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Numerical Solutions of a New Type of Fractional Coupled Nonlinear Equations*

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Abstract *In this paper, we investigate a new type of fractional coupled nonlinear equations. By introducing the fractional derivative that satisfies the Caputo's definition, we directly extend the applications of the Adomian decomposition method to the new system. As a result, with the aid of Maple, the realistic and convergent rapidly series solutions are obtained with easily computable components. Two famous fractional coupled examples: KdV and mKdV equations, are used to illustrate the efficiency and accuracy of the proposed method.*

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1 Introduction

Since the beginning of 1980s, the Adomian decomposition method[1−3] (ADM) has been effectively used to solve a wide class of differential equations of integer order containing linear and nonlinear, ordinary and partial equation(s). Compared with other traditional methods,[4−10] the obvious advantage of the method is that it can provide rapidly convergent series solution with easily computable components and no special techniques or assumptions are required.

For fractional derivative, although there exist a number of definitions in mathematic literature, their extensive applications in physics and engineering[11] are just in recent decades. With fractional derivative, the nonlinear oscillation of earthquake can be modelled;[12] the fluid dynamic traffic model can eliminate the deficiency arising from the assumption of continuum traffic flow;[12] in addition, phenomena in electromagnetics, acoustics electrochemistry and material science can be well described.[13−16] As we all know, for fractional differential equations, there are only limited approaches, such as Laplace transform method,[16] the Fourier transform method,[17] the iteration method,[15] and the operational method.[18] What's to our disappointment is that each of them is only suitable for special equations. Physicists and mathematicians have tried every effort to find an effective unified method to solve the fractional equation(s).

In this paper, we consider the numerical solutions of a new type of fractional coupled nonlinear equations (FCNLEs) in this form

$$\frac{\partial^\alpha u}{\partial t^\alpha} = L_1(u,v) + N_1(u,v)\,, \qquad t>0\,,$$
$$\frac{\partial^\beta v}{\partial t^\beta} = L_2(u,v) + N_2(u,v)\,, \qquad t>0 \tag{1}$$

by the ADM, where L_i and N_i $(i=1,2)$ are the linear and nonlinear functions of u and v, respectively, α and β are the parameters that describe the order of the fractional derivative. Different expressions of FCNLEs can be obtained when one of the parameters α, β varies. The study to Eq. (1) is very necessary and significant that is because their special cases contain many important mathematical physics equations such as the coupled KdV,[7,19,20] mKdV[19] equations, Burgers equations[21] and many coupled reaction-diffusion equations,[22,23] and so on. Recently, the coupled KdV and mKdV equations of integer order have been investigated and the numerical solutions by Kaya and E. Inan with the ADM[19] were obtained, the exact solutions of them have been derived by Lu *et al.*[20] and Fan,[7] respectively. However, up to now, not much has been done for the new type of FCNLEs. In this paper, by introducing the fractional derivatives – Caputo derivative,[24] we will study the problem by the ADM in detail.

The paper is organized as follows. In Sec. 2, some necessary description and analysis on the fractional calculus as well as the ADM for FCNLEs are given. In Sec. 3, two famous fractional coupled examples: KdV and mKdV equations are given to illustrate the effectiveness and accuracy of the proposed method. Finally, conclusions are followed.

2 Description and Analysis on the Fractional Calculus and ADM

2.1 *Description on the Fractional Calculus*

For the concept of fractional derivative, there exist many mathematical definitions.[14−16,24,25]

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In this paper, the two most commonly definitions: the Caputo derivative and its reverse operator Riemann–Liouville integral are used. That is because Caputo fractional derivative[24] allows traditional assumption of initial and boundary conditions. The Caputo fractional derivative is defined as

$$D^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)}\int_0^t (t-\tau)^{n-\alpha-1} f^{(n)}(\tau)\,\mathrm{d}\tau\,, \quad (n-1<\mathrm{Re}(\alpha)\le n\,, \quad n\in N)\,, \tag{2}$$

and the Riemann–Liouville fractional integral is defined as

$$J^\alpha f(x) = \frac{1}{\Gamma(\alpha)}\int_0^x (x-t)^{\alpha-1} f(t)\,\mathrm{d}t\,, \qquad x>0\,. \tag{3}$$

Here, we also need two basic properties about them

$$D^\alpha J^\alpha f(x) = f(x)\,,$$
$$J^\alpha D^\alpha f(x) = f(x) - \sum_{k=0}^{\infty} f^k(0^+)\frac{x^k}{k!}\,, \qquad x>0\,. \tag{4}$$

More details on the fractional derivative and integral can consult Ref. [16].

Remark In this paper, we need to discuss the fractional derivative for the type of FCNLEs. When $\alpha\in R^+$ we just need to copy (2), when $\alpha = n\in N$, the fractional derivative reduces to the commonly used derivative. That is to say

$$D_t^\alpha u(x,t) = \frac{\partial^\alpha u(x,t)}{\partial t^\alpha} = \begin{cases} \dfrac{1}{\Gamma(n-\alpha)}\displaystyle\int_0^t (t-\tau)^{n-\alpha-1}\frac{\partial^n u(x,\tau)}{\partial\tau^n}\mathrm{d}\tau\,, & n-1<\alpha<n\,, \\ \dfrac{\partial^n u(x,t)}{\partial t^n}\,, & \alpha = n\in N\,. \end{cases} \tag{5}$$

2.2 *Analysis on the ADM*

Consider the operator form of the type of nonlinear Eq. (1)

$$D_t^\alpha u = L_1(u,v) + N_1(u,v)\,, \qquad t>0\,,$$
$$D_t^\beta v = L_2(u,v) + N_2(u,v)\,, \qquad t>0\,, \tag{6}$$

where the operators D_t^α and D_t^β stand for the fractional derivative and are defined as in Eq. (5). Assuming the initial condition as

$$u(x,0) = f(x)\,, \quad v(x,0) = g(x)\,. \tag{7}$$

Applying the operators J^α and J^β, the inverse operators of D^α and D^β on both sides of the corresponding sub-equation in Eq. (6) and using the initial condition (7), yield

$$u = F(x,t) + J^\alpha L_1(u,v) + J^\alpha N_1(u,v)\,,$$
$$v = G(x,t) + J^\beta L_2(u,v) + J^\beta N_2(u,v)\,, \tag{8}$$

where $F(x,t)$ and $G(x,t)$ are arising from the initial conditions and integral operators.

Remark It must be noticed that when setting $0<\alpha\le 1$, $0<\beta\le 1$, we can obtain $J^\alpha D^\alpha u = u(x,t) - u(x,0)$. That is to say, $F(x,t)\equiv f(x) = u(x,0)$, $G(x,t)\equiv g(x) = v(x,0)$. Generally, take $n<\alpha\le n+1$, $n<\beta\le n+1$ as an example, we have $J^\alpha D^\alpha u = u(x,t) - \sum_{i=0}^n (t^i/i!)u_t^{(i)}(x,0)$. That is to say, we need another initial boundary condition $u_t^{(i)}(x,0)$, then we have $F(x,t) = u(x,0) + \sum_{i=0}^n (t^i/i!)u_t^{(i)}(x,0)$. With bigger values of α and β, more initial boundary conditions are needed. Correspondingly, the final solutions will also become more complex. In order to avoid the heavy calculations, we only choose $0<\alpha\le 1$, $0<\beta\le 1$ to study the paper in the following.

According to the ADM,[1−3] the solutions are represented as infinite series like

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t)\,, \quad v(x,t) = \sum_{n=0}^{\infty} v_n(x,t)\,, \tag{9}$$

and the nonlinear terms $N_1(u,v)$ and $N_2(u,v)$ are decomposed as

$$N_1(u,v) = \sum_{n=0}^{\infty} A_n\,, \quad N_2(u,v) = \sum_{n=0}^{\infty} B_n\,, \tag{10}$$

where A_n and B_n are the so-called Adomian polynomials and have the forms of

$$A_n = \frac{1}{n!}\Big[\frac{\mathrm{d}^n}{\mathrm{d}\lambda^n}N_1\Big(\sum_{k=0}^{\infty}\lambda^k u_k, \sum_{k=0}^{\infty}\lambda^k v_k\Big)\Big]_{\lambda=0}\,,$$
$$B_n = \frac{1}{n!}\Big[\frac{\mathrm{d}^n}{\mathrm{d}\lambda^n}N_2\Big(\sum_{k=0}^{\infty}\lambda^k u_k, \sum_{k=0}^{\infty}\lambda^k v_k\Big)\Big]_{\lambda=0}\,. \tag{11}$$

It is important to note that A_0 only depends on u_0, A_1 only depends on u_0 and u_1, etc. Moreover, the sum of the subscripts in each term of A_n is equal to n. The same holds for the Adomian polynomials B_n. However, these formulas are easy to be computed with the aid of *Maple*.

Substituting the decomposition series (9) and (10) into Eq. (8), we can obtain the iterations recursive relations of the numerical solutions

$$u_0 = f(x)\,, \quad u_{n+1} = J^\alpha L_1(u_n,v_n) + J^\alpha A_n\,,$$
$$v_0 = g(x)\,, \quad v_{n+1} = J^\beta L_2(u_n,v_n) + J^\beta B_n\,. \tag{12}$$

In the following, we will give two famous fractional coupled examples to illustrate the effectiveness of the method.

3 Applications of ADM with Two Famous Fractional Coupled Examples

3.1 *Fractional Coupled KdV Equations*

According to the above analyses, we take the coupled KdV equations written in an operator form

$$D_t^\alpha u = a u_{xxx} + 6a u u_x + 2b v v_x\,, \qquad (0<\alpha\le 1)\,,$$
$$D_t^\beta v = -v_{xxx} - 3u v_x\,, \qquad (0<\beta\le 1)\,. \tag{13}$$

It is well known that the exact solutions when $\alpha = 1$, $\beta = 1$, are

$$u(x,t) = -\frac{1+a}{3+6a}k^2 + 4k^2\frac{\mathrm{e}^{k\xi}}{(1+\mathrm{e}^{k\xi})^2}\,,$$
$$v(x,t) = \sqrt{\frac{-24a}{b}}\frac{k^2\,\mathrm{e}^{k\xi}}{(1+\mathrm{e}^{k\xi})^2}\,, \tag{14}$$

constructed by Lu and Wang[20] and $\xi = x - ak^2/1 + 2at$. Assuming the initial conditions for Eq. (13) as

$$u(x,0) = f(x) = -\frac{1+a}{3+6a}k^2 + 4k^2\frac{\mathrm{e}^{kx}}{(1+\mathrm{e}^{kx})^2}\,,$$
$$v(x,0) = g(x) = \sqrt{\frac{-24a}{b}}\frac{k^2\,\mathrm{e}^{kx}}{(1+\mathrm{e}^{kx})^2}\,, \tag{15}$$

where $a \neq -1/2$, $ab < 0$ and k is an arbitrary constant.

Using the initial condition (15) and applying the operators J^α and J^β, the inverse operators of D_t^α and D_t^β, on the corresponding sub-equation of Eq. (13), according to Eq. (8), yields

$$u(x,t) = u(x,0) + aJ^\alpha u_{xxx} + J^\alpha[6auu_x + 2bvv_x]\,,$$
$$v(x,t) = v(x,0) - J^\beta v_{xxx} - 3J^\beta uv_x\,, \tag{16}$$

The Adomian polynomials for the nonlinear terms $6uu_x + 2bvv_x$, $-3uv_x$ are taken as

$$6auu_x + 2bvv_x = 6a\sum_{n=0}^{+\infty}\sum_{k=0}^{n}u_k\frac{\partial}{\partial x}u_{n-k} + 2b\sum_{n=0}^{+\infty}\sum_{k=0}^{n}v_k\frac{\partial}{\partial x}v_{n-k}\,, \quad -3uv_x = -3\sum_{n=0}^{+\infty}\sum_{k=0}^{n}u_k\frac{\partial}{\partial x}v_{n-k}\,. \tag{17}$$

Substituting Eqs. (15) and (17) into Eq. (16) yields the following iterations recursive formulas

$$u_0 = f(x)\,, \quad u_{n+1} = aJ^\alpha u_{nxxx} + J^\alpha\Big[\sum_{k=0}^{n}\Big(6au_k\frac{\partial}{\partial x}u_{n-k} + 2bv_k\frac{\partial}{\partial x}v_{n-k}\Big)\Big]\,,$$
$$v_0 = g(x)\,, \quad v_{n+1} = -J^\beta v_{nxxx} - 3J^\beta\Big[\sum_{k=0}^{n}u_k\frac{\partial}{\partial x}v_{n-k}\Big]\,. \tag{18}$$

For the convenience of the reader, we just list the first few terms of the decomposition series

$$u_0 = f(x)\,, \quad v_0 = g(x)\,, \quad u_1 = aJ^\alpha[u_{0xxx}] + 6aJ^\alpha[u_0u_{0x}] + 2bJ^\alpha[v_0v_{0x}] = f_1(x)\frac{t^\alpha}{\Gamma(\alpha+1)}\,,$$
$$v_1 = -J^\beta[v_{0xx}] - J^\beta[u_0v_{0x}] = g_1(x)\frac{t^\beta}{\Gamma(\beta+1)}\,,$$
$$u_2 = aJ^\alpha[u_{1xxx}] + 6aJ^\alpha[u_0u_{1x} + u_{0x}u_1] + 2bJ^\alpha[v_0v_{1x} + v_{0x}v_1] = f_2(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + f_3(x)\frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)}\,,$$
$$v_2 = -J^\beta[v_{1xxx}] - 3J^\beta[u_0v_{1x} + u_1v_{0x}] = g_2(x)\frac{t^{2\beta}}{\Gamma(2\beta+1)} + g_3(x)\frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)}\,,$$

where

$$f(x) = -\frac{1+a}{3+6a}k^2 + 4k^2\frac{\mathrm{e}^{kx}}{(1+\mathrm{e}^{kx})^2}\,, \quad g(x) = \sqrt{\frac{-24a}{b}}\frac{k^2\,\mathrm{e}^{kx}}{(1+\mathrm{e}^{kx})^2}\,,$$
$$f_1(x) = a(f_{xxx} + 6f_xf) - 2bgg_x\,, \quad g_1(x) = -g_{xxx} + 3fg_x\,, \quad f_2(x) = a(f_{1xxx} + 6f_1f_x + 6ff_{1x})\,,$$
$$g_2(x) = g_{1xxx} + 3fg_{1x}\,, \quad f_3(x) = 2b(g_xg_1 + gg_{1x})\,, \quad g_3(x) = -3f_1g_x\,.$$

Then we can have the numerical solutions of Eq. (13) in series form as

$$u(x,t) = f + f_1(x)\frac{t^\alpha}{\Gamma(\alpha+1)} + f_2(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + f_3(x)\frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)} + \cdots\,, \tag{19}$$
$$v(x,t) = g + g_1(x)\frac{t^\beta}{\Gamma(\beta+1)} + g_2(x)\frac{t^{2\beta}}{\Gamma(2\beta+1)} + g_3(x)\frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)} + \cdots\,. \tag{20}$$

In order to verify whether the ADM for the fractional coupled equations leads to higher accuracy, we draw the figures of the numerical solutions (19) and (20) with $\alpha = 1/2$, $\beta = 1/3$, as well as the exact solutions (14) derived by Lu and Wang[20] when $\alpha = \beta = 1$. Comparing Figs. 1(a) and 1(b) with Figs. 2(a) and 2(b), we can see that the solutions obtained by different methods are nearly the same. So we conclude that a good approximation is achieved by using N-term approximation of the ADM solutions.

Remark We evaluate the numerical solutions by using N-term approximation. The accuracy of the numerical solutions obtained depends on how many terms we choose. A higher level of accuracy for the ADM solutions can be achieved with much more terms. In the meanwhile, the overall errors can be made smaller by adding new terms of the decomposition series.

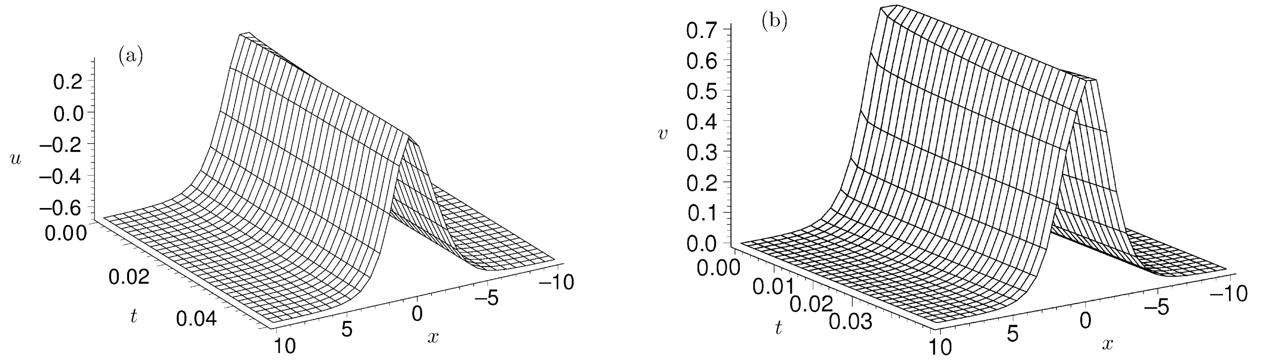

Fig. 1 Explicit numerical solutions for KdV Eq. (13). (a) $u(x,t)$ as in Eq. (19); (b) $v(x,t)$ as in Eq. (20), with $\alpha = 1/2$, $\beta = 1/3$.

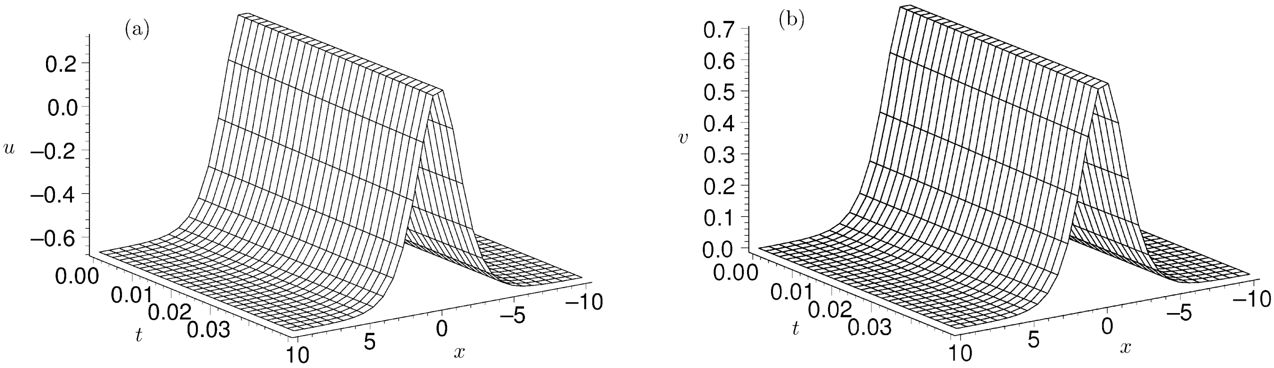

Fig. 2 Exact solutions (14) for Eq. (13). (a) $u(x,t)$; (b) $v(x,t)$ with $\alpha = 1$, $\beta = 1$.

3.2 *Fractional Coupled mKdV Equations*

In this section, we will take the other fractional coupled equations as an example to illustrate the feasibleness and accuracy of the method for the FCNLEs. As the main steps are nearly the same, we just give some necessary expressions and the final results.

The coupled mKdV equations are given in the operator form

$$\begin{aligned}
D_t^{\alpha} u &= \frac{1}{2} u_{xxx} - 3u^2 u_x + \frac{3}{2} v_{xx} + 3(uv)_x - 3\lambda u_x \,, \qquad (0 < \alpha \le 1)\,, \\
D_t^{\beta} v &= -v_{xxx} - 3v v_x - 3u_x v_x + 3u^2 v_x + 3\lambda v_x \,, \qquad (0 < \beta \le 1)
\end{aligned} \tag{21}$$

with the initial conditions

$$u(x,0) = \frac{b}{2k} + k\tanh(kx)\,, \qquad v(x,0) = \frac{\lambda}{2}\Big(1 + \frac{k}{b}\Big) + b\tanh(kx)\,.$$

Implement the ADM to Eq. (21) and repeat the above similar steps. Omitting the heavy calculations, we just give the first three terms of the recursive relations

$$\begin{aligned}
u_0 &= f(x)\,, \quad v_0 = g(x)\,, \\
u_1 &= \frac{1}{2} J^{\alpha}[u_{0xxx}] - 3J^{\alpha}[u_0^2 u_{0x}] + \frac{3}{2} J^{\alpha}[v_{0xx}] + 3J^{\alpha}[(u_0 v_0)_x] - 3\lambda J^{\alpha}[u_{0x}] = f_1(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)}\,, \\
v_1 &= -J^{\beta}[v_{0xxx}] - 3J^{\beta}[v_0 v_{0x}] - 3J^{\beta}[u_{0x} v_{0x}] + 3J^{\beta}[u_0^2 v_{0x}] + 3\lambda J^{\beta}[v_{0x}] = g_1(x)\frac{t^{\beta}}{\Gamma(\beta+1)}\,, \\
u_2 &= \frac{1}{2} J^{\alpha}[u_{0xxx}] - 3J^{\alpha}[u_0^2 u_{1x} + 2u_0 u_{0x} u_1] + \frac{3}{2} J^{\alpha}[v_{0xx}] + 3J^{\alpha}[(u_0 v_1 + u_1 v_0)_x] - 3\lambda J^{\alpha}[u_{1x}] \\
&= f_2(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + f_3(x)\frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)}\,, \\
v_2 &= -J^{\beta}[v_{1xxx}] - 3J^{\beta}[(v_0 v_1)_x] - 3J^{\beta}[u_{0x} v_{1x} + u_{1x} v_{0x}] + 3J^{\beta}[u_0^2 v_{1x} + 2u_0 u_1 v_{0x}] + 3\lambda J^{\beta}[v_{1x}] \\
&= g_2(x)\frac{t^{2\beta}}{\Gamma(2\beta+1)} + g_3(x)\frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)}\,,
\end{aligned}$$

where

$$f(x)=\frac{b}{2k}+k\tanh(kx)\,,\quad g(x)=\frac{\lambda}{2}\Big(1+\frac{k}{b}\Big)+b\tanh(kx)\,,$$

$$f_1(x)=\frac{1}{4}(2f_{xxx}-6f_xf^2+3g_{xx}+6f_xg+6fg_x-6\lambda f_x)\,,\quad g_1(x)=-g_{xxx}-3gg_x-3f_xg_x+3f^2g_x+3\lambda g_x\,,$$

$$f_2(x)=\frac{1}{2}f_{1xxx}-3\lambda f_{1x}-6ff_xf_1-3f^2f_{1x}+3f_{1x}g+3f_1g_x\,,$$

$$g_2(x)=-g_{1xxx}-3g_xg_1-3g_{1x}g-3f_xg_{1x}+3f^2g_{1x}+3\lambda g_{1x}\,,$$

$$f_3(x)=f_xg_1+fg_{1x}+\frac{3}{2}g_{1xx}\,,\quad g_3(x)=3g_x-f_{1x}+2ff_1\,.$$

So the final numerical results are

$$u(x,t)=f+f_1(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)}+f_2(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}+f_3(x)\frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)}+\cdots\,, \tag{22}$$

$$v(x,t)=g+g_1(x)\frac{t^{\beta}}{\Gamma(\beta+1)}+g_2(x)\frac{t^{2\beta}}{\Gamma(2\beta+1)}+g_3(x)\frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)}+\cdots\,. \tag{23}$$

As we know, when $\alpha=\beta=1$, equation (21) has the kink-type soliton solutions

$$u(x,t)=\frac{b}{2k}+k\tanh(k\xi)\,,\quad v(x,t)=\frac{\lambda}{2}\Big(1+\frac{k}{b}\Big)+b\tanh(k\xi)\,, \tag{24}$$

constructed by Fan,[7] where

$$\xi=x+\frac{1}{4}\Big(-4k^2-6\lambda+\frac{6k\lambda}{b}+\frac{3b^2}{k^2}\Big)t\,,\quad k\neq 0\,,\quad b\neq 0\,.$$

The effectiveness and accuracy of the numerical results can be seen from the comparison figures.

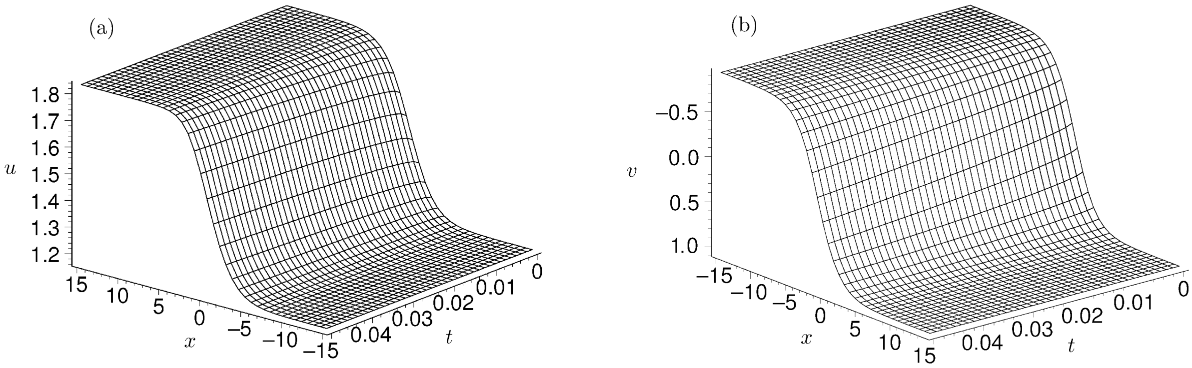

Fig. 3 Explicit numerical solutions for Eq. (21). (a) $u(x,t)$ as in Eq. (22); (b) $v(x,t)$ as in Eq. (23), with $\alpha=1/2$, $\beta=1/3$, $\lambda=1$, $k=1/3$.

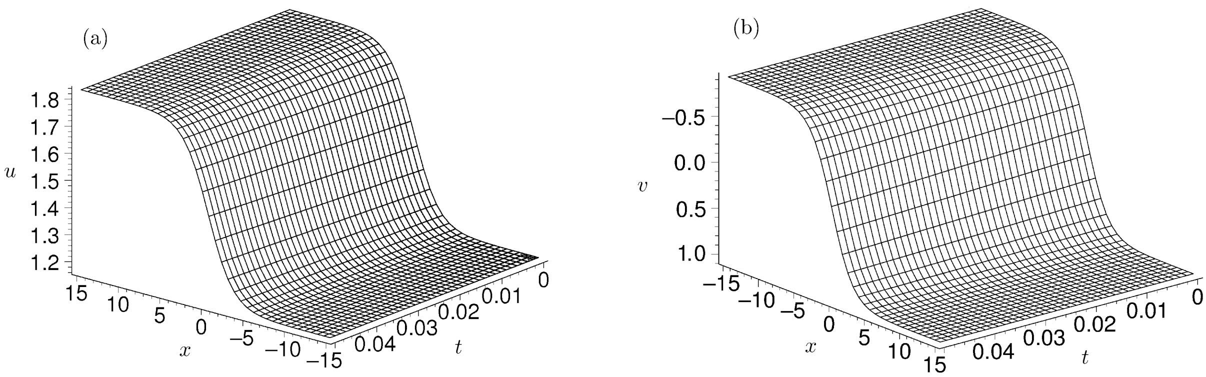

Fig. 4 Exact solutions (24) for Eq. (21). (a) $u(x,t)$; (b) $v(x,t)$ with $\alpha=1$, $\beta=1$, $\lambda=0.1$, $k=1/3$.

Figure 3 and 4 show the numerical solutions (22) and (23) with $\alpha=1/2$, $\beta=2/3$ and the exact ones (24) with $\alpha=\beta=1$ when $\lambda=0.1$, $b=1$, $k=1/3$, respectively. From these figures, we can know that the series solutions converge rapidly, so we say that a good approximation has been achieved.

4 Conclusion

In this paper, we have investigated a new type of FCNLEs. Based on the assumption that the fractional derivative satisfies the Caputo derivative, the ADM has been successfully extended to derive the explicit numerical solutions of the system. The study to the FCNLEs is very interesting and significant that is because their special cases contain many important mathematical physics models and coupled reaction-diffusion equations, such as KdV, mKdV equations as many reaction-diffusion equations, etc. The study shows: the ADM is a powerful and effective technique to handle a wide class of equations, in particular, the FCNLEs; the method is straightforward without any restrictive assumptions and special techniques; the continuity of the solutions depends on fractional derivative and the convergent speed is related with terms. In addition, the technique can also be extended to a generalized type of coupled equations with time and space fractional derivatives and the coupled systems with integer order. Whether we can introduce other new feasible derivative operators or algorithms to solve the problems and whether we can adopt other techniques to accelerate the convergent speed of the ADM solutions, these questions will be further studied.

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