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Numerical solutions of coupled Burgers equations with time- and space-fractional derivatives

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Abstract

In this paper, by introducing the fractional derivative in the sense of Caputo, the Adomian decomposition method is directly extended to study the coupled Burgers equations with time- and space-fractional derivatives. As a result, the realistic numerical solutions are obtained in a form of rapidly convergent series with easily computable components. The figures show the effectiveness and good accuracy of the proposed method.

Keywords: Adomian decomposition method; Fractional coupled nonlinear equations; Fractional calculus; Numerical solution

1. Introduction

Since Adomian firstly proposed the decomposition method [1] at the beginning of 1980s, the algorithm has been widely and effectively used for solving the analytic solutions of physically significant equations arranging from linear to nonlinear, from ordinary differential to partial differential, from integer to fractional, etc., [1–8]. With this method, we don't need to take any special technique and can easily obtain the realistic solution in the form of a rapidly convergent infinite series with each term computed conveniently.

As we all know, for the nonlinear equations of integer order, there exist many methods used to derive the explicit solutions [1,9–15]. However, for the fractional differential equations, there are only limited approaches, such as Laplace transform method [16], the Fourier transform method [17], the iteration method [18] and the operational method [19]. In recent ten years, the fractional differential equations have been attracted great attention and widely been used in the areas of physics and engineering [20]. Particularly in some interdisciplinary fields, the fractional derivatives are considered to be a very powerful and useful tool [16–18,20,21]. With the help of fractional derivatives, phenomena in electromagnetics, acoustics electrochem-

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istry and material science can be elegantly described [16–18,20,21]. With the help of fractional derivatives, the nonlinear oscillation of earthquake can be well modelled [21]; with fractional derivatives, the fluid dynamic traffic model can eliminate the deficiency arising from the assumption of continuum traffic flow [21].

The study to coupled Burgers equations is very significant for that the system is a simple model of sedimentation or evolution of scaled volume concentrations of two kinds of particles in fluid suspensions or colloids, under the effect of gravity [22]. It has been studied by many authors by different methods [23–25]. Especially recently, Dehghan et al. have obtained a good numerical results by using Adomian–Pade technique [25]. However, as we know, the study for the coupled Burgers equations with time- and space-fractional derivatives of this form

$$\begin{aligned} \frac{\partial^{\alpha_1} u}{\partial t^{\alpha_1}} &= \frac{\partial^2 u}{\partial x^2} + 2u\frac{\partial^{\alpha_2} u}{\partial x^{\alpha_2}} - \frac{\partial(uv)}{\partial x}, \\ \frac{\partial^{\beta_1} v}{\partial t^{\beta_1}} &= \frac{\partial^2 v}{\partial x^2} + 2v\frac{\partial^{\beta_2} v}{\partial x^{\beta_2}} - \frac{\partial(uv)}{\partial x}, \end{aligned} \tag{1}$$

by the Adomian method (ADM) has not been investigated. Here α_i and β_i $(i = 1,2)$ are the parameters standing for the order of the fractional time and space derivatives, respectively, and they satisfy $0 < \alpha_i, \beta_i \leqslant 1$ $(i = 1,2)$ and $t > 0$. In fact, different response systems can be obtained when at less one of the parameters varies. When $\alpha_i = \beta_i = 1$, the fractional equations reduce to the classical coupled Burgers equation. We introduce Caputo fractional derivative and extend the ADM to derive explicit and numerical solutions of the coupled Burgers equations with time- and space-fractional derivatives. The solutions obtained by us are calculated in the form of convergent series with easily computable components.

The paper is organized as follows. In Section 2, some necessary details on the fractional calculus are provided. In Section 3, the coupled Burgers equations with time- and space- fractional derivatives are studied with the ADM and figures are used to show the efficiency as well as the accuracy of the approximate results achieved. Finally, conclusions are followed.

2. Description of the fractional calculus

There are several mathematical definitions about fractional derivative [16,18]. Here, we adopt the two usually used definitions: the Caputo and its reverse operator Riemann–Liouville. That is because Caputo fractional derivative allows traditional initial condition assumption and boundary conditions. More details one can consults Ref. [16]. In the following, we will give the necessary notation and basic definition.

Definition 1. The Riemann–Liouville fractional integral operator of order $\alpha \geqslant 0$, for a function $f \in C_\mu$ $(\mu \geqslant -1)$ is defined as

$$\begin{aligned} &J^\alpha f(x) = \frac{1}{\Gamma(\alpha)}\int_0^x (x-t)^{\alpha-1} f(t)\mathrm{d}t, \quad \alpha > 0, \ x > 0, \\ &J^0 f(x) = f(x). \end{aligned} \tag{2}$$

For the convenience of establishing the results for the fractional coupled Burgers equations, we give two basic properties

$$\begin{aligned} &J^\alpha J^\beta f(x) = J^{\alpha+\beta} f(x), \\ &J^\alpha J^\beta f(x) = J^\beta J^\alpha f(x). \end{aligned} \tag{3}$$

For expression (2), when $f(x) = x^\beta$ we get another expression that will be used later

$$J^\alpha x^\beta = \frac{\Gamma(\beta+1)}{\Gamma(\alpha+\beta+1)} x^{\alpha+\beta}. \tag{4}$$

Definition 2. The fractional derivative of $f \in C_{-1}^n$ in the Caputo sense is defined as

$$D^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} f^{(n)}(\tau)\mathrm{d}\tau, \quad (n-1 < Re(\alpha) \leqslant n, n \in N). \tag{5}$$

According to Caputo's derivative, we can easily obtain the following expression:

$$D^\alpha K = 0, \quad K \text{ is a constant},$$
$$D^\alpha t^\beta = \begin{cases} 0, & \beta \leqslant \alpha - 1, \\ \frac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)} t^{\beta-\alpha}, & \beta > \alpha - 1. \end{cases} \tag{6}$$

Details on Caputo's derivative can be found in Ref. [16]. Here we just give the expressions used later: the linear relationship and the so-called Leibnitz rule, that's

$$D^\alpha(f(t)g(t)) = \sum_{k=0}^{\infty} \binom{\alpha}{k} f^{(k)}(t) D^{\alpha-k} f(t),$$
$$D^\alpha(\lambda f(t) + \mu g(t)) = \lambda D^\alpha f(t) + \mu D^\alpha g(t), \tag{7}$$

where λ, μ are constants, $f(t)$ is continuous in $[a, t]$ and $g(t)$ has $n+1$ continuous derivatives in $[a, t]$.

In addition, we also need the following two relations:

$$D^\alpha J^\alpha f(x) = f(x), \quad J^\alpha D^\alpha f(x) = f(x) - \sum_{k=0}^{\infty} f^k(0^+) \frac{x^k}{k!}, \qquad x > 0,$$

where $f \in C_\mu^m, \mu \geqslant -1, \alpha \geqslant 0$ and $\beta \geqslant 0$.

Remark. In this paper, we need to discuss the coupled Burgers equations with time- and space-fractional derivatives. When $\alpha \in R^+$ we just copy (5), when $\alpha = n \in N$ fractional derivative reduces to the commonly used derivative. That's to say

$$D_t^\alpha u(x,t) = \frac{\partial^\alpha u(x,t)}{\partial t^\alpha} = \begin{cases} \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} \frac{\partial^n u(x,\tau)}{\partial \tau^n} \mathrm{d}\tau, & n-1 < \alpha < n, \\ \frac{\partial^n u(x,t)}{\partial t^n}, & \alpha = n \in N. \end{cases} \tag{8}$$

The form of the space-fractional derivative is similar to the above and we just omit it here.

3. Applications of the ADM

Consider the coupled Burgers equations with time- and space-fractional derivatives Eq. (1). For convenience, we just discuss the following case $\alpha_1 = \alpha_2 = \alpha$ and $\beta_1 = \beta_2 = \beta$. The other cases are similar. In order to solve numerical solutions for Eq. (1) by using ADM, we write it in the operator form

$$D_t^\alpha u = L_{2x} u + 2u D_x^\alpha u - L_x(uv), \quad 0 < \alpha \leqslant 1,$$
$$D_t^\beta v = L_{2x} v + 2v D_x^\beta v - L_x(uv), \quad 0 < \beta \leqslant 1, \tag{9}$$

where $L_{nx} = \frac{\partial^n}{\partial x^n}$, the operators $D_t^\alpha, D_t^\beta, D_x^\alpha$ and D_x^β stand for the fractional derivative and are defined as in (8). Take the initial condition as

$$u(x,0) = f(x), \quad v(x,0) = g(x). \tag{10}$$

Applying the operator J^α and J^β, the inverse of D^α and D^β, respectively, on corresponding sub-equation of Eq. (9), using the initial condition (10), yields

$$u(x,t) = f(x) + J^\alpha L_{2x} u + 2J^\alpha \Phi_1(u) - J^\alpha \Psi(u,v),$$
$$v(x,t) = g(x) + J^\beta L_{2x} v + 2J^\beta \Phi_2(v) - J^\beta \Psi(u,v), \tag{11}$$

where $\Phi_1(u) = u D_x^\alpha u, \Phi_2(v) = v D_x^\beta v$ and $\Psi(u,v) = L_x(uv)$. Following Adomian decomposition method [1], the solutions are represented as infinite series like

$$u(x,t) = \sum_{n=0} \infty u_n(x,t), v(x,t) = \sum_{n=0} \infty v_n(x,t). \tag{12}$$

The nonlinear operators $\Phi_1(u)$, $\Phi_2(v)$ and $\Psi(u,v)$ are decomposed in these forms

$$\Phi_1(u) = \sum_{n=0} \infty A_n, \quad \Phi_2(v) = \sum_{n=0} \infty B_n, \quad \Psi(u,v) = \sum_{n=0} \infty C_n, \tag{13}$$

where A_n, B_n and C_n are the so-called Adomian polynomials and have the form

$$\begin{aligned}
A_n &= \frac{1}{n!}\left[\frac{\mathrm{d}^n}{\mathrm{d}\lambda^n}\Phi_1\left(\sum_{k=0}^{\infty}\lambda^k u_k\right)\right]_{\lambda=0} = \frac{1}{n!}\frac{\mathrm{d}^n}{\mathrm{d}\lambda^n}\left[\left(\sum_{k=0}^{\infty}\lambda^k u_k\right)\left(D_x^{\alpha}\left(\sum_{k=0}^{\infty}\lambda^k u_k\right)\right)\right]_{\lambda=0}, \\
B_n &= \frac{1}{n!}\left[\frac{\mathrm{d}^n}{\mathrm{d}\lambda^n}\Phi_2\left(\sum_{k=0}^{\infty}\lambda^k v_k\right)\right]_{\lambda=0} = \frac{1}{n!}\frac{\mathrm{d}^n}{\mathrm{d}\lambda^n}\left[\left(\sum_{k=0}^{\infty}\lambda^k v_k\right)\left(D_x^{\beta}\left(\sum_{k=0}^{\infty}\lambda^k v_k\right)\right)\right]_{\lambda=0}, \\
C_n &= \frac{1}{n!}\left[\frac{\mathrm{d}^n}{\mathrm{d}\lambda^n}\Psi\left(\sum_{k=0}^{\infty}\lambda^k u_k, \sum_{k=0}^{\infty}\lambda^k v_k\right)\right]_{\lambda=0} = \frac{1}{n!}\frac{\mathrm{d}^n}{\mathrm{d}\lambda^n}\left[L_x\left(\left(\sum_{k=0}^{\infty}\lambda^k u_k\right)\left(\sum_{k=0}^{\infty}\lambda^k v_k\right)\right)\right]_{\lambda=0}.
\end{aligned} \tag{14}$$

In fact, with the aid of *Maple*, these Adomian polynomials (14) can be easily calculated. Here we give the expressions

$$A_n = \sum_{k=0}^{n} u_k D_x^{\alpha} u_{n-k}, \quad B_n = \sum_{k=0}^{n} v_k D_x^{\beta} v_{n-k}, \quad C_n = \sum_{k=0}^{n} \frac{\partial}{\partial x}(u_k v_{n-k}). \tag{15}$$

Then substituting the decomposition series (12) and (13) into Eq. (11), yields the following recursive formulas:

$$\begin{aligned}
&u_0 = f(x), \ \ u_{n+1} = J^{\alpha} L_{2x} u_n + 2J^{\alpha} A_n - J^{\alpha} C_n, \quad n \geqslant 0, \\
&v_0 = g(x), \ \ v_{n+1} = J^{\beta} L_{2x} v_n + 2J^{\beta} B_n - J^{\beta} C_n, \quad n \geqslant 0.
\end{aligned} \tag{16}$$

In the following, according to the above steps, we will derive the numerical solutions for the coupled Burgers equations with time- and space-fractional derivatives in details.

3.1. Numerical solutions of the time-fractional coupled Burgers equations

Consider the following form of time-fractional coupled Burgers equations:

$$\begin{cases} D_t^{\alpha} u = L_{2x} u + 2u L_x u - L_x(uv) \quad (0 < \alpha \leqslant 1), \\ D_t^{\beta} v = L_{2x} v + 2v L_x v - L_x(uv) \quad (0 < \beta \leqslant 1), \end{cases} \tag{17}$$

with the initial condition

$$\begin{cases} u(x,0) = f(x) = \sin x, \\ v(x,0) = g(x) = \sin x. \end{cases} \tag{18}$$

The exact solutions of (17) for the special case $\alpha = \beta = 1$ is

$$\begin{cases} u(x,t) = \mathrm{e}^{-t} \sin x, \\ v(x,t) = \mathrm{e}^{-t} \sin x. \end{cases} \tag{19}$$

In order to obtain the numerical solutions of Eq. (17), substituting the initial condition (18) and using the Adomian polynomials (15) into the expression (16), we can calculate the results. For simplify, we only give the first few terms of series:

$$\begin{aligned}
u_0 &= f(x), v_0 = g(x), \\
u_1 &= J^{\alpha} L_{xx} u_0 + 2J^{\alpha} A_0 - J^{\alpha} C_0 \\
&= J^{\alpha}[u_{0xx}] + 2J^{\alpha}[u_0 u_{0x}] - J^{\alpha}[u_0 v_{0x} + u_{0x} v_0]
\end{aligned}$$

$$
\begin{aligned}
&= f_1(x)\frac{t^\alpha}{\Gamma(\alpha+1)},\\
v_1 &= J^\beta L_{xx}v_0 + 2J^\beta B_0 - J^\beta C_0\\
&= J^\beta[v_{0xx}] + 2J^\beta[v_0v_{0x}] - J^\beta[u_0v_{0x} + u_{0x}v_0]\\
&= g_1(x)\frac{t^\beta}{\Gamma(\beta+1)},\\
u_2 &= J^\alpha L_{xx}u_1 + 2J^\alpha A_1 - J^\alpha C_1\\
&= J^\alpha[u_{1xx}] + 2J^\alpha[u_0u_{1x} + u_{0x}u_1] - J^\alpha[u_0v_{1x} + u_{1x}v_0]\\
&= f_2(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + f_3(x)\frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)},\\
v_2 &= J^\beta L_{xx}v_1 + 2J^\beta B_1 - J^\beta C_1\\
&= J^\beta[v_{1xx}] + 2J^\beta[v_0v_{1x} + v_{0x}v_1] - J^\beta[u_0v_{1x} + u_{1x}v_0]\\
&= g_2(x)\frac{t^{2\beta}}{\Gamma(2\beta+1)} + g_3(x)\frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)},
\end{aligned}
$$

where

$$
\begin{aligned}
&f(x) = \sin x, \quad f_1(x) = f_{xx} + 2ff_x - fg_x - f_xg, \quad f_2(x) = f_{1xx} + 2ff_{1x} + 2f_1f_x - f_{1x}g, f_3(x) = -f_xg_1,\\
&g(x) = \sin x, \quad g_1(x) = g_{xx} + 2gg_x - fg_x - f_xg, \quad g_2(x) = g_{1xx} + 2gg_{1x} + 2g_1g_x - f_xg_1, \quad g_3(x) = -f_{1x}g.
\end{aligned}
$$

Then we can have the numerical solutions of time-fractional coupled Burgers equation (17) in series form

$$
u(x,t) = f + f_1(x)\frac{t^\alpha}{\Gamma(\alpha+1)} + f_2(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + f_3(x)\frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)} + \cdots, \tag{20}
$$

$$
v(x,t) = g + g_1(x)\frac{t^\beta}{\Gamma(\beta+1)} + g_2(x)\frac{t^{2\beta}}{\Gamma(2\beta+1)} + g_3(x)\frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)} + \cdots. \tag{21}
$$

In order to verify the efficiency and accuracy of the proposed ADM for the time-fractional coupled Burgers equations, we draw figures for the numerical solutions with $\alpha = \frac{1}{2}, \beta = \frac{1}{4}$ as well as the exact solutions (19) when $\alpha = \beta = 1$. Fig. 1 stands for the numerical solutions of (20) and (21). Fig. 2 shows the exact solutions (19). From the figs, we can know the series solutions converge rapidly and we nearly cannot tell the difference between solutions obtained by different methods. That's to say a good approximation is achieved by using N-term approximation of the ADM solutions.

Remark. The accuracy of the numerical solutions obtained depends on how many terms we choose. The more terms we calculate, the smaller the error becomes. That's to say: in order to reduce the overall errors, what we need is to add new terms to the decomposition series.

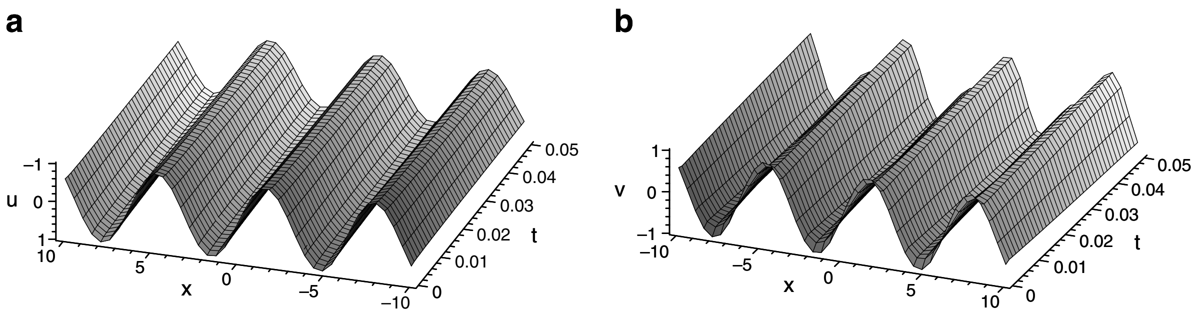

Fig. 1. Explicit numerical solutions for Eq. (17): (a) (20) $u(x,t)$, (b) (21) $v(x,t)$, with $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{4}$.

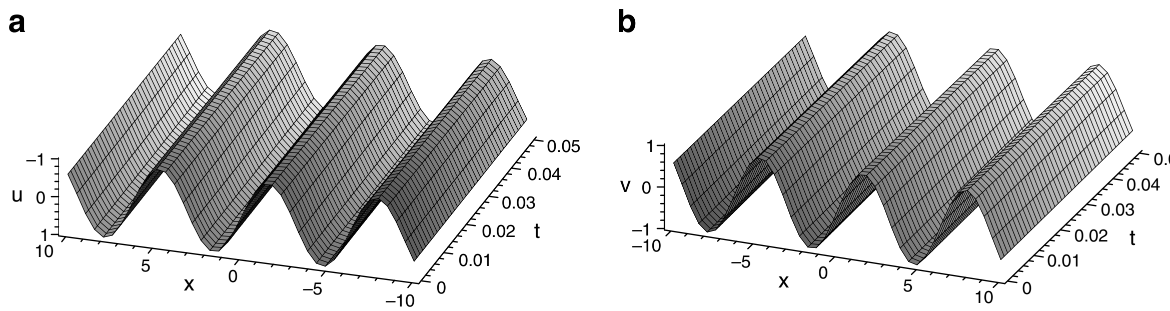

Fig. 2. Exact solutions (18) for Eq. (17): (a) $u(x,t) = e^{-t}\sin x$, (b) $v(x,t) = e^{-t}\sin x$, with $\alpha = 1$ and $\beta = 1$.

3.2. Numerical solutions for the space-fractional coupled Burgers equations

In this section, we will take the space-fractional coupled equations as another example to illustrate the efficiency of the method. As the main method is the same as the above, we will omit the heavy calculation and only give some necessary expressions.

Considering the operator form of the space-fractional coupled Burgers equations

$$\begin{cases} D_t u = L_{2x}u + 2uD_x^{\alpha}u - L_x(uv), & (0 < \alpha \leqslant 1), \\ D_t v = L_{2x}v + 2vD_x^{\beta}v - L_x(uv), & (0 < \beta \leqslant 1). \end{cases} \tag{22}$$

Assuming the initial condition as

$$\begin{cases} u(x,0) = f(x) = x^2, \\ v(x,0) = g(x) = x^3. \end{cases} \tag{23}$$

Remark. In fact, arbitrary function in the initial condition (23) can be chosen. In order to avoid the difficult of fractional differentiation computation, we just set it as the simple form x^n.

In order to estimate the numerical solutions of Eq. (22), substituting (12), (13) and the initial condition (23) into (16), we can get the Adomian solutions. Here we only give the first few terms of series solutions:

$$\begin{aligned}
u_0 &= x^2, \quad v_0 = x^3, \\
u_1 &= JL_{xx}u_0 + 2JA_0 - JC_0 \\
&= J[u_{0xx}] + 2J[u_0D_x^{\alpha}u_{0x}] - J[u_0v_{0x} + u_{0x}v_0] \\
&= (2 - 5x^4 + f_1x^{4-\alpha})t, \\
v_1 &= JL_{xx}v_0 + 2JB_0 - JC_0 \\
&= J[v_{0xx}] + 2J[v_0D_x^{\beta}v_{0x}] - J[u_0v_{0x} + u_{0x}v_0] \\
&= (6x - 5x^4 + g_1x^{6-\beta})t, \\
u_2 &= JL_{xx}u_1 + 2JA_1 - JC_1 \\
&= J[u_{1xx}] + 2J[u_0D_x^{\alpha}u_{1x} + D_x^{\alpha}u_{0x}u_1] - J[u_0v_{1x} + u_{1x}v_0] \\
&= \frac{t^2}{2}(f_2x^{6-2\alpha} + f_3x^{6-\alpha} + f_4x^{2-\alpha} + f_5x^{7-\beta} + 20x^6 + 10x^5 - 72x^2), \\
v_2 &= JL_{xx}v_1 + 2JB_1 - JC_1 \\
&= J[v_{1xx}] + 2J[v_0D_x^{\beta}v_{1x} + D_x^{\beta}v_{0x}v_1] - J[u_0v_{1x} + u_{1x}v_0] \\
&= \frac{t^2}{2}(g_2x^{9-2\beta} + g_3x^{7-\beta} + g_4x^{4-\beta} + g_5x^{6-\alpha} + 20x^6 + 10x^5 - 72x^2),
\end{aligned}$$

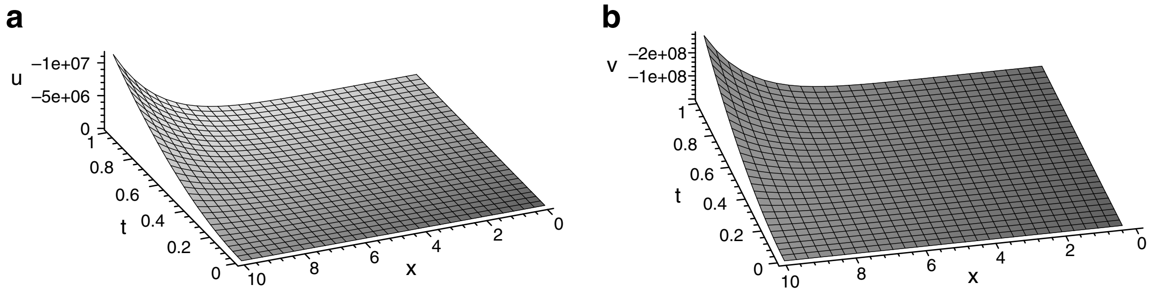

Fig. 3. Approximate numerical solutions for Eq. (22): (a) (24) $u(x,t)$, (b) (25) $v(x,t)$, with $\alpha=\frac{1}{4}$ and $\beta=\frac{1}{3}$.

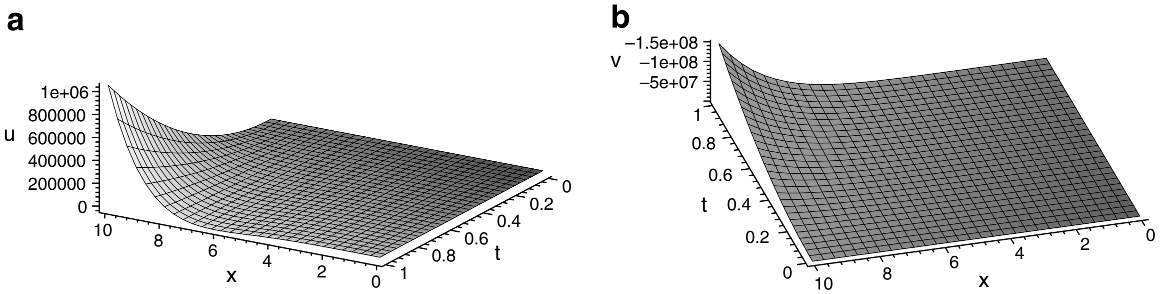

Fig. 4. Approximate numerical solutions $u(x,t)$ and $v(x,t)$ for Eq. (22) with $\alpha=\beta=1$. (a) $u(x,t)$, (b) $v(x,t)$.

where

$$f(x)=x^2,\quad f_1(x)=\frac{4}{\Gamma(3-\alpha)},\quad f_2(x)=\left[\frac{4}{\Gamma(3-\alpha)}+\frac{2\Gamma(5-\alpha)}{\Gamma(5-2\alpha)}\right]f_1,$$
$$f_3(x)=(\alpha-4)f_1-\frac{20}{\Gamma(3-\alpha)}-\frac{240}{\Gamma(5-\alpha)},\quad f_4(x)=(4-\alpha)(3-\alpha)f_1+\frac{8}{\Gamma(3-\alpha)},\quad f_5(x)=-2g_1,$$
$$g(x)=x^3,\quad g_1(x)=\frac{12}{\Gamma(4-\beta)},\quad g_2(x)=\left[\frac{12}{\Gamma(4-\beta)}-\frac{2\Gamma(7-\beta)}{\Gamma(7-2\beta)}\right]g_1,$$
$$g_3(x)=-2g_1-\frac{60}{\Gamma(4-\beta)}-\frac{240}{\Gamma(5-\beta)},\quad g_4(x)=(6-\beta)(5-\beta)g_1+\frac{12}{\Gamma(2-\beta)}+\frac{72}{\Gamma(4-\beta)},$$
$$g_5(x)=(\alpha-4)f_1.$$

Then we obtain the numerical solutions of space-fractional equation (22) in series form

$$u(x,t)=x^2+(2-5x^4+f_1x^{4-\alpha})t+\frac{t^2}{2}(f_2x^{6-2\alpha}+f_3x^{6-\alpha}+f_4x^{2-\alpha}+f_5x^{7-\beta}+20x^6+10x^5-72x^2)+\cdots,\tag{24}$$

$$v(x,t)=x^3+(6x-5x^4+g_1x^{6-\beta})t+\frac{t^2}{2}(g_2x^{9-2\beta}+g_3x^{7-\beta}+g_4x^{4-\beta}+g_5x^{6-\alpha}+20x^6+10x^5-72x^2)+\cdots.\tag{25}$$

Fig. 3a and b shows the numerical solutions (24) and (25) for the space-fractional equation (22) with $\alpha=\frac{1}{4}$ and $\beta=\frac{1}{3}$. Fig. 4a and b is the figure for Eq. (22) with $\alpha=\beta=1$. From figures we can see that the Adomian solutions converge rapidly, which indicate that good results are achieved.

4. Conclusion

In this paper, combining the Caputo fractional derivative, the ADM has been successfully extended to derive the explicit numerical solutions for the time- and space-fractional coupled Burgers equations with initial condition. The above procedure shows that: (1) the ADM is an efficient and powerful method in solving a wide class of equations, in particular, coupled fractional order equations; (2) the method is straightforward without any restrictive assumptions and special techniques; (3) the continuity of the solution depends on the time- and space-fractional derivatives and the convergent speed is related with terms. Whether we can introduce other new feasible derivative operator or algorithms to solve differential equations and whether existing other techniques that can accelerate the convergent speed for the ADM solution, we hope these questions will be further studied.

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