Generation of a localized hollow laser beam using crossed nonlinear optical crystals

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A nonlinear optical approach to generate a localized hollow laser beam and its array using crossed nonlinear ZnSe crystals is proposed. We calculate the intensity distributions of the hollow beam and its propagating properties in free space, and study the dependences of the dark spot size of the hollow beam on the incident beam waist radius and the crystal length. Our studies show that the longitudinal size of the hollow beam can be reduced by ~660 times than that generated by a single nonlinear crystal. Such laser beam with a dark hollow region and a large intensity gradient can be used to trap cold neutral atoms, molecules and microscopic particles.

1. Introduction

A localized hollow laser beam (LHB) is a kind of laser beam consisting of a dark central region that is fully surrounded by regions of high intensity [1]. Since the LHB has a three-dimension (3D) closed dark hollow region and a large intensity gradient, it can be used to trap microscopic particles and neutral atoms. So, the LHB has many important and extensive applications in modern optics and atomic (or molecular) optics [2–6]. In particular, the intensity gradient near the focal point of a LHB may be used to efficiently cool neutral atoms by LHB-induced Sisyphus cooling [7].

LHBs have been generated by a number of approaches, e.g. holograms [8], conical refraction [9–12], uniaxial crystal [13], interferometer [14], photon sieve [15], optical fiber [16], polarized light [17], focusing multi-ring hollow Gaussian beam [18], to form a desirable hollow axial structure. The concept of a LHB has also been recently extended to plasmonic structures [19,20], the plasmonic bottle beam can be used to optically sort particles by trapping those with a specific size. Nowadays, nonlinear optical methods have been proposed to generate non-diffraction beams [21–23]. Many self-action effects have been thoroughly investigated, including self-phase modulation [24], self-focusing [25–27] and self-bending [28–30]. Our approach is motivated by the self-bending (SB) effect of ZnSe nonlinear material. ZnSe is a kind of materials with large nonlinear refractive index, and it can induce focusing and self-bending [28–30]. Our approach is motivated by the self-bending (SB) effect of ZnSe nonlinear material. ZnSe is a kind of materials with large nonlinear refractive index, and it can induce focusing and self-bending [28–30].

In this paper, we propose a scheme – using crossed ZnSe nonlinear crystals – to generate a LHB and its array, and validate this scheme theoretically. The intensity distributions of the LHB in different cases of the symmetric parameters are calculated. The ratio of the longitudinal size of LHB can be tuned by altering the relative position of two crystals. As well, the aspect ratio of the LHB is calculated to study the shape of the beam. The LHB array formed by more nonlinear ZnSe crystals is also discussed.

2. Principle description

The schematic setup to generate a LHB is shown in Fig. 1. Two orthogonal incident Gaussian laser beams (GBs) propagate along the x and z direction, respectively. The GB 1 (or GB 2 ) is collimated by the lenses L 1 (or L 2 ) and then passes through a nonlinear dielectric material ZnSe crystal C 1 (or C 2 ). Due to the nonlinearity of the ZnSe crystals, the phases of two GBs will be modulated. In each optical path, a hollow beam can be generated after the nonlinear crystal C 1 (or C 2 ). Two beams are modulated by two crystals, and then superimposed to form a LHB. Such a LHB has a 3D closed hollow region, and the longitudinal size of the hollow beam is much smaller than that generated by a single nonlinear crystal [31].

In theory, the refractive index n of the ZnSe crystal can be described by the relationship

\[ n = n_0 + n_2 |U|^2 \]  

(1)

where \( n_0 \) is the linear refractive index, \( n_2 \) is the nonlinear refractive index, and \( |U| \) is the intensity of the incident beam.
Where $n_0$ and $n_2$ are the linear and nonlinear refractive indexes, respectively, $U$ is the complex amplitude of the light field.

We set $L$ as the length of the nonlinear ZnSe crystal along the propagating direction and set the center of the superposition of two crossed beams as the coordinate origin. The 2D complex amplitude $U(y_1, z_1, -(L + x_{10}))$ of the incident well-collimated GB can be described by

$$U(y_1, z_1, -(L + x_{10})) = U_0(y_1, z_1) \exp\left(i \phi_0(y_1, z_1, -(L + x_{10}))\right)$$  \hspace{1cm} (2)

where $x_{10}$ is the initial position of the crystal $C_1$ and $U_0(y_1, z_1)$ can be expressed as

$$U_0(y_1, z_1) = U_{\text{in}} \exp\left(-\frac{y_1^2 + z_1^2}{w_0^2}\right)$$  \hspace{1cm} (3)

where $U_{\text{in}}$ is the light field complex amplitude, which depends on both the waist radius and power of the incident GB. The complex amplitude $U_1(y_1, z_1, x_{10})$ of the light field at the output face of the nonlinear ZnSe crystal $C_1$ can be given by

$$U_1(y_1, z_1, -x_{10}) = U_0(y_1, z_1, -(L + x_{10})) \times \exp\left[i (\phi_0(y_1, z_1, -x_{10}) + n(y_1, z_1)kL)\right]$$  \hspace{1cm} (4)

According to the Huygens–Fresnel diffraction theory, the light field complex amplitude $U_1(x, y, z)$ in the propagating position $x$ after the nonlinear ZnSe crystal $C_1$ can be written as

$$U_1(x, y, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp\left(i k (x - x_{10})\right)
\exp\left(i \frac{(x - x_{10})^2 + (y - y_{10})^2}{2(x - x_{10})}\right)
U_1(y_1, z_1, -x_{10}) \, dy_1 \, dz_1.$$  \hspace{1cm} (5)

Based on the analysis above, the corresponding light field complex amplitude $U_2(x, y, z)$, reshaped by $C_2$, in the transverse at propagation position $z$ in free space can be written as

$$U_2(x, y, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp\left(i k (z - z_{20})\right)
\exp\left(i \frac{(x - x_{20})^2 + (y - y_{20})^2}{2(z - z_{20})}\right)
U_2(x_2, y_2, -z_{20}) \, dx_2 \, dy_2.$$  \hspace{1cm} (6)

where $z_{20}$ is the initial position of the crystal $C_2$ and $U_2(x, y, z_{20})$ can be expressed as

$$U_2(x_2, y_2, -z_{20}) = U(\phi_0(x_2, y_2, -z_{20}))$$
$$+ n(x_2, y_2)kL)$$  \hspace{1cm} (7)

As a result, the intensity $I(x, y, z)$ of the light field can be expressed as

$$I(x, y, z) = |U_1(x, y, z)|^2 + |U_2(x, y, z)|^2$$  \hspace{1cm} (8)

3. Results and Discussions

To study the propagation characteristics of the LHB in free space, we define the special parameter, the dark spot size (DSS) as the full width at half maximum (FWHM) of the radial-intensity distribution inside the notch of the LHB [3]. Based on the theoretical analyses above, the intensity distributions of the LHB moving the relative position between two nonlinear ZnSe crystals can be calculated by using Eqs. (2)–(8). In the numerical calculation, the waist radius of the two incident GBs is taken as $w_1 = w_2 = 300 \mu m$, the light field complex amplitude is $U_0 = 3.3 \times 10^4 \sqrt{W/m}$ and the wavelength is $\lambda = 447.6 \text{ nm}$ experimentally from a dye laser. With these light parameters, the significant nonlinear effect of the ZnSe crystal can be excited. For the ZnSe crystal, the length is $L = 450 \mu m$, $n_0 = 2.8$ and $n_2 = 6.1 \times 10^{-14} \text{ m}^2/\text{W}$. According to the calculated results in Ref. [31], an optimal hollow beam can be obtained at a distance $z_{\text{optimal}} = 0.62 \text{ m}$ away from the ZnSe crystal under these parameters, the beam is kept to be a ring beam and to be hollow at the beam center for a long distance with $0.8 \text{ m}$ in the propagating direction, which looks like a hollow ellipsoid with a large elliptical ratio. From our calculation, the intensity distributions of the superposed light beam after crystal $C_1$ at the different $z$ position and at $x_{10} = x_{\text{optimal}} = -0.62 \text{ m}$ are shown in Fig. 2. The left and center columns are the 1D intensity distributions along the $x$ and $z$ direction, respectively, the right column is 2D intensity distributions in $xz$ plane. In Fig. 2, the position of the crystal $C_1$ is fixed while the position of the crystal $C_2$ moves along the $z$ direction. The superposed light field appears as a pipe in the $z$ direction where the position of the crystal $C_2$ is $z_{20} = -0.2 \text{ m}$ (as shown in Fig. 2(m)).

As the position of the crystal $C_2$ approaches the optimal position $z_{20} = z_{\text{optimal}} = -0.62 \text{ m}$ gradually, the light intensity in the $x$ and $z$ direction will have the same hollow profile distributions (as shown in Fig. 2(d), (j)). At the position of the crystal $C_2 z_{20} = x_{10} = -0.62 \text{ m}$, an optimal 3D closed LHB is formed — it is nearly square (as shown in Fig. 2(p)), and the DSS (1.2 mm) of this LHB is much less than DSS (0.8 m) that generated by a single ZnSe crystal. So the ratio of the longitudinal size of LHB can be easily changed by altering the relative position of two crystals. The smaller the longitudinal size is, the higher the intensity gradient of the LHB is, so the blue-detuned LHB can be used to trap and cool neutral atoms by intensity-gradient induced Sisyphus cooling. When the position of the crystal $C_2$ is larger than $0.62 \text{ m}$, the light beam gradually displays a profile with a non-zero central intensity and larger size, and the superposed light beam looks like a pipe in the $x$ direction (as shown in Fig. 2(r)). In this case, the superposed light beam is dominated by the light beam after $C_1$.

As a result, we study the relationship of the LHB size with the waist radius of the incident GB and the length of the nonlinear ZnSe crystal. Firstly, we assume that two waist radii of both GBs are the same, i.e., $w_1 = w_2$. When the positions of the two crystals $C_1$ and $C_2$ are fixed at $x_{10} = z_{20} = -0.62 \text{ m}$, an optimal closed LHB can be obtained. Fig. 3 shows the 2D intensity distributions of the LHBs under the different waist radius of incident GBs. It is evident that all the LHBs are nearly square in $xoz$ plane and the DSS is gradually increasing while increasing the waist radius $w$. The detailed information about each LHB’s size — DSS at $x$ (or $z$) direction is concluded in Fig. 4. The DSS is linearly related...
Fig. 2. Calculated and normalized intensity distributions of the superposed beam at various $z$ distances. The left and center columns are the 1D intensity distributions, the right column is 2D intensity distributions in $xoz$ plane.
Fig. 3. Calculated and normalized intensity distributions in the case of different waist radius $w = w_1 = w_2$ of incident GBs, the length of each crystal is $L_1 = L_2 = 450 \mu m$.

Fig. 4. Dependence of the DSS on the waist radius $w$ of two incident GBs.

to the waist radius $w$. As the waist radius $w$ is increasing from 150 $\mu m$ to 400 $\mu m$, the LHB size is increasing from 0.6 mm to 1.64 mm.

If the waist radius $w_1$ and $w_2$ of the GBs are not the same, the results will be changed. In our numerical simulations, the $w_1$ is fixed, $w_1 = 300 \mu m$. While the $w_2$ is increasing from 150 $\mu m$ to 400 $\mu m$, the dependence of the 2D intensity distributions of the LHB on the $w_2$ is shown in Fig. 5. For $w_2 < 300 \mu m$, a dark hollow beam is formed whose DSS$_x$ is smaller than DSS$_z$ (see Fig. 5(a)–(c)). For $w_2 > 300 \mu m$, the DSS$_x$ is gradually larger than the DSS$_z$ (see Fig. 5(e) and (f)). In particular, $w_1 = w_2 = 300 \mu m$, a square dark hollow beam is shown in Fig. 5(d). The relationship between the aspect ratio DSS$_x$/DSS$_z$ and $w_2/w_1$ is plotted in Fig. 6. It can be recognized as a linear line.

The principle of the beam-shaping in this scheme is based on the nonlinear interaction between the laser light field and the nonlinear crystal. The interaction situation is also related to the length $L$ of the nonlinear crystal. So we study the influence of the length of ZnSe crystal on the LHB’s shape. To simplify, the length $L_2$ of the crystal $C_2$ is not changed, $L_2 = 450 \mu m$, and $w_1 = w_2 = 300 \mu m$, both the crystals $C_1$ and $C_2$ are 0.62 m away from the superposed region. The 2D intensity

Fig. 5. Calculated and normalized intensity distributions in the incident GBs with the invariable waist ratio $w_1 = 300 \mu m$ and the different waist ratio $w_2$ at distance $x = z = 0.62$ m.
Fig. 6. Dependence of the aspect ratio $DSS_x/DSS_z$ on the waist radius ratio $w_2/w_1$, $w_1 = 300 \mu m$.

distributions of the LHBs in the case of different length $L_1$ of the crystal $C_1$ are shown in Fig. 7. When the length $L_1$ is smaller than $L_2$, the $DSS_z$ of the LHB is smaller than the $DSS_z$ (see Fig. 7(a) and (b)), the aspect ratio $DSS_x/DSS_z$ is $>1.0$. For $L_1 = L_2 = 450 \mu m$, as shown in Fig. 7(c), the 2D intensity distribution of the LHB is square. In contrast, a LHB of which $DSS_z$ is smaller than $DSS_x$ is produced for $L_1$ larger than $L_2$, as shown in Fig. 7(d)–(f). The relationship between the aspect ratio $DSS_x/DSS_z$ and $L_2/L_1$ is plotted in Fig. 8. The aspect ratio is increasing with the increasing of the crystal length ratio.

What is more, with the optical system illuminated by four incoherent incident light waves with a cross angle $\pi/2$, a 1D array of the LHB will be formed. The calculated intensity distribution of a 1D LHB array is shown in Fig. 9. In order to obtain the array of the LHB, we choose appropriate length of the crystals as well as the waist radius of the incident GBs. We find that the generated LHB is nearly parallel to each other with a DSS of 2.44 mm when the crystal length is $L_1 = 450 \mu m$ and the waist radius of the incident beam is $w_1 = 1.0 \ mm$.

Fig. 7. Calculated and normalized intensity distributions in the case of different length $L_1$ of ZnSe crystal $C_1$ for $L_2 = 450 \mu m$ and $w_1 = w_2 = 300 \mu m$.

Fig. 8. Dependence of the aspect ratio $DSS_x/DSS_z$ on the length ratio $L_2/L_1$, $L_2 = 450 \mu m$.

4. Conclusions

In conclusion, we have proposed a nonlinear optical method to generate a LHB by using crossed nonlinear ZnSe crystals. Our study shows that the longitudinal size of LHB can be changed from 0.8 m to 1.2 mm, which is reduced by $\sim 660$ times compared to that generated by a single nonlinear crystal. The dependence of the DSS of LHB on the waist radius of the incident beam, and of the aspect ratio of $DSS_x/DSS_z$ on the waist radius ratio $w_2/w_1$ and length ratio $L_2/L_1$ are linear. In addition, the LHB arrays can be formed under the appropriate crystal length and the waist radius of the incident laser beam. Such a LHB has a 3D closed dark hollow region and a large intensity gradient; it can be used to trap cold neutral atoms, molecules and microscopic particles. In particular, the intensity gradient of a LHB may be used to efficiently cool neutral atoms by LHB-induced Sisyphus cooling [7].
Acknowledgments

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References