

Measuring orbital angular momentums of light based on petal interference patterns

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Abstract: We demonstrate an interferometric method to measure the topological charges of the vortex beams carrying orbital angular momentums (OAMs). The petal interference patterns are generated by combining modulated vortex beams and an unmodulated incident Gaussian beam reflected by a spatial light modulator. The number of petals is in agreement with the value of OAM that the modulated beam carries, by which we analyze the characteristic of interference patterns of integer OAM beams, including intensity profiles, phase profiles, and hologram structures. We also uncover the principle of how radial parameter *l* influences the hollow radius of OAM beams. Beams carrying non-integer orbital angular momentums are visualized with our method, from which we observe the evolution of a speckle generated by the decimal part of holograms. A kind of hologram is designed to prove that the petal near the singularity line is separated owing to the diffraction enhancement. All the experiment results agree well with the simulated results.

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1. Introduction

In recent years, optical vortex beams carrying orbital angular momentums (OAMs) have attracted considerable attention [1,2], owing to their applications in optical information communications [3–5], particle manipulation [6–10], optical imaging [11] and etc. Light beams with an azimuthal phase structure $\exp(il\varphi)$ carry OAM of $l\hbar$ per photon [12]. The presence of this phase factor causes the null intensity at the point of screw wavefront dislocation. The beams can be generated in the laboratory by many devices, such as spiral phase plates [13,14], forked holograms [15] and mode converters [16].

Although OAM beams have a helical phase structure, we can only observe a ring structure on intensity profiles. One issue is how to measure the value of OAM of the light, which has been performed in several kinds of methods. The first kind is the interference approach, yielding up phase information of the OAM beams. In 1996, Padgett et al. used a Mach-Zehnder interferometer to make a Gaussian beam interfere with a collinear OAM beam, observing spiral fringes whose number is in agreement with the value of OAM [17], and then Leach et al. added a Dove prism in each arm of a Mach-Zehnder interferometer to sort OAM at the single photon-level [18]. A robust method was proposed by reconfiguring the Mach-Zehnder interferometer as a Sagnac interferometer to provide high sorting efficiency and stability [19]. In 2011, Lavery et al. presented a compact, robust interferometer to remove many of the previously required degrees of freedom [20]. Almost all interferometric methods have to split a beam into two parts modulated by different optical devices. The interference optical path, however, should be precisely controlled and carefully preserved, since any tiny fluctuation or unbalance in polarization, phase difference and intensity associated with the laser beams could lead to the instability of the interference pattern. The second kind mentioned here used the diffraction effects through apertures [21–23]. An elegant example of this was that Hickmann et al. generated a truncated triangular optical lattice to diffract beams with OAM through a triangular aperture, acquiring the number and orientation of OAM [21].

Vol. 1, No. 2 | 15 Oct 2018 | OSA CONTINUUM 452

integer and non-integer average OAM of a laser beam [24,25]. Furthermore, some other methods such as optical transformation [26,27] and complex holograms [28,29] were proposed to measure the value of OAM.

In this paper, we propose a simple interferometric method to measure the OAM of optical vortex beams. The basic principle is that due to the incomplete phase modulation of the phase-only reflective spatial light modulator (SLM), the reflected beam contains unmodulated portion of an incident Gaussian beam that can interfere with the modulated OAM beam to generate petal patterns. The number of petals is in agreement with the value of OAM the modulated beam carries, by which we analyze the characteristic of interference patterns of integer OAM beams, including intensity profiles, phase profiles, and hologram structures. To confirm the power ratio of the unmodulated and modulated incident Gaussian beam, petal patterns of four different power ratios in both experiment and simulation are also discussed. Furthermore, we obviously visualize the evolution of light beams carrying non-integer OAM by using the interferometric method with holograms designed. Although the optical vortex structure has been discussed many times, we discover something novel and make efforts to confirm our thoughts. This work extends and consolidates previous studies on the measurement of integer OAM beams and the evolution of fractional OAM beams.

2. Basic principles

First, we describe the generation of OAM beams. For a paraxial monochromatic Gaussian laser beam, the transversal distribution of its electric field at z = 0 can be expressed as

$$E_0(x, y, 0) = \exp\left(-\frac{x^2 + y^2}{a_0^2}\right)$$
(1)

where ω_0 is the beam waist.

The laser beam is incident on the SLM screen. After reflected by the hologram, the generated OAM beam carries a phase factor of $\exp(il\varphi)$ where φ is the azimuthal angle acquired from the holograms, and the light field is therefore given by

$$E_0(x, y, 0) = \exp\left(-\frac{x^2 + y^2}{\omega_0^2}\right) \exp(il\varphi)$$
⁽²⁾

In the observation plane with a distance of z after the SLM, the field distribution can be calculated according to Fresnel diffraction integral, as follows

$$E(x, y, z) = \frac{\exp(ikz)}{i\lambda z} \exp\left(ik\frac{x^2 + y^2}{2z}\right) \int_{-\infty}^{\infty} E_0 \exp\left[\frac{ik}{2z} \left(x_0^2 + y_0^2 - 2xx_0 - 2yy_0\right)\right] dx_0 dy_0$$
(3)

where $k = 2\pi/\lambda$ is the wave vector.

Next, there is interference between the generated OAM beam and the unmodulated incident Gaussian beam on the screen because of their coherence. Thus, we define a parameter η denoting the power ratio of unmodulated and modulated incident Gaussian beam

$$\eta = \frac{I_{\text{Gauss}}}{I_{\text{OAM}}} \tag{4}$$

with I_{Gauss} and I_{OAM} denoting the intensity of unmodulated Gaussian beam and modulated OAM beam, respectively.



Fig. 1. Generation of an OAM beam and an interference pattern. The first row: a hologram (b) transforms an incident Gaussian beam (a) into an OAM beam (c). The second row: the unmodulated weak Gaussian beam (d) reflected by the SLM interferes with an integer OAM beam, generating a petal pattern (e). The third row: the same as the second row, but for a fractional OAM beam (f) and the interference pattern (g).

We can observe the interference patterns on the CCD. The three schematic diagrams are shown in Fig. 1. The Gaussian beam [Fig. 1(a)] is transformed to an OAM beam [Fig. 1(c)] by a hologram designed for the quantum number of the OAM of light l = 3 [Fig. 1(b)], which is uploaded on the SLM. Then, the modulated beam interferes with the unmodulated weak Gaussian beam [Fig. 1(d)] reflected by the SLM, generating an interference pattern [Fig. 1(e)]. We note an interesting phenomenon that there are three petals in the interference pattern, different from the ring structure of the OAM beam, and the three petals correspond to the hologram with l = 3. So this rule can be used as a criterion of whether it is an integer or fractional angular momentum of light, and of its specific quantum number. The last row illustrates that an OAM beam with l = 3.5 [Fig. 1(f)] interferes with the unmodulated weak Gaussian beam. On the petal pattern [Fig. 1(g)], we observe the phase information and visualize the evolution of fractional OAM beams. Similar behavior are observed for all holograms. Therefore, OAM beams can be generated by holograms on SLM and we can observe petal patterns on CCD without any other optical operator.

It is important to note that there are subtle difference in the phase profiles of modulated OAM beams and hybrid beams constituting of these OAM beams and the unmodulated incident Gaussian beam. Figure 2 shows examples of phase profiles of l = 10 and l = 6.5. The figure of OAM beam of l = 10 [Fig. 2(a)] shows the disorder in the center of the phase profile, denoting the undefined phase of an OAM beam. The center of the phase profile of the hybrid beam [Fig. 2(b)] only has a little disorder, mainly keeping red with an addition of the fixed phase of unmodulated Gaussian beam. The figure of l = 6.5 [Fig. 2(c)] shows the broken symmetry in the center of the phase profile due to the singularity line. The significant impact of the unmodulated Gaussian beam on the phase profile of the hybrid beam is shown in Fig. 2(d).



Fig. 2. Phase profiles of an OAM beam and a hybrid beam of l = 10 and 6.5. The OAM beam is generated by the modulation of an incident Gaussian beam with different holograms, and the hybrid beam constitutes of the modulated OAM beam and a unmodulated incident Gaussian beam.

3. Experiment setup

Our experimental setup for visualizing the interference patterns is shown in Fig. 3. A Gaussian beam is generated from a He-Ne laser ($\lambda = 632.8$ nm). Two spherical lens and an aperture are used to collimate the light beam and to remove any other higher-order diffracted terms. The light beam then passes through a half wave plate that changes the polarization direction of the beam to satisfy the requirement of SLM. Next, the beam splitter (BS) divides the beam into two parts, one of which can be reflected and transformed into an OAM beam. In our experiment, the SLM (Holoeye) is a phase-only reflective liquid crystal device with a resolution of 1920 × 1080 pixels and 8.0µm of pixel pitch. Finally, the OAM beam and the unmodulated Gaussian beam are imaged onto a CCD camera where interference patterns can be observed.



Fig. 3. Experiment setup to produce the interference patterns. Acronyms are $\lambda/2$: half wave plate; L1, L2: spherical lens; A: aperture; BS: beam splitter; SLM: spatial light modulator; CCD: CCD camera.

4. Experimental results and discussions

Using the proposed method, an arbitrary OAM beam and an interference pattern can be generated. The series of graphs in Fig. 4 shows interference patterns of weak unmodulated Gaussian beam and generated OAM beams of l = 10, 20, 30, 40, 50 and their negative values. The first and second row illustrates the experimental and simulated intensity profiles of hybrid OAM beams, respectively. Corresponding phase profiles of hybrid OAM beams in simulations are shown in the third row. The fourth row shows holograms we use on the SLM in both experiments and simulations.

The holograms are designed for the idea that both integer and fractional OAMs change smoothly so that we can visualize the evolution of all OAM beams, the same as [3]. The phase in holograms changes continually from 0 to 2π and undergoes a step of 2π , and the cycle repeats *l* times where *l* denotes the value of OAM.

Some features in Fig. 4 arouse great sights. Taking l = 5 as a simple example, we observe the petal structure instead of a ring structure in the intensity profile of the hybrid beam in experiment due to the interference between unmodulated and modulated beams. The number of petals equals five, agreeing with the value of OAM. The simulation result of l = 5 is in accordance with the experiment result except the slightly stronger diffraction. In addition, the phase profiles in simulations also reveal the value of OAM and the rotation of the light carrying OAM. All these interesting traits are showed in the other figures in Fig. 4.

Overviewing all experimental results, values of OAM designed on the holograms are in correspondence with the numbers of petals in the patterns, by which we can measure the value of OAM. The number of cycles in the phase profiles also agrees with the value of OAM. The hollow radius in intensity profiles increases with the rise of quantum number l, which can prove that l is the radial parameter of vortex beams. We merely cite the principle behind l = 50 as typical of the common sense. Due to fifty phase cycles from 0 to 2π , each sector on the circle possess only a small angle. The beams with different phase are so close that coherent cancellation emerges in the inner circle. However, beams in the outer circle have enough area to form complete intensity distribution modulated by the large-quantum-number hologram. To sum up, the phase profiles in Fig. 4 have a bigger circular area with the quantum number l arising, causing the same phenomenon that intensity profiles in both experiments and simulations have an increasing hollow area. The red circular area in the center of phase profiles denotes the phase of hybrid beam, where the disorder results from the phase singularity of the OAM beams. These figures of phase profiles also indicate the rotation of OAM beams, for positive values counter-clockwise and for negative values clockwise [30].

To confirm the power ratio of unmodulated and modulated incident Gaussian beams, we add a plane mirror and a variable attenuator along one of the two beams split by a beam splitter to reflect the Gaussian beam and to adjust the intensity of the beam, and keep other optical devices constant. After detecting the power of two beams respectively, we find out the change of interference patterns as the ratio of unmodulated and modulated beam decreases. Taking l = 4 as an example, Fig. 5 illustrates the interference patterns of $\eta = 100\%$, 66.7%, 25%, 12.5%, in which η denotes the power ratio of the unmodulated Gaussian beam, as shown in Eq. (4).

Both experiments and simulations show that when $\eta = 100\%$, interference patterns have four spiral petals and the strong Gaussian beam separates these petals. As the power ratio decreases, the Gaussian beam becomes increasingly weak so that four petals gradually fold and finally become a ring structure that means the effect of interference on the OAM beam can be almost ignored. With the comparison between the intensity profiles of hybrid beams and these four interference patterns, we confirm that the power ratio of unmodulated and modulated incident Gaussian beams is about 25%. It has to be mentioned that the Gaussian beam in simulations has an additional term of $\exp(i\pi/2)$ because of the different propagation distances of two beams split by the BS, namely the phase of the unmodulated Gaussian beam $\pi/2$ earlier than the other Gaussian beam going to be modulated. Although the figure of $\eta =$

| Research Article | Vol. 1, No. 2 15 Oct 2018 OSA CONTINUUM 456 |
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| OSA CONTINU | |

100% in simulation agrees with the experiment, other three figures in the second row don't exactly match the experiments. The petal rotation of corresponding figures becomes greater with η decreasing, the possible reason of which is that the variable attenuator changes not only the intensity of the unmodulated Gaussian beam, but also the phase of the beam.



Fig. 4. The first and second row: intensity profiles of different OAM beams in both experiments and simulations; the third row: corresponding phase profiles in simulations; the fourth row: holograms of different OAMs; the fifth-eighth rows: corresponding cases of negative values.



Fig. 5. Experimental and simulated interference patterns of the OAM beam (l = 4) under four different power ratios of unmodulated and modulated incident Gaussian beams.

Next, we discuss the generation of fractional OAM beams. It is well-established that fractional OAM beams can be described by introducing a singularity line into the phase holograms. Holograms used in researches to generate fractional OAM beams are mainly forked gratings. The advantage of employing this holographic approach is that the envisaged beam is easily isolated in the first-order diffraction from the other higher modes, because the combination of phase patterns and a plane wave carrier will bring the Gaussian beam phase information and separate the unmodulated Gaussian beam, respectively. However, the holograms we use are phase patterns without overlapping with a plane wave carrier because there is no need to separate the unmodulated Gaussian beam, which is just what we want. Furthermore, these holograms are generated from another idea that the phase changes continually except the singularity line on the left. To ensure the accuracy of the designed holograms, we calculate the *z*-component of the OAM density of the beams [31,32]:

$$j_{z} = \left(\vec{r} \times \vec{p}\right)_{z} = \left(\vec{r} \times \frac{i\varepsilon_{0}\omega}{2} \left(u^{*}\nabla u - u\nabla u^{*}\right)\right)_{z}$$
(5)

where $r = (x^2 + y^2)^{1/2}$, ε_0 is the permittivity in vacuum, ω is the frequency of the laser beam, k is the wave vector, and u is the complex scalar function describing the distribution of the field amplitude. Figure 6 shows the relationship between the measured OAMs of fractional OAM beams and the designed OAMs for holograms. This result is to ensure the reliability of our discussion.



Fig. 6. The relationship between the measured OAMs and the designed OAMs.

Vol. 1, No. 2 | 15 Oct 2018 | OSA CONTINUUM 458

OSA CONTINUUM



Fig. 7. The first and second row: intensity profiles of fractional OAM beams in both experiments and simulations; the third row: phase profiles in simulations; the fourth row: holograms of fractional OAMs; the fifth-eighth rows: corresponding cases of negative values.

Then, we analyze interference patterns and holograms of generated OAM beams of l = 4.0, 4.2, 4.4, 4.6, 4.8, 5.0 and their negative values, as shown in Fig. 7. Fractional holograms have a special feature that there is a singularity line in a certain position, leading to the split ring structures in intensity profiles. The above side of the singularity line always keeps white which denotes the phase of 2π , but the other side becomes gloomy with the value of OAM increasing. When the singularity line is the same as other steps, the value of OAM becomes an integer.

Both experimental and simulated patterns illustrate the generation of a new petal. The pattern of l = 4.2 shows that an emerging petal is formed outside the inner ring, and the petal

becomes stronger and comes into the inner ring in the figure of l = 4.4. The singularity line occurs obviously when l = 4.6 and gradually disappears with a new petal formed when l = 4.8. These figures show the development of a new petal, meaning the OAM plus one (l = 5.0). The reason why the petal is formed in the higher-order diffraction is that the decimal part of phase modulation in the inner ring is too weak to be observed. Only in the outer ring can the modulation be visualized with the enough area, which can also explain the large radial hollow area in the pattern of l = 50.

Apart from this, two interesting features can be observed in the intensity profiles of the patterns. The first one is that all petals are pushed to another adjacent petals and only the petal above the singularity line keeps still, which is also showed in the holograms, verifying the interference theory. The second one is that the petal (top left corner) seems to be separated owing to the diffraction enhancement induced by the singularity line when the value of OAM is fractional, which will be proved by another kind of hologram in the following discussion. The whole discussion above can be visualized in the phase profiles of simulated patterns.

After reversing the gray scale, we get the holograms for l = -4.0, -4.2, -4.4, -4.6, -4.8, -5.0. Consequently, we acquire the intensity profiles and phase profiles for negative fractional OAM beams in experiments and simulations. To our surprise, the experimental and simulated figures (the fifth and sixth row in Fig. 7) have differences from positive OAM patterns. First, the emerging petal is produced above the singularity line, and the diffraction enhancement separating a petal to two speckles locates below the line, both of which are reasonable because of the reverse of holograms. However, the biggest difference is the location of petals and the conjunction of the top petal and the emerging one, which results from the incomplete symmetry of the phases in the holograms.

To further confirm the nature of fractional OAM patterns, we design a kind of hologram which changes the phase from 0 to π as a cycle instead of the 2π cycle in fractional holograms mentioned above. The intensity profiles have a peculiar property due to the phase singularities, which enable the diffraction along the singularity lines become unexpectedly strong.



Fig. 8. The first and second row: intensity profiles of different beams in both experiments and simulations; the third row: holograms of different beams.

The strong diffraction brought by singularity lines lead to the separation of beam patterns. As depicted in Fig. 8, we can visualize the effect of diffraction and the evolution of different beams. There are four and five speckles in the figures of m = 4.0 and 5.0. With the value of m increasing, only the left speckle and the emerging one change. The intensity of first-order

diffraction speckles along the singularity line become increasingly high and the weak zerothorder speckles separate from the origin petal when m = 4.4, except the biggest zeroth-order speckle. These separated speckles produce new diffraction in two directions and finally are turned into a new strong speckle and its diffraction speckles, proving the discussion above that the emerging petal is generated from outer ring. It is important to mention that the diffraction produced by the singularity lines is so strong that the weak Gaussian beam has no effect on the pattern.

5. Conclusion

We have shown that arbitrary OAM beams can be measured with the interference between a vortex beam and a weak unmodulated Gaussian beam reflected by the SLM. The interference patterns have petals in correspondence with the values of OAM. The power ratio of unmodulated and modulated Gaussian beam has been discussed. Fractional OAM beams can also be generated by similar holograms with a singularity line. Our work has demonstrated the evolution of fractional OAM beams based on the petal patterns and has uncovered the principle of how radial parameter l influences the hollow radius of OAM beams. Both experiments and simulations reveal that the fractional OAM beams are generated by uneven modulation owing to the decimal part of holograms, and the diffraction enhancement separates a petal into two speckles.

The simple interferometric method is attractive for measurement of OAM beams owing to the use of only one hybrid beam including weak incident Gaussian beam and modulated OAM beam. The visualization of evolution apparently shows the nature of fractional OAM beams. The holograms used in this paper have potential application in the manipulation of particles.

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Research Article

OSA CONTINUUM

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