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A new general algebraic method with symbolic computation to construct new doubly-periodic solutions of the (2 + 1)-dimensional dispersive long wave equation

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Abstract

For constructing more new exact doubly-periodic solutions in terms of rational form Jacobi elliptic function of nonlinear evolution equations, a new direct and unified algebraic method, named Jacobi elliptic function rational expansion method, is presented and implemented in a computer algebraic system. Compared with most of the existing Jacobi elliptic function expansion methods, the proposed method can be expected to obtain new and more general formal solutions. We choose a (2 + 1)-dimensional dispersive long wave equation to illustrate the method.

Keywords: (2 + 1)-dimensional dispersive long wave equation; Jacobi elliptic functions; Travelling wave solution; Soliton solution; Periodic solution

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1. Introduction

In the past decades, both mathematicians and physicists have devoted considerable effort to the study of solitons and related issue of the construction of solutions to nonlinear evolution equation. Except for the traditional methods such as inverse scattering method [1], Bäcklund transformation [2,3], Darboux transformation [2], Hirota's method [4], these exist some direct and unified algebraic methods such as tanh function method [5,6], various extended tanh-function methods [7–10], Jacobi elliptic function expansion method [11], various extended Jacobi elliptic function expansion methods [12,13]. These algebraic methods have the power to give a clear picture of the relation between different terms of nonlinear wave equations and are to simplify the routine calculation of the method or obtain more general solutions. This owes to that the success of the symbolic mathematical computation discipline is striking, the availability of computer symbolic system like *Maple* or *Mathematica* which allows us to perform complicated and tedious algebraic calculation that these algebraic methods included on a computer.

In [11] Liu et al. presented a Jacobi elliptic function expansion method that used three Jacobi elliptic functions to express exact solutions of some nonlinear evolution equations. Fan [12] extended Jacobi elliptic function method to some nonlinear evolution equations and, in particular, special-type nonlinear equations for constructing their doubly periodic wave solutions. Such equations cannot be directly dealt with by the method and require some kinds of pre-possessing techniques. Yan [13] further developed an extended Jacobi elliptic function expansion method. In this paper, based on above ideas, by means of a new general ansätz than ones in the above methods, a new Jacobi elliptic function rational expansion method is presented and is more powerful than above exiting Jacobi elliptic function expansion methods [11–13] to uniformly construct more new exact doubly-periodic solutions in terms of rational formal Jacobi elliptic functions solutions of nonlinear evolution equations (NLEEs). For illustration, we apply the proposed method to (2 + 1)-dimensional dispersive long wave equations (DLWE), which reads

$$u_{yt} + v_{xx} + (uu_x)_y = 0, \tag{1.1.1}$$

$$v_t + u_x + (uv)_x + u_{xxy} = 0. \tag{1.1.2}$$

The DLWE was first obtained by Boiti et al. [14] as a compatiblility condition for a "weak" Lax pair. Paquin and Winternitz [15] have given a Kac–Moody–Virasoro type Lie algebra for that DLWE. Lou [16] has shown that DLWE cannot pass the painlevé test both in the ARS algorithm and in the WTC approach. Wang et al. [17] used the homogeneous balance method to construct the exact solution of the DLWE system.

This paper is organized as follows. In Section 2, we summarize the Jacobi elliptic function rational expansion method. In Section 3, we apply the generalized method to (2 + 1)-dimensional dispersive long wave equation and bring out many solutions. Conclusions will be presented in finally.

2. Summary of the Jacobi elliptic function rational expansion method

In the following we would like to outline the main steps of our general method:

Step 1. For a given nonlinear evolution equation system with some physical fields $u_i(x,y,t)$ in three variables x, y, t,

$$F_i(u_i, u_{it}, u_{ix}, u_{iy}, u_{itt}, u_{ixt}, u_{iyt}, u_{ixx}, u_{iyy}, u_{ixy}, \ldots) = 0, \tag{2.1}$$

by using the wave transformation

$$u_i(x,y,t) = u_i(\xi), \qquad \xi = x + ly - \lambda t, \tag{2.2}$$

where l and λ are constants to be determined later. Then the nonlinear evolution Eq. (2.1) is reduced to a nonlinear ordinary differential equation (ODE):

$$G_i(u_i, u_i', u_i'', \ldots) = 0. \tag{2.3}$$

Step 2. We introduce a new ans ätz in terms of finite Jacobi elliptic function rational expansion in the following forms:

$$u_i(\xi) = a_{i0} + \sum_{j=1}^{m_i}\left(a_{ij}\frac{\mathrm{sn}^j(\xi)}{(\mu_1\mathrm{sn}(\xi)+\mu_2\mathrm{cn}(\xi)+1)^j} + b_{ij}\frac{\mathrm{sn}^{j-1}(\xi)\mathrm{cn}(\xi)}{(\mu_1\mathrm{sn}(\xi)+\mu_2\mathrm{cn}(\xi)+1)^j}\right). \tag{2.4}$$

Notice that

$$\begin{aligned}\frac{\mathrm{d}u_i}{\mathrm{d}\xi} = {} & \sum_{j=1}^{m_i}\frac{\mathrm{dn}(\xi)(-\mathrm{cn}(\xi)b_{ij}\mathrm{sn}^{j-2}(\xi)\mu_2 + a_{ij}\mathrm{sn}^{j-1}(\xi)j\mathrm{cn}(\xi) + \mathrm{cn}(\xi)b_{ij}\mathrm{sn}^{j-2}(\xi)j\mu_2)}{(\mu_1\mathrm{sn}(\xi)+\mu_2\mathrm{cn}(\xi)+1)^{(j+1)}} \\ & + \sum_{j=1}^{m_i}\frac{\mathrm{dn}(\xi)(a_{ij}\mathrm{sn}^{j-1}(\xi)j\mu_2 - b_{ij}\mathrm{sn}^{j-1}(\xi)\mu_1 - b_{ij}\mathrm{sn}^{j-2}(\xi) + b_{ij}\mathrm{sn}^{j-2}(\xi)j - b_{ij}\mathrm{sn}^j(\xi)j)}{(\mu_1\mathrm{sn}(\xi)+\mu_2\mathrm{cn}(\xi)+1)^{(j+1)}},\end{aligned} \tag{2.5}$$

where $\mathrm{sn}\xi$, $\mathrm{cn}\xi$, $\mathrm{dn}\xi$, $\mathrm{ns}\xi$, $\mathrm{cs}\xi$, and $\mathrm{ds}\xi$ etc. are Jacobi elliptic functions, which are double periodic and posses the following properties:

1. Properties of triangular function

$$\mathrm{cn}^2\xi + \mathrm{sn}^2\xi = \mathrm{dn}^2\xi + m^2\mathrm{sn}^2\xi = 1, \tag{2.6.1}$$

$$\mathrm{ns}^2\xi = 1 + \mathrm{cs}^2\xi, \qquad \mathrm{ns}^2\xi = m^2 + \mathrm{ds}^2\xi. \tag{2.6.2}$$

2. Derivatives of the Jacobi elliptic functions

$$\mathrm{sn}'\xi = \mathrm{cn}\xi\mathrm{dn}\xi, \qquad \mathrm{cn}'\xi = -\mathrm{sn}\xi\mathrm{dn}\xi, \qquad \mathrm{dn}'\xi = -m^2\mathrm{sn}\xi\mathrm{cn}\xi, \tag{2.7.1}$$

$$\mathrm{ns}'\xi = -\mathrm{ds}\xi\mathrm{cs}\xi, \qquad \mathrm{ds}'\xi = -\mathrm{cs}\xi\mathrm{ns}\xi, \qquad \mathrm{cs}'\xi = -\mathrm{ns}\xi\mathrm{ds}\xi, \tag{2.7.2}$$

where m is a modulus. The Jacobi–Glaisher functions for elliptic function can be found in Refs. [18–20].

Step 3. The underlying mechanism for a series of fundamental solutions such as polynomial, exponential, solitary wave, rational, triangular periodic, Jacobi and Weierstrass doubly periodic solutions to occur is that differ effects that act to change wave forms in many nonlinear equations, i.e. dispersion, dissipation and nonlinearity, either separately or various combination are able to balance out. We define the degree of $u_i(\xi)$ as $D[u_i(\xi)] = n_i$, which gives rise to the degrees of other expressions as

$$D[u_i^{(\alpha)}] = n_i + \alpha, \qquad D[u_i^{\beta}(u_j^{(\alpha)})^s] = n_i\beta + (\alpha + n_j)s. \tag{2.8}$$

Therefore we can get the value of m_i in Eq. (2.4). If n_i is a nonnegative integer, then we first make the transformation $u_i = \omega^{n_i}$.

Step 4. Substitute Eq. (2.4) into Eq. (2.3) along with Eqs. (2.5) and (2.7) and then set all coefficients of $\mathrm{sn}^i(\xi)\mathrm{n}^j(\xi)$ ($i = 1, 2, \ldots$; $j = 0, 1$) to be zero to get an over-determined system of nonlinear algebraic equations with respect to λ, l, μ_1, μ_2, a_{i0}, a_{ij} and b_{ij} ($i = 1, 2, \ldots$; $j = 1, 2, \ldots, m_i$).

Step 5. Solving the over-determined system of nonlinear algebraic equations by use of Maple, we would end up with the explicit expressions for λ, l, μ_1, μ_2, k, a_{i0}, a_{ij} and b_{ij} ($i = 1, 2, \ldots$; $j = 1, 2, \ldots, m_i$).

From which λ, l, μ, a_{i0}, a_{ij} and b_{ij} ($i = 1, 2, \ldots$; $j = 1, 2, \ldots, m_i$) can be obtained. In this way, we can get double periodic solutions with Jacobi elliptic function.

Since

$$\lim_{m\to 1}\mathrm{sn}\xi = \tanh\xi, \qquad \lim_{m\to 1}\mathrm{cn}\xi = \mathrm{sech}\,\xi, \qquad \lim_{m\to 1}\mathrm{dn}\xi = \mathrm{sech}\,\xi, \tag{2.9.1}$$

$$\lim_{m\to 1}\mathrm{ns}\xi = \coth\xi, \qquad \lim_{m\to 1}\mathrm{cs}\xi = \mathrm{csch}\,\xi, \qquad \lim_{m\to 1}\mathrm{ds}\xi = \mathrm{csch}\,\xi, \tag{2.9.2}$$

$$\lim_{m\to 0}\mathrm{sn}\xi = \sin\xi, \qquad \lim_{m\to 0}\mathrm{cn}\xi = \cos\xi, \qquad \lim_{m\to 0}\mathrm{dn}\xi = 1, \tag{2.9.3}$$

$$\lim_{m\to 0}\mathrm{ns}\xi = \csc\xi, \qquad \lim_{m\to 0}\mathrm{cs}\xi = \cot\xi, \qquad \lim_{m\to 0}\mathrm{ds}\xi = \csc\xi. \tag{2.9.4}$$

u_i degenerate respectively as the following form:

1. Solitary wave solutions:

$$u_i(\xi) = a_{i0} + \sum_{j=1}^{m_i} \left(a_{ij} \frac{\tanh^j(\xi)}{(\mu_1 \tanh(\xi) + \mu_2 \mathrm{sech}(\xi) + 1)^j} + b_{ij} \frac{\tanh^{j-1}(\xi)\mathrm{sech}(\xi)}{(\mu_1 \tanh(\xi) + \mu_2 \mathrm{sech}(\xi) + 1)^j} \right). \tag{2.10}$$

2. Triangular function formal solution:

$$u_i(\xi) = a_{i0} + \sum_{j=1}^{m_i} \left(a_{ij} \frac{\sin^j(\xi)}{(\mu_1 \sin(\xi) + \mu_2 \cos(\xi) + 1)^j} + b_{ij} \frac{\sin^{j-1}(\xi)\cos(\xi)}{(\mu_1 \sin(\xi) + \mu_2 \cos(\xi) + 1)^j} \right). \tag{2.11}$$

So the Jacobi elliptic function rational expansion method is more powerful than the method by Liu et al. [11], the method by Fan [12] and the method extended by Yan [13]. The solutions which contain solitary wave solutions, singular solitary solutions and triangular function formal solutions can be gotten by the extended method.

Remark 1. If we replace the Jacobi elliptic functions sn(ξ), cn(ξ) in the ansätz (2.4) with other pairs of Jacobi elliptic functions such as sn(ξ) and dn(ξ); ns(ξ) and cs(ξ); ns(ξ) and ds(ξ); sc(ξ) and nc(ξ); dc(ξ) and nc(ξ); sd(ξ) and nd(ξ); cd(ξ) and nd(ξ) Refs. [18–20]. It is necessary to point out that above combinations only require solving the recurrent coefficient relation or derivative relation for the terms of polynomial for computation closed. Therefore other new double periodic wave solutions, solitary wave solutions, and triangular functional solutions can be obtained for some system. For simplicity, we omit them here.

3. Exact solutions of the (2 + 1)-dimensional dispersive long wave equation (DLWE)

Let us consider the (2 + 1)-dimensional dispersive long wave equation (DLWE), i.e.,

$$\begin{cases} u_{yt} + v_{xx} + (uu_x)_y = 0, \\ v_t + u_x + (uv)_x + u_{xxy} = 0. \end{cases} \tag{3.1}$$

According to the above method, to seek travelling wave solutions of Eq. (3.1), we make the transformation

$$u(x,y,t)=\phi(\xi),\qquad v(x,y,t)=\sigma(\xi),\qquad \xi=x+ly-\lambda t, \tag{3.2}$$

where l and λ are constants to be determined later, and thus Eq. (3.1) becomes

$$\begin{cases} -\lambda l\phi''+\sigma''+l\phi'^2+l\phi\phi''=0,\\ -\lambda\sigma'+\phi'+(\phi\sigma)'+l\phi'''=0. \end{cases} \tag{3.3}$$

According to Step 1 in Section 2, by balancing $\phi'''(\xi)$ and $(\sigma(\xi)\phi(\xi))'$ in Eq. (3.3) and by balancing $\sigma''(\xi)$ and $\phi(\xi)\phi''(\xi)$ in Eq. (3.3), we suppose that Eq. (3.3) has the following formal solutions:

$$\begin{cases} \phi(\xi)=a_0+a_1\frac{\mathrm{sn}(\xi)}{\mu_1\mathrm{sn}(\xi)+\mu_2\mathrm{cn}(\xi)+1}+b_1\frac{\mathrm{cn}(\xi)}{\mu_1\mathrm{sn}(\xi)+\mu_2\mathrm{cn}(\xi)+1},\\ \sigma(\xi)=A_0+A_1\frac{\mathrm{sn}(\xi)}{\mu_1\mathrm{sn}(\xi)+\mu_2\mathrm{cn}(\xi)+1}+B_1\frac{\mathrm{cn}(\xi)}{\mu_1\mathrm{sn}(\xi)+\mu_2\mathrm{cn}(\xi)+1}\\ \qquad +A_2\frac{\mathrm{sn}^2(\xi)}{(\mu_1\mathrm{sn}(\xi)+\mu_2\mathrm{cn}(\xi)+1)^2}+B_2\frac{\mathrm{sn}(\xi)\mathrm{cn}(\xi)}{(\mu_1\mathrm{sn}(\xi)+\mu_2\mathrm{cn}(\xi)+1)^2}, \end{cases} \tag{3.4}$$

where a_0, a_1, b_1, A_0, A_1, A_2, B_1, B_2 are constants to be determined later.

With the aid of *Maple*, substituting (3.4) along with (2.6) and (2.7) into (3.3), yields a set of algebraic equations for $\mathrm{sn}^i(\xi)\mathrm{cn}^j(\xi)$ ($i=0,1,\ldots;j=0,1$). Setting the coefficients of these terms $\mathrm{sn}^i(\xi)\mathrm{cn}^j(\xi)$ to zero yields a set of over-determined algebraic equations with respect to a_0, a_1, b_1, A_0, A_1, B_1, A_2, B_2, l and λ.

By use of the *Maple* soft package "Charsets" by Dongming Wang, which is based on the Wu-elimination method [22], solving the over-determined algebraic equations, we get the following results:

Case 1

$$\begin{aligned} &a_1=\pm 2m,\qquad \lambda=a_0,\qquad A_2=-2\frac{m^2(A_0+1)}{m^2+1},\\ &\mu_1=\mu_2=b_1=A_1=B_1=B_2=0,\qquad l=\frac{A_0+1}{m^2+1}. \end{aligned} \tag{3.5}$$

Case 2

$$\begin{aligned} &b_1=\pm 2im,\qquad A_2=-2m^2-2m^2A_0,\qquad l=A_0+1,\\ &\lambda=a_0,\qquad \mu_1=\mu_2=a_1=A_1=B_1=B_2=0. \end{aligned} \tag{3.6}$$

Case 3

$$\begin{aligned} &a_1=\pm m,\qquad l=A_0+1,\qquad \lambda=a_0,\qquad A_2=-m^2A_0-m^2,\qquad b_1=\pm im,\\ &B_2=\pm(im^2A_0+im^2),\qquad \mu_1=\mu_2=A_1=B_1=0. \end{aligned} \tag{3.7}$$

Case 4

$$a_0 = \frac{\pm(\mu_1^3 - \mu_1) + \sqrt{m^2 + \mu_1^4 - \mu_1^2 - \mu_1^2 m^2}\lambda}{\sqrt{m^2 + \mu_1^4 - \mu_1^2 - \mu_1^2 m^2}},$$

$$a_1 = \pm\sqrt{m^2 + \mu_1^4 - \mu_1^2 - \mu_1^2 m^2}, \qquad b_1 = \pm\sqrt{-m^2 + \mu_1^2},$$

$$A_0 = -\frac{-l\mu_1^4 + 2l\mu_1^2 m^2 - \mu_1^2 + m^2 - lm^2}{-\mu_1^2 + m^2}, \qquad A_1 = -lm^2\mu_1 + 2l\mu_1^3 - l\mu_1,$$

$$A_2 = -lm^2 - l\mu_1^4 + l\mu_1^2 + l\mu_1^2 m^2, \qquad B_1 = \pm\frac{l\sqrt{-m^2 + \mu_1^2}\mu_1(-1 + \mu_1^2)}{\sqrt{m^2 + \mu_1^4 - \mu_1^2 - \mu_1^2 m^2}},$$

$$B_2 = \pm l\sqrt{m^2 + \mu_1^4 - \mu_1^2 - \mu_1^2 m^2}\sqrt{-m^2 + \mu_1^2}. \tag{3.8}$$

Case 5

$$a_0 = \frac{\sqrt{m^2 + \mu_1^4 - \mu_1^2 - \mu_1^2 m^2}\mu_1 + \mu_1 m^2\sqrt{m^2 + \mu_1^4 - \mu_1^2 - \mu_1^2 m^2}}{-m^2 - \mu_1^4 + \mu_1^2 + \mu_1^2 m^2}$$
$$+ \frac{-2\sqrt{m^2 + \mu_1^4 - \mu_1^2 - \mu_1^2 m^2}\mu_1^3 - \lambda m^2 - \mu_1^4\lambda + \mu_1^2\lambda + \mu_1^2 m^2\lambda}{-m^2 - \mu_1^4 + \mu_1^2 + \mu_1^2 m^2},$$

$$a_1 = -2\sqrt{m^2 + \mu_1^4 - \mu_1^2 - \mu_1^2 m^2}, \qquad b_1 = B_1 = B_2 = 0,$$

$$A_0 = -\frac{\mu_1^2 - m^2 - \mu_1^4 + \mu_1^2 m^2 - 6l\mu_1^2 m^2 + 3l\mu_1^4 + m^4 l - 2l\mu_1^6 + 3lm^2\mu_1^4 + lm^2}{-m^2 - \mu_1^4 + \mu_1^2 + \mu_1^2 m^2},$$

$$A_1 = -2l\mu_1 - 2lm^2\mu_1 + 4l\mu_1^3, \qquad A_2 = -2lm^2 + 2l\mu_1^2 m^2 - 2l\mu_1^4 + 2l\mu_1^2. \tag{3.9}$$

Case 6

$$a_0 = \pm\frac{(2\mu_2 m^2 - 2m^2\mu_2^3 - \mu_2 + 2\mu_2^3)\sqrt{2\mu_2^2 m^2 - m^2 - \mu_2^4 m^2 - \mu_2^2 + \mu_2^4}}{-2\mu_2^2 m^2 + m^2 + \mu_2^4 m^2 + \mu_2^2 - \mu_2^4}$$
$$+ \frac{m^2\lambda + m^2\lambda\mu_2^4 - 2m^2\mu_2^2\lambda + \lambda\mu_2^2 - \lambda\mu_2^4}{-2\mu_2^2 m^2 + m^2 + \mu_2^4 m^2 + \mu_2^2 - \mu_2^4},$$

$$b_1 = \pm 2\sqrt{2\mu_2^2 m^2 - m^2 - \mu_2^4 m^2 - \mu_2^2 + \mu_2^4}, \qquad A_2 = 2l\mu_2^2 m^2 - 2lm^2 - 2l\mu_2^2,$$

$$A_0 = \frac{2\mu_2^2 m^2 - m^2 - \mu_2^2 - \mu_2^4 l - \mu_2^4 m^2 + \mu_2^4 + \mu_2^4 m^2 l + lm^2 - 2l\mu_2^2 m^2}{-2\mu_2^2 m^2 + m^2 + \mu_2^4 m^2 + \mu_2^2 - \mu_2^4},$$

$$B_1 = -2l\mu_2, \qquad a_1 = A_1 = B_1 = 0. \tag{3.10}$$

Case 7

$$a_1 = \pm 2\sqrt{1-m^2}, \qquad \mu_1 = \pm 1, \qquad \mu_2 = \pm 1, \qquad B_2 = 2lm^2 - 2l - A_1,$$
$$A_2 = 2lm^2 - 2l, \qquad B_1 = 2lm^2 - 2l - A_1, \qquad b_1 = 0,$$
$$\lambda = \frac{m^2 - 1 + a_0\sqrt{1-m^2}}{\sqrt{1-m^2}}, \qquad A_0 = -1 + \frac{1}{2}A_1 - lm^2. \tag{3.11}$$

From (3.2) and (3.4) and Cases 1–7, we obtain the following solutions for Eq. (3.1).

Family 1. From Eq. (3.5), we obtain the following rational formal doubly periodic solutions for the DLWE, as follows:

$$u_1(x,y,t) = a_0 \pm 2m\,\mathrm{sn}(\xi), \tag{3.12.1}$$

$$v_1(x,y,t) = A_0 - 2\frac{m^2(A_0+1)\mathrm{sn}^2(\xi)}{m^2+1}, \tag{3.12.2}$$

where $\xi = x + ly - \lambda t$, a_0, A_0, l and λ are determined by (3.5).

Family 2. From Eq. (3.6), we obtain the following rational formal doubly periodic solutions for the DLWE, as follows:

$$u_2(x,y,t) = a_0 \pm 2im\,\mathrm{cn}(\xi), \tag{3.13.1}$$

$$v_2(x,y,t) = A_0 - (2m^2 + 2m^2A_0)\mathrm{sn}^2(\xi), \tag{3.13.2}$$

where $\xi = x + ly - \lambda t$, a_0, A_0, l and λ are determined by (3.6).

Family 3. From Eq. (3.7), we obtain the following rational formal doubly periodic solutions for the DLWE, as follows:

$$u_3(x,y,t) = a_0 \pm m\,\mathrm{sn}(\xi) \pm im\,\mathrm{cn}(\xi), \tag{3.14.1}$$

$$v_3(x,y,t) = A_0 - (m^2A_0 + m^2)\mathrm{sn}^2(\xi) \pm (im^2A_0 + im^2)\mathrm{sn}(\xi)\mathrm{cn}(\xi), \tag{3.14.2}$$

where $\xi = x + ly - \lambda t$, a_0, A_0, l and λ are determined by (3.7).

Family 4. From Eq. (3.8), we obtain the following rational doubly periodic solutions for the DLWE, as follows:

$$u_4(x,y,t) = \pm\frac{\sqrt{m^2+\mu_1^4-\mu_1^2-\mu_1^2m^2}\,\mathrm{sn}(\xi)}{\mu_1\mathrm{sn}(\xi)+1} \pm \frac{\sqrt{-m^2+\mu_1^2}\,\mathrm{cn}(\xi)}{\mu_1\mathrm{sn}(\xi)+1}$$
$$+ \frac{\pm(\mu_1^3-\mu_1) + \sqrt{m^2+\mu_1^4-\mu_1^2-\mu_1^2m^2}\,\lambda}{\sqrt{m^2+\mu_1^4-\mu_1^2-\mu_1^2m^2}}, \tag{3.15.1}$$

$$v_4(x,y,t) = -\frac{-l\mu_1^4 + 2l\mu_1^2m^2 - \mu_1^2 + m^2 - lm^2}{-\mu_1^2 + m^2} + \frac{(-lm^2\mu_1 + 2l\mu_1^3 - l\mu_1)\mathrm{sn}(\xi)}{\mu_1\mathrm{sn}(\xi) + 1} \pm \frac{l\sqrt{-m^2 + \mu_1^2}\mu_1(-1 + \mu_1^2)\mathrm{cn}(\xi)}{\sqrt{m^2 + \mu_1^4 - \mu_1^2 - \mu_1^2m^2}(\mu_1\mathrm{sn}(\xi) + 1)} + \frac{(-lm^2 - l\mu_1^4 + l\mu_1^2 + l\mu_1^2m^2)\mathrm{sn}^2(\xi)}{(\mu_1\mathrm{sn}(\xi) + 1)^2} \pm \frac{l\sqrt{m^2 + \mu_1^4 - \mu_1^2 - \mu_1^2m^2}\sqrt{-m^2 + \mu_1^2}\mathrm{sn}(\xi)\mathrm{cn}(\xi)}{(\mu_1\mathrm{sn}(\xi) + 1)^2}, \tag{3.15.2}$$

where $\xi = x + ly - \lambda t$, μ_1, l and λ are arbitrary constants.

Family 5. From Eq. (3.9), we obtain the following rational doubly periodic solutions for the DLWE, as follows:

$$u_7(x,y,t) = a_0 + 2\frac{\sqrt{m^2 + \mu_1^4 - \mu_1^2 - \mu_1^2m^2}\mathrm{sn}(\xi)}{\mu_1\mathrm{sn}(\xi) + 1}, \tag{3.16.1}$$

$$v_7(x,y,t) = A_0 + \frac{(-2l\mu_1 - 2lm^2\mu_1 + 4l\mu_1^3)\mathrm{sn}(\xi)}{\mu_1\mathrm{sn}(\xi) + 1} + \frac{(-2lm^2 + 2l\mu_1^2m^2 - 2l\mu_1^4 + 2l\mu_1^2)\mathrm{sn}^2(\xi)}{(\mu_1\mathrm{sn}(\xi) + 1)^2}, \tag{3.16.2}$$

where $\xi = x + ly - \lambda t$, a_0 and A_0 are determined by (3.9), μ_1, l and λ are arbitrary constants.

Family 6. From Eq. (3.10), we obtain the following rational doubly periodic solutions for the DLWE, as follows:

$$u_5(x,y,t) = a_0 \pm 2\frac{\sqrt{2\mu_2^2m^2 - m^2\mu_2^4m^2 - \mu_2^2 + \mu_2^4}\mathrm{cn}(\xi)}{\mu_2\mathrm{cn}(\xi) + 1}, \tag{3.17.1}$$

$$v_5(x,y,t) = A_0 - 2\frac{l\mu_2\mathrm{cn}(\xi)}{\mu_2\mathrm{cn}(\xi) + 1} + \frac{(2l\mu_2^2m^2 - 2lm^2 - 2l\mu_2^2)\mathrm{sn}^2(\xi)}{(\mu_2\mathrm{cn}(\xi) + 1)^2}, \tag{3.17.2}$$

where $\xi = x + ly - \lambda t$, a_0 and A_0 are determined by (3.10), μ_2, l and λ are arbitrary constants.

Family 7. From Eq. (3.11), we obtain the following rational doubly periodic solutions for the DLWE, as follows:

$$u_5(x,y,t) = a_0 \pm 2\frac{\sqrt{1-m^2}\mathrm{sn}(\xi)}{\mathrm{cn}(\xi) \pm \mathrm{sn}(\xi) \pm 1}, \tag{3.18.1}$$

$$v_5(x,y,t) = -1 - \frac{1}{2}A_1 - lm^2 \pm \frac{A_1\mathrm{sn}(\xi)}{\mathrm{cn}(\xi) \pm \mathrm{sn}(\xi) \pm 1} \pm \frac{(2lm^2 - 2l + A_1)\mathrm{cn}(\xi)}{\mathrm{cn}(\xi) \pm \mathrm{sn}(\xi) \pm 1} + \frac{(-2l + 2lm^2)\mathrm{sn}^2(\xi)}{(\mathrm{cn}(\xi) \pm \mathrm{sn}(\xi) \pm 1)^2} + \frac{(-2lm^2 + 2l - A_1)\mathrm{sn}(\xi)\mathrm{cn}(\xi)}{(\mathrm{cn}(\xi) \pm \mathrm{sn}(\xi) \pm 1)^2}, \tag{3.18.2}$$

where $\xi = x + ly - \lambda t$, λ is determined by (3.11), a_0, A_1 and l are arbitrary constants.

Remark 2

(1) The solutions (3.12) reproduce the solution (15) in [21], when $A_0 = \frac{2(C_1+1)(m^2+1)}{2+2m^2-\lambda^2} - 1$.
(2) The other solutions obtained here, to our knowledge, are all new families of doubly periodic solutions of the DLWE.

4. Summary and conclusions

In summary, based on a computer algebraic system, we have proposed an algebraic method named Jacobi elliptic function rational expansion method to construct a series of new doubly-periodic solutions in term of rational form of some nonlinear evolution equations. The method is more powerful than the method proposed recently by Liu [11], improved by Fan [12] and extended by Yan [13]. As an application of the method, we found four families of new formal doubly-periodic solutions of the (2 + 1)-dimensional dispersive long wave equation. The algorithm can be also applied to many nonlinear evolution equations in mathematical physics. Further work about various extensions and improvement of Jacobi function method needs us to find the more general ansätzes or the more general subequation. which is under our current research focus.

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