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Three-Wave Resonant Interaction in Optical Fibres on a Continuous-Wave Background *

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We predict that a three-wave resonant interaction (TWRI) for the excitations created from a continuous-wave background is possible in nonlinear optical fibres with a centro-symmetry. We show that in normal dispersion regime and near the zero-dispersion point of a single-mode optical fibre, the phase-matching condition for the TWRI can be satisfied by suitably choosing the wavevectors and frequencies of the exciting waves. The nonlinear envelope equations for the TWRI are derived by using a method of multiple-scales, and their explicit solutions for sum- and difference-frequency mixing are provided and discussed.

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Wave resonant interaction is a classical chapter in nonlinear optics.[1] For a passive optical medium there exists a common belief that a three-wave resonant interaction (TWRI) is not possible theoretically in optical fibre because the second-order susceptibility $\chi^{(2)}$ vanishes for centro-symmetric materials such as silica. Since second-harmonic generation (SHG) as a special case of TWRI was first observed in optical fibres,[2] parametric resonances have attracted much attention in recent years (see Ref. [3] and references therein). Experimentally an SHG in a Ge-doped silica optical fibre with the conversion efficiency up to 10% after irradiation by a laser beam during several hours has been realized.[4] There are numerous works devoted to the experimental and theoretical study which mostly focused on the breaking of a centro-symmetry of the system.[5,6]

Recently we proposed a new mechanism for realizing an SHG in optical fibres with a centro-symmetry based on the resonant interaction of two exciting waves from a cw background.[7] In this work we generalize the idea in Ref. [7] to consider a resonance among three waves, i.e. a TWRI of exciting waves on a cw background without needing any breaking of centro-symmetry. The idea is as follows. If we consider a nonlinear optical fibre ($\chi^{(2)}=0$) working in normal dispersion regime, a plane wave (i.e. the cw background) is modulationally stable. The excited waves considered here are generated from a cw background and the interaction between them has a character of a quadratic nonlinearity. Assuming that the system works near at zero-dispersion (ZD) point, the third-order dispersion of the fibre must be taken into account. Because of the third-order dispersion the linear dispersion relation of the excitation displays two branches, which provides a possibility for fulfilling the phase-matching condition of the a TWRI by suitably choosing the wavevectors and frequencies of the exciting waves. We derive a set of TWRI equations by using a method of multiple-scales and provide some explicit solutions for both the quasi-stationary and non-stationary cases. We note that although the nonlinear dynamics of dark solitons generated from a cw background in optical fibres near the ZD point has been investigated intensively,[8,9] a possible TWRI between these excitations is overlooked. In this Letter, we show that the three-wave soliton in the TWRI process is another example of soliton propagation in optical fibres.

Using the slowly varying envelope and paraxial approximations, the dimensionless envelope amplitude $u(z,t)$ of the electric field in optical fibre satisfies the modified (2+1)-dimensional Nonlinear Schrödinger equation[8]

$$iu_z - \frac{1}{2}\alpha u_{tt} + |u|^2 u = i\beta u_{ttt}, \tag{1}$$

where the subscripts z and t represent partial derivatives. Time t in the reference frame moving with the group velocity is measured in units of the pulse duration T, the longitudinal z and transverse x coordinate are normalized to $T/k^{(1)}$ and $[T/(k^{(1)}k^{(2)})]^{1/2}$. The parameters $\alpha = k^{(2)}/(Tk^{(1)})$ and $\beta = k^{(3)}/(6Tk^{(1)})$ denote the dimensionless second-order dispersion and third-order dispersion, respectively. Here k is the propagation constant and $k^{(j)} = \partial^j k/\partial\omega^j$ $(j=1,2,3)$. Equation (1) has a cw solution $u = u_0\exp(i|u_0|^2 z)$, with u_0 an arbitrary constant. Assuming that $u = u_0[1+a(z,t)]\exp[iu_0^2 z + i\phi(z,t)]$, Eq. (1) becomes the following system of equations:

$$-\phi_z + 2u_0^2 a - \frac{\alpha}{2}a_{tt} + \beta\phi_{ttt} - a\phi_z + \frac{\alpha}{2}\phi_t^2 + 3u_0^2 a^2 + 3\beta a_{tt}\phi_t + 3\beta a_t\phi_{tt} + \beta a\phi_{ttt} + \frac{\alpha}{2}a\phi_t^2 + u_0^2 a^3 - \beta\phi_t^3 - \beta a\phi_t^3 = 0, \tag{2}$$

$$a_z - \frac{\alpha}{2}\phi_{tt} - \beta a_{ttt} - \alpha a_t\phi_t - \frac{\alpha}{2}a\phi_{tt} + 3\beta\phi_t\phi_{tt} + 3\beta a_t\phi_t^2 + 3\beta a\phi_t\phi_{tt} = 0. \tag{3}$$

It is obvious that the set of nonlinear coupled equations are of *quadratic* nonlinearity. Based on Eqs. (2)

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and (3), an analysis of the linear stability of the cw solution against a small perturbation shows that the cw solution is modulational stable if the fibre is working in normal dispersion regime (i.e. $\alpha > 0$). Assuming a and ϕ varying with the form $\sim \exp(i\omega t - ikz)$, we can obtain the linear dispersion relation

$$k = k_{\pm}(\omega) = \beta\omega^3 \pm \omega\sqrt{\alpha\Big(u_0^2 + \frac{\alpha}{4}\omega^2\Big)}, \qquad (4)$$

which is shown in Fig. 1. We see that both $k_+(\omega)$ and $k_-(\omega)$ are acoustic, i.e. $k_{\pm}(0) = 0$.

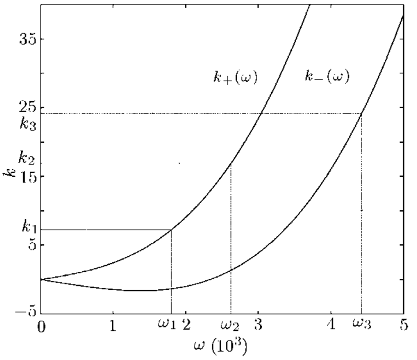

Fig. 1. The linear dispersion relation and phase matching of a TWRI for the excitations on the cw background. The phase-matching condition can be satisfied if (ω_1, k_1) and (ω_2, k_2) are chosen from one dispersion branch $k_+(\omega)$ and (ω_3, k_3) from $k_-(\omega)$. The parameters used in the figure are $\alpha = 1.854 \times 10^{-6}$, $\beta = 5.0612 \times 10^{-10}$, and $u_0 = 1.25$.

We are interested in a possible TWRI for the excitations created on the cw background. The frequencies of the three interaction waves are labelled by ω_1, ω_2 and ω_3, where $\omega_1 \le \omega_2 < \omega_3$ and $\omega_3 = \omega_1 + \omega_2$. A necessary condition for the TWRI is to fulfil the phase-matching condition $k_3 = k_2 + k_1$. Considering Eq. (4) we find that this is indeed possible if the system parameters α, β and cw amplitude u_0 are chosen properly, with the points (ω_1, k_1) and (ω_3, k_3) taking from the curve $k_-(\omega)$ and the point (ω_2, k_2) from the curve $k_+(\omega)$ to satisfy

$$k_-(\omega_3) = k_-(\omega_1) + k_+(\omega_2). \qquad (5)$$

Figure 1 shows the linear dispersion relation of the system and the phase-matching condition (5) for the TWRI. The parameters are provided from standard single-mode optical fibres, i.e. $\lambda_{\rm ZD}$ (the ZD point wavelength) $= 1.27\,(\mu$m). Near $\lambda_{\rm ZD}$, $k^{(1)} = 5 \times 10^{-9}\,({\rm s\,m^{-1}})$, $k^{(2)} = 9 \times [(1.27 - \lambda_0(\mu{\rm m})] \times 10^{-26}\,({\rm s^2\,m^{-1}})$, $k^{(3)} = 2.3 \times \sqrt{\lambda_0(\mu{\rm m})}[\lambda_0(\mu{\rm m}) - 1] \times 10^{-40}\,({\rm s^3\,m^{-1}})$, and n_2 (Kerr coefficient)$= 1.2 \times 10^{-22}({\rm m/V})^2$. The wavelength of the carrier-wave and the pulse duration of the electric field are chosen to be $\lambda_0 = 1.064$ (μm) and $T = 10^{-12}$ (s), respectively. Thus we obtain $\alpha = 1.8 \times 10^{-6}$ and $\beta = 5.06 \times 10^{-10}$. In the figure the dimensionless amplitude of the electric-field background is taken as $u_0 = 1.25$, which corresponds to the dimensional electric field $E_0 = (2|k^{(1)}|cA_{\rm eff}/T\omega_0 n_2)^{1/2}u_0 = 1.2 \times 10^4\,{\rm V/m}$, when taking the effective cross area $A_{\rm eff}$ of the fibre as $20\,\mu{\rm m}^2$, where $\omega_0 = 2\pi c/\lambda_0$, and c is the speed of light in vacuum. Three interacting waves are taken as $(\omega_1, k_1) = (1.80 \times 10^3\,{\rm s^{-1}},\ 7.24\,{\rm m^{-1}})$, $(\omega_2, k_2) = (2.62 \times 10^3\,{\rm s^{-1}},\ 16.93\,{\rm m^{-1}})$, $(\omega_3, k_3) = (4.42 \times 10^3\,{\rm s^{-1}},\ 24.17\,{\rm m^{-1}})$, respectively.

We now derive the nonlinear envelope equations for the TWRI using a method of multiple-scales. Introducing the following asymptotic expansion $a = \varepsilon a^{(1)} + \varepsilon^2 a^{(2)} + \varepsilon^3 a^{(3)} + \cdots$, and $\phi = \varepsilon\phi^{(1)} + \varepsilon^2\phi^{(2)} + \varepsilon^3\phi^{(3)} + \cdots$, where ε is a small ordering parameter, a and ϕ are the functions of the fast variables z and t as well as the slow variables εz and εt. From Eqs. (2) and (3) we obtain

$$a_z^{(i)} - \beta a_{ttt}^{(i)} - \frac{\alpha}{2}\phi_{tt}^{(i)} = m^{(i)}, \qquad (6)$$

$$2u_0^2 a^{(i)} - \frac{\alpha}{2}a_{tt}^{(i)} - \phi_z^{(i)} + \beta\phi_{ttt}^{(i)} = n^{(i)}. \qquad (7)$$

The explicit expressions of $m^{(i)}$ and $n^{(i)}$ $(i = 1, 2\cdots)$ are omitted here.

In the leading order $(i = 1)$, Eqs. (6) and (7) are linear equations which admit the solution $\phi^{(1)} = \phi_{1j}\exp(i\theta_j) + $ c.c., $a^{(1)} = a_{1j}\exp(i\theta_j) + $ c.c. with $\theta_j = \omega_j t - k_j x$, $a_{1j} = ib_j = -i(\alpha\omega_j)/(k_j - \beta\omega_j^3)$. In the case of TWRI we take the leading solution as a superposition of three components: $\phi^{(1)} = \phi_{11}\exp(i\theta_1) + \phi_{12}\exp(i\theta_2) + \phi_{13}\exp(i\theta_3) + $ c.c., $a^{(1)} = a_{11}\exp(i\theta_1) + a_{12}\exp(i\theta_2) + a_{13}\exp(i\theta_3) + $ c.c., where c.c. represents the corresponding complex conjugation. The frequencies and wavevectors of the three interaction waves (k_1, ω_1), (k_2, ω_2), (k_3, ω_3) are selected according to the phase-matching condition (5) (see Fig. 1). The envelopes $a_{1j} = ib_j\phi_{1j}$ with $b_j = -\alpha\omega_j/(k_j - \beta\omega_j^3)$ $(j = 1, 2, 3)$ are the functions of the slow variables εz and εt.

In the order $(i = 2)$, we can obtain the closed equations governing the envelopes ϕ_{1j}. Then by taking $\varepsilon\phi_{1j} = \phi_j$ and returning to the original variables we obtain

$$\frac{\partial\phi_1}{\partial z} = -i\lambda_1\phi_2^*\phi_3\exp^{-i\Delta kz}, \qquad (8)$$

$$\frac{\partial\phi_2}{\partial z} = -i\lambda_2\phi_1^*\phi_3\exp^{-i\Delta kz}, \qquad (9)$$

$$\frac{\partial\phi_3}{\partial z} = -i\lambda_3\phi_1\phi_2\exp^{i\Delta kz}, \qquad (10)$$

where $\Delta k = k_1 + k_2 - k_3$ is a possible phase mismatch. The explicit expressions of the coefficients λ_1, λ_2, and λ_3 are omitted here. Envelope equations (8)–(10) can be solved exactly.[1] By writing ϕ_1, ϕ_2, and ϕ_3 as $\phi_1 = f\exp(-i\varphi_f)$, $\phi_2 = g\exp(-i\varphi_g)$, and $\phi_3 = h\exp(-i\varphi_h)$, where f, g, h, φ_f, φ_g, and φ_h are the real functions, Eqs. (8)–(10) become $\dfrac{\partial f}{\partial z} = -\lambda_1 gh\sin\theta$, $\dfrac{\partial g}{\partial z} = -\lambda_2 fh\sin\theta$, $\dfrac{\partial h}{\partial z} = \lambda_3 gh\sin\theta$, $f\dfrac{\partial\varphi_f}{\partial z} = \lambda_1 gh\cos\theta$, $g\dfrac{\partial\varphi_g}{\partial z} = \lambda_1 fh\cos\theta$, and $h\dfrac{\partial\varphi_h}{\partial z} = \lambda_3 fg\cos\theta$, with the relative phase angle $\theta = \varphi_h - \varphi_f - \varphi_g + \Delta kz$. There are three conservative quantities for these equations: $f^2/\lambda_1 + h^2/\lambda_3 = m_1$, $g^2/\lambda_2 + h^2/\lambda_3 = m_2$ and $f^2/\lambda_1 - g^2/\lambda_2 = m_3$, where

m_1, m_2 and m_3 are the integration constants. Here λ_1, λ_2 and λ_3 are the coupling coefficients which carries the sign of wave energy. With these relations we obtain

$$\int_{z_1}^{z_2} dz = \frac{1}{2}\int_{H(z_1)}^{H(z_2)} dh^2\Big[\lambda_3^2\lambda_1\lambda_2\Big(m_1-\frac{h^2}{\lambda_3}\Big)\cdot\Big(m_2-\frac{h^2}{\lambda_3}\Big)h^2-\Big(\Gamma_h-\frac{1}{2}\Delta k h^2\Big)^2\Big]^{-1/2} \quad (11)$$

$$\int_{z_1}^{z_2} dz = \frac{1}{2}\int_{G(z_1)}^{G(z_2)} dg^2\Big[\lambda_2^2\lambda_1\lambda_3\Big(m_2-\frac{g^2}{\lambda_2}\Big)\cdot\Big(m_3+\frac{g^2}{\lambda_2}\Big)g^2-\Big(\Gamma_g-\frac{1}{2}\Delta k g^2\Big)^2\Big]^{-1/2} \quad (12)$$

with $H(z)\equiv h^2(z)$, $G(z)\equiv g^2(z)$, $\Gamma_h=\lambda_3 fgh\cos\theta+\frac{1}{2}\Delta k h^2$, and $\Gamma_g=\lambda_2 fgh\cos\theta-\frac{1}{2}\Delta k g^2$. The integral equations (11) and (12) give the general solution $H(z_2)$ and $G(z_2)$ at the distance z_2 for arbitrary inputs power $F(z_1)(F(z)=f^2(z)), G(z_1)$ and $H(z_1)$ at distance z_1.

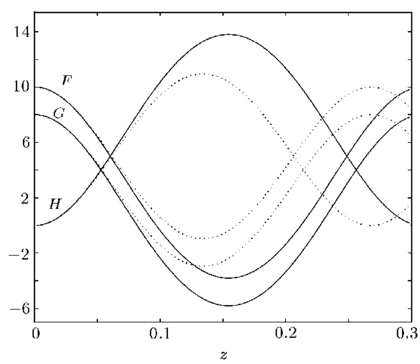

Fig. 2. Energy conversion for the sum-frequency mixing with the initial boundary condition $F(0)=10$, $G(0)=8$, and $H(0)=0$. The system parameters are chosen to be $\alpha=1.854\times10^{-6}$, $\beta=5.061\times10^{-10}$, $u_0=1.25$ and phase mismatch $\Delta k=1.0$ (solid curve). The dotted curves show the effect of increasing phase mismatch $\Delta k=3.0$.

As is known, a TWRI can be classified according to the initial or boundary conditions. If the two input waves are at the two lower frequencies ω_1 and ω_2, it is sum-frequency mixing. We first consider this case by assuming $H(z_1)=0$. Then we have $m_1=F(z_1)/\lambda_1$, $m_2=G(z_1)/\lambda_2$, $m_3=F(z_1)/\lambda_1-G(z_1)/\lambda_2$, and $\Gamma_h=0$. The general expression for the magnitude of the generated wave h can be obtained by integrating Eq. (11),

$$H=\sqrt{\lambda_3 m_1}A_{H-}^2 sn^2[(\lambda_1\lambda_2\lambda_3 m_1 A_{H+}^2 z^2)^{1/2},\gamma_H], \quad (13)$$

where γ_H is the modulus of the elliptic function sn, given by $\gamma_H=A_{H-}^2/A_{H+}^2$ with $A_{H+}^2, A_{H-}^2=\frac{1}{2}[(1+\sigma+\epsilon)\pm\sqrt{(1+\sigma+\epsilon)^2-4\sigma}]$, where $\epsilon=\Big(\frac{1}{2}\Delta k\Big)^2/(m_1\lambda_1\lambda_2\lambda_3)$ is responsible for the properties of the fibre and $\sigma=m_2/m_1$. The results for the energy conversion of sum-frequency mixing have been plotted in Fig. 2, from which we can see that there is a periodic energy conversion among three waves. The bold curves show the generation of H with phase mismatch $\Delta k=1$ from initial values of $F=10$. The dashed curves show the effect of the increment of $\Delta k=2$. It is clear that the energy conversion efficiency decreases with increasing phase mismatch Δk.

Another case is difference-frequency mixing, in which ω_1 (or ω_2) is generated from ω_2 (or ω_1) and higher frequency ω_3. Then initially one has $G(z_1)=0$ and in this case we have $m_1=F/\lambda_1+H/\lambda_3$, $m_2=H/\lambda_3$, $m_3=F/\lambda_1$, and $\Gamma_g=0$. From the integration equation (12) about generated wave g we obtain the general result in the case of difference-frequency mixing

$$H=\sqrt{\lambda_2 m_1}\;A_{G-}^2\cdot\frac{\gamma_G^2 sn^2[(\lambda_1\lambda_2\lambda_3 m_1 A_{G+}^2 z^2)^{1/2}/\gamma_G,\gamma_G]}{1-\gamma_G^2 sn^2[(\lambda_1\lambda_2\lambda_3 m_1 A_{G+}^2 z^2)^{1/2}/\gamma_G,\gamma_G]}, \quad (14)$$

where $A_{G+}^2, A_{G-}^2=\frac{1}{2}[\pm(-1+\sigma-\epsilon)+\sqrt{(-1+\sigma-\epsilon)^2+4\sigma}]$, and $\gamma_H=A_{G-}^2/A_{G+}^2$. Figure 3 shows the energy conversion for difference-frequency mixing.

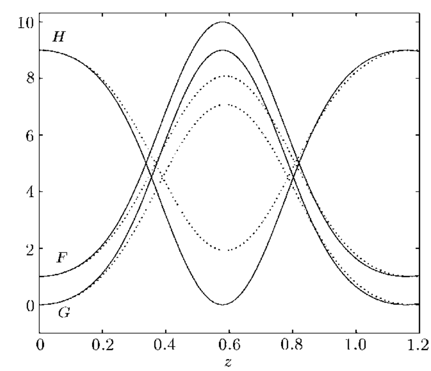

Fig. 3. Energy conversion for difference-frequency mixing with the initial boundary conditions $F(0)=1$, $H(0)=9$, $G(0)=0$. The system parameters are chosen to be $\alpha=1.854\times10^{-6}$, $\beta=5.061\times10^{-10}$, $u_0=1.25$ and phase mismatch $\Delta k=1.0$ (solid curve). The dotted curves shows the effect of increasing phase mismatch $\Delta k=3.0$.

Note that the quasi-stationary approximation used in deriving Eqs. (8)–(10) is valid only for infinitely large plane wave excitations. For the excitations of a narrower width, the propagation of the excitations will be of a non-stationary character. The energy conversion for the TWRI by ultrashort pulses can be greatly reduced by walk-off. Using a similar approach as the same as that for deriving Eqs. (8)–(10) but now assuming that the envelopes depends also on the slowly varying time variable εt, we can obtain

$$\frac{\partial\phi_1}{\partial z}+\frac{1}{v_1}\frac{\partial\phi_1}{\partial t}=-i\lambda_1\phi_3\phi_1^*\exp(i\Delta kx), \quad (15)$$

$$\frac{\partial\phi_2}{\partial z}+\frac{1}{v_2}\frac{\partial\phi_2}{\partial t}=-i\lambda_2\phi_3\phi_2^*\exp(i\Delta kx), \quad (16)$$

$$\frac{\partial \phi_3}{\partial z} + \frac{1}{v_3}\frac{\partial \phi_3}{\partial t} = -i\lambda_3 \phi_1 \phi_2 \exp(-i\Delta k x), \tag{17}$$

where $v_j = d\omega_j/dk_j$ $(j = 1, 2, 3)$ are the group velocities of jth waves, and λ_j $(j = 1, 2, 3)$ are the same as those in Eqs. (8)–(10). The above equations are completely integrable and can be solved by the inverse scattering transform.[12] Under the phase-matching condition $\Delta k = 0$ one type of three-wave soliton solution reads as

$$\phi_1 = \Big(\frac{\beta_{21}\beta_{31}}{v_2 v_3 \lambda_2 \lambda_3}\Big)^{1/2} \frac{4\eta_1}{D}[\exp[-i(\varphi_1 - 2\xi_1 z_1)]] \times [\exp(-2\eta_2 z_2) - \frac{\zeta_2 - \zeta_1}{\zeta_2^* - \zeta_1}\exp(2\eta_2 z_2)], \tag{18}$$

$$\phi_2 = \Big(\frac{\beta_{21}\beta_{23}}{v_1 v_3 \lambda_1 \lambda_3}\Big)^{1/2} \frac{4\eta_2}{D}[\exp[-i(\varphi_2 - 2\xi_2 z_2)]] \times [\exp(2\eta_1 z_1) - \frac{\zeta_2^* - \zeta_1^*}{\zeta_2^* - \zeta_1}\exp(-2\eta_1 z_1)], \tag{19}$$

$$\phi_3 = \Big(\frac{1}{\lambda_1 \lambda_2 v_1 v_2}\Big)^{1/2} \frac{16\eta_1\eta_2\beta_{21}}{D(\xi_2 - \xi_1)(\beta_{23}\beta_{31})^{1/2}} \cdot \exp[i(\varphi_1 + \varphi_2 - 2\xi_1 z_1 - 2\xi_2 z_2)], \tag{20}$$

with

$$D = \exp[2(z_1\eta_1 + z_2\eta_2)] + \exp[2(z_1\eta_1 - z_2\eta_2)] + \Big|\frac{\zeta_2 - \zeta_1}{\zeta_2^* - \zeta_1}\Big|^2 \exp[2(-z_1\eta_1 + z_2\eta_2)] + \exp[-2(z_1\eta_1 + z_2\eta_2)], \tag{21}$$

where $z_1 = z - v_1 t - z_{10}$, $z_2 = z - v_2 t - z_{20}$, $\zeta_1 = 2(\xi_1 + i\eta_1)/\beta_{23}$, $\zeta_2 = 2(\xi_2 + i\eta_2)/\beta_{31}$, $\beta_{ij} = v_j - v_i (v_1 > v_3 > v_2)$ $(j = 1, 2, 3)$. $\xi_1, \xi_2, \eta_1, \eta_2, z_{10}, z_{20}, \varphi_1$, and φ_2 are constants.

We can easily obtain the asymptotic form of the solution. The initial shapes of the fundamental waves before the interaction starts are

$$\phi_1 = 2\eta_1\Big(\frac{\beta_{21}\beta_{31}}{v_2 v_3 \lambda_2 \lambda_3}\Big)^{1/2} \mathrm{sech}(2\eta_1 z_1)\exp(i\varphi_1), \tag{22}$$

$$\phi_2 = 2\eta_2\Big(\frac{\beta_{21}\beta_{23}}{v_1 v_3 \lambda_1 \lambda_3}\Big)^{1/2} \mathrm{sech}(2\eta_2 z_2)\exp(i\varphi_2), \quad \phi_3 = 0. \tag{23}$$

After collision, the asymptotic profiles ϕ_1, ϕ_2, ϕ_3 become

$$\phi_1 = 2\eta_1\Big(\frac{\beta_{21}\beta_{31}}{v_2 v_3 \lambda_2 \lambda_3}\Big)^{1/2} \mathrm{sech}(2\sigma_1 z_1 - z_{10} - \delta) \cdot \exp(i\varphi_1 + \delta'), \tag{24}$$

$$\phi_2 = 2\eta_2(\frac{\beta_{21}\beta_{23}}{v_1 v_3 \lambda_1 \lambda_3})^{1/2} \mathrm{sech}(2\sigma_2 z_2 - z_{10} + \delta) \exp(i\varphi_2 - \delta'), \quad \phi_3 = 0, \tag{25}$$

where δ and δ' are defined by $(\zeta_1 - \zeta_2)/(\zeta_1^* - \zeta_2) = e^{-\delta} e^{i\delta'}$. We find that the soliton solutions describe two initially separated fundamental waves preserving their shape on nonlinear interaction with each other and exactly preserving the same shape after separation is regained. In the colliding region, a new soliton ϕ_3 is produced as seen in Fig. 4.

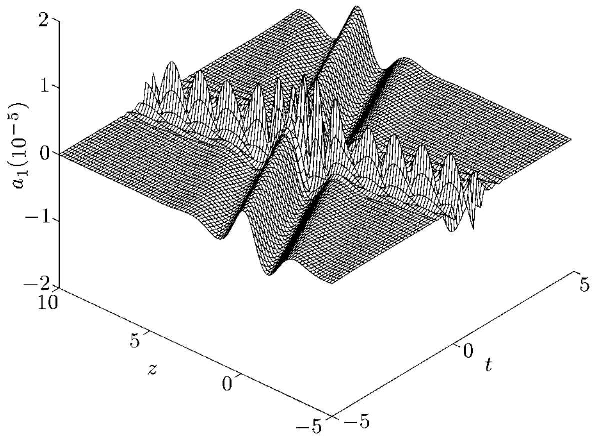

Fig. 4. Three-wave soliton interaction in optical fibres. The parameters are chosen to be $\xi_1 = 1$, $\xi_2 = 1$, $\eta_1 = 0.5$, $\eta_2 = 0.2$, $z_{10} = 1$, $z_{20} = 3$, $\varphi_1 = 1$, and $\varphi_2 = 1$.

In summary, we have proposed a new mechanism of the TWRI of the excitations on a cw background in nonlinear optical fibres without any breaking of a centro-symmetry. We have shown that in the normal dispersion regime and near the ZD point of a centro-symmetric single-mode optical fibre, the phase-matching condition of a TWRI can be fulfilled by a suitable selection of the wavevectors and frequencies of three exciting waves. We have also derived the nonlinearly coupled envelope equations for the TWRI by using a method of multiple-scales, and their explicit solutions are provided and discussed.

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