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A series of new soliton-like solutions and double-like periodic solutions of a (2 + 1)-dimensional dispersive long wave equation

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Abstract

In this paper, we extend the algebraic method proposed by Fan (Chaos, Solitons & Fractals 20 (2004) 609) and the improved extended tanh method by Yomba (Chaos, Solitons and Fractals 20 (2004) 1135) to uniformly construct a series of soliton-like solutions and double-like periodic solutions for nonlinear partial differential equations (NPDE). Some new soliton-like solutions and double-like periodic solutions of a (2 + 1)-dimensional dispersive long wave equation are obtained.

1. Introduction

There has been a great amount of activity aiming to find methods for exact solutions of nonlinear partial differential equations (NPDE). The exact analytic solutions of NPDE may enable help physicists and engineers to better understand the phenomena that a given NPDE describes, for these exact solution may help them to discuss and examine the sensitivity of the model to some physical parameters. In the past decades, there has been significant progression in the development of these methods such as inverse scattering method [1], Bäcklund transformation [2,3], Darboux transformation [4], Hirota bilinear method [5], homogeneous balance method [6] and various tanh methods [7–11]. Among those, the tanh method provides a straightforward and effective algorithm to obtain such particular solutions for a large of nonlinear equations. Recently much research work has been concentrated on the various extensions and applications of the tanh method [8–11] to simplify the routine calculation of the method and to find more general travelling wave solutions, more general soliton-like solutions and so on.

Recently, Fan [12] developed a new algebraic method, to seek more new solitary wave solutions of NPDEs that can be expressed as a polynomial in a elementary which satisfies a more general Riccati equation. The solutions obtained include polynomial solutions, exponential solutions, rational solutions, triangular periodic wave solutions, hyperbolic, and soliton solutions, Jacobi and Weierstrass doubly periodic wave solutions.

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More recently, Yomba [13] improved extended tanh method and obtained new soliton-like solution of the $(2+1)$-dimensional dispersive long wave equation.

The present work is motivated by the desire to generalize the work made in [12] and in [13] by proposing a more general ansatz so that it can be used to obtained more types and general formal solutions which contain not only the results obtained by using the method [12] and [13] but also other types of solutions. For illustration, we apply the generalized method to solve $(2+1)$-dimensional dispersive long wave equation and successfully construct a series of new and more general soliton-like solutions and double-like periodic solutions. The more general soliton-like solutions and double-like periodic solutions obtained by the generalized method contain some arbitrarily differentiable functions and some arbitrary constants, which may enable one to discuss the behaviour of solutions as a function of these arbitrary differentiable functions and some arbitrary constants and this also provide enough freedom to build up solutions that may correspond to a particular physical situation, or initial condition have some desired features, which means a great variation in the solutions.

Our paper is organized as follows. In the following Section 2, the detail derivation of the generalized algebraic method will be given. The applications of the generalized method to $(2+1)$-dimensional dispersive long wave equation are illustrated in Section 3. The conclusion is then given in the final Section 4.

2. Summary of the generalized method

In the following we would like to outline the main steps of our general method:

Step 1. For a given nonlinear partial differential equation (NPDE) system with some physical fields $u_i(x,y,t)(i=1,\dots,n)$ in three variables x, y, t,

$$F_i(u_i,u_{it},u_{ix},u_{iy},u_{itt},u_{ixt},u_{iyt},u_{ixx},u_{iyy},u_{ixy},\dots)=0. \tag{2.1}$$

We express the solutions of the NPDE by the new more general ansatz

$$u_i(\xi)=a_{i0}+\sum_{j=1}^{m_i}a_{ij}\phi^j, \tag{2.2}$$

where m_i is an integer to be determined by balance the highest-order derivative terms with the nonlinear terms in Eq. (2.1), the new variable $\phi=\phi(\xi)$ satisfying:

$$\phi'=\frac{\mathrm{d}\phi}{\mathrm{d}\xi}=\sqrt{\sum_{\rho=0}^{r}h_\rho\phi^\rho}, \tag{2.3}$$

and $a_{i0}=a_{i0}(x,y,t)$, $a_{ij}=a_{ij}(x,y,t)$ $(i=1,2,\dots;j=1,2,\dots,m_i)$ and $\xi=\xi(x,y,t)$ are all differentiable functions to be determined later. Here h_0, h_1, h_2, h_3, h_4 are constants.

The ansatz proposed here is more general than the method [12] by Fan and the improved method [13] by Yomba. Firstly, compared with the method [12], the restriction on $\xi(x,y,t)$ as merely a linear function x,y,t and the restriction on the coefficients a_{i0}, a_{ij} $(i=1,2,\dots;j=1,2,\dots,m_i)$ as constants are removed. Secondly, compared with the improved method [13] by Yomba, Eq. (2.3) that the new variable $\phi=\phi(\xi)$ satisfies is more general, so more types of solutions would be expected for some equations.

Step 2. Substitute Eq. (2.2) into Eq. (2.1) along with Eq. (2.3) and then set all coefficients of $\phi^p(\sqrt{\sum_{\rho=0}^{r}h_\rho\phi^\rho})^q$ $(q=0,1;p=0,1,2,\dots)$ to be zero to get an over-determined partial differential equations with respect to a_{i0}, a_{ij}, $(i=1,2,\dots;j=1,2,\dots,m_i)$ and ξ.

Step 3. Solving the over-determined partial differential equations by use of Maple, we would end up with the explicit expressions for a_{i0}, a_{ij} $(i=1,2,\dots;j=1,2,\dots,m_i)$ and ξ or the constrains among them.

Step 4. By using the results obtained in the above steps, we can derive a series of fundamental-like solutions such as polynomial-like, exponential-like, solitary-like wave, rational-like, triangular-like periodic, Jacobi and Weierstrass doubly-like periodic solutions. Because we are interested in solitary-like wave, Jacobi and Weierstrass doubly-like periodic solutions and tan-like and cot-like type solutions appearing in pairs with tanh-like and coth-like type solutions respectively, therefore polynomial-like, rational-like, triangular-like periodic solutions are omitted in this paper. By considering the different values of h_0, h_1, h_2, h_3 and h_4, Eq. (2.3) has many kinds of solitary-like wave, Jacobi and Weierstrass doubly-like periodic solutions which are listed as follows.

(i) Solitary-like wave solutions.

a. Bell shaped soliton-like solutions.

$$\phi = \sqrt{-\frac{h_2}{h_4}}\operatorname{sech}(\sqrt{h_2}\xi), \quad h_0 = h_1 = h_3 = 0, \quad h_2 > 0, \quad h_4 < 0, \tag{2.4}$$

$$\phi = -\frac{h_2}{h_3}\operatorname{sech}^2\left(\frac{\sqrt{h_2}}{2}\xi\right), \quad h_0 = h_1 = h_4 = 0, \quad h_2 > 0. \tag{2.5}$$

b. Kink shaped soliton-like solutions.

$$\phi = \sqrt{-\frac{h_2}{2h_4}}\tanh\left(\sqrt{-\frac{h_2}{2}}\xi\right), \quad h_0 = \frac{h_2^2}{4h_4}, \quad h_1 = h_3 = 0, \quad h_2 < 0, \quad h_4 > 0. \tag{2.6}$$

c. Soliton-like solutions.

$$\phi = \frac{h_2\operatorname{sech}^2\left(\frac{1}{2}\sqrt{h_2}\xi\right)}{2\sqrt{h_2h_4}\tanh\left(\frac{1}{2}\sqrt{h_2}\xi\right) - h_3}, \quad h_0 = h_1 = 0, \quad h_2 > 0. \tag{2.7}$$

(ii) Jacobi and Weierstrass doubly-like periodic solutions.

$$\phi = \sqrt{\frac{-h_2m^2}{h_4(2m^2-1)}}\operatorname{cn}\left(\sqrt{\frac{h_2}{2m^2-1}}\xi\right), \quad h_4 < 0, \quad h_2 > 0, \quad h_0 = \frac{h_2^2m^2(1-m^2)}{h_4(2m^2-1)^2}, \tag{2.8}$$

$$\phi = \sqrt{\frac{-m^2}{h_4(2-m^2)}}\operatorname{dn}\left(\sqrt{\frac{h_2}{2-m^2}}\xi\right), \quad h_4 < 0, \quad h_2 > 0, \quad h_0 = \frac{h_2^2(1-m^2)}{h_4(2-m^2)^2}, \tag{2.9}$$

$$\phi = \sqrt{\frac{-h_2m^2}{h_4(m^2+1)}}\operatorname{sn}\left(\sqrt{-\frac{h_2}{m^2+1}}\xi\right), \quad h_4 > 0, \quad h_2 < 0, \quad h_0 = \frac{h_2^2m^2}{h_4(m^2+1)^2}, \tag{2.10}$$

where m is a modulus.

$$\phi = \wp\left(\frac{\sqrt{h_3}}{2}\xi, g_2, g_3\right), \quad h_2 = 0, \quad h_3 > 0, \tag{2.11}$$

where $g_2 = -4\frac{h_1}{h_3}$ and $g_3 = -4\frac{h_0}{h_3}$ are called invariants of Weierstrass elliptic function. The Jacobi elliptic functions are doubly periodic and possess properties of triangular functions:

$$\operatorname{sn}^2\xi + \operatorname{cn}^2\xi = 1, \quad \operatorname{dn}^2\xi = 1 - m^2\operatorname{sn}^2\xi,$$

$$(\operatorname{sn}\xi)' = \operatorname{cn}\xi\operatorname{dn}\xi, \quad (\operatorname{cn}\xi)' = -\operatorname{sn}\xi, \quad (\operatorname{dn}\xi)' = -m^2\operatorname{sn}\xi\operatorname{cn}\xi.$$

When $m \to 1$, the Jacobi functions degenerate to the hyperbolic functions, i.e.

$$\operatorname{sn}\xi \to \tanh\xi, \quad \operatorname{cn}\xi \to \operatorname{sech}\xi,$$

when $m \to 0$, the Jacobi functions degenerate to the triangular functions, i.e.

$$\operatorname{sn}\xi \to \sin\xi, \quad \operatorname{cn}\xi \to \cos\xi.$$

The more detailed notations for the Weierstrass and Jacobi elliptic functions can be found in Refs. [14,15].

Remarks. For the generalization of the ansatz, naturally more complicated computation is expected than ever before. Even if the availability of computer symbolic systems like *Maple* and *Mathematica* allow us to perform the complicated and tedious algebraic calculation and differentiation on a computer, in general it is very difficult, sometime impossible, to solve the set of over-determined partial differential equations in step 3. As the calculation goes on, in order to drastically simplify the work or make the work feasible, we often choose special function forms for a_{i0}, a_{ij} $(i = 1, 2, \ldots; j = 1, 2, \ldots, m_i)$ and ξ, on a trial-and-error basis.

3. Exact soliton like solutions of the (2 + 1)-dimensional dispersive long wave equation

Let us consider the (2 + 1)-dimensional dispersive long wave (DLWE) equation,

$$\begin{cases} u_{yt} + v_{xx} + (uu_x)_y = 0, \\ v_t + u_x + (uv)_x + u_{xxy} = 0, \end{cases} \tag{3.1}$$

which was first derived by Boiti et al. [16] as a compatibility for a "weak" Lax pair. Recently considerable effort has been devoted to the study of this system. For more detail about results about this system, the reader is advised to see the remarkable achievements in Refs. [13,16–22].

By balancing the highest-order contributions from both the linear and nonlinear terms in Eq. (3.1), we suppose that Eq. (3.1) has the following formal solutions,

$$\begin{cases} u(x,y,t) = a_0 + a_1\phi, \\ v(x,y,t) = A_0 + A_1\phi + A_2\phi^2, \end{cases} \tag{3.2}$$

where $a_0 = a_0(y,t)$, $a_1 = a_1(y,t)$, $A_0 = A_0(y,t)$, $A_1 = A_1(y,t)$, $A_2 = A_2(y,t)$ and $\xi = kp + q(k = k(x), p = p(y,t)$ and $q = q(y,t))$ are all differentiable functions, and $\phi = \phi(\xi)$ satisfies (2.3).

With the aid of *Maple*, substituting (3.2) along with (2.3) into (3.1), yields a set of partial differential equations for $\phi^i\left(\sqrt{\sum_{\rho=0}^4 h_\rho\phi^\rho}\right)^j$, $(i = 0,1,\ldots; j = 0,1)$. Setting the coefficients of these terms $\phi^i\left(\sqrt{\sum_{\rho=0}^4 h_\rho\phi^\rho}\right)^j$ to zero yields a set of over-determined partial differential equations with respect to a_0, a_1, A_0, A_1, A_2, k, p and q.

By use of the *Maple*, solving the over-determined partial differential equations, we get the following results.

$$\begin{aligned} &k = C_1x + C_2, \quad q = \frac{\pm\int F_1(y)dy + F_2(t)\sqrt{h_4}C_3}{\sqrt{h_4}C_3}, \quad A_2 = F_1(y), \\ &a_0 = \frac{\pm 8h_4^{\frac{3}{2}}\frac{d}{dt}F_2(t) + C_3^2h_3}{4C_3h_4}, \quad a_1 = C_3, \quad A_1 = \frac{h_3F_1(y)}{2h_4}, \\ &A_0 = \frac{4h_4h_2F_1(y) - h_3^2F_1(y) - 8h_4^2}{8h_4^2}, \quad p = \frac{1}{2}\frac{C_3}{\sqrt{h_4}C_1}. \end{aligned} \tag{3.3}$$

From (3.2) and (3.3), we obtain the following solutions for Eq. (3.1).

Family 1. From Eq. (3.3), when $h_0 = h_1 = h_3 = 0$, $h_2 > 0$ and $h_4 < 0$, we obtain the following soliton-like solution for the DLWE equation, as follows.

$$u_1 = \pm 2\frac{\sqrt{h_4}\frac{d}{dt}F_2(t)}{C_3} + C_3\sqrt{-\frac{h_2}{h_4}}\operatorname{sech}(\sqrt{h_2}\xi), \tag{3.4.1}$$

$$v_1 = \frac{4h_4h_2F_1(y) - 8h_4^2}{8h_4^2} - \frac{F_1(y)h_2\operatorname{sech}^2(\sqrt{h_2}\xi)}{h_4}, \tag{3.4.2}$$

where $\xi = kp + q$, k, p and q are determined by Eq. (3.3).

Family 2. From Eq. (3.3), when $h_1 = h_3 = 0$, $h_0 = \frac{h_2^2}{4h_4}$, $h_2 < 0$ and $h_4 > 0$, we obtain the following soliton-like solution for the DLWE equation, as follows.

$$u_2 = \pm 2\frac{\sqrt{h_4}\frac{d}{dt}F_2(t)}{C_3} + \frac{1}{2}C_3\sqrt{-2\frac{h_2}{h_4}}\tanh\left(\frac{\sqrt{-2h_2}}{2}\xi\right), \tag{3.5.1}$$

$$v_2 = \frac{4h_4h_2F_1(y) - 8h_4^2}{8h_4^2} - \frac{F_1(y)h_2\tanh^2\left(\frac{\sqrt{-2h_2}}{2}\xi\right)}{2h_4}, \tag{3.5.2}$$

where $\xi = kp + q$, k, p and q are determined by Eq. (3.3).

Family 3. From Eq. (3.3), when $h_0 = h_1 = 0$, $h_2 > 0$ we obtain the following soliton-like solution for the DLWE equation, as follows.

$$u_3 = \frac{\pm 8h_4^{\frac{3}{2}} \frac{d}{dt} F_2(t) + C_3^2 h_3}{4C_3 h_4} + \frac{C_3 h_2 \operatorname{sech}^2\left(\frac{\sqrt{-2h_2}}{2}\xi\right)}{2\sqrt{h_2 h_4} \tanh\left(\frac{\sqrt{-2h_2}}{2}\xi\right) - h_3}, \tag{3.6.1}$$

$$v_3 = \frac{4h_4 h_2 F_1(y) - h_3^2 F_1(y) - 8h_4^2}{8h_4^2} + \frac{1}{2}\frac{h_3 F_1(y) h_2 \operatorname{sech}^2\left(\frac{\sqrt{-2h_2}}{2}\xi\right)}{h_4\left(2\sqrt{h_2 h_4}\tanh\left(\frac{\sqrt{-2h_2}}{2}\xi\right) - h_3\right)} + \frac{F_1(y) h_2^2 \operatorname{sech}^4\left(\frac{\sqrt{-2h_2}}{2}\xi\right)}{\left(2\sqrt{h_2 h_4}\tanh\left(\frac{\sqrt{-2h_2}}{2}\xi\right) - h_3\right)^2}, \tag{3.6.2}$$

where $\xi = kp + q$, k, p and q are determined by Eq. (3.3).

Family 4. From Eq. (3.3), when $h_1 = h_3 = 0$, $h_0 = \frac{h_2^2 m^2 (1-m^2)}{h_4 (2m^2-1)^2}$, $h_2 > 0$ and $h_4 < 0$, we obtain the following soliton-like solution for the DLWE equation, as follows.

$$u_4 = \pm 2\frac{\sqrt{h_4}\frac{d}{dt}F_2(t)}{C_3} + C_3\sqrt{-\frac{h_2 m^2}{h_4(2m^2-1)}}\operatorname{cn}\left(\sqrt{\frac{h_2}{2m^2-1}}\,\xi\right), \tag{3.7.1}$$

$$v_4 = \frac{4h_4 h_2 F_1(y) - 8h_4^2}{8h_4^2} - \frac{F_1(y) h_2 m^2 \operatorname{cn}^2\left(\sqrt{\frac{h_2}{2m^2-1}}\xi\right)}{h_4(2m^2-1)}, \tag{3.7.2}$$

where $\xi = kp + q$, k, p and q are determined by Eq. (3.3).

Family 5. From Eq. (3.3), when $h_1 = h_3 = 0$, $h_0 = \frac{h_2^2 (1-m^2)}{h_4 (2-m^2)^2}$, $h_2 > 0$ and $h_4 < 0$, we obtain the following soliton-like solution for the DLWE equation, as follows.

$$u_5 = \pm 2\frac{\sqrt{h_4}\frac{d}{dt}F_2(t)}{C_3} + C_3\sqrt{-\frac{m^2}{h_4(2-m^2)}}\operatorname{dn}\left(\sqrt{\frac{h_2}{2-m^2}}\,\xi\right), \tag{3.8.1}$$

$$v_5 = \frac{4h_4 h_2 F_1(y) - 8h_4^2}{8h_4^2} - \frac{F_1(y) m^2 \operatorname{dn}^2\left(\sqrt{\frac{h_2}{2-m^2}}\xi\right)}{h_4(2-m^2)}, \tag{3.8.2}$$

where $\xi = kp + q$, k, p and q are determined by Eq. (3.3).

Family 6. From Eq. (3.3), when $h_1 = h_3 = 0$, $h_0 = \frac{h_2 m^2}{h_4 (m^2+1)^2}$, $h_2 < 0$ and $h_4 > 0$, we obtain the following soliton-like solution for the DLWE equation, as follows.

$$u_6 = \pm 2\frac{\sqrt{h_4}\frac{d}{dt}F_2(t)}{C_3} + C_3\sqrt{-\frac{h_2 m^2}{h_4(m^2+1)}}\operatorname{dn}\left(\sqrt{-\frac{h_2}{m^2+1}}\xi\right), \tag{3.9.1}$$

$$v_6 = \frac{4h_4 h_2 F_1(y) - 8h_4^2}{8h_4^2} - \frac{F_1(y) h_2 m^2 \operatorname{dn}^2\left(\sqrt{-\frac{h_2}{m^2+1}}\,\xi\right)}{h_4(m^2+1)}, \tag{3.9.2}$$

where $\xi = kp + q$, k, p and q are determined by Eq. (3.3).

Remarks. The solution (3.6) can reproduce to the solution (13) in [13], when $h_2 = a,\ h_3 = b,\ h_4 = c,\ p = \pm\frac{2c_3}{\sqrt{h_4}}$, $F_2(t) = k_2(t)$ and $F_1(t) = k_1(t)$ and can reproduce to the solution (14) in [13], when $h_2 = a > 0$, $h_3 = b$, $h_4 = c > 0$, $h_3^2 - h_2 h_4 > 0$, $p = \pm\frac{2c_3}{\sqrt{h_4}}$, $F_2(t) = k_2(t)$, $F_1(t) = k_1(t)$. From above results we have not only recover all the solutions in [13] but also obtain some new soliton-like solutions and double-like periodic solutions.

4. Summary and conclusions

In summary, in fact, we naturally present a more general a ansatz, which read,

$$u_i(\xi) = a_{i0} + \sum_{j=1}^{m_i} \left\{ a_{ij}\phi^j + b_{ij}\phi^{-j} + f_{ij}\phi^{j-1}\sqrt{\sum_{\rho=0}^{r} h_\rho \phi^\rho} + k_{ij}\frac{\sqrt{\sum_{\rho=0}^{r} h_\rho \phi^\rho}}{\varphi^j} \right\}, \tag{4.1}$$

where a_{i0}, a_{ij}, b_{ij}, f_{ij}, $k_{ij}(i = 1, 2, \ldots; j = 1, 2, \ldots, m_i)$ and ξ are differentiable function to be determined later. When a_{i0}, a_{ij}, b_{ij}, f_{ij}, $k_{ij}(i = 1, 2, \ldots; j = 1, 2, \ldots, m_i)$ are constants and ξ is linear function with respect to x, y and t in the above ansaze, we have studied in Ref. [23]. Therefore, for some nonlinear equations, more types of solutions would be expected. Seeking new more general soliton-like and double-like periodic solutions of in the form Eq. (4.1) is still an interesting subject. This will be given in forthcoming paper.

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