Computers and Mathematics with Applications 76 (2018) 1938–1949

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Rogue wave and a pair of resonance stripe solitons to KP equation

Xiaoen Zhang [a], Yong Chen [a,b,*], Xiaoyan Tang [a]

[a] *Shanghai Key Laboratory of Trustworthy Computing, East China Normal University, Shanghai, 200062, China*
[b] *Department of Physics, Zhejiang Normal University, Jinhua 321004, China*

ARTICLE INFO

Article history:
Received 23 March 2018
Received in revised form 21 June 2018
Accepted 29 July 2018
Available online 14 August 2018

Keywords:
Hirota bilinear method
KP equation
Rogue wave
Stripe soliton

ABSTRACT

The rogue wave and a pair of resonance stripe solitons to KP equation are discovered. First, based on the bilinear method, some lump solutions are obtained containing six parameters, four of which must cater to the non-zero conditions so as to insure the solution analytic and rationally localized. Second, a one-stripe-soliton-lump solution is presented and the interaction shows that the lump soliton can be drowned or swallowed by the stripe soliton, conversely, the lump soliton is spit out from the stripe soliton. Finally, a new ansatz of combination of positive quadratic functions and hyperbolic functions is introduced, and thus a novel nonlinear phenomenon is explored. It is interesting that a rogue wave can be excited. It is observed that the rogue wave, possessing a peak wave profile, arises from one of the resonance stripe solitons, moves to the other, and then disappears. Therefore, a rogue wave can be generated by the interaction between the lump soliton and the pair of resonance stripe solitons. However, compared with classic rouge wave, the dynamics of above nonlinear waves are quite different, which are graphically demonstrated.

1. Introduction

In 1964, Draper wrote "Stories abound of monstrous waves; every sailor has his tale of how a great wave arose from nowhere and hit his ship leaving a trail of damaged lifeboats and shattered crockery" in his paper titled "Freakocean waves" [1]. Thereafter, a great number of marine disasters aroused by rouge waves are reported in the following decades [2–4]. Rogue waves are large and spontaneous ocean surface waves that occur in the sea and are a threat even to large ships and ocean liners [5]. Recently, rogue wave phenomena have become one of the most active and important research areas on both experimental observations and theoretical analysis, since they exist not only in ocean but also in various other fields, such as optics [6], atmosphere [7], Bose–Einstein condensates [8], superfluid [9], capillary flow [10] and even finance [11]. However, due to the complexity of the geographical environment and the limitations of observation instruments, it is difficult to lucubrate the rogue wave in the ocean. Mathematically, scientists focus on investigating the dynamical properties and mechanisms of the mysterious rogue wave via nonlinear partial differential equations. From the view point of mathematics, rouge wave solutions appear in the class of nonlinear Schrodinger (NLS) equation, while in Physics, they describe steep waves local both in time and space with their amplitudes more than triple times of the height of the background field. In 1983, Peregrine first presented a simplest wave solution of the NLS equation by the famous inverse scattering method [12]. This solution consists of second order rational polynomials and exponential functions, approaches a nonzero constant background as time goes to positive and negative infinity, but develops a localized hump with its peak amplitude three times of the

* Corresponding author at: Shanghai Key Laboratory of Trustworthy Computing, East China Normal University, Shanghai, 200062, China.
E-mail address: ychen@sei.ecnu.edu.cn (Y. Chen).

https://doi.org/10.1016/j.camwa.2018.07.040
0898-1221/

constant background during its lifetime. As can be easily seen, the evolution of the solution fits two basic characteristics of a rogue wave: momentariness and steepness. With the successive appearance of rouge waves in different fields in recent years, Peregrine soliton came into our sight again, acting as a rogue wave solution. Now, to investigate every kind of rouge wave solution of nonlinear equations has become a focus issue for scientists working in different fields. For instance, inspired by the work of Peregrine, people have great interests in the rogue wave solutions of the NLS equation and their dynamical properties, in particular, whether the characteristic properties of the theoretical rogue wave solutions coincide with those in the real physics experiments, the existence of high order rogue waves, and how to classify rogue waves. Akhmediev et al. observed rogue waves in the wave tank in 2011 [13], and one year later, they obtained high order (up to the fifth order) rogue waves experimentally. The results indicate that the characteristics of the rogue waves observed in the experiment are in good agreement with the rational wave solutions. Consequently, it is not only experimentally proved the existence of rogue waves and even high order rogue waves, but also illustrated that rogue waves in the real world can be well described by analytic solutions. In 2013, Akhmediev gave a complete classification of the high order (up to the sixth order) rogue waves of the NLS equation [14].

It is a natural thinking that apart from NLS type equations, whether other important mathematics physical equations, especially integrable systems [15,16], for example the KP equation, have rogue wave solutions. Moreover, rogue waves have been so far studied mostly in one dimensional, but in reality, ocean surface waves are in two dimensions. Therefore, one has to investigate rogue waves in two dimensional nonlinear models [17], and even in more spatiotemporal dimensions. Another thinking is that as for the methods to find a rogue wave solution, apart from the classic Darboux transformation, can other methods famous for integrable systems be used to construct rogue wave solutions. The Hirota direct method was successfully applied to describe the MRW solutions by Ohta and Yang [18,19]. In 2010, Dubard and Matveev discovered the multi-rogue wave (MRW) solutions, which can describe the famous Three Sister waves observed in ocean by higher Peregrine breather and the new comprehension of (2+1)-dimensional rogue wave through the NLS-KP equation.

Based on the generalized DT, Wang and Chen derived a unified Nth-order rogue wave solution for the AB system and discovered the 'four-peak' shaped rogue wave [20]. The AB system serves as model equations to describe marginally unstable baroclinic wave packets in geophysical fluids [21] and ultra-short optical pulse propagation in nonlinear optics [22]. Meanwhile, two-dimensional dark, intermediate counterparts of rogue waves and even their interaction and superposition were found for the two-dimensional coupled Yajima–Oikawa system by using the bilinear and KP hierarchy reduction methods [23]. Apart from, the rogue wave in reduced Jimbo–Miwa equation and (2+1)-dimensional KdV equation are also derived with the Hirota bilinear method [24–26].

In contrast, lump solution is a special kind of rational solution [27,28], rationally localized in all directions, while rogue wave solution is a particularly interesting class of lump-like solutions. In 2002, Lou et al. studied the lump solution with the variable separation method [29]. Recently, Ma proposed the positive quadratic function to obtain lump solution. Special examples of lump solutions have been found, for the KPI equation [30–32], BKP equation [33], p-gKP and p-gBKP equations [34] and Boussinesq equation [35]. In addition, collisions may happen among different solitons. There are two kinds of collisions, either elastic or inelastic [36]. It is reported that lump solutions will keep their shapes, amplitudes, velocities after the collision with other soliton solutions, which means that the collision is completely elastic [37]. However, many collisions are completely inelastic [38]. For instance, Becker et al. studied the inelastic collision of solitary waves in an isotropic Bose–Einstein condensates [39]; Tan discussed the rational breather wave swallowed by kink wave [40]; Tang showed the lump solution drowned by a stripe solution [41]. Collisions will change essentially under different conditions.

In this paper, we construct the rogue wave to the KP equation [42]

$$(u_t + h_1 u u_x + h_2 u_{xxx} + h_3 u_x)_x + h_4 u_{yy} + h_5 u_{zz} = 0, \tag{1}$$

It has a wide range of applications in plasma [43,44]. Based on the bilinear method, some lump solutions are presented in Section 2, containing six parameters, which are analytical and rationally localized. In Section 3, the lump soliton and one stripe soliton are presented and their interaction shows that the lump soliton can be drowned or swallowed by the stripe soliton. We then extend the method proposed in [30] to a new combination of positive quadratic functions and hyperbolic functions. It is very interesting that a novel nonlinear phenomenon: a rogue wave and a pair of resonance stripe solitons is excited. It is observed that the rogue wave, possessing a peak wave profile, appears on one line of the resonance stripe solitons, moves to the other, and then disappears. Therefore, a rogue wave is originated from the interaction between the lump soliton and the pair of resonance stripe solitons. It should be pointed out that, besides some common characters, the rogue waves presented in this paper have some different properties from the traditional rogue waves. Our results show that a two-dimensional rogue wave, excited from two resonance stripe solitons, has a zero background, and as time flows, it reaches its maximum amplitude, decays gradually and finally disappears, which can describe the ocean rogue wave more essentially from the realistic and physical point of view. It is remarkable that the previously obtained two dimensional rogue waves are actually line rogue waves, but rogue waves obtained here are localized in two dimensions.

2. Lump solution to KP equation

When $h_1 = -1, h_2 = -\frac{1}{3}, h_3 = 1, h_4 = 1, h_5 = -\frac{2}{3}$, Eq. (1) becomes

$$(u_t - u u_x - \frac{1}{3} u_{xxx} + u_x)_x + u_{yy} - \frac{2}{3} u_{zz} = 0. \tag{2}$$

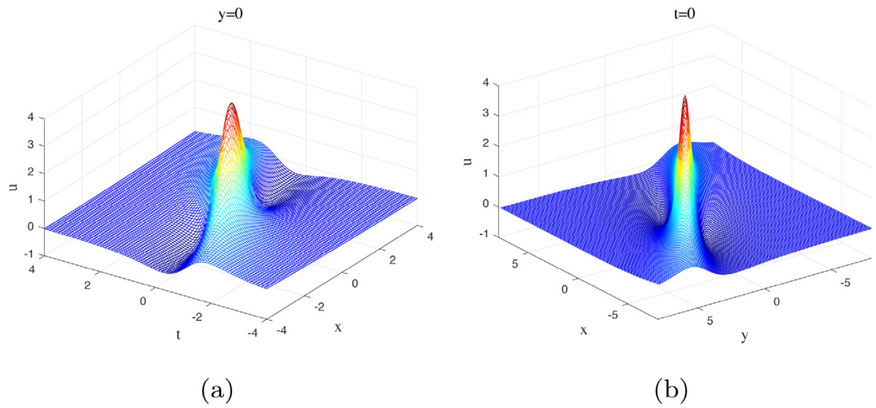


Fig. 1. (Color online) Plots of lump solution(8) with the parameters $a_1 = 1.5$, $a_2 = 2.1$, $a_4 = 0$, $a_5 = -0.2$, $a_6 = 0.8$, $a_8 = 0$, and (a) $y = 0$, (b) $t = 0$.

which can be converted into a bilinear with the transformation $u = 4(\ln f)_{xx}$,

$$(D_xD_t - \frac{1}{3}D_x^4 + D_x^2 + D_y^2 - \frac{2}{3}D_z^2)f \cdot f = 0, \tag{3}$$

When $z = x$, Eq. (3) can be expanded as

$$\begin{aligned}&(D_xD_t - \frac{1}{3}D_x^4 + \frac{1}{3}D_x^2 + D_y^2)f \cdot f \\ &= 2f_{xt}f - 2f_xf_t - \frac{2}{3}f_{xxxx}f + \frac{8}{3}f_{xxx}f_x - 2f_{xx}^2 + \frac{2}{3}f_{xx}f - \frac{2}{3}f_x^2 + 2f_{yy}f - 2f_y^2.\end{aligned} \tag{4}$$

Assume

$$f = g^2 + h^2 + a_9, \quad g = a_1x + a_2y + a_3t + a_4, \quad h = a_5x + a_6y + a_7t + a_8, \tag{5}$$

where $a_i, (i = 1, 2, \ldots, 9)$ are parameters to be determined. By substituting f into Eq. (4), with a direct calculation, these parameters can be expressed:

$$\begin{cases} a_3 = \dfrac{-a_1^3 + (3a_6^2 - a_5^2 - 3a_2^2)a_1 - 6a_5a_6a_2}{3a_5^2 + 3a_1^2}, \; a_9 = \dfrac{(a_1^2 + a_5^2)^3}{(a_1a_6 - a_2a_5)^2}, \\ a_7 = \dfrac{-a_5^3 + (3a_2^2 - a_1^2 - 3a_6^2)a_5 - 6a_1a_6a_2}{3a_5^2 + 3a_1^2}, \end{cases} \tag{6}$$

with the conditions

$$a_1a_5 \neq 0 \quad \text{and} \quad a_1a_6 - a_2a_5 \neq 0 \tag{7}$$

in order to insure f analytical and positive.

Consequently, the solution of u can be written, through the transformation $u = 4(\ln f)_{xx}$ as

$$\begin{aligned} u = &\frac{8(a_1^2 + a_5^2)}{(a_1x + a_2y + a_3t + a_4)^2 + (a_5x + a_6y + a_7t + a_8)^2 + a_9} \\ &- \frac{16((a_1x + a_2y + a_3t + a_4)a_1 + (a_5x + a_6y + a_7t + a_8)a_5)^2}{((a_1x + a_2y + a_3t + a_4)^2 + (a_5x + a_6y + a_7t + a_8)^2 + a_9)^2}. \end{aligned} \tag{8}$$

Which describes lump waves, as shown in Figs. 1 and 2 with a particular parameter value.

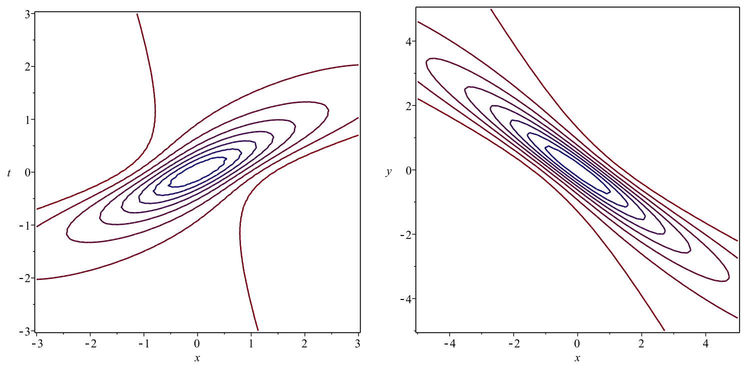
3. The interaction between lump soliton and one stripe soliton

In this section, assume f as a combination of two positive quadratic functions and one exponential function, that is,

$$f_1 = m_1^2 + n_1^2 + l_1 + a_9, \tag{9}$$

where

$$m_1 = a_1x + a_2y + a_3t + a_4, n_1 = a_5x + a_6y + a_7t + a_8, l_1 = ke^{k_1x + k_2y + k_3t}.$$



(a) (b)

Fig. 2. (Color online) Corresponding density plots of the lump solutions in Fig. 1.

Substitute Eq. (9) into Eq. (4), the parameters in (9) are calculated as follows:

$$a_2=\frac{a_1k_2\mp a_5k_1^2}{k_1},\ a_3=\frac{3a_1k_1^4\pm 6a_5k_1^2k_2-a_1k_1^2-3a_1k_2^2}{k_1^2},\ a_6=\frac{a_5k_2\pm a_1k_1^2}{k_1},$$
$$a_7=\frac{3a_5k_1^4\mp 6a_1k_1^2k_2-a_5k_1^2-3a_5k_2^2}{3k_1^2},\ a_9=\frac{a_1^2+a_5^2}{k_1^2},\ k_3=\frac{k_1^4-k_1^2-3k_2^2}{3k_1}. \tag{10}$$

With the lower sign is needed in the following analysis.

Based on the transformation $u=4(\ln f_1)_{xx}$, the solution of Eq. (4) in this case reads

$$u=\frac{4(2a_1^2+2a_5^2+k_1^2l)}{f_1}-\frac{4(2a_1m+2a_5n+k_1l)^2}{f_1^2}, \tag{11}$$

where

$$f_1=(a_1x+\frac{a_1k_2+a_5k_1^2}{k_1}y+\frac{3a_1k_1^4-6a_5k_1^2k_2-a_1k_1^2-3a_1k_2^2}{k_1^2}t+a_4)^2+ke^{k_1x+k_2y+\frac{k_1^4-k_1^2-3k_2^2}{3k_1}t}$$
$$+(a_5x+\frac{a_5k_2-a_1k_1^2}{k_1}y+\frac{3a_5k_1^4+6a_1k_1^2k_2-a_5k_1^2-3a_5k_2^2}{3k_1^2}t+a_8)^2+\frac{a_1^2+a_5^2}{k_1^2},$$
$$m_1=a_1x+\frac{a_1k_2+a_5k_1^2}{k_1}y+\frac{3a_1k_1^4-6a_5k_1^2k_2-a_1k_1^2-3a_1k_2^2}{k_1^2}t+a_4,$$
$$n_1=a_5x+\frac{a_5k_2-a_1k_1^2}{k_1}y+\frac{3a_5k_1^4+6a_1k_1^2k_2-a_5k_1^2-3a_5k_2^2}{3k_1^2}t+a_8,$$
$$l_1=ke^{k_1x+k_2y+\frac{k_1^4-k_1^2-3k_2^2}{3k_1}t}.$$

By choosing appropriate values of these parameters, collisions between the lump soliton and one stripe soliton are shown in Figs. 3 and 4:

It is observed from Fig. 4(a) that, lump soliton is separated with the stripe soliton, when t goes to 0, the lump soliton begins to be swallowed by the stripe soliton with its energy transferring into the stripe soliton gradually, until it is completely swallowed by the stripe soliton, when two kinds of solitons roll into one.

Next we give the detailed description about the collision. When choosing $a_1=\frac{4}{5}, a_4=0, a_5=\frac{6}{5}, a_8=0, k=2, k_1=-3, k_2=0$, then the solution of Eq. (11) is changed into

$$u=\frac{4\left(\frac{104}{25}+18e^{-3x-8t}\right)}{\left(\frac{4}{5}x-\frac{18}{5}y+\frac{104}{15}t\right)^2+\left(\frac{6}{5}x+\frac{12}{5}y+\frac{52}{15}t\right)^2+2e^{-3x-8t}+\frac{52}{225}}$$
$$-\frac{4\left(\frac{104}{25}x+\frac{2704}{75}t-6e^{-3x-8t}\right)^2}{\left(\left(\frac{4}{5}x-\frac{18}{5}y+\frac{104}{15}t\right)^2+\left(\frac{6}{5}x+\frac{12}{5}y+\frac{52}{15}t\right)^2+2e^{-3x-8t}+\frac{52}{225}\right)^2}, \tag{12}$$

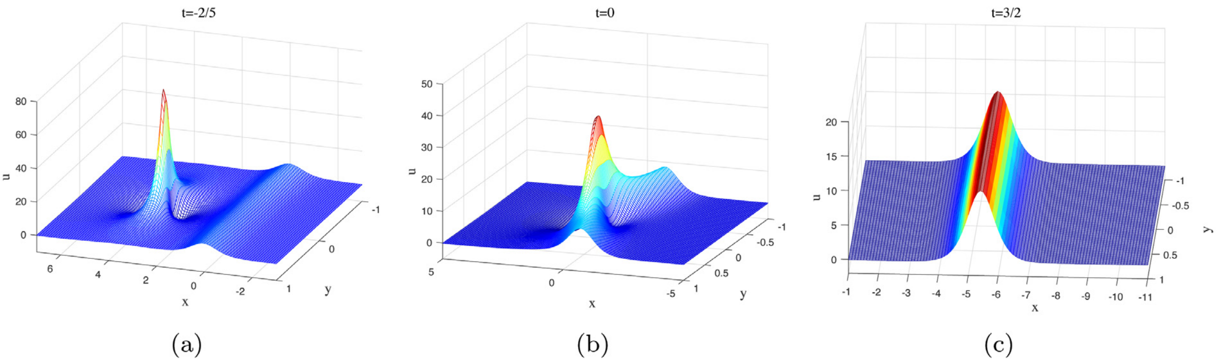


Fig. 3. (Color online) Evolutional plots of Eq. (11) by choosing $a_1 = \frac{4}{5}, a_4 = 0, a_5 = \frac{6}{5}, a_8 = 0, k = 2, k_1 = -3, k_2 = 0,$ at times (a) $t = -\frac{2}{5}$, (one stripe soliton and lump soliton), (b) t = 0, (lump soliton and stripe soliton begin to collide) and (c) $t = \frac{3}{2}$, (lump soliton is swallowed by the stripe soliton).

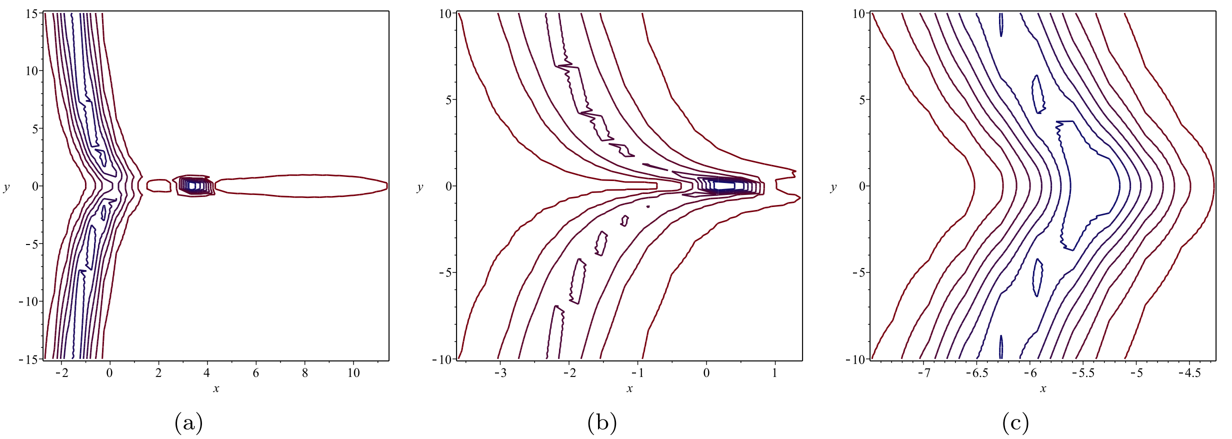


Fig. 4. (Color online) Corresponding density plots of the collision between lump and stripe soliton as shown in Fig. 3.

the speed of the stripe soliton is $v_s = \frac{-8}{3}$ and the speed of the lump soliton is $v_l = -\frac{26}{3}$, the speed of v_l is faster than the v_s and their directions are all along the negative x axis and the speed along the y axis is zero. So we can discuss the collision under $y = 0$. Before the collision, such as $t = -\frac{2}{5}$, then Eq. (12) is changed into

$$u = \frac{70200\left(16875e^{-3x+\frac{16}{5}}x^2 - 94500xe^{-3x+\frac{16}{5}} - 3900x^2 + 130425e^{-3x+\frac{16}{5}} + 27040x - 46436\right)}{\left(5850x^2 + 5625e^{-3x+\frac{16}{5}} - 40560x + 70954\right)^2}$$

then the amplitude to stripe soliton is $u = 5.3555$ at $x = 0.18579$ and the amplitude to lump soliton is $u = 71.7690$ at$x = 3.46756$. After the collision, such as $t = \frac{3}{2}$, then Eq. (12) is changed into

$$u = 936\frac{\left(2025e^{-3x-12}x^2 + 55350xe^{-3x-12} - 468x^2 + 378000e^{-3x-12} - 12168x - 79040\right)}{\left(234x^2 + 225e^{-3x-12} + 6084x + 39572\right)^2}$$

then the amplitude of this solution is $u = 10.5705$ at $x = -5.374$, so it is obvious that the amplitude of lump soliton is much larger than the stripe soliton before the collision but its amplitude decreases rapidly after the collision until it is swallowed by the stripe soliton completely. Then the common speed is the speed of the stripe soliton $v_c = \frac{8}{3}$. The evolution dynamics behavior is shown in Fig. 5.

In return, choosing an opposite propagation speed of the stripe soliton, then one can see that the lump soliton is not visible, and as times goes on, lump soliton appears and then separates from the stripe soliton finally (see Fig. 6 and Fig. 7).

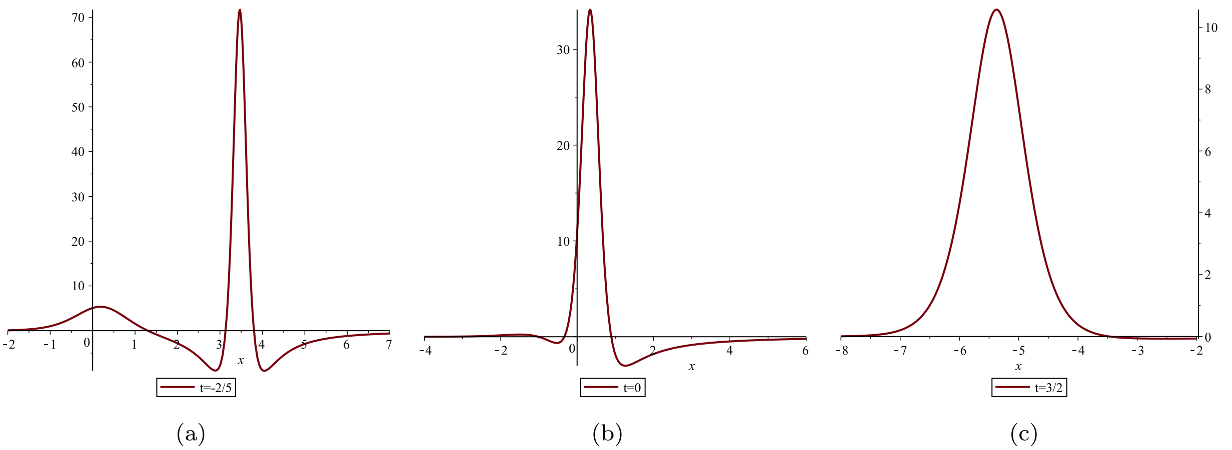


Fig. 5. The evolutional plots of Eq. (11) by choosing $y = 0$.

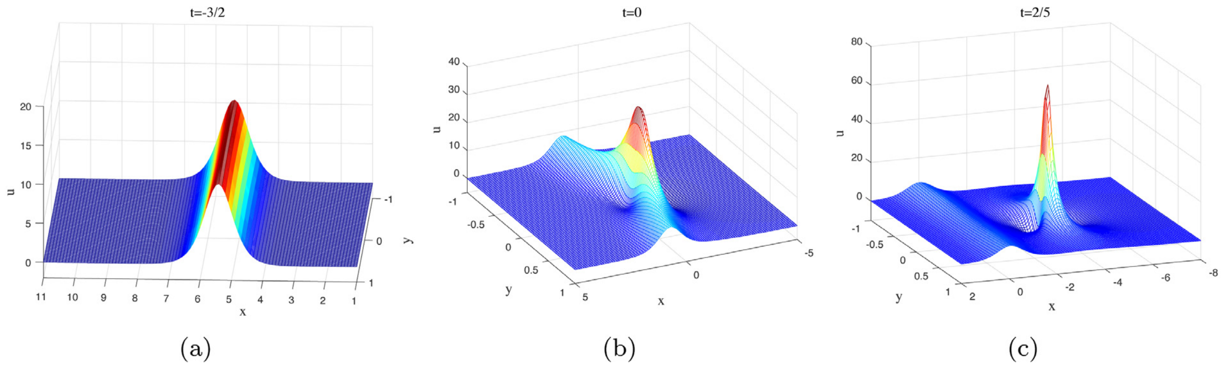


Fig. 6. (Color online) Evolution plots of Eq. (11) by choosing $a_1 = 0.8, a_4 = 0, a_5 = 1.2, a_8 = 0, k = 2, k_1 = 3, k_2 = 0$, at times (a) t $= -1.5$, (only one stripe), (b) t $= 0$, (a lump soliton appears) and (c) t $= 0.4$, (the lump soliton gets away the stripe soliton).

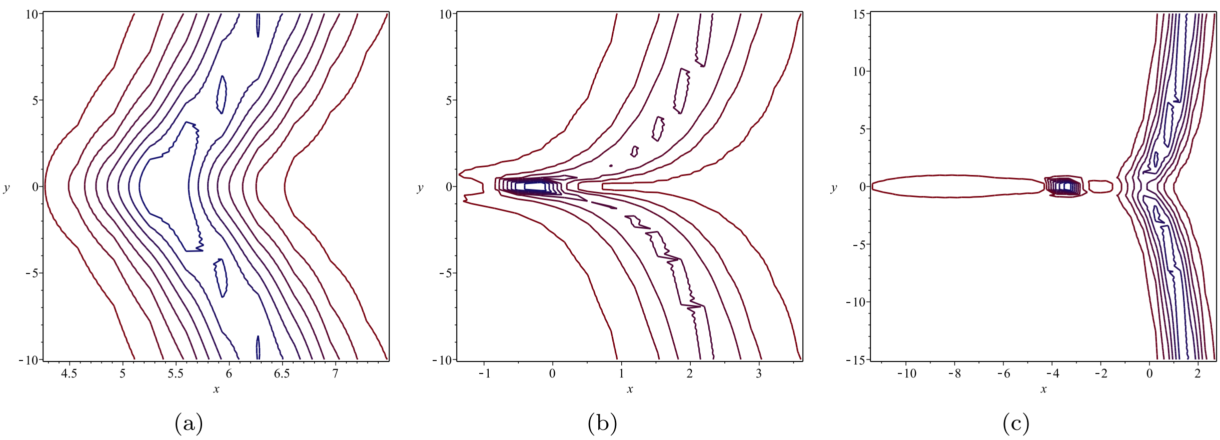


Fig. 7. (Color online) Corresponding density plots of Fig. 6.

4. Rogue wave and a pair of resonance solitons

Now, take f as the combination of the positive quadratic functions and two exponential functions inspired from the N-soliton solutions of bilinear form, that is,

$$f_2 = m_2^2 + n_2^2 + kg_2 + k_4h_2 + k_8g_2h_2, \tag{13}$$

where

$$m_2 = a_1x + a_2y + a_3t + a_4, n_2 = a_5x + a_6y + a_7t + a_8, g_2 = e^{k_1x+k_2y+k_3t}, h_2 = e^{k_5x+k_6y+k_7t}.$$

Substituting Eq. (13) into Eq. (5), we can get 7 classes of solutions of the parameters:

$$(1)\ a_1 = \frac{a_6}{k_1}, a_3 = -\frac{3a_2^2k_1^2 - 3a_6^2k_1^2 + a_6^2}{3k_1a_6}, a_7 = -2a_2k_1, a_9 = \frac{a_6^2}{k_1^4}, k_2 = \frac{a_2k_1^2}{a_6},$$

$$k_6 = \frac{a_2k_1^2}{a_6}, k_8 = 0$$

$$(2)\ a_1 = -\frac{a_6}{k_1}, a_3 = \frac{3a_2^2k_1^2 - 3a_6^2k_1^2 + a_6^2}{3k_1a_6}, a_7 = 2a_2k_1, a_9 = \frac{a_6^2}{k_1^4}, k_2 = -\frac{a_2k_1^2}{a_6},$$

$$k_6 = -\frac{a_2k_1^2}{a_6}, k_8 = 0$$

$$(3)\ a_1^2 + 3a_2^2 - 3a_6^2 = 0, a_3 = 0, a_7 = -\frac{2a_6a_2}{a_1}, a_9 = \frac{9(a_2^4 - 2a_2^2a_6^2 + a_6^4)}{a_6^2}, k_5 = -k_1,$$

$$k_1^2a_1^2 - 1 = 0,\ \ k_2 = \frac{a_2k_1a_6}{a_1}, k = \frac{9k_8(a_2^2 - a_6^2)^2}{a_6^2k_4}, k_6 = -k_2$$

$$(4)\ a_3 = \frac{3a_1^2k_5^2 - a_1^2 - 3a_2^2}{3a_1}, a_6 = -k_1^2, a_7 = -2a_2k_5, a_9 = \frac{a_1^2}{k_5^2}, k = \frac{a_1^2k_8}{k_5^2k_4}, k_1 = -k_5,$$

$$k_2 = -\frac{a_2k_5}{a_1}, k_6 = \frac{a_2k_5}{a_1},$$

$$(5)\ a_3 = \frac{3a_1^2k_5^2 - a_1^2 - 3a_2^2}{3a_1}, a_6 = -k_5k_1, a_7 = 2a_2k_5, a_9 = \frac{a_1^2}{k_5^2}, k = \frac{a_1^2k_8}{k_5^2k_4}, k_1 = -k_5,$$

$$k_2 = -\frac{a_2k_5}{a_1}, k_2 = -k_6,$$

$$(6)\ a_1 = -\frac{a_6}{k_1}, a_3 = \frac{3a_2^2k_1^2 - 3a_6^2k_1^2 + a_6^2}{3k_1a_6}, a_4 = \frac{2a_6}{k_1^2}, a_7 = 2a_2k_1, a_9 = \frac{a_6^2}{k_1^4}, k_2 = -\frac{a_2k_1^2}{a_6},$$

$$k_1 = -k_5, k_6 = -k_2, k = \frac{a_6^2k_8}{k_1^4k_4},$$

$$(7)\ a_1 = \frac{a_6}{k_1}, a_3 = -\frac{3a_2^2k_1^2 - 3a_6^2k_1^2 + a_6^2}{3k_1a_6}, a_4 = -\frac{2a_6}{k_1^2}, a_7 = -2a_2k_1, a_9 = \frac{a_6^2}{k_1^4}, k_2 = \frac{a_2k_1^2}{a_6},$$

$$k_1 = -k_5, k_6 = -k_2, k = \frac{a_6^2k_8}{k_1^4k_4}.$$

It is found that the above 7 classes of solutions can change the exponential functions into a hyperbolic cosine function. So we reinstall f as follows:

$$f_3 = m_3^2 + n_3^2 + k\cosh(k_1x + k_2y + k_3t) + a_9, \tag{14}$$

where

$$m_3 = a_1x + a_2y + a_3t + a_4, n_3 = a_5x + a_6y + a_7t + a_8,$$

and, substituting Eq. (14) into Eq. (4) leads to the following relation:

$$\begin{aligned} a_6 &= \frac{a_2a_5 \pm a_1^2k_1 \pm a_5^2k_1}{a_1}, a_9 = \frac{k^2k_1^4 + 4a_1^4 + 8a_1^2a_5^2 + 4a_5^4}{4k_1^2(a_1^2+a_5^2)}, k_2 = \frac{k_1(a_2 \pm a_5k_1)}{a_1}, \\ k_3 &= \frac{k_1(a_1^2k_1^2 - 3a_5^2k_1^2 \mp 6a_2a_5k_1 - a_1^2 - 3a_2^2)}{3a_1^2}, a_3 = \frac{3a_1^2k_1^2 + 3a_5^2k_1^2 - a_1^2 - 3a_2^2}{3a_1}, \\ a_7 &= -\frac{3a_1^2a_5k_1^2 + 3a_5^3k_1^2 \pm 6a_1^2a_2k_1 \pm 6a_2a_5^2k_1 + a_1^2a_5 + 3a_2^2a_5}{3a_1^2} \end{aligned} \quad (15)$$

Since these two classes of solutions are similar, we only choose one of it, then the final solution of u can be written as

$$\begin{aligned} u &= \frac{4(2a_1^2 + 2a_5^2 + k\cosh(k_1x + k_2y + k_3t)k_1^2)}{f_3} \\ &- 4\frac{(2a_1m_3 + 2a_5n_3 + kk_1\sinh(k_1x + k_2y + k_3t))^2}{f_3^2}, \end{aligned} \quad (16)$$

where

$$\begin{aligned} f_3 &= (a_1x + a_2y + \frac{3a_1^2k_1^2 + 3a_5^2k_1^2 - a_1^2 - 3a_2^2}{3a_1}t + a_4)^2 + \frac{k^2k_1^4 + 4a_1^4 + 8a_1^2a_5^2 + 4a_5^4}{4k_1^2(a_1^2+a_5^2)} \\ &+ (a_5x + \frac{a_1^2k_1 + a_5^2k_1 + a_2a_5}{a_1}y - \frac{3a_1^2a_5k_1^2 + 3a_5^3k_1^2 + 6a_1^2a_2k_1 + 6a_2a_5^2k_1 + a_1^2a_5 + 3a_2^2a_5}{3a_1^2}t + a_8)^2 \\ &+ k\cosh(k_1x + \frac{k_1(a_5k_1 + a_2)}{a_1}y + \frac{k_1(a_1^2k_1^2 - 3a_5^2k_1^2 - 6a_2a_5k_1 - a_1^2 - 3a_2^2)}{3a_1^2}t), \end{aligned}$$

$$\begin{aligned} n_3 &= a_5x + \frac{a_1^2k_1 + a_5^2k_1 + a_2a_5}{a_1}y - \frac{3a_1^2a_5k_1^2 + 3a_5^3k_1^2 + 6a_1^2a_2k_1 + 6a_2a_5^2k_1 + a_1^2a_5 + 3a_2^2a_5}{3a_1^2}t + a_8, \\ m_3 &= a_1x + a_2y + \frac{3a_1^2k_1^2 + 3a_5^2k_1^2 - a_1^2 - 3a_2^2}{3a_1}t + a_4. \end{aligned}$$

According to the expressions of f_3, n_3, m_3, the asymptotic property of the lump solution and a pair of resonance solitons are analyzed. Taking

$$\xi_1 = m_3, \quad \xi_2 = n_3, \quad \xi_3 = k_1x + \frac{k_1(a_5k_1 + a_2)}{a_1}y + \frac{k_1(a_1^2k_1^2 - 3a_5^2k_1^2 - 6a_2a_5k_1 - a_1^2 - 3a_2^2)}{3a_1^2}t,$$

it is proved that

$$\lim_{t=\pm\infty} \frac{\xi_1^2}{\xi_2^2} = \frac{(3a_1^2k_1^2 + 3a_5^2k_1^2 - a_1^2 - 3a_2^2)^2a_1^2}{(3a_1^2a_5k_1^2 + 3a_5^3k_1^2 + 6a_1^2a_2k_1 + 6a_2a_5^2k_1 + a_1^2a_5 + 3a_2^2a_5)^2}.$$

Because as $t \to \pm\infty$, ξ_1, ξ_2 are of the same order, we only need to compare ξ_2^2 and $\cosh\xi_3$. If ξ_3 is supposed as a constant, then $\xi_2 = \xi_3\frac{a_5}{k_1} + a_1k_1y - \frac{4}{3}a_5k_1^2t - 2a_2k_1t$, one has

$$\lim_{t=\pm\infty} \frac{\xi_2^2}{\cosh(\xi_3)} = 0,$$

so we can come to a conclusion that when $t \to \pm\infty$, there are only a pair of resonance solitons; when t is little, the lump soliton is more clear, which can be seen in Fig. 8.

Fig. 8(a) depicts a pair of resonance solitons and the lump soliton is in a invisible place, similar to a ghoston, when $t = -0.8$, the lump soliton appears gradually, which is born from one of the resonance stripe soliton. Because of the energy conservation, the shapes of these two resonance solitons change at the same time. In the same location on one position appears a lump soliton and on the other a sunk envelope. When $t = 0$, there exists a rogue wave, derived from the lump soliton, located in the middle of these two resonance solitons and linked them with each other. Then the lump soliton begins to transfer, until it attaches to the other stripe soliton successfully. Finally it goes out of our vision (see Fig. 9).

To show this whole progress more delicately, the sectional drawing and vertical views are exhibited in Figs. 10 and 11, respectively. Obviously, the wave amplitude at t = 0 is about five times than that at t = ±3, and its appearing time is quite short. In this sense, the lump solution can also be called rogue wave, which is aroused by the interaction between the lump soliton and a pair of resonance solitons.

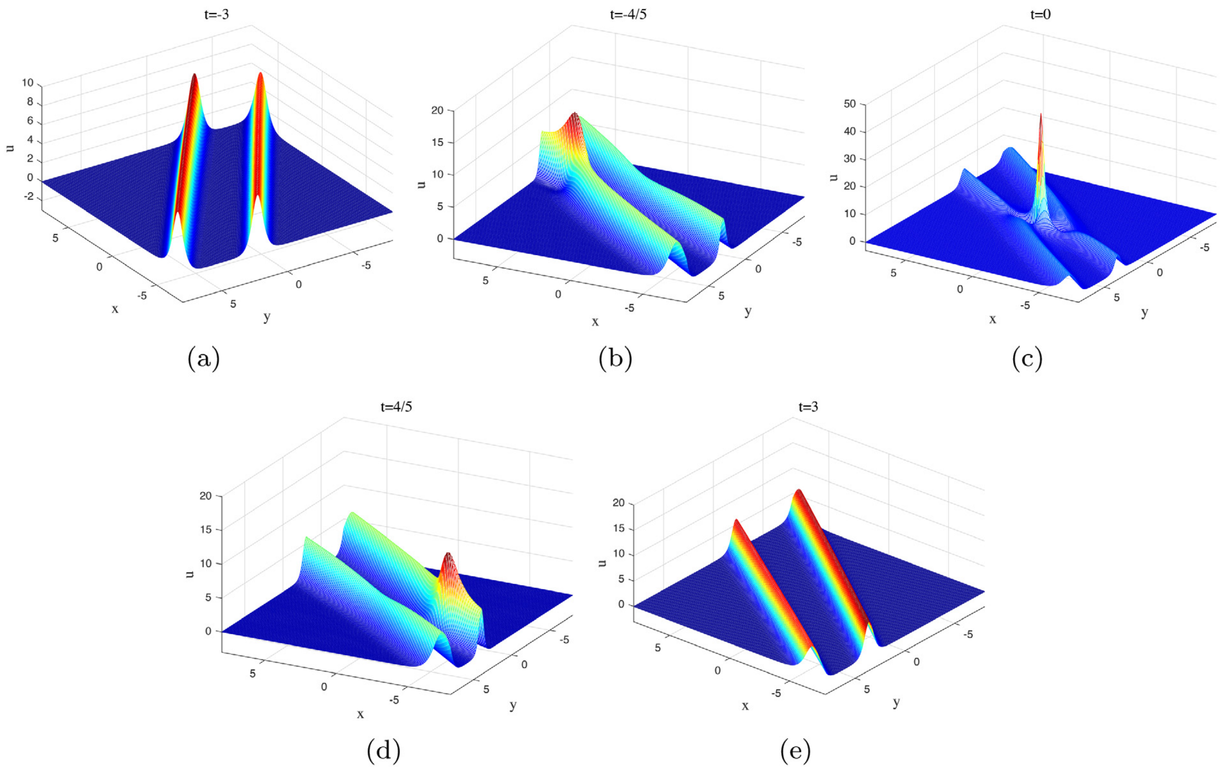


Fig. 8. (Color online) Evolution plots of Eq. (16) by choosing $a_1 = 5$, $a_2 = 2$, $a_4 = 0$, $a_5 = 2$, $a_8 = 0$, $k = 1.8$, $k_1 = 2.5$, at times (a) t = −3, (b) t = −0.8, (c) t = 0, (d) t = 0.8, (e) t = 3.

5. Conclusion

Based on the Hirota direct method and a new ansatz, the lump solution, the lump and one stripe soliton, and the rogue wave and a pair of resonance stripe solitons are discovered for KP equation. The new ansatz is a combination of positive quadratic functions and hyperbolic functions. However not every Eq. (1) has a lump soliton, for instance, when $h_1 = h_2 = h_3 = h_4 = h_5 = 1$, the bilinear form

$$(D_xD_t + D_x^4 + D_x^2 + D_y^2 + D_z^2)f \cdot f = 0. \tag{17}$$

Then setting $z = x$ and assuming f as (4), the quadratic function of f is determined to be

$$f = (a_1x + a_2y + \frac{-a_1^3 + (-2a_5^2 - a_2^2 + a_6^2)a_1 - 2a_5a_6a_2}{a_1^2 + a_5^2} + a_4)^2 - \frac{3(a_1^2 + a_5^2)^3}{(a_1a_6 - a_2a_5)^2}$$
$$+ (a_5x + a_6y + \frac{-a_5^3 + (-2a_1^2 - a_6^2 + a_2^2)a_5 - 2a_1a_6a_2}{a_1^2 + a_5^2})^2.$$

It is evident that singularity cannot be avoided due to the second part of f is always negative, so lump soliton does not exist, let alone its collision with other solitons.

Second, the collision between the lump soliton and one stripe soliton is presented in Figs. 3(c) and 6(c), respectively. With the opposite of the stripe soliton, showing two different physics phenomena, fusion and fission. As for the fusion interaction, at the beginning, the lump soliton keeps its shapes, energy, spreading with a steady rate, but when it meets the stripe soliton, it begins to be swallowed gradually until out of our horizon. But fission is an opposite interaction. At first, there is only one stripe soliton, when t approximates zero, the lump soliton is born from the stripe soliton gradually until it separated from the stripe soliton completely. Both progress will lose much more energy than the usual collision.

Third, it is observed that the rogue wave, possessing a peak wave profile, arises from one of the resonance stripe solitons, moves to the other, and then disappears. Therefore, a rogue wave is originated from the interaction between the lump soliton and the pair of resonance stripe solitons. It should be pointed out that, besides some common characters, the rogue wave presented in this paper has some different properties from the traditional rogue waves, which are stated as follows:

(1) Our results show that a two-dimensional rogue wave can be generated from two stripe solitons, as shown in Fig. 8. The whole progress completely satisfies the character that "appear from nowhere and disappear without a trace". Moreover,

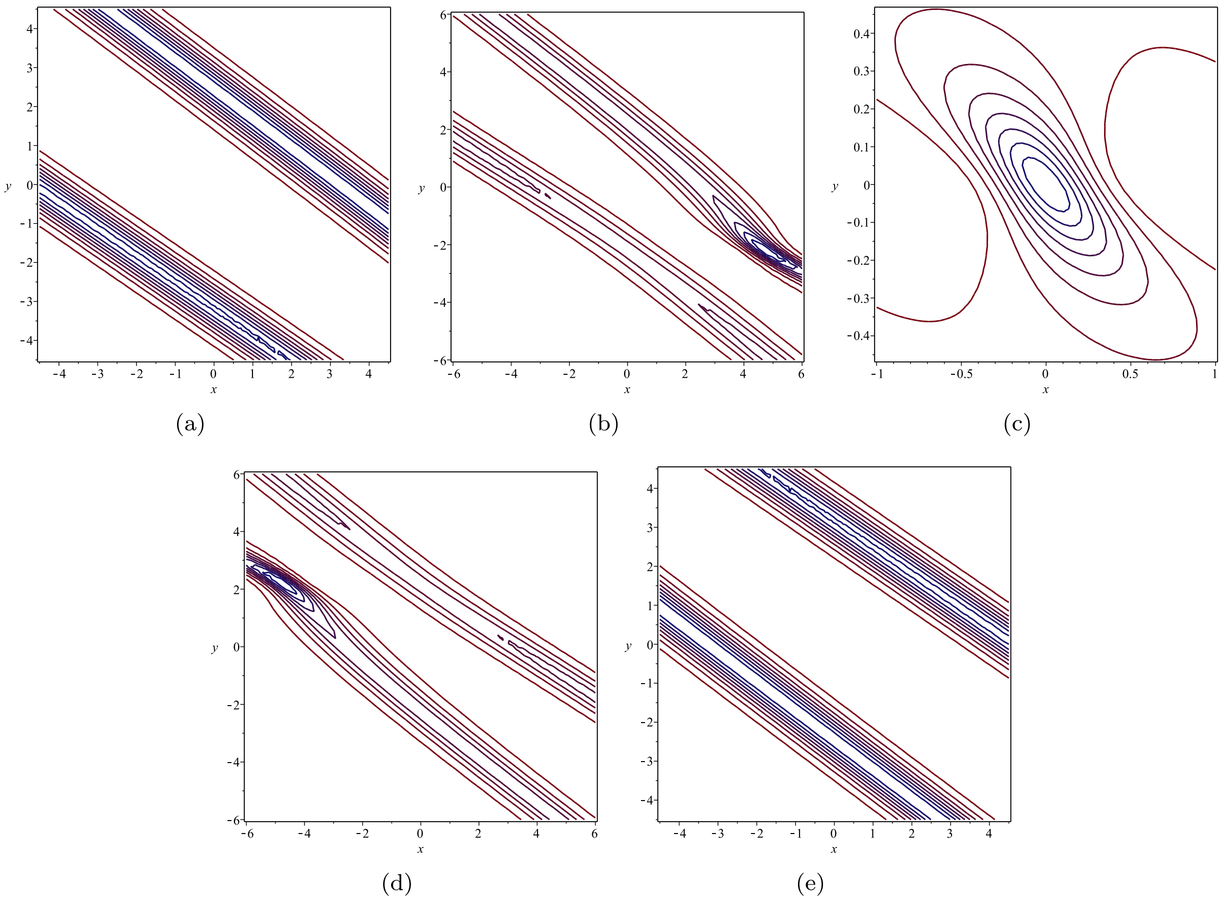


Fig. 9. (Color online) Corresponding density plots of Fig. 8.

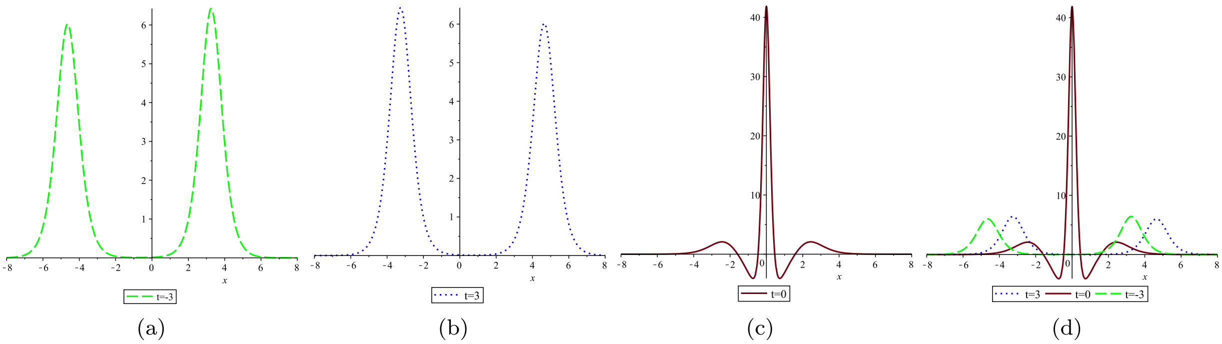


Fig. 10. Sectional views of Eq. (16), at times (a) $t = -3$ (green), (b) $t = 3$ (blue), (c) $t = 0$ (red), (d) is the whole profile at (a), (b) and (c). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

it demonstrates that two dimensional rogue wave, different from the one dimensional case, cannot appear independently, as it can only be generated via solitary waves.

(2) It should be emphasized that the rogue wave studied in most references at present, for instance, the one dimensional rogue wave, although localized in space and time, it has a nonzero (usually a constant) background. Consequently, it is hard to be applied in real physics, such as in ocean. However, the rogue wave depicted in Fig. 8 has a zero background. That is to say, a localized wave package appears as two resonance solitons with a zero background before interaction, reaches the maximum amplitude (almost several times of the amplitude of the resonance soliton) during the interaction, then decays gradually and disappears after the interaction.

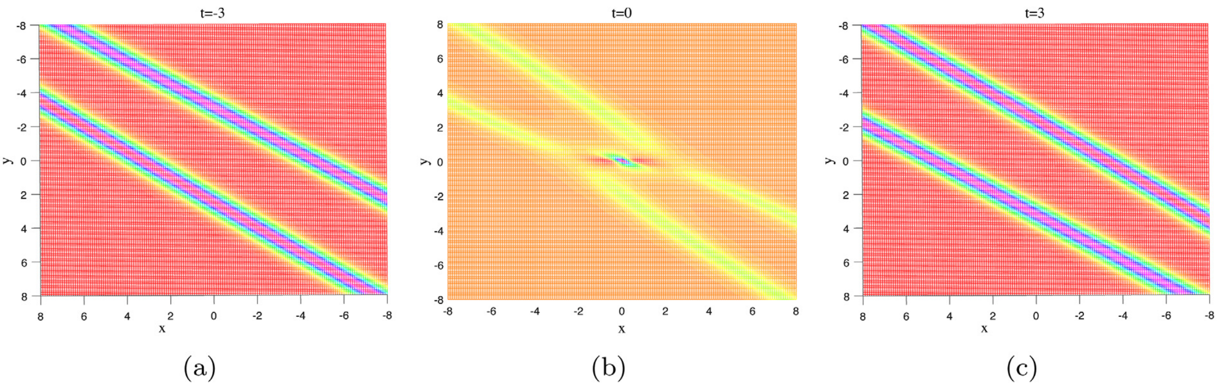


Fig. 11. (Color online) Vertical views of the Eq. (16), at times (a) $t = -3$, (b) $t = 0$ and (c) $t = 3$.

(3) It is remarkable that previously obtained solutions of two dimensional rogue waves are actually linear as they are only local in one dimension along the characteristic line. However, the rogue wave expressed by Eq. (16) is localized in two dimensions independently.

In addition, since the KP equation can describe the evolution of two dimensional water wave packets, these rogue-wave solutions could have interesting implications for dynamics two dimensional water waves.

Acknowledgments

The project is supported by the Global Change Research Program of China (No. 2015CB953904), National Natural Science Foundation of China (No. 11675054, 11435005, 11675055), and Shanghai Collaborative Innovation Center of Trustworthy Software for Internet of Things, China (No. ZF1213).

References

[1] L. Draper, Freak ocean waves, Weather 21 (1966) 2–4.
[2] B.L. Guo, L.X. Tian, L.M. Ling, Z.Y. Yan, Rogue Wave and Mathematical Theory, Zhejiang science and technology Press, 2015.
[3] S.P. Kjeldsen, Dangerous wave groups, Norw. Marit. Res. 12 (1984) 4–16.
[4] D.A.G. Walker, P.H. Taylor, R.E. Taylor, The shape of large surface waves on the open sea and the draupner new year wave, Appl. Ocean Res. 26 (2004) 73–83.
[5] E. Pelinovsky, C. Kharif, A. Slunyaev, Rogue Waves in the Ocean-Review and Progress, Springer, Berlin, 2010.
[6] D.R. Solli, C. Ropers, P. Koonath, B. Jalali, Optical rogue waves, Nature 450 (2007) 1054–1058.
[7] L. Stenflo, M. Marklund, Rogue waves in the atmosphere, J. Plasma Phys. 76 (2009) 293–295.
[8] Y.V. Bludov, V.V. Konotop, N. Akhmediev, Matter rogue waves, Phys. Rev. A 80 (2009) 033610.
[9] V.B. Efimov, A.N. Ganshin, G.V. Kolmakov, P.V.E. Mcclintock, L.P. Mezhov-Deglin, Rogue waves in superfluid helium, Eur. Phys. J.-Spec. Top. 185 (2010) 181–193.
[10] M. Shats, H. Punzmann, H. Xia, Capillary rogue waves, Phys. Rev. Lett. 104 (2010) 104503.
[11] Z.Y. Yan, Vector financial rogue waves, Phys. Lett. A 375 (2011) 4274–4279.
[12] D.H. Peregrine, Water waves, nonlinear Schrödinger equations and their solutions, J. Aust. Math. Soc. B 25 (1983) 16–43.
[13] A. Chabchoub, N.P. Hoffmann, N. Akhmediev, Rogue wave observation in a water wave tank, Phys. Rev. Lett. 106 (2011) 204502.
[14] D.J. Kedziora, A. Ankiewicz, N. Akhmediev, Classifying the hierarchy of nonlinear-schrödinger-equation rogue-wave solutions, Phys. Rev. E 88 (2013) 013207.
[15] X.X. Xu, An integrable coupling hierarchy of the Mkdv-integrable systems, its hamiltonian structure and corresponding nonisospectral integrable hierarchy, Appl. Math. Comput. 216 (2010) 344–353.
[16] X.Y. Li, Y.X. Li, H.X. Yang, Two families of liouville integrable lattice equations, Appl. Math. Comput. 217 (2011) 8671–8682.
[17] Y. Ohta, J.K. Yang, Rogue waves in the Davey–Stewartson I equation, Phys. Rev. E 86 (2012) 036604.
[18] Y. Ohta, J.K. Yang, Dynamics of rogue waves in the Davey–Stewartson II equation, J. Phys. A 46 (2013) 105202.
[19] Y. Ohta, J.K. Yang, Genera high-order rogue wvae and their dynamics in the nonlinear Schrödinger equation, Pro. R. Soc. London. Sec. A 468 (2012) 1716–1740.
[20] X. Wang, Y.Q. Li, F. Huang, Y. Chen, Rogue wave solutions of AB system, Commun. Nonlinear Sci. Numer. Simul. 20 (2015) 434–442.
[21] B. Tan, J.P. Boyd, Envelope solitary waves and periodic waves in the AB equations, Stud, Appl. Math. 109 (2002) 67–87.
[22] R.K. Dodd, J.C. Eilbeck, J.D. Gibbon, H.C. Morris, Solitons and Nonlinear Wave Equations, Academic Press, London, 1982.
[23] J.C. Chen, Y. Chen, B.F. Feng, K.I. Maruno, Rational solutions to two-and one-dimensional multicomponent Yajima-Oikawa systems, Phys. Lett. A 379 (2015) 1510–1519.
[24] X.E. Zhang, Y. Chen, X.Y. Tang, Rogue wave and a pair of resonance stripe solitons to a reduced generalized (3+1)-dimensional KP equation, arXiv:1610.09507v1.
[25] X.E. Zhang, Y. Chen, Rogue wave and a pair of resonance stripe solitons to a reduced (3+1)-dimensional Jimbo–Miwa equation, Commun. Nonlinear Sci. Numer. Simul. 52 (2017) 24–31.
[26] X.E. Zhang, Y. Chen, Deformation rogue wave to the (2+1)-dimensional KdV equation, Nonlinear Dynam. 90 (2017) 755–763.

[27] Y. Zhang, H.H. Dong, X.E. Zhang, H.W. Yang, Rational solutions and lump solutions to the generalized (3+1)-dimensional Shallow Water-like equation, Comput. Math. Appl. 73 (2017) 246–252.
[28] C.J. Wang, Spatiotemporal deformation of lump solution to (2+1)-dimensional KdV equation, Nonlinear Dynam. 84 (2016) 697–702.
[29] S.Y. Lou, X.Y. Tang, Nonlinear Mathematical Physics Method, Academic Press, Beijing, 2006.
[30] W.X. Ma, Lump solutions to the kadomtsev-petviashvili equation, Phys. Lett. A 379 (2015) 1975–1978.
[31] S.V. Manakov, V.E. Zakharov, L.A. Bordag, A.R. Its, V.B. Matveev, Two-dimensional solitons of the Kadomtsev–Petviashvili equation and their interaction, Phys. Lett. A 63 (1977) 205–206.
[32] R.S. Johnson, S. Thompson, A solution of the inverse scattering problem for the Kadomtsev–Petviashvili equation by the method of separation of variables, Phys. Lett. A 66 (1978) 279–281.
[33] J.Y. Yang, W.X. Ma, Lump solutions to the Bkp equation by symbolic computation, Internat. J. Modern Phys. B 30 (2016) 1640028.
[34] W.X. Ma, Z.Y. Qin, X. Lü, Lump solutions to dimensionally reduced p-gKP and p-gBKP equations, Nonlinear Dynam. 84 (2016) 923–931.
[35] H.C. Ma, A.P. Deng, Lump solution of (2+1)-dimensional boussinesq equation, Commun. Theor. Phys. 65 (2016) 546–552.
[36] H.W. Yang, X. Chen, M. Guo, Y.D. Chen, A new ZK-BO equation for three-dimensional algebraic Rossby solitary waves and its solution as well as fission property, Nonlinear Dynam. 91 (2018) 2019–2032.
[37] A.S. Fokas, D.E. Pelinovsky, C. Sulem, Interaction of lumps with a line soliton for the dsii equation, Physica D 152–153 (2001) 189–198.
[38] C.J. Wang, Z.D. Dai, C.F. Liu, Interaction between kink solitary wave and rogue wave for (2+1)-dimensional burgers equation, Mediterr. J. Math. 13 (2016) 1087–1098.
[39] C. Becker, K. Sengstock, P. Schmelcher, P.G. Kevrekidis, R.C. Gonzalez, Inelastic collisions of solitary waves in anisotropic Bose–Einstein condensates: slingshot events and expanding collision bubbles, New J. Phys. 15 (2013) 113028.
[40] W. Tan, Z.D. Dai, Dynamics of kinky wave for (3+1)-dimensional potential Yu-Toda-Sasa-Fukuyama equation, Nonlinear Dynam. 85 (2016) 817–823.
[41] Y.N. Tang, S.Q. Tao, Q. Guan, Lump solitons and the interaction phenomena of them for two classes of nonlinear evolution equations, Comput. Math. Appl. 72 (2016) 2334–2342.
[42] J.M. Tu, S.F. Tian, M.J. Xu, X.Q. Song, T.T. Zhang, Bäcklund transformation, infinite conservation laws and periodic wave solutions of a generalized (3+1)-dimensional nonlinear wave in liquid with gas bubbles, Nonlinear Dynam. 83 (2016) 1199–1215.
[43] U.K. Samanta, A. Saha, P. Chatterjee, Bifurcations of dust ion acoustic travelling waves in a magnetized dusty plasma with a q-nonextensive electron velocity distribution, Phys. Plasmas 20 (2013) 022111.
[44] A. Saha, N. Pal, P. Chatterjee, Bifurcation and quasiperiodic behaviors of ion acoustic waves in magnetoplasmas with nonthermal electrons featuring tsallis distribution, Braz. J. Phys. 45 (2015) 325–333.