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Ultraslow optical solitons in a four-level tripod atomic system

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Abstract

We investigate possible formation and propagation of localized, shape-preserving nonlinear optical pulse in a resonant, lifetime-broadened four-level tripod atomic system via electromagnetically induced transparency (EIT). We prove both analytically and numerically that although in anomalous dispersion regimes near resonance a superluminal optical soliton may appear, such soliton suffers serious absorption. However, by choosing appropriate parameters to make the system work in normal dispersion regimes and within an EIT transparency window, ultraslow optical solitons with very low light intensity can form and propagate stably in the system.

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Keywords: Electromagnetically induced transparency; Ultraslow optical solitons

1. Introduction

Solitons, i.e. localized nonlinear excitations that are shape-preserving during propagation, have been studied in many branches of physics and states of matter ranging from solid media, such as optical fibers (optical solitons [1]), to Bose–Einstein condensed atomic vapors (matter wave solitons [2,3]). Optical solitons are of special interest because of their important applications for information processing and transmission [1]. However, most optical solitons found up to now are produced in passive media such as glass-based optical fibers, in which far-off resonance excitation schemes are employed in order to avoid serious optical absorption. Because the nonlinear effect in such passive media is extremely weak, to form an optical soliton a high input light intensity and a long propagation distance are required. In addition, optical solitons generated in this way generally travel with a speed very close to c (the light speed in vacuum).

In recent years, considerable interest has focused on the wave propagation in highly resonant optical media via electromagnetically induced transparency (EIT) [4]. Due to the quantum interference effect induced by a control field, the propagation of a weak probe field in a three-level atomic system displays many striking features, including a large suppression of optical absorption, a significant reduction of probe field group velocity [5,6], and a giant enhancement of Kerr nonlinearities [7–9], which are beneficial to certain important nonlinear optical processes under weak very drive conditions, such as highly efficient four-wave mixing and precision spectroscopy [10–15]. Based on these interesting features, it has been shown recently that there exists a class of optical solitons, called *ultraslow optical solitons*, in highly resonant three-level [16–18] and four-level atomic systems [19,20]. The formation of matched ultraslow soliton pairs have also been predicted [21–23]. These fruitful explorations have enriched largely the research field of the *resonant nonlinear optics* in coherent media [24].

In a recent work, Han et al. has considered the nonlinear optical pulse propagation in a four-level tripod atomic system [25]. The authors claimed that a superluminal (or called

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fast light [26]) optical soliton can propagate stably in the system. In the present work, we shall show that their conclusion is incorrect. Actually, a superluminal optical soliton obtained in an anomalous dispersion regime near resonance suffers a serious absorption because such soliton works outside of EIT transparency window. However, by choosing appropriate parameters the system may support an ultraslow optical soliton with very low light intensity when working in a normal dispersion regime near resonance. Since in this case the system works within the EIT transparency window, the ultraslow optical soliton can propagate stably in the system.

The Letter is arranged as follows. In Section 2, the Hamiltonian, the Maxwell–Schrödinger equations, and the linear optical property of the system are presented. In Section 3, by using a standard method of multiple-scales an asymptotic expansion on the Maxwell–Schrödinger equations is made and a nonlinear envelope equation governing the time-evolution of probe field envelope is derived. In Section 4 ultraslow optical soliton solutions are given and their physical properties are discussed in detail. The stability of the ultra-slow optical solitons and their collision are also studied by means of numerical simulations. Section 5 contains a discussion and summary of our main results of this work.

2. Model and the solution in linear regime

The system under study is a resonant, lifetime-broadened four-level tripod atomic system [25] with upper energy level $|0\rangle$ and lower energy levels $|1\rangle$, $|2\rangle$, and $|3\rangle$ (see Fig. 1). A weak, pulsed probe optical field (with pulse duration τ_0) of center frequency $\omega_p/(2\pi)$ is coupled to $|1\rangle \to |0\rangle$ transition, a strong and continuous-wave (cw) pumping optical field of frequency $\omega_c/(2\pi)$ is coupled to $|2\rangle \to |0\rangle$ transition, and a strong and cw control field of frequency $\omega_d/(2\pi)$ is coupled to $|3\rangle \to |0\rangle$ transition, respectively.

The electric-field vector of the whole optical field can be written as $\mathbf{E} = \sum_{l=p,c,d} \mathbf{e}_l \mathcal{E}_l \exp[i(\mathbf{k}_l \cdot \mathbf{r} - \omega_l t)] + \text{c.c.}$, where $|\mathbf{k}_l| = \omega_l/c$ $(l = p, c, d)$ and $\mathbf{e}_l$ is the unit vector in the lth polarization direction. The Hamiltonian of the system reads $\hat{H} = \hat{H}_0 + \hat{H}'$, where $\hat{H}_0$ describes an free atom and $\hat{H}'$ describes the interaction between the atom and the optical field. In Schrödinger picture, the state vector of the systems is expressed by $|\Psi(z,t)\rangle_s = \sum_{j=0}^{3} C_j(z,t)|j\rangle$, where $|j\rangle$ is the eigenstate of $\hat{H}_0$. Under electric-dipole and rotating-wave approximations, the Hamiltonian takes the form

$$\begin{aligned}\hat{H} = \sum_{j=0}^{3} \varepsilon_j |j\rangle\langle j| - \hbar\big\{&\Omega_1 \exp\big[i(k_p z - \omega_p t)\big]|0\rangle\langle 1| \\ &+ \Omega_2 \exp\big[i(\mathbf{k}_c \cdot \mathbf{r} - \omega_c t)\big]|0\rangle\langle 2| \\ &+ \Omega_3 \exp\big[i(\mathbf{k}_d \cdot \mathbf{r} - \omega_d t)\big]|0\rangle\langle 3| + \text{H.c.}\big\}, \end{aligned} \tag{1}$$

where ε_j is the energy of state $|j\rangle$, $\Omega_1 = \mathbf{e}_p \cdot \mathbf{p}_{01}\mathcal{E}_p/\hbar$, $\Omega_2 = \mathbf{e}_c \cdot \mathbf{p}_{02}\mathcal{E}_c/\hbar$ and $\Omega_3 = \mathbf{e}_d \cdot \mathbf{p}_{03}\mathcal{E}_d/\hbar$ are the half Rabi frequencies of the probe, pumping, and control fields, respectively. $\mathbf{p}_{0l}$ is the electric dipole matrix element associated with the transition $|0\rangle \leftrightarrow |l\rangle$ $(l = 1, 2, 3)$. The detunings are given by $\Delta_1 = (\varepsilon_0 - \varepsilon_1)/\hbar - \omega_p = (\varepsilon_0 - \varepsilon_2)/\hbar - \omega_c$ and $\Delta_2 = (\varepsilon_0 - \varepsilon_3)/\hbar - \omega_d$. For simplicity and without loss of generality, the wavevector direction of the probe field has been chosen along the z axis, i.e. $\mathbf{k}_p = k_p \mathbf{e}_z$.

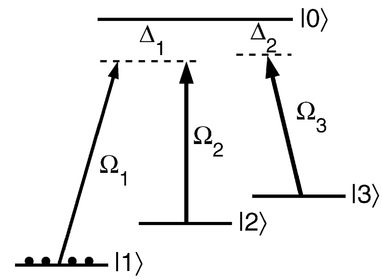


Fig. 1. Energy level structure and excitation scheme of the lifetime-broadened four-level tripod atomic system with upper energy level $|0\rangle$ and the lower energy levels $|1\rangle$, $|2\rangle$, and $|3\rangle$. Δ_1 and Δ_2 are detunings. Ω_1, Ω_1, and Ω_3 are the half Rabi frequencies of the probe, pumping, and control fields, respectively.

In order to investigate the time evolution of the system, it is more convenient to employ an interaction picture, which is obtained by making the transformation $C_j = A_j \exp\{i[\mathbf{k}_j \cdot \mathbf{r} - (\varepsilon_j/\hbar + \lambda_j)t]\}$, with $\mathbf{k}_0 = k_p\mathbf{e}_z$, $\mathbf{k}_1 = 0$, $\mathbf{k}_2 = k_p\mathbf{e}_z - \mathbf{k}_c$, $\mathbf{k}_3 = k_p\mathbf{e}_z - \mathbf{k}_d$, $\lambda_0 = 0$, $\lambda_1 = \lambda_2 = \Delta_1$, and $\lambda_3 = \Delta_2$. Then we obtain the Hamiltonian in the interaction picture

$$\begin{aligned}\hat{H}_{\text{int}} = &-\hbar\big[\Delta_1|1\rangle\langle 1| + \Delta_1|2\rangle\langle 2| + \Delta_2|3\rangle\langle 3|\big] \\ &- \hbar\big[\Omega_1|0\rangle\langle 1| + \Omega_2|0\rangle\langle 2| + \Omega_3|0\rangle\langle 3| + \text{H.c.}\big].\end{aligned} \tag{2}$$

Using the Schrödinger equation $i\hbar\partial|\Psi(t)\rangle_{\text{int}}/\partial t = H_{\text{int}}|\Psi(t)\rangle_{\text{int}}$ with $|\Psi(t)\rangle_{\text{int}} = (A_1, A_2, A_3, A_4)^T$ (T represents transpose) and the Maxwell equation $\nabla^2\mathbf{E} - (1/c^2)\partial^2\mathbf{E}/\partial t^2 = [1/(\epsilon_0 c^2)]\partial^2\mathbf{P}/\partial t^2$ with $\mathbf{P} = \mathcal{N}_a\{\mathbf{p}_{01}A_0A_1^* \exp[i(k_p z - \omega_p t)] + \mathbf{p}_{02}A_0A_2^* \exp[i(\mathbf{k}_c \cdot \mathbf{r} - \omega_c t)] + \mathbf{p}_{03}A_0A_3^* \exp[i(\mathbf{k}_d \cdot \mathbf{r} - \omega_d t)] + \text{c.c.}\}$, one can readily obtain the Maxwell–Schrödinger (MS) equations governing the motion of atomic state amplitudes and the time-dependent probe field

$$\left(i\frac{\partial}{\partial t} + d_0\right)A_0 + \Omega_1 A_1 + \Omega_2 A_2 + \Omega_3 A_3 = 0, \tag{3a}$$

$$\left(i\frac{\partial}{\partial t} + d_2\right)A_2 + \Omega_2^* A_0 = 0, \tag{3b}$$

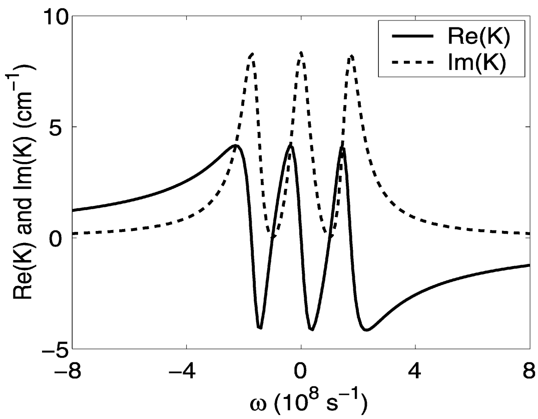
$$\left(i\frac{\partial}{\partial t} + d_3\right)A_3 + \Omega_3^* A_0 = 0, \tag{3c}$$

$$i\left(\frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{\partial t}\right)\Omega_1 + \kappa A_0 A_1^* = 0, \tag{3d}$$

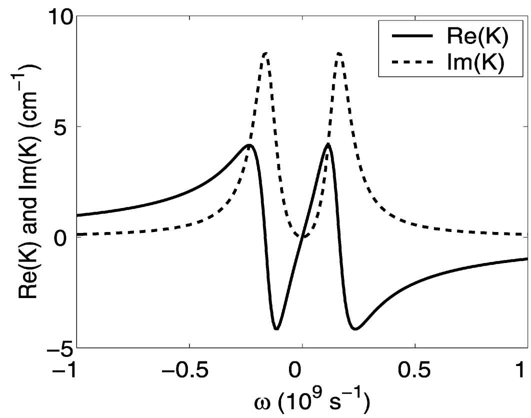
$$|A_0|^2 + |A_1|^2 + |A_2|^2 + |A_3|^2 = 1, \tag{3e}$$

where $d_0 = i\gamma_0$, $d_2 = \Delta_1 + i\gamma_2$, and $d_3 = \Delta_2 + i\gamma_3$ with γ_l describing the corresponding decay rate of the energy level $|l\rangle$ $(l = 0, 2, 3)$. $\kappa = \mathcal{N}_a\omega_p|\mathbf{p}_{10}|^2/(2\epsilon_0 c\hbar)$ is the coupling constant between the electric field and the atomic state vector, with $\mathcal{N}_a$ being the atomic concentration and ϵ_0 the vacuum dielectric constant.

Before solving the MS equations (3a)–(3e), we first examine the linear property of the system, which may provide a useful starting point for the weak nonlinear theory developed in the following sections. We assume that atoms are initially populated in the ground state $|1\rangle$ and the probe field is infinitesimal.



(a)



(b)

Fig. 2. The dispersion curve $\mathrm{Re}(K)$ (solid line) and absorption curve $\mathrm{Im}(K)$ (dashed line) of the probe field as functions of ω, obtained with $\Delta_1 = -\Delta_2 \neq 0$ (panel (a)) and $\Delta_1 = \Delta_2 = 0$ (panel (b)). The system parameters have been given in the text.

In this case the population of the ground state $|1\rangle$ is not depleted during time evolution, i.e. $A_1 \approx 1$. Taking Ω_1 and A_j ($j = 0, 2, 3$) as being proportional to $\exp[i(K(\omega)z - \omega t)]$, one obtains readily the linear dispersion relation

$$K(\omega) = \frac{\omega}{c} - \frac{\kappa}{(\omega + i\gamma_0) - \frac{|\Omega_2|^2}{\omega + d_2} - \frac{|\Omega_3|^2}{\omega + d_3}}. \tag{4}$$

In most operation conditions $K(\omega)$ can be Taylor expanded around the center frequency of the probe field, i.e. $\omega = 0$. We obtain $K(\omega) = K_0 + K_1\omega + \frac{1}{2}K_2\omega^2 + \cdots$, where $K_j = [\partial^j K(\omega)/\partial\omega^j]|_{\omega=0}$ ($j = 0, 1, 2, \ldots$), which can be obtained from Eq. (4) explicitly. The physical interpretation of the dispersion coefficients K_j is rather clear. $K_0 = \phi + i\alpha/2$ gives the phase shift ϕ per unit length and absorption coefficient α, $K_1 = 1/V_g$ determines the group velocity V_g of the probe pulse, and K_2 represents the group-velocity dispersion that contributes to probe pulse's shape change and an additional loss.

The panel (a) of Fig. 2 shows the dispersion (solid line) and absorption (dashed line) curves of the probe field Ω_1, which are characterized respectively by $\mathrm{Re}(K)$ and $\mathrm{Im}(K)$. The parameters are taken as $\gamma_0 \simeq 1.2 \times 10^8\ \mathrm{s}^{-1}$, $\gamma_2 \simeq \gamma_3 \simeq 1.0 \times 10^2\ \mathrm{s}^{-1}$, $\kappa = 1.0 \times 10^9\ \mathrm{cm}^{-1}\,\mathrm{s}^{-1}$, $\Omega_2 = \Omega_3 = 1.0 \times 10^8\ \mathrm{s}^{-1}$, $\Delta_1 = -1.0 \times 10^8\ \mathrm{s}^{-1}$, and $\Delta_2 = 1.0 \times 10^8\ \mathrm{s}^{-1}$. Notice that two EIT transparency windows (called double dark resonance [27]) in the absorption curve $\mathrm{Im}(K)$ appear due to the quantum interference effect induced by the pumping field Ω_2 and the control field Ω_3. Interestingly, if one takes $\Delta_1 = \Delta_2 = 0$ without changing other parameters, only one transparency window occurs, as shown in panel (b) of the figure.

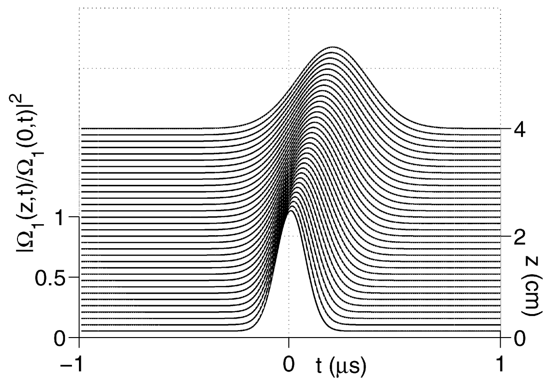


Fig. 3. The deformation of the dimensionless probe wave intensity $|\Omega_1(z,t)/\Omega_1(0,t)|^2$ due to dispersion. The initial condition is a Gaussian pulse with the form $\Omega_1(0,t) = \Omega_1(0,0)\exp[-t^2/(2\tau_0^2)]$ with $\tau_0 = 1.0 \times 10^{-7}$ s and other parameters being the same as those used in Fig. 2(b).

The dispersion curves in the figure can be divided into normal and anomalous dispersion regimes. In the normal dispersion regimes, i.e. $\partial\,\mathrm{Re}(K)/\partial\omega > 0$, we have $V_g < c$ and thus the probe field is a slow light; in the anomalous dispersion regimes, i.e. $\partial\,\mathrm{Re}(K)/\partial\omega < 0$, one has $V_g > c$ or even negative and hence the probe field is superluminal [26]. From the figure we see that in the normal anomalous (anomalous) dispersion regimes the probe field has a negligible (strong) absorption. Consequently, the probe field is nearly transparent (opaque) for the slow-light (superluminal) propagation when working in the normal (anomalous) dispersion regimes.

Although in the normal dispersion regimes the absorption of the probe field can be largely suppressed, the dispersion effect of the system may be significant for the probe pulse with a shorter temporal width. Fig. 3 shows the dimensionless probe field intensity $|\Omega_1(z,t)/\Omega_1(0,t)|^2$ during propagation in the linear level. The initial condition is a Gaussian pulse with the form $\Omega_1(0,t) = \Omega_1(0,0)\exp[-t^2/(2\tau_0^2)]$ with $\tau_0 = 1.0 \times 10^{-7}$ s and other parameters being the same as those used in Fig. 2(b). We see that due to the dispersion effect of the system the probe pulse spreads rapidly. The main contribution for the pulse deformation in the linear case is mainly due to the group-velocity dispersion of the system.

3. Asymptotic expansion and nonlinear envelope equation

Because an EIT-based system has giant Kerr nonlinearity [7–9], it is natural to use such nonlinearity to balance the dispersion effect of the system. In this section, we apply standard multiple-scale perturbation theory [1] to solve equations (3a)–(3d) and investigate the possible formation and propagation of a shape-preserving nonlinear probe pulse in the system. In order to make nonlinear effect significant, one may increase the intensity of the probe field and hence the population in the ground state $|1\rangle$ will be depleted. Thus we make the following asymptotic expansion $A_j = \delta_{1j} + \sum_{n=1}^{\infty} \epsilon^n A_j^{(n)}$ ($j = 0, 1, 2, 3$) and $\Omega_1 = \sum_{n=1}^{\infty} \epsilon^n \Omega_1^{(n)}$, where ϵ is a small parameter characterizing the small population depletion of the ground state. To obtain a divergence-free expansion, all quantities on the right-hand side of the asymptotic expansion are considered as functions of the multi-scale variables $z_j = \epsilon^j z$ ($j = 0$ to 2) and $t_j = \epsilon^j t$ ($j = 0, 1$). Substituting the expansion and the multi-scale variables into Eqs. (3a)–(3e), we obtain a chain of linear, but inhomogeneous equations for $A_j^{(l)}$ and $\Omega_1^{(l)}$, i.e.

$$\left(i\frac{\partial}{\partial t_0} + d_0\right)A_0^{(l)} + \Omega_1^{(l)} + \Omega_2 A_2^{(l)} + \Omega_3 A_3^{(l)} = M^{(l)}, \tag{5a}$$

$$\left(i\frac{\partial}{\partial t_0} + d_2\right)A_2^{(l)} + \Omega_2^* A_0^{(l)} = N^{(l)}, \tag{5b}$$

$$\left(i\frac{\partial}{\partial t_0} + d_3\right)A_3^{(l)} + \Omega_3^* A_0^{(l)} = Q^{(l)}, \tag{5c}$$

$$i\left(\frac{\partial}{\partial z_0} + \frac{1}{c}\frac{\partial}{\partial t_0}\right)\Omega_1^{(l)} + \kappa A_0^{(l)} = R^{(l)}. \tag{5d}$$

The explicit expressions of $M^{(l)}$, $N^{(l)}$, $Q^{(l)}$ and $R^{(l)}$ are omitted here for saving space. Eqs. (5a)–(5d) can be further expressed in the following convenient form

$$\hat{L}\Omega_1^{(l)} = S^{(l)}, \tag{6a}$$

$$A_0^{(l)} = \frac{1}{\kappa}\left[R^{(l)} - i\left(\frac{\partial}{\partial z_0} + \frac{1}{c}\frac{\partial}{\partial t_0}\right)\Omega_1^{(l)}\right], \tag{6b}$$

$$A_2^{(l)} = \left(i\frac{\partial}{\partial t_0} + d_2\right)^{-1}\left[N^{(l)} - \Omega_2^* A_0^{(l)}\right], \tag{6c}$$

$$A_3^{(l)} = \left(i\frac{\partial}{\partial t_0} + d_3\right)^{-1}\left[Q^{(l)} - \Omega_3^* A_0^{(l)}\right], \tag{6d}$$

with

$$\hat{L} = \kappa - i\left[\left(i\frac{\partial}{\partial t_0} + d_0\right) - |\Omega_2|^2\left(i\frac{\partial}{\partial t_0} + d_2\right)^{-1} - |\Omega_3|^2\left(i\frac{\partial}{\partial t_0} + d_3\right)^{-1}\right]\left(\frac{\partial}{\partial z_0} + \frac{1}{c}\frac{\partial}{\partial t_0}\right),$$

and

$$S^{(l)} = -\left[\left(i\frac{\partial}{\partial t_0} + d_0\right) - |\Omega_2|^2\left(i\frac{\partial}{\partial t_0} + d_2\right)^{-1} - |\Omega_3|^2\left(i\frac{\partial}{\partial t_0} + d_3\right)^{-1}\right]R^{(l)} + \kappa\left[M^{(l)} - \Omega_2\left(i\frac{\partial}{\partial t_0} + d_2\right)^{-1}N^{(l)} - \Omega_3\left(i\frac{\partial}{\partial t_0} + d_3\right)^{-1}Q^{(l)}\right].$$

Eqs. (6a)–(6d) can be solved order by order. The leading order ($l = 1$) solution is just that obtained in the linear regime, described already in the last section. The expression of Ω_1 has the form $F\exp\{i[K(\omega)z_0 - \omega t_0]\}$, with F being a yet to be determined envelope function depending on the slow variables t_1 and z_j ($j = 1, 2$).

At the second order ($l = 2$), a divergence-free condition for the second-order solution requires

$$i\left(\frac{\partial F}{\partial z_1} + \frac{1}{V_g}\frac{\partial F}{\partial t_1}\right) = 0, \tag{7}$$

where $V_g = 1/K_1$ is the group velocity of the envelope F.

To the third order ($l = 3$), a divergence-free condition yields the equation for F

$$i\frac{\partial F}{\partial z_2} - \frac{K_2}{2}\frac{\partial^2 F}{\partial t_1^2} - W\exp(-\alpha_1 z_2)F|F|^2 = 0, \tag{8}$$

with $\alpha_1 = \epsilon^{-2}\alpha$, and

$$W = -\frac{\kappa}{G|G|^2}\left(1 + \frac{|\Omega_2|^2}{\Delta_1^2 + \gamma_2^2} + \frac{|\Omega_3|^2}{\Delta_2^2 + \gamma_3^2}\right),$$

where $G = i\gamma_0 - |\Omega_2|^2/(\Delta_1 + i\gamma_2) - |\Omega_3|^2/(\Delta_2 + i\gamma_3)$. Here, the real and the imaginary parts of W contribute to the Kerr (self-phase modulation) effect and the nonlinear absorption of the system, respectively.

Combining equations (7) and (8) and returning to original variables, we obtain the nonlinear envelope equation

$$i\left(\frac{\partial}{\partial z} + \frac{\alpha}{2}\right)U - \frac{K_2}{2}\frac{\partial^2 U}{\partial \tau^2} - W|U|^2 U = 0, \tag{9}$$

where $\tau = t - z/V_g$ and $U = \epsilon F e^{-i\alpha z/2}$.

4. Ultraslow optical solitons

4.1. Ultraslow soliton solutions

Eq. (9) is a Ginzberg–Landau equation, which has complex coefficients and thus generally does not allows soliton solutions. However, if a practical set of system parameters can be found so that the imaginary part of these coefficients can be made much smaller than their corresponding real part, it can be approximated as a nonlinear Schrödinger (NLS) equation and thus it is possible to obtain a shape-preserving soliton solution that can propagate for a rather long distance without significant distortion. This is just the case for the present system when working in the normal dispersion regimes (i.e. within the EIT transparency window).

For convenience of following discussions, we write Eq. (9) into the dimensionless form

$$i\frac{\partial u}{\partial s} + \frac{\partial^2 u}{\partial \sigma^2} + 2u|u|^2 = id_0 u, \tag{10}$$

where $s = -z/(2L_D)$, $\sigma = \tau/\tau_0$, $u = U/U_0$, and $d_0 = L_D/L_A$. Here $L_D = \tau_0^2/\tilde{K}_2$ is characteristic dispersion length, $L_A = 1/\alpha$ is characteristic absorption length, and $U_0 = (1/\tau_0) \times \sqrt{\tilde{K}_2/\tilde{W}}$ is characteristic Rabi frequency of the probe field. The symbol tilde above these quantities denotes the real part of the corresponding coefficients. Notice that in order to form solitons the characteristic Rabi frequency U_0 is obtained by setting $L_D = L_{NL}$ (i.e. the dispersion and the nonlinearity are balanced each other), where L_{NL} is the characteristic nonlinearity length defined by $L_{NL} = 1/(\tilde{W}U_0^2)$.

If $d_0 \ll 1$ (i.e. L_D is much less than L_A), the term on the right-hand side of Eq. (10) is a high-order one and thus can be neglected within the propagation distance up to L_D. In this case Eq. (10) reduces to the standard NLS equation $i\partial u/\partial s + \partial^2 u/\partial\sigma^2 + 2u|u|^2 = 0$, which is completely integrable and allows multi-soliton solutions. A single soliton solution reads $u = \operatorname{sech}\sigma \exp(is)$, or in terms of field

$$\begin{aligned}\Omega_1 &= Ue^{i\phi z}\\ &= \frac{1}{\tau_0}\sqrt{\frac{\tilde{K}_2}{\tilde{W}}}\operatorname{sech}\left[\frac{1}{\tau_0}\left(t - \frac{z}{\tilde{V}_g}\right)\right]\exp\left[i\phi z - i\frac{z}{2L_D}\right], \qquad (11)\end{aligned}$$

which describes a fundamental bright soliton travelling with propagating velocity $\tilde{V}_g$.

Now we consider an practical cold atomic (e.g. ^{87}Rb) system that can be realized by a typical alkali atomic vapor at very low temperature. The parameters suitable to this system can be chosen as $\gamma_0 \simeq 1.2\times 10^8\ \mathrm{s}^{-1}$, $\gamma_2 \simeq \gamma_3 \simeq 1.0\times 10^2\ \mathrm{s}^{-1}$. We take $\kappa = 1.0\times 10^9\ \mathrm{cm}^{-1}\,\mathrm{s}^{-1}$, $\Omega_2 = 1.0\times 10^8\ \mathrm{s}^{-1}$, $\Omega_3 = 1.3\times 10^8\ \mathrm{s}^{-1}$, $\Delta_1 = -8.0\times 10^5\ \mathrm{s}^{-1}$, and $\Delta_2 = 1.0\times 10^6\ \mathrm{s}^{-1}$. With these parameters we get $K_0 = (0.227 + i0.006)\ \mathrm{cm}^{-1}$, $K_1 = (1.68 + i0.09)\times 10^{-6}\ \mathrm{cm}^{-1}\,\mathrm{s}$, $K_2 = (2.50 + i0.21)\times 10^{-11}\ \mathrm{cm}^{-1}\,\mathrm{s}^2$, and $W = (3.81 + i0.11)\times 10^{-16}\ \mathrm{cm}^{-1}\,\mathrm{s}^2$. We see that the imaginary part of these coefficients are indeed much smaller than their corresponding real part.

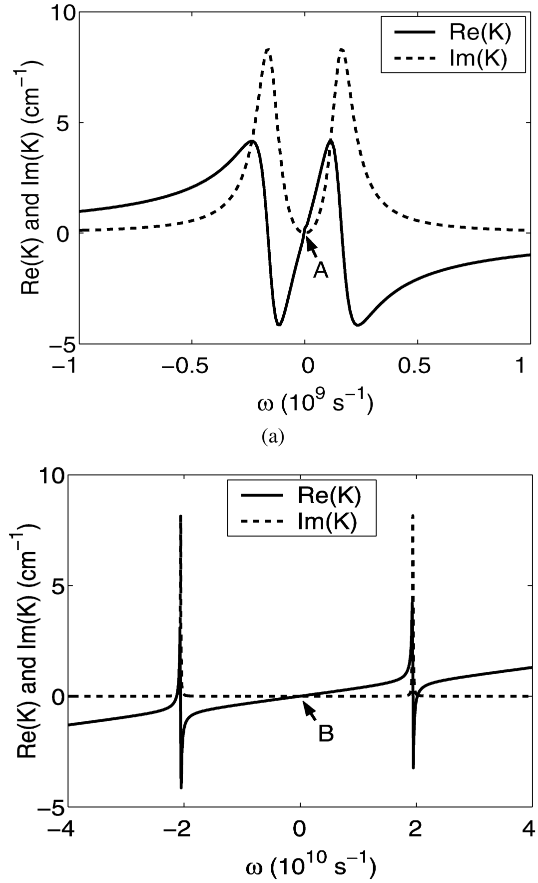
After taking $\tau_0 = 5.0\times 10^{-6}$ s, we obtain $U_0 = 5.0\times 10^7\ \mathrm{s}^{-1}$, $L_D = L_{NL} = 1.0$ cm, and $L_\mathrm{A} = 157$ cm. Because the dispersion and nonlinear lengths are much smaller than the absorption length, the absorption of the probe field can be safely neglected within the distance up to the dispersion and nonlinearity lengths. In this case the system can be described reasonably by the standard NLS equation (10) without the d_0 term and hence the soliton with the form (11) for the probe field Rabi frequency is physically possible in the system.

With the above parameters we obtain the propagating velocity of the soliton

$$\tilde{V}_g = 2.0\times 10^{-5}c. \qquad (12)$$

So the *soliton obtained travels with an ultraslow propagating velocity*. The spatial width of such ultraslow soliton is $\tau_0\tilde{V}_g = 2.98$ cm. The bright soliton with the form (11) is obtained under the condition $\mathrm{Re}(\tilde{K}_2\tilde{W}) > 0$. Note that the system also supports dark solitons when the parameters are chosen to satisfy the condition $\mathrm{Re}(\tilde{K}_2\tilde{W}) < 0$.

The input power of the ultra-slow optical soliton in the normal dispersion regimes described by Eq. (11) can be easily calculated by Poynting's vector. By a simple calculation we obtain the average flux of energy over carrier-wave period, which is given by $\bar{P} = \bar{P}_{\max}\operatorname{sech}^2[(t - z/\tilde{V}_g)/\tau_0]$, with the peak power $\bar{P}_{\max} = 2\varepsilon_0 c n_p S_0 |\mathcal{E}_p|^2_{\max} = 2\varepsilon_0 c n_p S_0 (\hbar/|\mathbf{p}_{13}|)^2 \tilde{K}_2/(\tau_0^2\tilde{W})$. Here $n_p = 1 + cK/\omega_p$ is the refractive index and S_0 is the cross-section area of the probe field. Using the above parameters and $S_0 \approx 1.0\times 10^{-4}\ \mathrm{cm}^2$, we obtain $\bar{P}_{\max} = 3.5\times 10^{-3}$ mW. *Thus to produce such ultraslow soliton very low input light intensity is needed*. However, in non-resonant media such as optical fibers, ps or fs laser pulses are usually needed to reach a very high peak power to bring out the enough nonlinear effect required for soliton formation.



(a)

(b)

Fig. 4. The dispersion curve $\mathrm{Re}(K)$ (solid line) and absorption curve $\mathrm{Im}(K)$ (dashed line) of the probe field as functions of ω obtained with the parameters given just below Eq. (11) (panel (a)) and given by [25] (panel (b)). The A and B are points where the optical solitons are obtained in the present work and in Ref. [25], respectively.

We stress that the above result is obtained in a normal dispersion regime, i.e. the system parameters have been chosen in an EIT transparency window. Shown in Fig. 4(a) is the dispersion (solid line) and absorption (dashed line) curves of the probe field by using the above parameters. The ultraslow optical soliton (11) is obtained in the normal dispersion regime near resonance, indicated by the point A in the figure, where the absorption is extremely low. Note that although a superluminal soliton may be also generated in the anomalous dispersion regime (i.e. the regions with negative slope of $\mathrm{Re}(K)$). However, such soliton decays very rapidly due to the very large absorption of the optical filed.

4.2. *Stability of the ultraslow optical solitons*

Now we discuss the stability of the ultraslow optical soliton by using numerical simulations. In Fig. 5(a), we have plotted the waveshape of $|\Omega_1/U_0|^2$ as a function of t/τ_0 and $z/(2L_D)$ with the parameters given just below the expression (11). The solution is obtained by numerically solving Eq. (9)

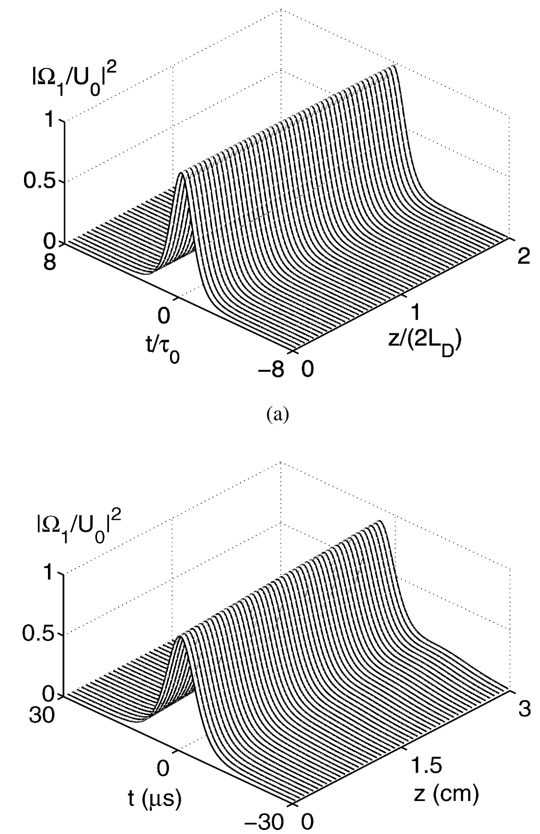


Fig. 5. (a): The shape-preserving (soliton) waveshape of $|\Omega_1/U_0|^2$ as a function of t/τ_0 and $z/(2L_D)$ with the parameters given just below Eq. (11). The solution is numerically obtained from Eq. (9) with full complex coefficients. The initial condition is given by $\Omega_1(0,t)/U_0 = \mathrm{sech}(t/\tau_0)$. (b): The evolution of $|\Omega_1/U_0|^2$ obtained by directly integrating equations (3a) and (3e) without any approximation. The initial condition is taken the same as that used in panel (a).

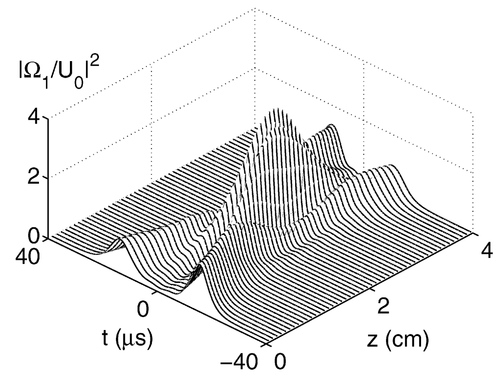


Fig. 6. The waveshape during the collision between two ultraslow optical solitons. The initial condition is given by $\Omega_1(0,t)/U_0 = \mathrm{sech}[(t-3.0)/\tau_0] + 1.2\,\mathrm{sech}[1.2(t+3.0)/\tau_0]$.

with full complex coefficients. The initial condition is given by $\Omega_1(0,t)/U_0 = \mathrm{sech}(t/\tau_0)$. We see that the shape of the soliton undergoes no apparent deformation because the imaginary part of the coefficients is very small. In order to make further confirmation on the soliton solution and check its stability, we have made a directly numerical integration based on Eqs. (3a)–(3e) without using any approximation. The result is shown in Fig. 5(b). The initial condition is taken the same as that used in Fig. 5(a). We see that the probe pulse suffers no apparent distortion except a very small wave radiation when propagating to 3 cm distance.

We have also numerically investigated the interaction between two ultraslow optical solitons. Assume initially we have two solitons created in the system. As time goes on they collide and then depart each other. Shown in Fig. 6 is the waveshape of the two solitons during their collision. The initial condition is taken by $\Omega_1(0,t)/U_0 = \mathrm{sech}[(t-3.0)/\tau_0] + 1.2\,\mathrm{sech}[1.2(t+3.0)/\tau_0]$. We see that the solitons are fairly stable during the collision.

Notice that the optical solitons in the same system have been studied recently by Han et al. [25]. For comparison, we have also calculated the coefficients using the parameters given in [25], i.e. $\gamma_0 \simeq 1.2\times10^8\ \mathrm{s}^{-1}$, $\gamma_2 \simeq \gamma_3 \simeq 0$, $\kappa = 1.0\times10^9\ \mathrm{cm}^{-1}\,\mathrm{s}^{-1}$, $\Omega_2 = 1.0\times10^8\ \mathrm{s}^{-1}$, $\Omega_3 = 2.0\times10^{10}\ \mathrm{s}^{-1}$, $\Delta_1 = \Delta_2 \simeq 1.2\times10^9\ \mathrm{s}^{-1}$, and $\tau_0 = 1.0\times10^{-11}$ s. We get $\phi \simeq -3.00\times10^{-3}\ \mathrm{rad\,cm}^{-1}$, $\alpha \simeq -2.16\times10^{-6}\ \mathrm{cm}^{-1}$, $K_1 = (3.58 + i0.0002)\times10^{-11}\ \mathrm{cm}^{-1}\,\mathrm{s}$, $K_2 = (3.01 + i0.15)\times10^{-23}\ \mathrm{cm}^{-1}\,\mathrm{s}^2$, and $W = (7.53 + i0.0027)\times10^{-24}\ \mathrm{cm}^{-1}\,\mathrm{s}^2$. Thus, we have $L_D = L_{NL} = 3.3$ cm, $\tilde{V}_g = 0.93c$, and $U_0 = 2.0\times10^{11}\ \mathrm{s}^{-1}$. The corresponding linear dispersion and absorption curves of the probe field are shown in Fig. 4(b) by using these parameters. Base on these results one can obtain the following conclusions: (i) The system works in a normal dispersion regime, not an anomalous regime as claimed by the authors. Hence the soliton obtained is a slow but not superluminal one. (ii) The system works in the point B of Fig. 4(b), i.e. far away from the resonance due to the large Ω_3, Δ_1, and Δ_2. This is not interesting for a resonant system. (iii) The magnitudes of $\tilde{K}_2$ and $\tilde{W}$ are respectively twelve and eight orders smaller than the corresponding coefficients given in our above scheme. (iv) The soliton formation in this case requires $U_0 \simeq 2000\Omega_2$, which violates the weak probe-field assumption and the EIT condition (i.e. $|\Omega_1| \ll |\Omega_2|, |\Omega_3|$). Consequently, the claim by the authors of Ref. [25] that they have obtained a superluminal optical soliton in the same system is incorrect. By a careful inspection on the calculations presented in Ref. [25], we found that the authors mistook a sign in the MS equations, which results in a wrong linear dispersion relation and a wrong group velocity expression of the probe field. In fact, as mentioned above although a superluminal optical soliton can be generated in the anomalous dispersion regimes, it cannot propagate to a significant distance due to the very large optical absorption.

5. Summary

In this work, we have made a detailed investigation on the possible formation and propagation of ultraslow optical solitons in a resonant, lifetime-broadened four-level tripod atomic system via electromagnetically induced transparency. We have shown that a superluminal optical soliton suffers a serious absorption during propagation because it works outside of the EIT transparency window. Thus the claim and result presented in Ref. [25] are incorrect. However, the system supports an ultraslow optical soliton if working in normal dispersion regimes near resonance. The ultraslow optical soliton can propagate stably for a fairly long distance because in this case the system works inside of the EIT transparency window. In addition, such ultraslow optical soliton can be generated by using very low input light intensity. Different from the ultraslow optical solitons in the N-type systems suggested earlier [20], the dissipation of the present tripod system is smaller because there is only one upper energy level. Due to the robust propagating property, the ultraslow optical solitons in such systems may have potential applications in optical information processing and transmission.

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