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Interaction phenomenon to (1+1)-dimensional Sharma–Tasso–Olver–Burgers equation

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ABSTRACT

The paper aims to explore the existence of diverse lump and interaction solutions to the (1+1)-dimensional Sharma–Tasso–Olver–Burgers equation. Through the Cole–Hopf transformation with the nonzero seed solution, the remarkable richness of exact solutions are exhibited, including lump, lump-periodic, lump-multiple solitary wave and periodic-multiple solitary wave. Some specific interaction phenomena are analyzed by the limit behavior. The generated rogue wave, half periodic kink (HPK) and breather-like solutions are displayed by some visual figures, respectively.

1. Introduction

The study of nonlinear phenomena plays an important role in various fields of mathematical physics and engineering [1–6]. Lump solutions, rationally localized in all directions in space, can be generated from solitons by taking long wave limits. Many examples of lump solutions and their interaction with solitons are found for many (2+1)-dimensional and (3+1)-dimensional integrable and nonintegrable systems [7–20], such as the Kadomtsev–Petviashvili (KP) [9], Sawada–Kotera (SK) [13] and Ito type systems [16]. Recently, Lou and Lin [21] find lump and lump-soliton solutions also exist in two (1+1)-dimensional nonintegrable KdV-type equations. Further, the similar solutions are discovered for the (1+1)-dimensional Ito equation [22] and Drinfel'd–Sokolov–Wilson equation [23]. To our knowledge, such kind of interaction solution among algebraic localized solutions (lumps) and exponentially localized line solitons has not been reported for other (1+1)-dimensional models.

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In this paper, we study the interaction solutions among the multiple solitary wave, lump and triangular periodic wave for the (1+1)-dimensional Sharma–Tasso–Olver–Burgers (STOB) equation

$$u_t + \alpha\left(3u_x^2 + 3u^2u_x + 3uu_{xx} + u_{xxx}\right) + \beta\left(2uu_x + u_{xx}\right) = 0. \tag{1}$$

The above equation will reduce to the Burgers equation for $\alpha = 0$ and to the STO equation for $\beta = 0$, respectively. For the sake of investigating the soliton molecules, the STOB equation is introduced in Ref. [24]. It has been verified the STOB equation is a new system possessing interesting and good structure and properties [24].

Through the Cole–Hopf transformation

$$u = \frac{f_x}{f} + u_0, \tag{2}$$

one can change Eq. (1) into a bilinear form

$$\begin{aligned} &3\alpha u_0^2 f_{xx} f + 3\alpha u_0 f_{xxx} f - f_t f_x - 3\alpha u_0^2 (f_x)^2 + \alpha f_{xxxx} f + f_{tx} f - 3\alpha u_0 f_{xx} f_x - \alpha f_{xxx} f_x \\ &+ 2\beta u_0 f_{xx} f - 2\beta u_0 f_x^2 + \beta f_{xxx} f - \beta f_{xx} f_x = 0. \end{aligned} \tag{3}$$

Here u_0 is a constant seed solution of Eq. (1). That is to say, if f solves the bilinear equation (3), then u given by the transformation (2) will solve the STOB equation (1). By means of the bilinear equation (3), we will discuss some fission and fusion phenomena, rogue wave, half periodic kink (HPK) and breathers-like solutions for the STOB equation.

2. Exact interaction solutions of Sharma-Tasso-Olver-Burgers equation

2.1. Interaction solution between lump and multiple solitary wave

Let us begin with

$$f = f_{L-S_N} = f_0 + X^2 + \sum_{i=1}^{N} e^{\eta_i} \tag{4}$$

and

$$X = a_1 x + a_2 t + a_0,\ \ \eta_i = k_i x + \omega_i t + \xi_{i,0}. \tag{5}$$

Here f_0, a_1, a_2, a_0, k_i, ω_i, $\xi_{i,0}$, $1 \le i \le N$ are parameters to be determined.

Substituting (4) into (3) leads to

$$u_0 = -\frac{\beta}{3\alpha},\ \ a_2 = \frac{\beta^2 a_1}{3\alpha},\ \ \omega_i = -\frac{k_i\left(3\alpha^2 k_i^2 - \beta^2\right)}{3\alpha}. \tag{6}$$

The interaction solution between lump (the quadratic term in (4)) and N-solitary wave (the exponential function terms in (4)) is generated through the transformation (2):

$$u = u_{L-S_N} = \frac{2a_1(a_1 x + \frac{\beta^2 a_1}{3\alpha} t + a_0) + \sum_{i=1}^{N} k_i e^{k_i x - \frac{k_i\left(3\alpha^2 k_i^2 - \beta^2\right)}{3\alpha} t + \xi_{i,0}}}{f_0 + \left(a_1 x + \frac{\beta^2 a_1}{3\alpha} t + a_0\right)^2 + \sum_{i=1}^{N} e^{k_i x - \frac{k_i\left(3\alpha^2 k_i^2 - \beta^2\right)}{3\alpha} t + \xi_{i,0}}} - \frac{\beta}{3\alpha}. \tag{7}$$

Note that $f = f_0 + X^2$ yields a pure algebraic solitary wave (lump) structure. At any given time t, we have the lump solution $u = \frac{2X}{f_0 + X^2} - \frac{\beta}{3\alpha} \to -\frac{\beta}{3\alpha}$ if and only if $|x| \to \infty$. On the one hand, it is readily observed that the STO equation (Eq. (1) with $\beta = 0$) only has the static lump solution ($a_2 = 0$). On the

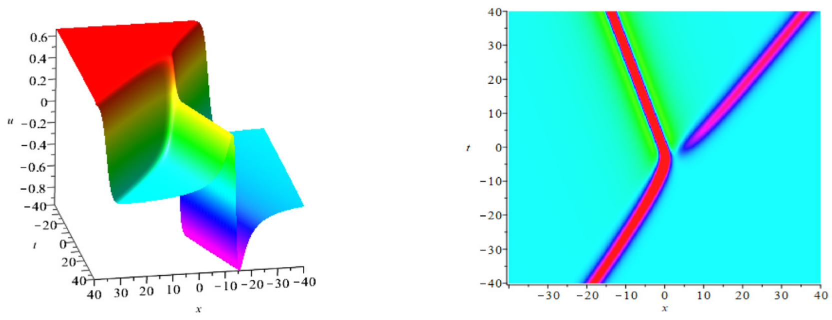


Fig. 1. Profiles of u_{L-S_1} and its field $z = u_x$ expressed by (8) with (9): (a) 3d plot of u; (b) density plot of z.

other hand, the STOB equation ($\alpha\beta \neq 0$) does not have such algebraic localized lump solution on the zero background (for $u_0 = 0$).

Let us see some special cases of the solution (7). For $N = 1$, the interaction solution u_{L-S_1} between lump and one solitary wave will produce a fission phenomena or a fusion phenomena. We have

$$u = u_{L-S_1} = \frac{2X + k_1 e^{\eta_1}}{f_0 + X^2 + e^{\eta_1}} - \frac{\beta}{3\alpha} \to \begin{cases} k_1 - \frac{\beta}{3\alpha}, & \eta_1 \to +\infty, \\ \frac{2X}{f_0 + X^2} - \frac{\beta}{3\alpha}, & \eta_1 \to -\infty. \end{cases} \tag{8}$$

That is to say, the lump will be truncated off in the region of $\eta_1 > 0$ or the lump only appears in the region of $\eta_1 < 0$. Fig. 1 displays a fission phenomena by selecting a special choice for the parameters:

$$\{f_0 = 10,\ a_1 = 2,\ a_0 = 1,\ k_1 = 1,\ \xi_{1,0} = 0,\ \alpha = 1,\ \beta = 1\}. \tag{9}$$

From Fig. 1, we observe the lump appears only for $t > 0$ and escapes from the kink-type solitary wave at $t \approx 0$.

For $N = 2$, we have the following solution

$$u = u_{L-S_2} = \frac{2X + k_1 e^{\eta_1} + k_2 e^{\eta_2}}{f_0 + X^2 + e^{\eta_1} + e^{\eta_2}} - \frac{\beta}{3\alpha}. \tag{10}$$

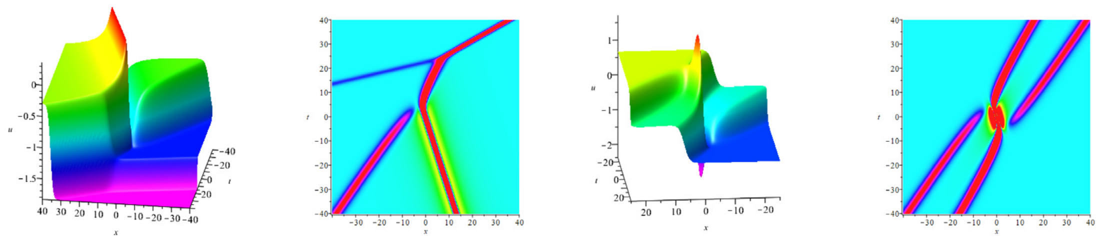
In the complex domain, if the wave number k_2 and the position parameter $\xi_{2,0}$ of the solitary wave are taken as the complex conjugate of $k_1 = k + \mathrm{i}\kappa$ and $\xi_{1,0} = p + \mathrm{i}\rho$, then the solution (10) will become a singular interaction solution among one lump and a complexiton:

$$u = \frac{2a_1^2 x + \frac{2\beta^2 a_1^2}{3\alpha} t + 2a_1 a_0 + 2e^{kx+\varpi t+p}\left[k\cos(\kappa x + \Omega t + \rho) - \kappa \sin(\kappa x + \Omega t + \rho)\right]}{\beta_0 + \left(a_1 x + \frac{\beta^2 a_1}{3\alpha} t + a_0\right)^2 + 2e^{kx+\varpi t+p}\cos(\kappa x + \Omega t + \rho)} - \frac{\beta}{3\alpha} \tag{11}$$

with $\varpi = \frac{k\left(9\alpha^2\kappa^2 - 3\alpha^2 k^2 + \beta^2\right)}{3\alpha}$, $\Omega = \frac{\kappa\left(3\alpha^2\kappa^2 - 9\alpha^2 k^2 + \beta^2\right)}{3\alpha}$. Next, we consider all the parameters are real. The solution (10) with $f_0 > 0$ is an analytical interaction solution between lump and two-solitary wave. Fig. 2(a) and (b) displays a fusion process between lump and an ordinary two-solitary wave with the following selection of the parameters:

$$\{f_0 = 100,\ a_1 = 7,\ a_0 = 2,\ k_1 = -1.5,\ k_2 = -1,\ \xi_{1,0} = -50,\ \xi_{2,0} = 0,\ \alpha = 1,\ \beta = 1\}. \tag{12}$$

In Fig. 2(a) and (b), the lump firstly aggregates with one solitary wave at $t \approx 0$. Then they are fused to the second solitary wave at $t \approx 20$.



(a) (b) (c) (d)

Fig. 2. (a) and (c): the 3d plot of u_{L-S_2} expressed by (10) with (12) and (14); (b) and (d): the corresponding density plot of field $z = u_x$ for (a) and (c).

Specifically, the solution (10) with $\eta_2 = -\eta_1$ (now called twin-solitary wave instead of two-solitary wave) will lead to an instanton or a rogue wave for

$$u = \frac{2X + k_1 e^{\eta_1} - k_1 e^{-\eta_1}}{f_0 + X^2 + e^{\eta_1} + e^{-\eta_1}} - \frac{\beta}{3\alpha} \to \begin{cases} k_1 - \frac{\beta}{3\alpha}, & \eta_1 \to +\infty, \\ -k_1 - \frac{\beta}{3\alpha}, & \eta_1 \to -\infty. \end{cases} \tag{13}$$

That is to say, the lump will be truncated off at both regions for $\eta_1 > 0$ and $\eta_1 < 0$. Thus the lump will become an instanton or a rogue wave if the instanton possesses a giant amplitude compared with that of the twin-solitary wave. Fig. 2(c) and (d) exhibits such kind of interaction behavior by selecting the parameters:

$$\{f_0 = 10,\ a_1 = 7,\ a_0 = 2,\ k_1 = -k_2 = -1,\ \xi_{1,0} = \xi_{2,0} = 0,\ \alpha = 1,\ \beta = 1\}. \tag{14}$$

From Fig. 2(d), one can find that the lump has disappeared before and after the time $t \approx 0$, while an instanton or a rogue wave has survived at $t \approx 0$.

2.2. Interaction solution between lump and triangular periodic wave

Consider

$$f = f_{L-P} = f_0 + X^2 + \lambda \sin(Y) \tag{15}$$

with $X = a_1 x + a_2 t + a_0$ and $Y = b_1 x + b_2 t + b_0$. Substituting (15) into (3) leads to

$$u_0 = -\frac{\beta}{3\alpha},\ a_2 = \frac{\beta^2 a_1}{3\alpha},\ b_2 = \frac{b_1\left(3\alpha^2 b_1^2 + \beta^2\right)}{3\alpha}. \tag{16}$$

The interaction solution between lump and triangular periodic wave is shown by

$$u = u_{L-P} = \frac{2a_1^2 x + \frac{2\beta^2 a_1^2}{3\alpha} t + 2a_1 a_0 + \lambda b_1 \cos\left(b_1 x + \frac{b_1\left(3\alpha^2 b_1^2 + \beta^2\right)}{3\alpha} t + b_0\right)}{f_0 + \left(a_1 x + \frac{\beta^2 a_1}{3\alpha} t + a_0\right)^2 + \lambda \sin\left(b_1 x + \frac{b_1\left(3\alpha^2 b_1^2 + \beta^2\right)}{3\alpha} t + b_0\right)} - \frac{\beta}{3\alpha}. \tag{17}$$

The solution (17) is analytic for $f_0 > |\lambda|$. Because the triangle periodic part "sin(Y)" can be neglected at $|X| \to \infty$, the periodic effect on the lump will concentrate on the local area $|X| \le \epsilon \approx 0$. Fig. 3(a) and (b) shows a periodic phenomenon in the evolution direction of one lump for the following special parameters:

$$\{f_0 = 20,\ a_1 = 2,\ a_0 = 10,\ \lambda = 8,\ b_1 = 1,\ b_0 = -0.5,\ \alpha = 0.4,\ \beta = 0.6\}. \tag{18}$$

As the parameter λ is closer to f_0, the solution (17) may produce a breather-like solution shown in Fig. 3(c) and (d), where the parameters are selected as

$$\{f_0 = 20,\ a_1 = 2,\ a_0 = 10,\ \lambda = 19.9,\ b_1 = 1,\ b_0 = -1,\ \alpha = 0.4,\ \beta = 0.6\}. \tag{19}$$

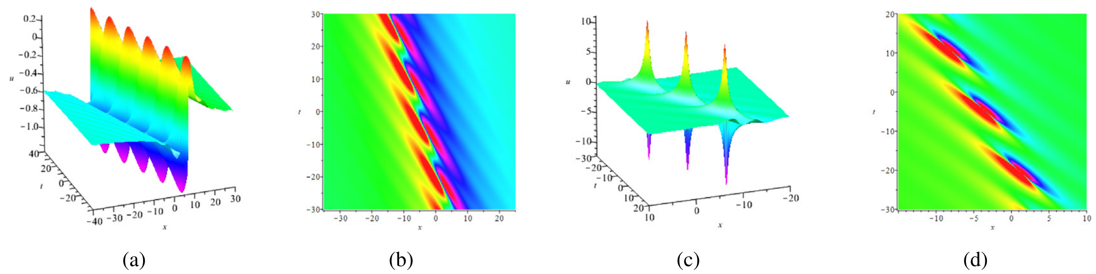


Fig. 3. (a) and (c): the 3d plot of u_{L-P} expressed by (17) with (18) and (19); (b) and (d): the corresponding density plot for (a) and (c).

2.3. Interaction solution between triangular periodic wave and multiple solitary wave

Assume

$$f = f_{P-S_N} = f_0 + \lambda \sin(Y) + \sum_{i=1}^{N} e^{\eta_i} \tag{20}$$

with $Y = b_1 x + b_2 t + b_0$ and $\eta_i = k_i x + \omega_i t + \xi_{i,0}$. Substituting (20) into (3) yields

$$u_0 = -\frac{\beta}{3\alpha},\ b_2 = \frac{b_1\left(3\alpha^2 b_1^2 + \beta^2\right)}{3\alpha},\ \omega_i = -\frac{k_i\left(3\alpha^2 k_i^2 - \beta^2\right)}{3\alpha}. \tag{21}$$

Now the corresponding interaction solution between triangular periodic wave and multiple solitary wave is written as

$$u = u_{P-S_N} = \frac{\lambda b_1 \cos\left[b_1 x + \frac{b_1\left(3\alpha^2 b_1^2+\beta^2\right)}{3\alpha} t + b_0\right] + \sum_{i=1}^{N} k_i e^{k_i x - \frac{k_i\left(3\alpha^2 k_i^2-\beta^2\right)}{3\alpha} t + \xi_{i,0}}}{f_0 + \lambda \sin\left[b_1 x + \frac{b_1\left(3\alpha^2 b_1^2+\beta^2\right)}{3\alpha} t + b_0\right] + \sum_{i=1}^{N} e^{k_i x - \frac{k_i\left(3\alpha^2 k_i^2-\beta^2\right)}{3\alpha} t + \xi_{i,0}}} - \frac{\beta}{3\alpha}. \tag{22}$$

The solution (22) is analytic for $f_0 > |\lambda|$. For $N = 1$, we have

$$u = u_{P-S_1} = \frac{\lambda b_1 \cos(Y) + k_1 e^{\eta_1}}{f_0 + \lambda \sin(Y) + e^{\eta_1}} - \frac{\beta}{3\alpha} \to \begin{cases} k_1 - \frac{\beta}{3\alpha}, & \eta_1 \to +\infty, \\ \frac{\lambda b_1 \cos(Y)}{f_0+\lambda \sin(Y)} - \frac{\beta}{3\alpha}, & \eta_1 \to -\infty. \end{cases} \tag{23}$$

The above solution u_{P-S_1} can be called the half periodic kink (HPK) because the periodic wave only occurs at the half plane $\eta_1 < 0$. By taking $b_0 = \delta + \frac{\pi}{2}$ and $k_1 = \frac{\beta}{3\alpha}$, one can reduce u_{P-S_1} to a special HPK

$$u = -\frac{1}{3\alpha} \frac{\beta f_0 + \lambda\beta \cos(Y_1) + 3\lambda\alpha b_1 \sin(Y_1)}{f_0 + \lambda \cos(Y_1) + e^{\frac{\beta}{3\alpha}x + \frac{2\beta^3}{27\alpha^2}t + \xi_{1,0}}} \tag{24}$$

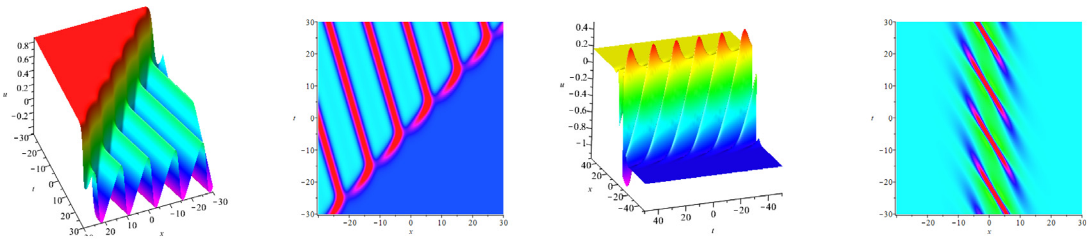
with $Y_1 = b_1 x + \frac{b_1\left(3\alpha^2 b_1^2+\beta^2\right)}{3\alpha} t + \delta$. It is clear that the solution (24) is equivalent to the HPK solution which was constructed by a series of resonances among three solitons in Ref. [24].

For $N = 2$, we discuss the solution u_{P-S_2} with $\eta_2 = -\eta_1$. There is

$$u = u_{P-S_2} = \frac{\lambda b_1 \cos(Y) + k_1(e^{\eta_1} - e^{-\eta_1})}{f_0 + \lambda \sin(Y) + e^{\eta_1} + e^{-\eta_1}} - \frac{\beta}{3\alpha} \to \begin{cases} k_1 - \frac{\beta}{3\alpha}, & \eta_1 \to +\infty, \\ -k_1 - \frac{\beta}{3\alpha}, & \eta_1 \to -\infty. \end{cases} \tag{25}$$

Hence the periodic wave arises along the area $|\eta_1| \le \varepsilon \approx 0$ of the twin-solitary wave. Fig. 4(a) shows an HPK solution u_{P-S_1} (23) with the specific parameters:

$$\{f_0 = 10,\ \lambda = 6,\ b_1 = 0.5,\ b_0 = 1,\ k_1 = 1,\ \xi_{1,0} = 0,\ \alpha = 1,\ \beta = 0.4\}. \tag{26}$$



(a) (b) (c) (d)

Fig. 4. (a) and (b): the 3d plot of u_{P-S_1} expressed by (23) with (26) and the corresponding density plot of field $z = u_x$; (c) and(d): the 3d plot of u_{P-S_2} expressed by (25) with (27) and the corresponding density plot of field $z = u_x$.

Fig. 4(b) tells us the periodic wave is truncated in the region of $x > \frac{71}{75}t$. Fig. 4(c) exhibits a periodic-twin solitary wave solution (25) with the selected parameters:

$$\{f_0 = 12,\ \lambda = 11,\ b_1 = 0.6,\ b_0 = 1,\ k_1 = -k_2 = 0.5,\ \xi_{1,0} = \xi_{2,0} = 0,\ \alpha = 1,\ \beta = 1\}. \tag{27}$$

Fig. 4(d) displays the periodic wave focused on the region $|12x + t| \leq \varepsilon \approx 0$.

3. Conclusions

For the (1+1)-dimensional STOB equation, rich interaction solutions including lump-multiple solitary wave, lump-periodic and periodic-multiple solitary wave were obtained. By analyzing the limit behavior of lump-one solitary wave u_{L-S_1} and periodic-one solitary wave u_{P-S_1}, one saw the lump and the periodic wave just occurred on a half plane of (x,t), respectively. In addition, the lump-twin solitary wave type solution would generate an instanton or a rogue which is a localized wave decayed in all space and time directions. For the STOB equation, it is conclude that the nonzero seed solution u_0 plays an vital role in these interactions. On the zero background ($u_0 = 0$), one cannot obtain the lump solution discussed in this paper for the STOB equation (with $\alpha\beta \neq 0$). By selecting some special parameters, the periodic-one solitary wave u_{P-S_1} was reduced to the HPK solution which was constructed by a series of resonances among three solitons in Ref. [24].

Acknowledgments

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