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Exact solutions for a new class of nonlinear evolution equations with nonlinear term of any order

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Abstract

In this paper, we consider a new class of nonlinear partial differential equations with nonlinear term of any order, $u_{tt} + a_1u_{xx} + a_2u + a_3u^p + a_4u^{2p-1} = 0$, which contains some particular important equations. We give a new kind of transformation and a new generalized ansätze to treat this class of equations. As a result, many explicit exact solutions, which contain new kink-profile solitary-wave solutions, bell-profile solitary-wave solutions, periodic wave solutions and combined formal solitary-wave solutions, are obtained by the extended method. In addition, we also can derive rational solutions for this class of equations.

1. Introduction

In this paper, we consider a new class of nonlinear evolution equations with nonlinear term of any order in the form

$$u_{tt} + a_1u_{xx} + a_2u + a_3u^p + a_4u^{2p-1} = 0, \tag{1.1}$$

where a_i $(i = 1,2,3,4)$ and $p \neq 1$ are arbitrary constants. When p takes a different constant, a different equation would be constructed. It is easily to see that Eq. (1.1) contains some important nonlinear equations of mathematical physics (NEMPS) such as Duffing equation [1], Klein–Gordon equation [2], Landau–Ginburg–Higgs equation [3], Sin–Gordon equation [2,3] and ϕ^4 equation [1]. Eq. (1.1) also contains the nonlinear evolution equation in [4]

$$u_{tt} + au_{xx} + bu + cu^3 = 0. \tag{1.2}$$

Recently based on the well-known Riccati equation, Fan [5,6] proposed a so-called the extended tanh-method to find exact solution of certain nonlinear PDEs. In order to obtain new formal solutions, various ansätzes have been proposed. A further development, discussed by Yan and Zhang [7,8], improved considerably the method. To seek the exact solutions of Eq. (1.1), we cannot use the extended tanh-method directly. We obtain the improved method by making a proper transformation and a new generalized ansätze, so that we are able to deal with arbitrary balance constants. As a result, with the aid of *Mathematica* and *Wu* elimination method [9], we are able to obtain more general solution form and more solutions than tanh-method. The solutions obtained contain new kink-profile solitary-wave solutions, bell-profile solitary-wave solutions, rational solutions and singular solitary-wave solutions.

The plan of the paper is as follows. In Section 2, we describe briefly the improved method. In Section 3, we apply the improved method to Eq. (1.1) and bring out rich solutions. Conclusions will be presented finally.

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2. Extension of the method

In this section, we improve the method in [7,8]. For a given nonlinear evolution equations, say, in two variables x, t

$$F(u, u_t, u_x, u_{xt}, u_{tt}, \ldots) = 0, \tag{2.1}$$

we seek the following formal travelling wave solutions

$$u(x,t) = u(\xi), \quad \xi = x - \beta t, \tag{2.2}$$

where β is a constant to be determined later. Then Eq. (2.1) reduces to a nonlinear ordinary differential equation under (2.2)

$$G(u, u', u'', u''', \ldots) = 0, \tag{2.3}$$

where "′" denotes $(\mathrm{d}/\mathrm{d}\xi)$. In order to seek the travelling wave solutions of (2.3), we take the following improved transformations

$$u(\xi) = \sum_{i=1}^{m} \omega^{i-1}(\xi)[A_i\omega(\xi) + B_i\sqrt{R+\omega^2(\xi)}] + A_0, \tag{2.4}$$

and the new variable $\omega = \omega(\xi)$ satisfies

$$\omega' - (R+\omega^2) = \frac{\mathrm{d}\omega}{\mathrm{d}\xi} - (R+\omega^2) = 0, \tag{2.5}$$

where $A_0, A_i, B_i, (i = 1, 2, \ldots, m)$ and R are constants to be determined later, and m is an positive integer. However, when we balance the highest order partial derivative term and the nonlinear term in Eq. (2.1) or Eq. (2.3), we find that the constant m need not be a positive integer. In order to apply the method in [7,8] when m is equal to a fraction or a negative integer, we make the following transformation

(1) When $m = (q/p)$ (where $m = (q/p)$ is a fraction in lowest terms), we let

$$u(\xi) = \varphi^{q/p}(\xi), \tag{2.6}$$

then substitute Eq. (2.6) into Eq. (2.3) and return to determine the value of m by balance the highest order partial derivative term and the nonlinear term in new Eq. (2.3).

(2) When m is a negative integer, we let

$$u(\xi) = \varphi^{m}(\xi), \tag{2.7}$$

then substitute Eq. (2.7) into Eq. (2.3) and return to determine the value of m once again.

In general, the constant m can be changed into a positive integer by means of the above proper transformation. We summarize the extended method as follows:

Step 1. Determine the values of m of Eq. (2.4) by balancing the highest order partial derivative term and the nonlinear term in Eq. (2.1) (or (2.3)).
- (i) If m is a positive integer then *Step 2*;
- (ii) If $m = (q/p)$, we make the transformation (2.6) and then return to Step 1;
- (iii) If m is a negative integer, we make the transformation (2.7) and then return to Step 1.

Step 2. With the aid of *Mathematica*, substituting Eq. (2.4) along with the condition (2.5) into Eq. (2.3), we get a system of algebraic equations with respect to $\omega^i(R+\omega^2)^{j/2}$ ($j = 0$, 1; $i = 0, 1, 2, \ldots$).

Step 3. Collect all terms with the same power in $\omega^i(R+\omega^2)^{j/2}$ ($j = 0$, 1; $i = 0, 1, 2, \ldots$). Set the coefficients of the terms $\omega^i(R+\omega^2)^{j/2}$ ($j = 0$, 1; $i = 0, 1, 2, \ldots$) to zero to get a over-determined system of nonlinear algebraic equations with respect to the unknown variables β, R, A_0, A_i, B_i ($i = 1, 2, \ldots, m$).

Step 4. With the aid of *Mathematica*, applying *Wu*-elimination method [9] to solve the above over-determined system of nonlinear algebraic equations obtained in Step 3, we get the values of β, R, A_0, A_i, B_i ($i = 1, 2, \ldots, m$).

Step 5. It is well known that the general solutions of Eq. (2.5) are

(1) When $R < 0$,

$$\omega(\xi) = -\sqrt{-R}\tanh(\sqrt{-R}\xi), \quad \omega(\xi) = -\sqrt{-R}\coth(\sqrt{-R}\xi). \tag{2.8}$$

(2) When $R = 0$,

$$\omega(\xi) = \frac{1}{\xi}. \tag{2.9}$$

(3) When $R > 0$,

$$\omega(\xi) = \sqrt{R}\tan(\sqrt{R}\xi), \quad \omega(\xi) = -\sqrt{R}\cot(\sqrt{R}\xi). \tag{2.10}$$

Because

$$\pm\sqrt{R + [-\sqrt{-R}\tanh(\sqrt{-R}\xi)]^2} = \pm\sqrt{R}\,\mathrm{sech}(\sqrt{-R}\xi),$$
$$\pm\sqrt{R + [-\sqrt{-R}\coth(\sqrt{-R}\xi)]^2} = \pm\sqrt{-R}\,\mathrm{csch}(\sqrt{-R}\xi),$$
$$\pm\sqrt{R + [\sqrt{R}\tan\sqrt{R}\xi]^2} = \pm\sqrt{R}\sec(\sqrt{R}\xi),$$
$$\pm\sqrt{R + [-\sqrt{R}\cot\sqrt{R}\xi]^2} = \pm\sqrt{R}\csc(\sqrt{R}\xi),$$

the solutions obtained by the improved method must contain the formal solutions obtained by Fan [5,6] and by Bai [4].

Thus, according to Eqs. (2.2), (2.4), (2.6) or (2.7)–(2.10) and the conclusions in Step 4, we obtain more travelling wave solutions of Eq. (2.1).

3. Explicit exact solutions for Eq. (1.1)

Let us consider Eq. (1.1). According to the above steps, we firstly make the following formal travelling wave transformation:

$$u(x,t) = u(\xi), \quad \xi = x - \beta t, \tag{3.1}$$

where β is a constant to be determined.

Substituting Eq. (3.1) into Eq. (1.1), we get a system of ODE

$$(\beta^2 + a_1)u''(\xi) + a_2 u(\xi) + a_3 u^p(\xi) + a_4 u^{2p-1}(\xi) = 0. \tag{3.2}$$

According to Step 1 in Section 2, by balancing the highest order partial derivative term and the nonlinear term in Eq. (3.2), we get $m = 1/(p-1)$. Therefore we make the following transformation

$$u(\xi) = \varphi^{1/(p-1)}(\xi), \tag{3.3}$$

then substituting (3.3) into Eq. (3.2) reads

$$(a_1 + \beta^2)[(p-1)\varphi(\xi)\varphi''(\xi) + (2-p)\varphi'^2(\xi)] + (p-1)^2[a_2\varphi^2(\xi) + a_3\varphi^3(\xi) + a_4\varphi^4(\xi)] = 0. \tag{3.4}$$

According to Step 1 in Section 2, we suppose that Eq. (3.4) has the following formal solutions

$$\varphi(\xi) = A_0 + A_1\omega + B_1\sqrt{R + \omega^2} \tag{3.5}$$

and $\omega = \omega(\xi)$ satisfies Eq. (2.5), where A_0, A_1, B_1 are constants to be determined later.

With the aid of *Mathematica*, substituting Eq. (3.5) into Eq. (3.4) along with Eq. (2.5) and collecting all terms with the same power in $\omega^i(R+\omega^2)^{j/2}$ $(j = 0,1;\ i = 0,1,2,3,4)$, we get a system of equations with respect to $\omega^i(R+\omega^2)^{j/2}$. Setting the coefficients of $\omega^i(R+\omega^2)^{j/2}$ $(j = 0,1;\ i = 0,1,2,3,4)$ in the obtained system of equations to zero, we can deduce the following set of over-determined algebraic polynomial with the unknowns β, R, A_0, A_1, B_1 namely:

$$\begin{aligned} &A_0^3a_3(-1+p)^2 + A_0^4a_4(-1+p)^2 + 3A_0a_3B_1^2(-1+p)^2R + A_0^2(-1+p)^2(a_2 + 6a_4B_1^2R) + R(a_2B_1^2(-1+p)^2 \\ &\quad + R(-a_1(A_1^2(-2+p) - B_1^2(-1+p)) + a_4B_1^4(-1+p)^2 - (A_1^2(-2+p) - B_1^2(-1+p))\beta^2)) = 0, \end{aligned} \tag{3.6}$$

$$B_1(-1+p)(3A_0^2a_3(-1+p)+4A_0^3a_4(-1+p)+a_3B_1^2(-1+p)R+(2a_2(-1+p)\\ +R(a_1+4a_4B_1^2(-1+p)+\beta^2)))=0, \tag{3.7}$$

$$A_1(-1+p)(3A_0^2a_3(-1+p)+4A_0^3a_4(-1+p)+3a_3B_1^2(-1+p)R+2A_0(a_2(-1+p)\\ +R(a_1+6a_4B_1^2(-1+p)+\beta^2)))=0, \tag{3.8}$$

$$A_1B_1(2a_2(-1+p)^2+6A_0a_3(-1+p)^2+12A_0^2a_4(-1+p)^2+R(4a_4B_1^2(-1+p)^2+a_1(1+p)+(1+p)\beta^2))=0, \tag{3.9}$$

$$A_1^2(a_2(-1+p)^2+2a_1R+3(-1+p)^2(A_0(a_3+2A_0a_4)+2a_4B_1^2R)+2R\beta^2)+B_1^2(a_2(-1+p)^2+3A_0a_3(-1+p)^2\\ +6A_0^2a_4(-1+p)^2+R(2a_4B_1^2(-1+p)^2+a_1(-1+2p)+(-1+2p)\beta^2))=0, \tag{3.10}$$

$$B_1(-1+p)(a_3(3A_1^2+B_1^2)(-1+p)+2A_0(a_1+2a_4(3A_1^2+B_1^2)(-1+p)+\beta^2))=0, \tag{3.11}$$

$$A_1(-1+p)(a_3(A_1^2+3B_1^2)(-1+p)+2A_0(a_1+2a_4(A_1^2+3B_1^2)(-1+p)+\beta^2))=0, \tag{3.12}$$

$$2A_1B_1(2a_4(A_1^2+B_1^2)(-1+p)^2+a_1p+p\beta^2)=0, \tag{3.13}$$

$$A_1^4a_4(-1+p)^2+B_1^2(a_4B_1^2(-1+p)^2+p(a_1+\beta^2))+A_1^2(6a_4B_1^2(-1+p)^2+p(a_1+\beta^2))=0. \tag{3.14}$$

We have found the following eight cases of solution for Eqs. (3.6)–(3.14) with the aid of *Mathematica* and *Wu* elimination method [9]:

Case 1

$$A_0=a_3=B_1=0,\quad p=2,\quad \beta=\pm\sqrt{-\frac{a_2+2a_1R}{2R}},\quad A_1=\pm\sqrt{\frac{a_2}{a_4R}}.$$

Case 2

$$A_0=\pm\sqrt{\frac{a_2p}{4a_4}},\quad B_1=0,\quad A_1=\pm\sqrt{-\frac{a_2p}{4a_4R}},\quad \beta=\pm\sqrt{\frac{a_2(-1+p)^2-4a_1R}{4R}},\quad a_3=\pm\frac{(p+1)\sqrt{a_2a_4}}{\sqrt{p}}.$$

Case 3

$$p=\frac{5}{2},\quad B_1=0,\quad A_0=-\frac{5a_3}{14a_4},\quad A_1=\pm\sqrt{\frac{-5a_2}{8a_4R}},\quad a_3=\pm\sqrt{\frac{49a_2a_4}{10}},\quad \beta=\pm\sqrt{-a_1+\frac{9a_2}{16R}}.$$

Case 4

$$A_0=A_1=a_3=0,\quad \beta=\pm\sqrt{\frac{a_2(-1+p)^2-a_1R}{R}},\quad B_1=\pm\sqrt{\frac{-a_2p}{a_4R}}.$$

Case 5

$$A_1=0,\quad p=2,\quad A_0=\pm\sqrt{\frac{a_2}{2a_4}},\quad B_1=\pm\sqrt{\frac{a_2}{a_4R}},\quad \beta=\pm\sqrt{\frac{-2a_2-4a_1R}{4R}},\quad a_3^2=\frac{9a_2a_4}{2}.$$

Case 6

$$R=0,\quad A_0=a_2=a_3=0,\quad A_1=\pm B_1=\pm\sqrt{-\frac{p(a_1+\beta^2)}{4a_4(-1+p)^2}}.$$

Case 7

$$A_0=a_3=0,\quad p=2,\quad A_1=\pm B_1=\pm\sqrt{\frac{a_2}{a_4R}},\quad \beta=\pm\sqrt{-\frac{a_1R+2a_2}{R}}.$$

Case 8

$$A_0 = \pm\sqrt{\frac{a_2 p}{4a_4}}, \quad A_1 = \pm B_1 = \pm\sqrt{-\frac{a_2 p}{4a_4 R}}, \quad \beta = \pm\sqrt{\frac{a_2(-1+p)^2 - a_1 R}{R}}, \quad a_3^2 = \frac{a_2 a_4 (1+p)^2}{p}.$$

Therefore, according to Step 5, combining Eqs. (2.8)–(3.1) and (3.3)–(3.5) along with Cases 1–8, eight families of explicit exact travelling wave solutions of Eq. (1.1), which contain solitary wave solutions, periodic wave solutions and new travelling wave solution, rational solutions and singular solitary wave solutions, are found as follows:

Case 1

When $p = 2$, $R < 0$,

$$u_{11} = \mp\sqrt{\frac{-a_2}{a_4}}\tanh(\sqrt{-R}(x - \beta t + \xi_0)),$$
$$u_{12} = \mp\sqrt{\frac{-a_2}{a_4}}\coth(\sqrt{-R}(x - \beta t + \xi_0)),$$

When $p = 2$, $R > 0$,

$$u_{13} = \pm\sqrt{\frac{a_2}{a_4}}\tan(\sqrt{R}(x - \beta t + \xi_0)),$$
$$u_{14} = \pm\sqrt{\frac{a_2}{a_4}}\cot(\sqrt{R}(x - \beta t + \xi_0)),$$

where

$$\beta = \pm\sqrt{-\frac{a_2 + 2a_1 R}{2R}}, \quad a_4 \neq 0.$$

Case 2

When $R < 0$,

$$u_{21} = \left[\pm\sqrt{\frac{a_2 p}{4a_4}} \mp \sqrt{\frac{a_2 p}{4a_4 R}}\tanh(\sqrt{-R}(x - \beta t + \xi_0))\right]^{1/(p-1)},$$
$$u_{22} = \left[\pm\sqrt{\frac{a_2 p}{4a_4}} \mp \sqrt{\frac{a_2 p}{4a_4 R}}\coth(\sqrt{-R}(x - \beta t + \xi_0))\right]^{1/(p-1)},$$

When $R > 0$,

$$u_{23} = \left[\pm\sqrt{\frac{a_2 p}{4a_4}} \mp \sqrt{\frac{-a_2 p}{4a_4 R}}\tan(\sqrt{R}(x - \beta t + \xi_0))\right]^{1/(p-1)},$$
$$u_{24} = \left[\pm\sqrt{\frac{a_2 p}{4a_4}} \pm \sqrt{\frac{-a_2 p}{4a_4 R}}\cot(\sqrt{R}(x - \beta t + \xi_0))\right]^{1/(p-1)},$$

where

$$\beta = \pm\sqrt{\frac{a_2(-1+p)^2 - 4a_1 R}{4R}}, \quad a_3 = \pm\frac{(p+1)\sqrt{a_2 a_4}}{\sqrt{p}}, \quad a_4 \neq 0.$$

Case 3

When $p = 5/2$, $R < 0$,

$$u_{31} = \left[\frac{-5a_3}{14a_4} \pm \sqrt{\frac{5a_2}{8a_4}}\tanh(\sqrt{-R}(x - \beta t + \xi_0))\right]^{2/3},$$
$$u_{32} = \left[\frac{-5a_3}{14a_4} \pm \sqrt{\frac{5a_2}{8a_4}}\coth(\sqrt{-R}(x - \beta t + \xi_0))\right]^{2/3},$$

When $p = 5/2$, $R > 0$,

$$u_{33} = \left[\frac{-5a_3}{14a_4} \pm \sqrt{-\frac{5a_2}{8a_4}}\tan(\sqrt{R}(x - \beta t + \xi_0))\right]^{2/3},$$

$$u_{34} = \left[\frac{-5a_3}{14a_4} \mp \sqrt{-\frac{5a_2}{8a_4}}\cot(\sqrt{R}(x - \beta t + \xi_0))\right]^{2/3},$$

where

$$\beta = \pm\sqrt{-a_1 + \frac{9a_2}{16R}}, \quad a_3 = \pm\sqrt{\frac{49a_2a_4}{10}}, \quad a_4 \neq 0.$$

Case 4

When $a_3 = 0$, $R < 0$,

$$u_{41} = \left[\pm\sqrt{\frac{a_2p}{a_4}}\operatorname{sech}(\sqrt{-R}(x - \beta t + \xi_0))\right]^{1/(p-1)},$$

$$u_{42} = \left[\pm\sqrt{\frac{-a_2p}{a_4}}\operatorname{csch}(\sqrt{-R}(x - \beta t + \xi_0))\right]^{1/(p-1)},$$

When $a_3 = 0$, $R > 0$,

$$u_{43} = \left[\pm\sqrt{\frac{-a_2p}{a_4}}\sec\sqrt{R}(x - \beta t + \xi_0)\right]^{1/(p-1)},$$

$$u_{44} = \left[\pm\sqrt{\frac{-a_2p}{a_4}}\csc\sqrt{R}(x - \beta t + \xi_0)\right]^{1/(p-1)}.$$

where

$$\beta = \pm\sqrt{\frac{a_2(-1+p)^2 - a_1R}{R}}, \quad a_4 \neq 0.$$

Case 5

When $R < 0$,

$$u_{51} = \left[\pm\sqrt{\frac{a_2}{2a_4}} \pm \sqrt{\frac{-a_2}{a_4}}\operatorname{sech}(\sqrt{-R}(x - \beta t + \xi_0))\right]^{1/(p-1)},$$

$$u_{52} = \left[\pm\sqrt{\frac{a_2}{2a_4}} \pm \sqrt{\frac{a_2}{a_4}}\operatorname{csch}(\sqrt{-R}(x - \beta t + \xi_0))\right]^{1/(p-1)},$$

$$u_{53} = \left[\pm\sqrt{\frac{a_2}{2a_4}} \pm \sqrt{\frac{a_2}{a_4}}\sec(\sqrt{-R}(x - \beta t + \xi_0))\right]^{1/(p-1)},$$

$$u_{54} = \left[\pm\sqrt{\frac{a_2}{2a_4}} \pm \sqrt{\frac{a_2}{a_4}}\csc(\sqrt{-R}(x - \beta t + \xi_0))\right]^{1/(p-1)},$$

where

$$\beta = \pm\sqrt{\frac{-2a_2 - 4a_1R}{4R}}, \quad a_3^2 = \frac{9a_2a_4}{2}, \quad a_4 \neq 0.$$

Case 6

When $a_2 = a_3 = 0$, $R = 0$,

$$u_6 = \left[\pm\sqrt{-\frac{p(a_1 + \beta^2)}{a_4(-1+p)^2}}\left(\frac{1}{x - \beta t + \xi_0}\right)\right]^{1/(p-1)},$$

where β is arbitrary constant, $a_4 \neq 0$.

Case 7

When $a_3 = 0$, $p = 2$, $R < 0$,

$$u_{71} = \mp\sqrt{\frac{-a_2}{a_4}}\tanh(\sqrt{-R}(x - \beta t + \xi_0)) \pm \sqrt{\frac{a_2}{a_4}}\operatorname{sech}(\sqrt{-R}(x - \beta t + \xi_0)),$$

$$u_{72} = \mp\sqrt{\frac{-a_2}{a_4}}\coth(\sqrt{-R}(x - \beta t + \xi_0)) \pm \sqrt{\frac{-a_2}{a_4}}\operatorname{csch}(\sqrt{-R}(x - \beta t + \xi_0)),$$

When $a_3 = 0$, $p = 2$, $R > 0$,

$$u_{73} = \mp\sqrt{\frac{a_2}{a_4}}\tan(\sqrt{R}(x - \beta t + \xi_0)) \pm \sqrt{\frac{a_2}{a_4}}\sec(\sqrt{R}(x - \beta t + \xi_0)),$$

$$u_{74} = \mp\sqrt{\frac{a_2}{a_4}}\cot(\sqrt{R}(x - \beta t + \xi_0)) \pm \sqrt{\frac{a_2}{a_4}}\csc(\sqrt{R}(x - \beta t + \xi_0)),$$

where

$$\beta = \pm\sqrt{-\frac{a_1 R + 2a_2}{R}}, \quad a_4 \neq 0.$$

Case 8

When $R < 0$,

$$u_{81} = \left[\pm\sqrt{\frac{a_2 p}{4a_4}} \mp \sqrt{\frac{a_2 p}{4a_4}}\tanh(\sqrt{-R}(x - \beta t + \xi_0)) \mp \sqrt{-\frac{a_2 p}{4a_4}}\operatorname{sech}(\sqrt{-R}(x - \beta t + \xi_0))\right]^{1/(p-1)},$$

$$u_{82} = \left[\pm\sqrt{\frac{a_2 p}{4a_4}} \mp \sqrt{\frac{a_2 p}{4a_4}}\coth(\sqrt{-R}(x - \beta t + \xi_0)) \mp \sqrt{\frac{a_2 p}{4a_4}}\operatorname{csch}(\sqrt{-R}(x - \beta t + \xi_0))\right]^{1/(p-1)}.$$

When $R > 0$,

$$u_{83} = \left[\pm\sqrt{\frac{a_2 p}{4a_4}} \mp \sqrt{\frac{-a_2 p}{4a_4}}\tan(\sqrt{R}(x - \beta t + \xi_0)) \mp \sqrt{\frac{-a_2 p}{4a_4}}\sec(\sqrt{R}(x - \beta t + \xi_0))\right]^{1/(p-1)},$$

$$u_{84} = \left[\pm\sqrt{\frac{a_2 p}{4a_4}} \mp \sqrt{\frac{-a_2 p}{4a_4}}\cot(\sqrt{R}(x - \beta t + \xi_0)) \mp \sqrt{\frac{-a_2 p}{4a_4}}\csc(\sqrt{R}(x - \beta t + \xi_0))\right]^{1/(p-1)},$$

where

$$\beta = \pm\sqrt{\frac{a_2(-1 + p)^2 - a_1 R}{R}}, \quad a_3^2 = \frac{a_2 a_4 (1 + p)^2}{p}, \quad a_4 \neq 0.$$

Remark 1

(1) It is easy to see that the solutions obtained for Eq. (1.1) completely include the solutions in [4]: u_{11}, u_{12} and u_{72}, are just the solutions type 1–3 respectively in [4].

(2) To our knowledge, the rest obtained solutions of a new class of Eq. (1.1) were not be found before.

4. Conclusions

In this paper, firstly we present a new class of nonlinear evolution equations with nonlinear term of any order, which includes many important NEMPS, so we think that the studying to Eq. (1.1) is very significant. Secondly the extended tanh-method is improved by means of new general ansätze, therefore we make the method much more lucid and straightforward to write solutions. Thirdly by a proper translation to Eq. (1.1), we applied the improved method to find solutions for Eq. (1.1). As a result, rich explicit exact solutions, which contain new kink-profile solitary-wave solutions, bell-profile solitary-wave solutions, periodic wave solutions and combined formal solitary-wave solutions, are obtained. In addition, we also can derive rational solutions for Eq. (1.1). Presently we are making the method computerizable, which allow us to perform complicated and tedious algebraic calculation on a computer.

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