2}}{(\delta^2-1)^{1/2}}\frac{\delta\cosh(\xi)+\cos(\eta)}{\cosh(\xi)+\delta\cos(\eta)} \\ &+ \mathrm{i}\frac{(\alpha\sinh(\xi)+\beta\sin(\eta))}{\cosh(\xi)+\delta\cos(\eta)}\bigg]\exp(\mathrm{i}\varDelta), \quad (9)\\ a_1 = {}& -\frac{k_{2z}(z)}{4k_2(z)^2},\quad a_2 = -\frac{k_{2z}(z)\delta^2C_4^2}{2\alpha^2B_1(z)^2},\\ \varGamma(z) = {}& \frac{2B_{1z}(z)k_2(z)-B_1(z)k_{2z}(z)}{2k_2(z)B_1(z)},\end{aligned}$$

where

$$\xi = \frac{\delta\beta C_4k_2(z)}{\alpha}t + \frac{\delta C_4k_2(z)[C_4(\beta^2\delta^2-2\beta^2-\alpha^2)+2MC_5\beta(\alpha^2+\beta^2)^{1/2}(\delta^2-1)^{1/2})]}{4\alpha M(\alpha^2+\beta^2)^{1/2}(\delta^2-1)^{1/2}},$$
$$\eta = k_2(z)C_4t + \frac{C_4k_2(z)[C_4(\beta^3\delta^2+2\beta\delta^2\alpha^2-\alpha^2\beta)+2MC_5(\alpha^2+\beta^2)^{1/2}(\delta^2-1)^{1/2})]}{4\alpha^2M(\alpha^2+\beta^2)^{1/2}(\delta^2-1)^{1/2}},$$
$$\varDelta = k_2(z)t^2 + C_5k_2(z)t + \frac{k_2(z)[C_4(\alpha^2+\beta^2\delta^2)^2+2\alpha^2C_5^2(\alpha^2+\beta^2)(\delta^2-1)]}{8\alpha^2(\alpha^2+\beta^2)(\delta^2-1)},$$

where $\alpha \neq 0$, $\delta > 1$, $\beta, C_i, i = 1,\ldots,5$, are arbitrary constants, $M = \pm 1$, $k_2(z)$ and $B_1(z)$ are arbitrary nonzero functions of the propagation distance z.

The evolutions of solution Eq. (9) with some special parameters are shown in Figs. 4(a) and 4(b).

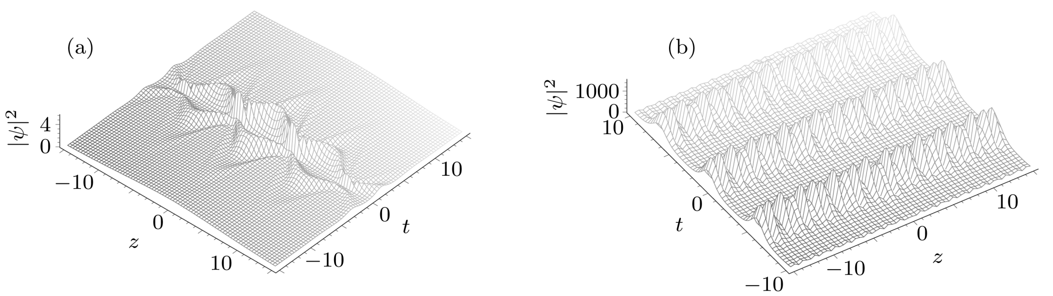

Fig. 4. Evolution of solutions ψ with the parameters: (a) $G_1 = 2, G_2 = G_3 = 1, G_4 = 3, G_5 = 0.8, k_2(z) = 7\,\mathrm{sech}(0.01z), M = 1, \delta = 0.01, \beta = 3, B_1(z) = -0.4\sin(2z)/(1-0.8\sin(z)^2)$; (b) $G_1 = 0.6, G_2 = 10, G_3 = 1, G_4 = 0.4, G_5 = 24, M = 1, \delta = 0.8, k_2(z) = 1-0.2\sin(0.15z), \beta = 3, B_1(z) = 20\,\mathrm{sech}(0.02z)/(\mathrm{sech}(0.1z)+\tanh(0.7z)^2)$.

Case 2

$$\psi = B_1(z)\left[-\frac{\delta\beta}{2M(1-\delta^2)^{1/2}}+\frac{M\beta(\delta\cosh(\xi)+\cos(\eta))}{(1-\delta^2)^{1/2}(\cosh(\xi)+\delta\cos(\eta))}+\frac{\mathrm{i}\beta\sin(\eta)}{\cosh(\xi)+\delta\cos(\eta)}\right]\exp(\mathrm{i}\varDelta), \tag{10}$$

$$a_1 = -\frac{k_{2z}(z)}{4k_2(z)^2},\quad a_2=-\frac{G_5^2k_{2z}(z)}{2B_1(z)^2\beta^2},\quad \varGamma(z)=\frac{2B_{1z}(z)k_2(z)-B_1(z)k_{2z}(z)}{2k_2(z)B_1(z)},$$

where

$$\xi = G_5t+\frac{1}{2}G_5G_4\ln(k_2(z))+G_3,\quad \eta=\frac{G_5^2}{4(1-\delta^2)^{1/2}Mk_2(z)}+G_1,$$

$$\varDelta = k_2(z)t^2+G_4k_2(z)t+\frac{2G_4^2k_2(z)^2(\delta^2-1)-\delta^2G_5^2+8G_2k_2(z)(\delta^2-1)}{8k_2(z)(\delta^2-1)},$$

$\delta\in(-1,1)$, $\beta\neq 0, G_i, i=1,\ldots,5$, are arbitrary constants, $M=\pm 1$, $k_2(z)$ and $B_1(z)$ are arbitrary nonzero functions of the propagation distance z.

The evolutions of solution (10) with some different parameters are shown in Figs. 5(a) and 5(b).

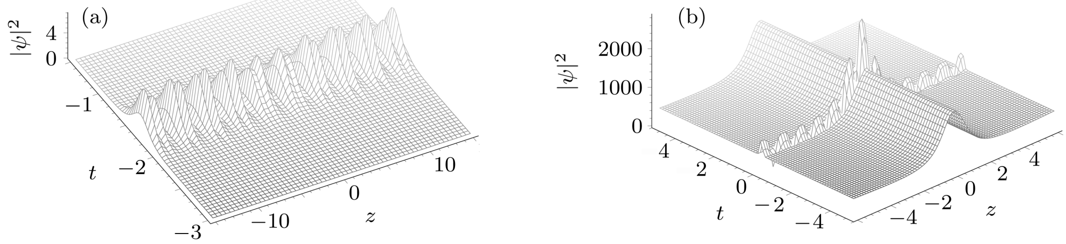

Fig. 5. Evolution of solutions ψ with the parameters: (a) $G_1=G_4=2, G_2=G_3=1, G_5=0.4, \delta=0.1, \beta=1.3, M=1, k_2(z)=1.5\sin(z), B_1(z)=-0.4\sin(2z)/(1-0.8\sin(z)^2)$; (b) $G_1=2, G_2=1, G_3=10, G_4=6, G_5=14, \delta=0.9, M=-1, \beta=0.3, k_2(z)=1-0.2\sin(0.1z), B_1(z)=2\,\mathrm{sech}(0.002z)/(\mathrm{sech}(0.1z)+\tanh(0.7z)^2)$.

3. Summary and discussion

In this paper, we have obtained some interesting soliton solutions of the one-dimensional generalized IHNLSE with variable coefficients by a direct method. As shown in the figures produced by computer simulation, these solutions possess abundant structures when arbitrary parameters are selected as some special forms, which present bright solitons, dark solitons, combined solitary wave solutions, dromions, dispersion-managed solitons, etc. These results may be useful in ultra-high-speed optical telecommunication.

References

[1] Hasegawa A and Tappert F 1973 *Appl. Phys. Lett.* **23** 142
[2] Mollenauer L F, Stolen R H and Gordon J P 1980 *Phys. Rev. Lett.* **45** 1095
[3] Li B, Chen Y and Li Y Q 2008 *Z. Naturforsch.* **63a** 763
[4] Yang R C, Hao R Y, Li L, Li Z H and Zhou G S 2004 *Opt. Commun.* **242** 285
[5] Yang R C, Hao R Y, Li L, Li Z H and Zhou G S 2005 *Phys. Rev.* E **71** 036616
[6] Tian B and Gao Y T 2005 *Phys. Lett.* A **342** 228
[7] Li L, Li Z H, Xu Z Y, Zhou G S and Spatschek K H 2002 *Phys. Rev.* E **66** 04661
[8] Kruglov V I, Peacock A C and Harvey J D 2003 *Phys. Rev. Lett.* **90** 21
[9] Li Z H, Li L, Tian H P and Zhou G S 2000 *Phys. Rev. Lett.* **84** 4096
[10] Li Z H, Li L, Tian H P, Zhou G S and Spatschek K H 2002 *Phys. Rev. Lett.* **89** 263901
[11] Jia M, Wang J Y and Lou S Y 2009 *Chin. Phys. Lett.* **26** 020201
[12] Li H M 2008 *Chin. Phys.* B **17** 759
[13] He J C, Chen Z Y and Huang N N 2009 *Acta Phys. Sin.* **58** 6063 (in Chinese)
[14] Li X Z, Zhang J L, Wang Y M and Wang M L 2004 *Acta Phys. Sin.* **53** 4045 (in Chinese)
[15] Li H M, Li Y S and Lin J 2009 *Chin. Phys.* B **18** 3657
[16] Lou S Y 1999 *Chin. Phys. Lett.* **16** 659
[17] He J S, Ji M and Li Y S 2007 *Chin. Phys. Lett.* **24** 2157

[18] Zhou R G 2007 *Chin. Phys. Lett.* **24** 598
[19] Zdravkovic S and Sataric M V 2007 *Chin. Phys. Lett.* **24** 1210
[20] Wang Y S, Li H Q and Song Y Z 2008 *Chin. Phys. Lett.* **25** 1538
[21] Hong W P 2001 *Opt. Commun.* **194** 217
[22] Li L, Li Z H, Xu Z Y, Zhou G S and Spatschek K H 2002 *Phys. Rev.* E **66** 04661
[23] Li L, Li Z H, Li S Q and Zhou G S 2004 *Opt. Commun.* **234** 169
[24] Li B 2005 *Int. J. Mod. Phys.* C **16** 1225
[25] Li B and Chen Y 2006 *Z. Naturforsch.* **60a** 768
[26] Li B, Zhang X F, Li Y Q, Chen Y and Liu W M 2008 *Phys. Rev.* A **78** 023608
[27] Lou S Y, Huang G X and Ruan H Y 1991 *J. Phys.* A **24** L587
[28] Tan Y K, Wu H Y, Zhang L and Zhou X T 2008 *Acta Phys. Sin.* **57** 3312 (in Chinese)
[29] Han J H, Shi L M, Wu G J, Zhang M and Zhang W L 2007 *Acta Phys. Sin.* **56** 5054 (in Chinese)
[30] Li W A, Chen H and Zhang G C 2009 *Chin. Phys.* B **18** 400
[31] Gao X Y, Hong X R, Wang C L and Duan W S 2008 *Chin. Phys.* B **17** 3378
[32] Chen C L, Zhang J and Li Y S 2007 *Chin. Phys.* **16** 2167
[33] Wu X F, Zhu J M and Ma Z Y 2007 *Chin. Phys.* **16** 2159
[34] Ma H C, Deng A P and Qin Z Y 2009 *Chin. Phys. Lett.* **26** 040201
[35] Li Z B and Liu Y P 2002 *Comput. Phys. Commun.* **148** 256
[36] Fan E 2000 *Phys. Lett.* A **277** 212
[37] Yan Z Y 2001 *Phys. Lett.* A **292** 100
[38] Yan Z Y and Zhang H Q 2001 *Phys. Lett.* A **285** 355
[39] Li B and Chen Y 2004 *Chaos, Solitons and Fractals* **21** 241
[40] Li B 2004 *Z. Naturforsch.* **59a** 919