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Chaos, Solitons and Fractals 17 (2003) 693–698

CHAOS
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Auto-Bäcklund transformation and exact solutions for modified nonlinear dispersive $mK(m,n)$ equations

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Accepted 11 October 2002

Communicated by Prof. M. Wadati

Abstract

Based on the idea of homogenous balance method and with the help of *Mathematica*, we obtain a new auto-Bäcklund transformation for modified nonlinear dispersive equation $mK(m,n)$. Then based on the Bäcklund transformation, some solitary patterns solution for $mK(m,n)$ equation are derived. In addition, we also obtain the general solutions for $mK(n,n)$ in higher dimensional spatial domains, even in N dimensional space.

1. Introduction

Since the soliton phenomena was first observed by Scott Russell in 1834 and KdV equation was solved by the inverse scattering method by Garder et al. in 1972 [1,2], the study of solutions and the related issue of the construction of solution to a wide of nonlinear equations has become one of the most exciting and extremely active areas of research and investigation. In recent years, the homogenous balance method (HB) has been widely applied to derive the nonlinear transformation and exact solutions (especially the solitary wave solutions) [4,6,7,9,10], and auto-Bäcklund transformations [5,7,8] as well as the similarity reductions [7,8] of nonlinear partial differential equations (PDEs) in mathematical physics. The Bäcklund transformations of nonlinear PDEs play an important role in solitary theory, which is an efficient method to obtain exact solutions of nonlinear PDE. The nonlinear iterative principle from Bäcklund transformations converts the problem of solving nonlinear PDE to purely algebraic calculations [1–3]. In Refs. [7,8], Fan extended HB method to search for Bäcklund transformation and similarity reductions of nonlinear PDE. So more solutions can be obtained by the improved HB method. However, they only dealt with the cases whose balance constants are positive integers. In this paper, we would further extend the HB method so that it can deal with the other cases whose balance constant is fraction or negative integer. To illustrate the extended HB method, we consider modified nonlinear dispersive equation $mK(m,n)$. Rosenau and Hyman [11] investigated the role of nonlinear dispersion in the formation of patterns in liquid drops by introducing and studying a family of nonlinear KdV like equation of the form

$$u_t + a(u^m)_x + (u^n)_{xxx} = 0, \quad m > 1, \quad 1 \leqslant n \leqslant 3, \tag{1.1}$$

they found that nonlinear dispersion can compactify solitary waves and generate compactions, and introduced a class of solitary waves with compact support, which they called compactions, that collide elastically and vanish identically outside a finite core region [11–15]. They discovered that solitary waves may compactify under the influence of nonlinear dispersion which is capable of causing deep qualitative changes in the native of genuinely nonlinear phenomena.

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0960-0779/03/$ - see front matter © 2003 Elsevier Science Ltd. All rights reserved.
doi:10.1016/S0960-0779(02)00485-X

Equations of this type with values of m and n are denoted by $K(m,n)$, In [11], four cases $m,n = 2,3$ were studied thoroughly. The studies continued in this direction and the general case where $m = n$ was examined and a general formula that satisfies (1.1)–(1.5) was derived. Several other paper by Rosenau [12–14] and by Rosenau and Hyman [11] investigated the new discovery thoroughly. Olver and Rosenau [12] investigated the tri-Hamiltonian duality between solitons and compactons. Ismail and Taha [16] implemented a finite difference method and a finite element method to study the two type $K(2,2)$ and $K(3,3)$ equations. Ludu and Draayer [17] introduced a useful work on patterns on liquid surfaces where cnoidal waves compactons and sealing wave discussed. In [18], Dinda and Remoissenet demonstrated the existence of a breacher with a compact support, i.e., a breather compacton, in a nonlinear Klein–Gordon lattice with a soft on site substrate potential.

For more details about the role of nonlinear dispersion in pattern formation and for more insight through the compacton behavior, the reader is advised to see the remarkable achievements in [11–17].

Wazwaz has devoted considerable effort to the study on $K(n,n)$ equation and make new developments in this regard [19–24]. In [23], two set of entirely new formulas that produce compactons and anticompacton for any integer n, $n \geqslant 1$ are established. Wazwaz present a general and unified approach for analyzing the genuinely nonlinear dispersive $mK(n,n)$ equation in one-, two- and three-dimensional spatial domain given by

$$u^{n-1}u_t + a(u^m)_x + (u^n)_{xxx} = 0, \tag{1.2}$$

$$u^{n-1}u_t + a(u^m)_x + b(u^n)_{xxx} = 0, \tag{1.3}$$

$$u^{n-1}u_t + a(u^m)_x + b(u^n)_{xxx} + k(u^n)_{yyy} = 0, \tag{1.4}$$

$$u^{n-1}u_t + a(u^m)_x + b(u^n)_{xxx} + k(u^n)_{yyy} + r(u^n)_{zzz} = 0, \tag{1.5}$$

where a, b, k, r are constants. Eqs. (1.2)–(1.5) differ from those studied in [11–22] only the addition of term u^{n-1} that multiplies u_t. Wazwaz formally shows how to construct compact and noncompact solutions in one-, two- and three-dimensional spatial domains. Two distinct general formulaes for each model, that are of substantial interest, are developed for all positive integers, n, $n > 1$.

The present work is motivated by the desire to extend the work made in [23] and with the use of some proper transformations and the extend HB method, we consider the general modified nonlinear dispersive equation $mK(m,n)$, we obtain all of the general compacton solution of the nonlinear $mK(n,n)$ equation in [23]. In particularly, we obtain a new auto-Bäcklund transformation for Eq. (1.3). Based on the auto-Bäcklund transformation , some solitary patterns solution for Eq. (1.3) are derived.

This paper is organized as follows. In Section 2, we derive the general solutions for $mK(n,n)$ equation. In Section 3, a new Bäcklund for $mK(m,n)$ is obtained, then based on the Bäcklund transformation, some solutions for $mK(2n,n)$ and $mK(3n,n)$ equation are obtained. Conclusions are given in the last section.

2. Solutions for $mK(n,n)$ equations

Let us consider the $mK(m,n)$ equations, i.e. Eq. (1.3). According to the idea of HB method [4–10], by balancing the highest order partial derivative term and the nonlinear term in Eq. (1.3), we obtain balance constant

$$p = \frac{2}{m-n}, \quad (m \neq n). \tag{2.1}$$

Therefore we separate two cases to discuss the $mK(m,n)$ equations, (i) $m = n$, (ii) $m \neq n$.

When $m = n$, Eq. (1.3) changes into

$$u^{n-1}u_t + a(u^n)_x + b(u^n)_{xxx} = 0. \tag{2.2}$$

We make transform

$$u(x,t) = v^{1/n}(x,t), \tag{2.3}$$

then substituting (2.3) into (2.2) yields

$$v_t + nav_x + nbv_{xxx} = 0. \tag{2.4}$$

We consider the travelling solutions of Eq. (2.4). Setting

$$v(x,t) = v(\xi), \quad \xi = k(x - ct), \tag{2.5}$$

then substituting (2.5) into Eq. (2.4) yields

$$(na - c)v' + nbk^2 v''' = 0, \tag{2.6}$$

where prime denotes $d/d\xi$.

The general solution of linear ordinary differential equation (2.6) is

$$v(\xi) = -\frac{k\sqrt{n}c_1}{\sqrt{c-an}}\exp\left[-\frac{\sqrt{c-an}}{k\sqrt{n}}\xi\right] + \frac{k\sqrt{n}c_2}{\sqrt{c-an}}\exp\left[\frac{\sqrt{c-an}}{k\sqrt{n}}\xi\right] + c_3, \tag{2.7}$$

where k, c_1, c_2, c_3 are arbitrary constants.Therefore from (2.3) and (2.7), we obtain a family of solutions of (1.2)

$$u(x,t) = \left\{-\frac{k\sqrt{n}c_1}{\sqrt{c-an}}\exp\left[-\frac{\sqrt{c-an}}{\sqrt{n}}(x-ct)\right] + \frac{k\sqrt{n}c_2}{\sqrt{c-an}}\exp\left[\frac{\sqrt{c-an}}{\sqrt{n}}(x-ct)\right] + c_3\right\}^{1/n}. \tag{2.8}$$

When setting the constants c_1, c_2, c_3 to be equal to various values, we obtain the following four families solutions for $mK(n,n)$ equations

$$u_1(x,t) = \left\{\frac{2k\sqrt{n}c_1}{\sqrt{an-c}}\sin\left[\frac{\sqrt{na-c}}{\sqrt{n}}(x-ct)\right]\right\}^{1/n}, \quad c_1 = c_2, \quad c_3 = 0, \tag{2.9}$$

$$u_2(x,t) = \left\{-\frac{2k\sqrt{n}c_1}{\sqrt{c-an}}\cos\left[\frac{\sqrt{na-c}}{\sqrt{n}}(x-ct)\right]\right\}^{1/n}, \quad c_1 = -c_2, \quad c_3 = 0. \tag{2.10}$$

In view of the arbitrariness of the constants k, c_1, c, the solutions of (2.9) and (2.10) cover the solutions in [14].

$$u_3(x,t) = \left\{\frac{2k\sqrt{n}c_1}{\sqrt{c-an}}\sinh\left[-\frac{\sqrt{c-an}}{\sqrt{n}}(x-ct)\right]\right\}^{1/n}, \quad c_1 = c_2, \quad c_3 = 0, \tag{2.11}$$

$$u_4(x,t) = \left\{-\frac{2k\sqrt{n}c_1}{\sqrt{c-an}}\cosh\left[-\frac{\sqrt{c-an}}{\sqrt{n}}(x-ct)\right]\right\}^{1/n}, \quad c_2 = -c_1, \quad c_3 = 0. \tag{2.12}$$

We set the nonlinear dispersive equation in an N-dimensional space

$$u^{n-1}u_t + a(u^n)_x + \sum_{i=1}^{N} b_i(u^n)_{x_i x_i x_i} = 0, \tag{2.13}$$

where $x_1 = x$, $u \equiv u(x_1, x_2, \ldots, x_N, t)$, and b_i are constants.

We now consider the travelling wave solutions of Eq. (2.13) in the form

$$u(x_1, x_2, \ldots, x_m, t) = u(\xi), \quad \xi = k(x_1 + x_2 + \cdots + x_N - ct). \tag{2.14}$$

Proceeding as before, we obtain the following general solutions of Eq. (2.13)

$$u(x,t) = \left\{-\frac{k\sqrt{n\sum_{i=1}^N b_i}c_1}{\sqrt{c-an}}\exp\left[-\frac{\sqrt{c-an}}{\sqrt{n\sum_{i=1}^N b_i}}\xi\right] + \frac{k\sqrt{n\sum_{i=1}^N b_i}c_2}{\sqrt{c-an}}\exp\left[\frac{\sqrt{c-an}}{\sqrt{n\sum_{i=1}^N b_i}}\xi\right] + c_3\right\}^{1/n}. \tag{2.15}$$

where k, c, c_1, c_2, c_3, $b_i (i = 1, \cdots, N)$ are arbitrary constants.

It is not difficult to verify that from the solution (2.15), when setting parameters to be equal to proper values, we can obtain all of the general compacton solutions of the nonlinear $mK(n,n)$ equation in [23].

3. Bäcklund transformation and solutions for $mK(m,n)$ equations

When $m \neq n$, from (2.1), p may be arbitrary constants. In order to apply the HB method under this condition , we firstly make the transformation

$$u(x,t) = v(x,t)^{2/(m-n)}, \tag{3.1}$$

then substituting transformation (3.1) into Eq. (3.2) yields

$$am(m-n)v^4v_x + 2bn(m^2-5mn+6n^2)v_x^3 - 3bn(m^2-4mn+3n^2)vv_xv_{xx} + (m-n)^2v^2(v_t+bnv_{xxx}) = 0. \tag{3.2}$$

Then by balancing the highest order partial derivative term and the nonlinear term in Eq. (3.2), we get the value of the balance constant $p = 1$. Therefore we seek for the Bäcklund transformation of Eq. (3.2) in the form

$$v = f'w_x + \phi. \tag{3.3}$$

Here and in the following context$' := \partial/\partial w$, $f^{(r)} = \partial^r/\partial w^r$, and $f = f(w)$, $w = w(x,t)$ is undetermined function and $\phi(x,t)$ is a special solution of Eq. (3.2).

With the help of Mathematica, substituting (3.3) into (3.2) yields (because the formula is so long, just one part of it is shown here)

$$\left[am(m-n)^2f'^4f'' + 2bn(m^2-5mn+6n^2)f''^3 + 3bn(m^2-4mn+3n^2)f'f''f^{(3)} + b(m-n)^2nf'^2f^{(4)}\right]w_x^6 + \cdots$$
$$= 0. \tag{3.4}$$

To simplify Eq. (3.4), setting the coefficient of w_x^6 to zero yields an ordinary differential equation for f

$$am(m-n)^2f'^4f'' + 2bn(m^2-5mn+6n^2)f''^3 + 3bn(m^2-4mn+3n^2)f'f''f^{(3)} + b(m-n)^2nf'^2f^{(4)} = 0. \tag{3.5}$$

Solving (3.5) we obtain a solution

$$f = \pm\sqrt{-\frac{2bn(m+n)}{a(m-n)^2}}\ln w. \tag{3.6}$$

Setting $\beta = \pm\sqrt{-2bn(m+n)/a(m-n)^2}$ (*note* : in the rest of this paper β denotes $\pm\sqrt{-2bn(m+n)/a(m-n)^2}$), then substituting (3.6) into (3.4), formula (3.4) can be simplified to a polynomial of $1/w^i$ $(i = 0,\dots,5)$, then setting the coefficients of $1/w^i$ $(i = 0,\dots,5)$, to zero yields a set of partial differential equations for $w(x,t)$

$$am(m-n)^2\phi^4\phi_x + 2bn(m^2-5mn+6n^2)\phi_x^3 - 3bn(m^2-4mn+3n^2)\phi\phi_x\phi_{xx} + (m-n)^2\phi^2(\phi_t+bn\phi_{xxx}) = 0, \tag{3.7}$$

$$\begin{aligned}&am(m-n)^2\phi^4w_{xx} + (m-n)^2\phi^2(w_{xt}+bnw_{xxxx}) + 4am(m-n)^2\phi^3w_x\phi_x + 3b(m-3n)n\phi_x(2(m-2n)w_{xx}\phi_x\\&\quad + (-m+n)w_x\phi_x x) + (m-n)\phi(2(m-n)w_x\phi_t + bn(-3(m-3n)w_{xxx}\phi_x - 3(m-3n)w_{xx}\phi_{xx}\\&\quad + 2(m-n)w_x\phi_{xxx})) = 0,\end{aligned} \tag{3.8}$$

$$\begin{aligned}&a^2m(m-n)^4\beta\phi^4w_x^2 + 8abm(m-n)^2n(m+n)\phi^3w_xw_{xx} + a(m-n)^2\phi^2((m-n)^2\beta w_tw_x + bn(3(m-n)^2\beta w_{xx}^2\\&\quad + 4(m-n)^2\beta w_xw_{xxx} + 12m(m+n)w_x^2\phi_x)) - b(m-n)n\phi(6bn(m^2-2mn-3n^2)w_{xx}w_{xxx} + (m-n)w_x\\&\quad \times(-4(m+n)w_{xt} - 4bn(m+n)w_{xxxx} + 9a(m^2-4mn+3n^2)\beta w_{xx}\phi_x) + 3a(m-3n)(m-n)^2\beta w_x^2\phi_{xx})\\&\quad + 2bn((m-n)^2(m+n)w_x^2\phi_t + 6bn(m^3-4m^2n+mn^2+6n^3)w_{xx}^2\phi_x - 3bn(m^3-3m^2n-mn^2+3n^3)\\&\quad \times w_x(w_{xxx}\phi_x + w_{xx}\phi_{xx}) + (m-n)^2w_x^2(3a(m^2-5mn+6n^2)\beta\phi_x^2 + bn(m+n)\phi_{xxx})) = 0,\end{aligned} \tag{3.9}$$

$$\begin{aligned}&-4am(m-n)^2(m+n)\phi^3w_x^3 + 6a(m-n)^2(3m-n)n\beta\phi^2w_x^2w_{xx} + (m-n)\phi w_x(2(-m^2+n^2)w_tw_x\\&\quad + 3bn(m^2-6mn-7n^2)w_{xx}^2 - bn(5m^2+6mn+n^2)w_xw_{xxx} + a(7m^3-15m^2n+17mn^2-9n^3)\beta w_x^2\phi_x)\\&\quad + (m+n)(2bn(m^2-5mn+6n^2)\beta w_{xx}^3 - 3bn(m^2-4mn+3n^2)\beta w_xw_{xx}w_{xxx} + w_x^2((m-n)^2\beta w_{xt}\\&\quad + bn((m-n)^2\beta w_{xxxx} - 3(m^2-8mn+15n^2)w_{xx}\phi_x)) + 3bn(m^2-4mn+3n^2)w_x^3\phi_{xx}) = 0,\end{aligned} \tag{3.10}$$

$$\begin{aligned}&2bnw_x^2((m-n)^2(m+n)\beta w_tw_x + 3a(m-n)^2(m^2+4mn-n^2)\beta\phi^2w_x^2 + bn(-m^3-5m^2n+17mn^2+21n^3)\phi w_xw_{xx}\\&\quad + bn(m+n)(12n^2\beta w_{xx}^2 + (m^2+4mn-5n^2)\beta w_xw_{xxx} + 2(m^2+4mn-9n^2)w_x^2\phi_x)) = 0,\end{aligned} \tag{3.11}$$

$$2b^2n^2(m+n)^2(m+3n)w_x^4(2\phi w_x + \beta w_{xx}) = 0. \tag{3.12}$$

From (3.1), (3.3), (3.7) and (3.12), we obtain a desired Bäcklund transformation of Eq. (1.3)

$$u = \left[\pm\sqrt{-\frac{2bn(m+n)}{a(m-n)^2}}\frac{\partial}{\partial x}\ln w + \phi\right]^{2/(m-n)}, \tag{3.13}$$

where w satisfies (3.7)–(3.12), ϕ is a solution of Eq. (3.7).

Now we use the Bäcklund transformation consisted of (3.13) and (3.7)–(3.12) to exploit some explicit exact solutions for Eq. (1.3). If we take initial solution of Eq. (3.7) as constant A, then (3.7)–(3.12) reduce to

$$w_{xt} + amA^2 w_{xx} + bnw_{xxxx} = 0, \tag{3.14}$$

$$a^2m(m-n)^3\beta A^3 w_x^2 + 8abm(m-n)n(m+n)A^2 w_x w_{xx} + a(m-n)^3\beta A(w_t w_x + bn(3w_{xx}^2 + 4w_x w_{xxx})) \\ - bn(6bn(m^2-2mn-3n^2)w_{xx}w_{xxx} - 4(m-n)(m+n)w_x(w_{xt} + bnw_{xxxx})) = 0, \tag{3.15}$$

$$-4am(m-n)^2(m+n)A^3w_x^3 + 6a(m-n)^2(3m-n)n\beta A^2 w_x^2 w_{xx} + (m-n)Aw_x(2(-m^2+n^2)w_t w_x \\ + bn(m+n)(3(m-7n)w_{xx}^2 - (5m+n)w_x w_{xxx})) + (m+n)\beta(2bn(m^2-5mn+6n^2)w_{xx}^3 \\ - 3bn(m^2-4mn+3n^2)w_x w_{xx} w_{xxx} + (m-n)^2 w_x^2(w_{xt} + bnw_{xxxx})) = 0, \tag{3.16}$$

$$2bnw_x^2((m-n)^2(m+n)\beta w_t w_x + 3a(m-n)^2(m^2+4mn-n^2)\beta A^2 w_x^2 + bn(-m^3-5m^2n+17mn^2+21n^3)Aw_x w_{xx} \\ + bn(m+n)(12n^2\beta w_{xx}^2 + (m^2+4mn-5n^2)\beta w_x w_{xxx})) = 0, \tag{3.17}$$

$$2b^2n^2(m+n)^2(m+3n)w_x^4(2Aw_x + \beta w_{xx}) = 0. \tag{3.18}$$

Now we assume that $w(x,t)$ is of the form

$$w(x,t) = C + D\exp^{k(x-\lambda t)}, \tag{3.19}$$

where $C \neq 0$, $D \neq 0$, k and λ are constants to be determined.

Substituting (3.19) into (3.14)–(3.18), we find that (3.19) satisfies Eqs. (3.14)–(3.18) under the following cases

Case 1

$$m = 2n, \quad \lambda_1 = \frac{4}{3}aA^2 n, \quad \beta_1 = \pm\sqrt{-\frac{6b}{a}}, \quad k_1 = \mp 2A\sqrt{-\frac{a}{6b}}, \tag{3.20}$$

Case 2

$$m = 3n, \quad \lambda_2 = aA^2 n, \quad \beta_2 = \pm\sqrt{-\frac{2b}{a}}, \quad k = \mp 2A\sqrt{-\frac{a}{2b}}. \tag{3.21}$$

From (3.20), the equation, $u^{n-1}u_t + a(u^{2n})_x + b(u^n)_{xxx} = 0$, i.e., $mK(2n,n)$ equation, has the following solutions

$$u_1 = \left\{ \frac{-2AD\exp\left[\mp 2A\sqrt{-\frac{a}{6b}}(x - \frac{4}{3}aA^2nt)\right]}{C + D\exp\left[\mp 2A\sqrt{-\frac{a}{6b}}(x - \frac{4}{3}aA^2nt)\right]} + A \right\}^{2/n}, \tag{3.22}$$

where A, C, D are arbitrary constants and $ab < 0$.

If setting $C = \pm D$, from (3.22) we can obtain the kink-profile solitary-wave solutions for $mK(2n,n)$ equation

$$u = \left\{ \mp A\tanh\left[\mp A\sqrt{-\frac{a}{6b}}\left(x - \frac{4}{3}aA^2nt\right)\right]\right\}^{2/n}. \tag{3.23}$$

From (3.21), the equation, $u^{n-1}u_t + a(u^{3n})_x + b(u^n)_{xxx} = 0$, i.e., $mK(3n,n)$ equation, has the following solutions

$$u_2 = \left\{ \frac{-2AD\exp\left[\mp 2A\sqrt{-\frac{a}{2b}}(x - aA^2nt)\right]}{C + D\exp[\mp 2A\sqrt{-\frac{a}{2b}}(x - aA^2nt)]} + A \right\}^{1/n}, \tag{3.24}$$

where A, C, D are arbitrary constants and $ab < 0$.

Setting $C = \pm D$, from (3.24) we can obtain the kink-profile solitary-wave solutions for $mK(3n,n)$ equation

$$u = \left\{ \mp A\tanh\left[\mp A\sqrt{-\frac{a}{2b}}(x - aA^2nt)\right]\right\}^{1/n}. \tag{3.25}$$

4. Conclusions

The phenomena of compactons shows a rich variety of concepts and properties that should be addressed and, therefore, more work should be invested in studying these newly developed structures [11–24]. Many scientific processes [17] other than fluid, such as super deformed nuclei, preformation of cluster in hydrodynamic models and the fission of liquid drops may be explained on the basis of the compacton concept. A general solutions for $mK(n,n)$ equation are obtained. At the same time, by use of the extended HB method, a new Bäcklund transformation for $mK(m,n)$ equations are obtained. To our knowledge, this type of Bäcklund transformation obtained has not been ever seen before in the literature. Then based on the Bäcklund transformation, some solutions for $mK(2n,n)$ equation and $mK(3n,n)$ equation are obtained. This method can also apply to other PDEs. In addition, this method is also computerizable, which allow us to perform complicated and tedious symbolic algebraic calculation on a computer.

Acknowledgements

The work is supported by the National Natural Science Foundation of China under the Grant no. 1007201, the National Key Basic Research Development Project Program under the Grant no. G1998030600 and Doctoral Foundation of China under the Grant no. 98014119.

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